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A Multi-fuzzy Set Theoretic Framework for Unanimity Measures

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Abstract This paper introduces and explores a range of distance measures defined on multi-fuzzy sets, emphasizing both their mathematical foundations and applicability to real-world decision environments. Classical distance metrics such as Minkowski, Hamming, and Euclidean measures are extended to the multi-fuzzy context, and their behaviour is analysed at both the set and element levels. The proposed formulations are rigorously analyzed at both the set level and element level to capture variations in structure and similarity more precisely. The study further examines how these measures are affected when multi-fuzzy sets are transformed via crisp functions or adjusted using fuzzy weight matrices. The Minkowski distance in the original multi-fuzzy sets dominates or bounds the corresponding distance in the multi-fuzzy weighted sets via fuzzy matrix transformation. In addition to these classical extensions, this paper introduces new deviation-based and normalised measures aimed at quantifying unanimity and consensus within group decision-making processes. By extending classical statistical notions such as mean, variance, and standard deviation into the multi-fuzzy domain, the authors develop refined methods for assessing agreement among individual judgments. These are further strengthened through the use of weighted criteria to reflect varying importance. A numerical case study is provided to demonstrate the practical effectiveness of the proposed approach in real-world consensus evaluation. By improving the accuracy of collective decision-making models, the research contributes to transparent, equitable, and evidence-based decision support systems in fields such as education, healthcare, and policy analysis. The study is primarily theoretical and validated through a limited case study; future work may involve empirical validation across larger datasets or multi–fuzzy–neutrosophic extensions.

Keywords Multi-fuzzy Set, Multi-fuzzy Measure, $\sigma-$distance Measure, Measure of Unanimity

1 Introduction

The evolution of set theory from classical to fuzzy set theory marked a significant role in handling imprecision and uncertainty built into real-world problems. Fuzzy set theory, introduced by Zadeh [1], allows for gradual membership of elements, thereby enabling a new representation of vague information. Significant research has been conducted on distance measures [2, 3, 4] and similarity measures [5]. Although traditional fuzzy sets are effective, they often struggle to model scenarios that involve multiple interrelated criteria; the multi-fuzzy set theory was developed as an extension. Several works have been developed by [6, 7] in multi-fuzzy set theory. This study examines the development and analysis of distance measures within the framework of multi-fuzzy set theory as an extension of classical and fuzzy set theories designed to more effectively model situations involving multiple, interrelated criteria. Building on the foundational definitions and operations of multi-fuzzy sets, the paper introduces a family of distance measures, namely, the Minkowski, Hamming, and Euclidean variants, which are adapted to suit multi-fuzzy membership values. The properties of these measures are rigorously analyzed to verify their compliance with the axioms of a valid distance metric. Particular attention is paid to their characteristics under standard set operations and their application to both individual elements and entire sets. In addition, the study explores how these measures respond when multi-fuzzy sets are weighted or transformed through fuzzy matrices [8, 9]. A key contribution of this work is the introduction of new quantitative tools, including normalized deviation measures, to assess unanimity in collective judgments. These measures, both weighted and unweighted, offer valuable insight into group consensus in decision-making contexts. By grounding these concepts in solid mathematical theory and providing numerical illustrations, the paper bridges theoretical development with practical application.

2 Multi-fuzzy Sets

In this section, the basic concept of a multi-fuzzy set has been reviewed.

Definition 2.1. [10] Let M and N be two multi-fuzzy sets of dimension n, defined over a finite universe X. These sets are expressed as:

$$M = \left\{\langle x, \mu_1^M(x), \mu_2^M(x), \ldots, \mu_n^M(x)\rangle : x \in X\right\},$$

$$N = \left\{\langle x, \mu_1^N(x), \mu_2^N(x), \ldots, \mu_n^N(x)\rangle : x \in X\right\},$$

where $\mu_i^M(x) \in [0,1]$ and $\mu_i^N(x) \in [0,1]$ denote the i^{th} membership degrees of the element $x \in X$, in the multi-fuzzy sets M and N, respectively.

We define the following relations and operations:

Definition 2.2. [10] Let M and N be two n-dimensional multi-fuzzy sets on a universe X with membership vectors $\boldsymbol{\mu}^M(x) = (\mu_1^M(x), \ldots, \mu_n^M(x))$ and $\boldsymbol{\mu}^N(x) = (\mu_1^N(x), \ldots, \mu_n^N(x))$.

1. $M \subseteq N$ if and only if
$$\mu_i^M(x) \leq \mu_i^N(x), \quad \forall x \in X,\ i = 1, \ldots, n;$$
2. $M = N$ if and only if
$$\mu_i^M(x) = \mu_i^N(x), \quad \forall x \in X,\ i = 1, \ldots, n;$$
3. The union of M and N is defined by
$$M \cup N = \left\{\langle x, \max(\boldsymbol{\mu}^M(x), \boldsymbol{\mu}^N(x))\rangle : x \in X\right\};$$
4. The intersection of M and N is defined by
$$M \cap N = \left\{\langle x, \min(\boldsymbol{\mu}^M(x), \boldsymbol{\mu}^N(x))\rangle : x \in X\right\}.$$

Definition 2.3. [11, 13] Let X be a universal set and let $\mathcal{P}(X)$ denote the power set of X. A real function

$$d : M^nFS(X) \times M^nFS(X) \to \mathbb{R}^+$$

is called a distance measure on a multi-fuzzy set $M^nFS(X)$, if d satisfies the following properties:

1. $d(M,N) = d(N,M), \forall M,N \in M^nFS(X)$;
2. $d(M,M) = 0, \forall M \in M^nFS(X)$;
3. $d(D,D^c) = \max_{M,N \in M^nFS(X)} d(M,N), \forall D \in \mathcal{P}(X)$;
4. $\forall M,N,P \in M^nFS(X)$ with $M \subseteq N \subseteq P$, $d(M,N) \leq d(M,P)$ and $d(N,P) \leq d(M,N)$.

Definition 2.4. [12] Let X be a finite set, and let M be a multi-fuzzy set of dimension n defined on X. Let A be a weight vector represented as:

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \text{where } a_i \in [0,1], \quad \sum_{i=1}^{n} a_i \leq 1.$$

Then the *multi-fuzzy weighted set* with respect to A is defined as:

$$[M]_A = \left\{\langle x,\ [\mu_1(x), \mu_2(x), \ldots, \mu_n(x)]A\rangle : x \in X\right\},$$

where the weighted membership is given by:

$$[\mu_1(x), \mu_2(x), ..., \mu_n(x)]\, A = \sum_{i=1}^{n} \mu_i(x)\, a_i.$$

Definition 2.5. A distance measure d is called a σ- distance measure on $M^nFS(X)$, if for any $M,N \in M^nFS(X)$ and $E \in \wp(X)$, $d(M,N) = d(M \cap E, N \cap E) + d(M \cap E^c, N \cap E^c)$ holds.

Definition 2.6. [14] Let A and B be two multi-fuzzy sets defined on a universe X with dimension i. Then the following distance measures are defined as follows:

$$D_H(A,B) = \sum_{x \in X} \sum_{n=1}^{i} |A^n(x) - B^n(x)|$$

$$D_E(A,B) = \left|\sum_{x \in X} \sum_{n=1}^{i} |A^n(x) - B^n(x)|^2\right|^{\frac{1}{2}}.$$

3 Measures Between Multi-fuzzy Sets

Let M and N be two multi-fuzzy sets of dimension n defined on a finite universe X. The **Minkowski measure** m_p **of order** p between M and N is defined as:

$$m_p(M,N) = \left(\sum_{x \in X} \sum_{i=1}^{n} \left|\mu_i^M(x) - \mu_i^N(x)\right|^p\right)^{\frac{1}{p}}, \ p \geq 1$$

is a parameter. The normalized multi-fuzzy Minkowski measure m_p^* is defined as:

$$m_p^*(M,N) = \left(\frac{1}{n|X|} \sum_{x \in X} \sum_{i=1}^{n} \left|\mu_i^M(x) - \mu_i^N(x)\right|^p\right)^{\frac{1}{p}}, \ p \geq 1$$

Alternatively, using vector norm notation, the Minkowski measure is defined as

$$m_p(M,N) = \left(\sum_{x \in X} \left\| \mu^M(x) - \mu^N(x) \right\|_p^p \right)^{\frac{1}{p}}, \ p \geq 1.$$

Special Cases:

- For $p = 1$, this corresponds to the multi-fuzzy Hamming measure

$$m_H(M,N) = \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^N(x) \right|$$

The normalized multi-fuzzy Hamming measure $m_H^*(M,N)$ is defined as:

$$m_H^*(M,N) = \frac{1}{n|X|} \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^N(x) \right|$$

where n is the dimension of the multi-fuzzy sets, and $|X|$ is the cardinality of the set X.

- For $p = 2$, this corresponds to the multi-fuzzy Euclidean measure

$$m_E(M,N) = \sqrt{\sum_{x \in X} \sum_{i=1}^{n} \left(\mu_i^M(x) - \mu_i^N(x) \right)^2}$$

The normalized Euclidean measure $m_E^*(M,N)$ between two multi-fuzzy sets M and N is defined as:

$$m_E^*(M,N) = \sqrt{\frac{1}{n|X|} \sum_{x \in X} \sum_{i=1}^{n} \left(\mu_i^M(x) - \mu_i^N(x) \right)^2}$$

where n is the dimension of the multi-fuzzy sets, and $|X|$ is the cardinality of the set X.

3.1 Measure Between Elements of a Multi-fuzzy Set

The **Minkowski measure of order** p between the elements x and y of a multi-fuzzy set M is defined as:

$$m_p(M(x), M(y), p)) = \left(\frac{1}{n} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^M(y) \right|^p \right)^{\frac{1}{p}}, \ p \geq 1.$$

Normalized Hamming measure between elements x and y as

$$m_H^*(M(x), M(y)) = \frac{1}{n} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^M(x) \right|$$

where n is the dimension of the multi-fuzzy set M.

Normalized Euclidean measure between elements x and y as

$$m_E^*(M(x), M(y)) = \sqrt{\frac{1}{n} \sum_{x \in X} \sum_{i=1}^{n} \left(\mu_i^M(x) - \mu_i^M(x) \right)^2}$$

where n is the dimension of the multi-fuzzy set M.

Theorem 3.1. *The Minkowski measure m_p is a distance measure.*

Proof.
Let $M, N \in M^n FS(X)$

(i)

$$\sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^N(x) \right|^p = \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^N(x) - \mu_i^M(x) \right|^p$$
$$\Rightarrow \quad m_p(M,N) = m_p(N,M).$$

(ii)

$$m_p(M,M) = 0, \quad \forall M \in M^n FS(X).$$

(iii) Let $E \in \wp(X)$, the power set of X, and let E^c denote the complement. Then:

$$\sum_{x \in X} \sum_{i=1}^{n} |E_i(x) - E_i^c(x)|^p \geq \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - (\mu_i^c)^M(x) \right|^p,$$
$$\forall M \in M^n FS(X)$$

Thus,

$$m_p(E, E^c) = \max_{M,N \in M^n FS(X)} m_p(M,N)$$

(iv) Suppose $M \subseteq N \subseteq P$. Then, $\forall x \in X$ and $i = 1, 2, \ldots, n$,

$$\mu_i^M(x) \leq \mu_i^N(x) \leq \mu_i^P(x).$$

Hence,

$$\begin{aligned} m_p(M,N) &= \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^N(x) \right|^p \\ &\leq \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^P(x) \right|^p \\ &= m_p(M,P) \end{aligned}$$

and similarly,

$$\begin{aligned} m_p(N,P) &= \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^N(x) - \mu_i^P(x) \right|^p \\ &\leq \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^P(x) \right|^p \\ &= m_p(M,P) \end{aligned}$$

Therefore,

$$m_p(M,N) + m_p(N,P) \leq 2\, m_p(M,P),$$

which establishes the triangle inequality for m_p under the assumption $M \subseteq N \subseteq P$.

Remark 3.2. The multi-fuzzy Hamming measure and Euclidean measure are distance measures.

Theorem 3.3. *The Minkowski measure m_p is not a σ-distance measure.*

Proof.
Let $M, N \in M^nFS(X)$ and $E \in \wp(X)$.
$m_p(M \cap E, N \cap E)$

$$= \sum_{x\in X}\sum_{i=1}^{n} \left|\min(\mu_i^M(x), E_i(x)) - \min(\mu_i^N(x), E_i(x))\right|$$
$$= \sum_{x\in X}\sum_{i=1}^{n} \Bigg|\frac{1}{2}\big[\mu_i^M(x) + E_i(x) - |\mu_i^M(x) - E_i(x)|\big] - \frac{1}{2}\big[\mu_i^N(x) + E_i(x) - |\mu_i^N(x) - E_i(x)|\big]\Bigg|$$
$$\leq \sum_{x\in X}\sum_{i=1}^{n} \left|\tfrac{1}{2}\big[\mu_i^M(x) + E_i(x) - |\mu_i^M(x) - E_i(x)|\big]\right| + \sum_{x\in X}\sum_{i=1}^{n} \left|\tfrac{1}{2}\big[\mu_i^N(x) + E_i(x) - |\mu_i^N(x) - E_i(x)|\big]\right|.$$

$m_p(M \cap E^c, N \cap E^c)$

$$= \sum_{x\in X}\sum_{i=1}^{n} \left|\min(\mu_i^M(x), E_i^c(x)) - \min(\mu_i^N(x), E_i^c(x))\right|$$
$$= \sum_{x\in X}\sum_{i=1}^{n} \Bigg|\frac{1}{2}\big[\mu_i^M(x) + E_i^c(x) - |\mu_i^M(x) - E_i^c(x)|\big] - \frac{1}{2}\big[\mu_i^N(x) + E_i^c(x) - |\mu_i^N(x) - E_i^c(x)|\big]\Bigg|$$
$$\leq \sum_{x\in X}\sum_{i=1}^{n} \left|\tfrac{1}{2}\big[\mu_i^M(x) + E_i^c(x) - |\mu_i^M(x) - E_i^c(x)|\big]\right| + \sum_{x\in X}\sum_{i=1}^{n} \left|\tfrac{1}{2}\big[\mu_i^N(x) + E_i^c(x) - |\mu_i^N(x) - E_i^c(x)|\big]\right|.$$

Hence,

$$m_p(M \cap E, N \cap E) + m_p(M \cap E^c, N \cap E^c) \neq m_p(M,N).$$

Thus, m_p does not satisfy the σ-additivity property, and hence, it is not a σ-distance measure.

Remark 3.4. Similarly, it can be shown that the Hamming measure m_H and Euclidean measure m_E are also not σ-distance measures.

Theorem 3.5. *Let $M, N \in M^n FS(X)$. Then the following identities hold:*
(i) $m_p(M, M \cup N) = m_p(N, M \cap N)$;
(ii) $m_p(M, M \cap N) = m_p(N, M \cup N)$.

Proof. $m_p(M, M \cup N)$

$$= \left(\sum_{x\in X}\sum_{i=1}^{n} \left|\mu_i^M(x) - \max\left(\mu_i^M(x), \mu_i^N(x)\right)\right|^p\right)^{1/p}$$
$$= \left(\sum_{x\in X}\sum_{i=1}^{n} \left|\min\left(\mu_i^M(x), \mu_i^N(x)\right) - \mu_i^M(x)\right|^p\right)^{1/p}$$
$$= \left(\sum_{x\in X}\sum_{i=1}^{n} \left|\mu_i^N(x) - \min\left(\mu_i^M(x), \mu_i^N(x)\right)\right|^p\right)^{1/p}$$
$$= m_p(N, M \cap N).$$

Similarly, interchanging roles of M and N, we get:

$$m_p(M, M \cap N) = m_p(N, M \cup N).$$

Theorem 3.6. $m_p(M,N) = m_p(M \cup N, M \cap N)$*, $\forall M, N \in M^nFS(X)$.*

Proof.
$m_p(M,N)$

$$= \left(\sum_{x\in X}\sum_{i=1}^{n} \left|\mu_i^M(x) - \mu_i^N(x)\right|^p\right)^{1/p}$$
$$= \left(\sum_{x\in X}\sum_{i=1}^{n} \left|\max(\mu_i^M(x), \mu_i^N(x)) - \min(\mu_i^M(x), \mu_i^N(x))\right|^p\right)^{1/p}$$
$$= m_p(M \cup N, M \cap N).$$

Theorem 3.7. $m_p(M,N) = m_p(M^c, N^c), \forall M, N \in M^nFS(X)$.

Proof.

$$m_p(M,N) = \sum_{x\in X}\sum_{i=1}^{n} \left|\mu_i^M(x) - \mu_i^N(x)\right|^p = \sum_{x\in X}\sum_{i=1}^{n} \left|(\mu_i^M)^c(x) - (\mu_i^N)^c(x)\right|^p = m_p(M^c, N^c).$$

Remark 3.8. The Minkowski measure $m_p(M \cup N, P)$ and $m_p(M, P)$ are not generally comparable.

Example 3.9. Let $M, N, P \in M^nFS(X)$ be defined such that $M \subset P$ and $N \nsubseteq P$. Then $m_p(M \cup N, P)$ may not be greater or less than $m_p(M, P)$ consistently.

Let $X = \{x_1, x_2\}$ and consider three 2-dimensional multi-fuzzy sets $M, N,$ and P defined on X as follows:

$$M = \{(x_1, 0.2, 0.3), (x_2, 0.6, 0.5)\},$$
$$N = \{(x_1, 0.7, 0.8), (x_2, 0.4, 0.6)\},$$
$$P = \{(x_1, 0.5, 0.6), (x_2, 0.7, 0.6)\}.$$

Clearly, $M \subset P$ but $N \nsubseteq P$.
The union $M \cup N$ is given by

$$M \cup N = \{(x_1, 0.7, 0.8), (x_2, 0.6, 0.6)\}.$$

Let $p = 2$. The Minkowski measure between M and P is

$$m_2(M, P) = \left(\sum_{x \in X} \sum_{i=1}^{2} \left| \mu_i^M(x) - \mu_i^P(x) \right|^2 \right)^{1/2}.$$

Substituting the values of each memebrship, we obtain

$$m_2(M, P) = \sqrt{0.20}.$$

Similarly, the Minkowski measure between $M \cup N$ and P is

$$m_2(M \cup N, P) = \sqrt{0.09}.$$

Hence,

$$m_2(M \cup N, P) < m_2(M, P).$$

On the other hand, by choosing a different multi-fuzzy set N with sufficiently large membership values, one can obtain

$$m_2(M \cup N, P) > m_2(M, P).$$

Example 3.10. Let $X = \{x_1, x_2\}$ and consider the following 2-dimensional multi-fuzzy sets defined on X:

$$M = \{(x_1, 0.2, 0.3), (x_2, 0.6, 0.5)\},$$

$$P = \{(x_1, 0.5, 0.6), (x_2, 0.7, 0.6)\},$$

and choose

$$N = \{(x_1, 1.0, 1.0), (x_2, 1.0, 1.0)\}.$$

Clearly, $M \subset P$ and $N \nsubseteq P$.

The union $M \cup N$ is

$$M \cup N = \{(x_1, 1.0, 1.0), (x_2, 1.0, 1.0)\}.$$

Let $p = 2$. The Minkowski measure between M and P is

$$m_2(M, P) = \sqrt{0.20}.$$

The Minkowski measure between $M \cup N$ and P is

$$m_2(M \cup N, P) = \sqrt{0.66}.$$

Hence,

$$m_2(M \cup N, P) > m_2(M, P).$$

Therefore, the Minkowski measures $m_p(M \cup N, P)$ and $m_p(M, P)$ are not generally comparable.

Remark 3.11. Similarly the Minkowski measure $m_p(M, P)$ and $m_p(M \cap N, P)$ are not generally comparable.

4 Minkowski Measure on Multi-fuzzy Set with Weighted Matrix

This section explores the behavior of the Minkowski measure when applied to multi-fuzzy sets, especially under the action of fuzzy matrices that generate fuzzy sets. Theorem 4.1 establishes that the Minkowski measure between multi-fuzzy sets dominates the distance between their weighted counterparts via a weighted fuzzy matrix.

Theorem 4.1. *Let $M, N \in M^n FS(X)$, and let*

$$A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \quad a_i \in [0, 1], \ \sum_{i=1}^{n} a_i = 1,$$

be a fuzzy weight matrix. Then

$$m_p([M]_A, [N]_A) \leq m_p(M, N),$$

where $[M]_A$ and $[N]_A$ denote the corresponding multi-fuzzy weighted sets.

Proof. Let $M = (\mu_1^M, \mu_2^M, \ldots, \mu_n^M)$ and $N = (\mu_1^N, \mu_2^N, \ldots, \mu_n^N)$. Then

$$\begin{aligned} m_p([M]_A, [N]_A) &= \left(\sum_{x \in X} \left| \sum_{i=1}^{n} a_i \big(\mu_i^M(x) - \mu_i^N(x) \big) \right|^p \right)^{\frac{1}{p}} \\ &\leq \left(\sum_{x \in X} \sum_{i=1}^{n} a_i^p \left| \mu_i^M(x) - \mu_i^N(x) \right|^p \right)^{\frac{1}{p}} \\ &\leq \left(\sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^N(x) \right|^p \right)^{\frac{1}{p}} \\ &= m_p(M, N), \end{aligned}$$

since $0 \leq a_i \leq 1$ implies $a_i^p \leq 1$ for all i. □

Corollary 4.2. *Multi-fuzzy weighted transformation is a non-expensive mapping.*

Let $M, N \in M^n FS(X)$, and let $A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$, where $a_i \in [0, 1], \sum_{i=1}^{n} a_i = 1$ be a fuzzy matrix. Then the mapping

$T_A(M)(x) = \sum\limits_{i=1}^{n} a_i \mu_i^M(x)$ is non-expansive with respect to the Minkowski measure m_p.

The distance measures developed in the above sections quantify the degree of dissimilarity between multi-fuzzy sets. However, in many decision-making and aggregation problems, the primary interest lies in the extent of unanimous agreement among the multiple membership dimensions. Unanimity measures are designed to capture the collective support provided by all components of a multi-fuzzy set for a given element. Since distance measures quantify disagreement, unanimity can naturally be modeled through similarity measures derived from such distances. Motivated by the properties of the Minkowski measure and its normalized forms, we now proceed to introduce unanimity measures for multi-fuzzy sets by transforming distance-based representations into agreement-based quantities.

5 Measures of Unanimity

Here, we propose two quantitative measures aimed at evaluating and supporting the process of achieving unanimous agreement among a group of individual judgments. These measures help evaluate the degree of agreement between multi-fuzzy sets and their collective average.

For this we consider the multi-fuzzy mean
$\bar{\mu} =< \bar{\mu}_1, \bar{\mu}_2, ..., \bar{\mu}_n >$, where $\bar{\mu}_i = \frac{1}{n} \sum_{j=1}^{m} \mu_i^M(x_j)$, $\forall i = 1, 2, \ldots, n$.

Definition 5.1. Normalized Hamming-Like Deviation Measure (NHM)

The normalized Hamming-like deviation measure is defined as:

$$m_H^*(M, \bar{M}) = \frac{1}{n|X|} \sum_{x \in X} \sum_{i=1}^{n} \left| \mu_i^M(x) - \mu_i^{\bar{M}} \right|$$

This is a total absolute deviation measure, summing all individual differences from the mean. Lower values imply greater unanimity.

Definition 5.2. Normaized Euclidean-like deviation measure (NEM)

$$m_E^*(M, \bar{M}) = \sqrt{\frac{1}{n|X|} \sum_{x \in X} \sum_{i=1}^{n} \left(\mu_i^M(x) - \mu_i^{\bar{M}} \right)^2}$$

Definition 5.3. Measure of unanimity based on normalized Hamming-Like-deviation measure is
$m_u^*(M, \bar{M}) = 1 - m_H^* \left(M, \bar{M} \right)$ and based on Euclidean measure is $m_U^*(M, \bar{M}) = 1 - m_E^* \left(M, \bar{M} \right)$.

Remark 5.4.

- If m_H^* or m_E^* is zero, all multi-fuzzy sets agree perfectly $\to$ unanimity.
- Larger values imply disagreement or divergence from the collective opinion.
- NHM is always less than NEM, so the measure of unanimity based on the Hamming-like deviation measure is stronger than the Euclidean-like deviation measure.

The unanimity measures introduced above treat all membership dimensions as equally important. However, in many practical decision-making situations, different criteria or attributes may contribute unequally to the overall agreement. To account for such heterogeneous importance, it is natural to incorporate a system of weights into the unanimity framework. Motivated by the concept of multi-fuzzy weighted sets and the non-expansive nature of the corresponding transformations, we now extend the proposed unanimity measures by introducing weighted measures of unanimity.

6 Weighted Measures of Unanimity

Let w_i be the weight attached to the criterion $\mu_i^M(x)$, where $i = 1, 2, \ldots, n$, $x \in X$, and

$$\sum_{i=1}^{n} w_i = 1.$$

Define the multi-fuzzy set M as:

$$M = \left\{ \left\langle x, \left(\zeta_1^M(x), \zeta_2^M(x), \ldots, \zeta_n^M(x) \right) \right\rangle : x \in X \right\},$$

where $\zeta_i^M(x) = w_i \left(\mu_i^M(x) \right)$, for all $i = 1, 2, \ldots, n$. Then, the multi-fuzzy weighted arithmetic mean (or multi-fuzzy weighted mean) is defined as:

$$\bar{M}_w = \left\langle \zeta_{w_1}^{\bar{M}}, \zeta_{w_2}^{\bar{M}}, \ldots, \zeta_{w_n}^{\bar{M}} \right\rangle,$$

where

$$\zeta_{w_i}^{\bar{M}}(x) = \sum_{j=1}^{m} \zeta_i^M(x_j)$$

Definition 6.1 (Normalized Weighted Hamming Deviation Measure(NWHM))**.** Let w_i be the weight attached to the criterion $\mu_i^M(x)$, where $i = 1, 2, \ldots, n$, $x \in X$, and $\sum_{i=1}^{n} w_i = 1$. Let $\zeta_i^M(x) = w_i \mu_i^M(x)$, for all $i = 1, 2, \ldots, n$. Then, the NWE measure is defined as:

$$m_{w_H}^*(M_w, \bar{M}_w) = \frac{1}{n|X|} \sum_{x \in X} \sum_{i=1}^{n} \left| \zeta_i^M(x) - \zeta_i^{\bar{M}} \right|.$$

This is a total absolute weighted deviation measure, summing all weighted individual differences from the weighted multi-fuzzy arithmetic mean. Lower values of m_{w_H} indicate greater unanimity among criteria.

Definition 6.2 (Normalized Weighted Euclidean Deviation Measure(NWEM))**.** Let w_i be the weight attached to the criterion $\mu_i^M(x)$, where $i = 1, 2, \ldots, n$, $x \in X$, and $\sum_{i=1}^{n} w_i = 1$. Let $\zeta_i^M(x) = w_i \mu_i^M(x)$, for all $i = 1, 2, \ldots, n$. The NWEM is defined as:

$$m_{w_E}^*(M_w, \bar{M}_w) = \sqrt{\frac{1}{n|X|} \sum_{x \in X} \sum_{i=1}^{n} \left(\zeta_i^M(x) - \zeta_i^{\bar{M}} \right)^2}.$$

Definition 6.3. The measure of unanimity based on the normalized weighted-like Hamming-deviation measure (NWHM) is
$m_{u_w}^*(M, \bar{M}) = 1 - m_{w_H}^* \left(M_w, \bar{M}_w \right)$ and based on weighted Euclidean measure is $m_{U_w}^*(M_w, \bar{M}_w) = 1 - m_{E_w}^* \left(M_w, \bar{M}_w \right)$.

7 Applications

A six-member hospital ethics committee must decide whether to approve the implementation of an AI-assisted diagnostic system in the hospital. The decision is evaluated based on four major criteria, and a unanimous decision is required. The four criteria are:

- Clinical Accuracy
- Patient Safety
- Data Privacy
- Cost and Resource Efficiency

Let $X = \{A, B, C, D, E, F\}$ represent the six members of the committee. Each member assigns a value (between 0 and 1) to each of the four criteria. Let:

$$M = \left\{ \langle x, \mu_1^M(x), \mu_2^M(x), \mu_3^M(x), \mu_4^M(x) \rangle \right\}$$

where $\mu_i^M(x)$ for $i = 1, 2, 3, 4$ denotes the membership value assigned to criterion i by member x.

The membership values for each element are given by

$$M = \{\langle A, (0.6, 0.7, 0.9, 0.5)\rangle, \langle B, (0.5, 0.7, 0.8, 0.4)\rangle, \langle C, (0.6, 0.8, 0.8, 0.7)\rangle, \langle D, (0.8, 0.7, 0.6, 0.4)\rangle, \langle E, (0.8, 0.7, 0.7, 0.5)\rangle, \langle F, (0.5, 0.8, 0.6, 0.6)\rangle\}.$$

The aggregated decision (average membership values across all members) is denoted as:

$$\bar{M} = \langle 0.63, 0.73, 0.73, 0.52 \rangle$$

Here

$$m_u^*(A, \bar{M}) = 0.9375,$$
$$m_u^*(B, \bar{M}) = 0.9125,$$
$$m_u^*(C, \bar{M}) = 0.9125,$$
$$m_u^*(D, \bar{M}) = 0.8875,$$
$$m_u^*(E, \bar{M}) = 0.9375,$$
$$m_u^*(F, \bar{M}) = 0.8975.$$

and

$$m_U^*(A, \bar{M}) = 0.9118,$$
$$m_U^*(B, \bar{M}) = 0.9037,$$
$$m_U^*(C, \bar{M}) = 0.8962,$$
$$m_U^*(D, \bar{M}) = 0.8764,$$
$$m_U^*(E, \bar{M}) = 0.9118,$$
$$m_U^*(F, \bar{M}) = 0.8938.$$

The Hamming-like deviation for each member from the collective decision $\bar{M}$ is calculated using:

$$m_H^*(M, \bar{M}) = \frac{1}{24} \sum_{i=1}^{4} \left| \mu_i^M(x) - \mu_i^{\bar{M}} \right|$$

We calculate the average deviation across all members as: $m_H^*(M, \bar{M}) = \frac{1}{24}\Big((0.03+0.03+0.17+0.02)+(0.13+0.03+0.07+0.12)+(0.03+0.07+0.07+0.18)+(0.17+0.03+0.13+0.12)+(0.17+0.03+0.03+0.02)+(0.13+0.07+0.13+0.08)\Big)$ $=\frac{1}{24}(0.25 + 0.35 + 0.35 + 0.45 + 0.25 + 0.41) = 0.086.$

Similarly, we calculated NEM as $m_E^*(M, \bar{M}) = 0.102$
Measure of Unanimity based on NHM and NEM are:

$$m_u^*(M, \bar{M}) = 1 - m_H^*(M, \bar{M}) = 1 - 0.086 = 0.914$$

and

$$m_U^*(M, \bar{M}) = 1 - m_E^*(M, \bar{M}) = 1 - 0.102 = 0.898.$$

NHM (0.914) indicates a higher level of unanimity, suggesting that there is a closer agreement between the individual membership values and the aggregated decision. NEM (0.898), on the other hand, accounts for squared deviations. It gives more weight to larger deviations, and therefore, slightly lower unanimity is expected. Even though the two measures indicate a strong level of agreement among the committee members. Since a unanimous decision ideally requires a unanimity measure close to 1, the result suggests that the committee is fully unanimous in approving the implementation of an AI-assisted diagnostic system in the hospital.
From m_u^* and m_U^*, member D shows the lowest agreement, while A and E show the highest.

8 Conclusions

In this paper, we have introduced and analyzed several multi-fuzzy measures defined on multi-fuzzy sets, along with their extensions to multi-fuzzy weighted sets. By studying how measures change when moving from crisp to multi-fuzzy domains, we better understand how these transformations affect the overall structure. Furthermore, by extending classical statistical concepts such as mean, variance, and standard deviation into the multi-fuzzy domain, we developed two new quantitative measures aimed at simplifying unanimous decision-making from individual judgments. These works not only enhance the theoretical foundations of multi-fuzzy set analysis but also offer practical tools for decision support in uncertain and complex environments. Despite the effectiveness of the proposed measures, the present study is restricted to a finite universe and fixed norms. Future research may focus on extending the framework to infinite or continuous universes, investigating alternative distance norms and aggregation operators, and applying the proposed measures to real-world multi-criteria decision-making and pattern recognition problems.

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