

# Contractive Conditions for Fixed Points in Complete Neutrosophic Fuzzy Metric Spaces

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**Abstract** Fixed point theory constitutes a fundamental pillar of nonlinear analysis and has found extensive applications in mathematical modeling, optimization, computer science, and engineering. Classical results such as Banach’s contraction principle have been generalized to various settings, including fuzzy metric spaces, cone metric spaces, and modular metric spaces. However, these frameworks often prove inadequate for modeling uncertainty involving indeterminacy and inconsistency. To address this limitation, neutrosophic fuzzy metric spaces (NFMSs) provide a powerful mathematical structure by integrating fuzzy distance measures with neutrosophic logic. In this paper, we establish several new fixed point theorems for single-valued mappings in complete neutrosophic fuzzy metric spaces under different generalized contractive conditions. The proposed contractions extend classical Banach-type, nonlinear, and rational-type contractions by incorporating neutrosophic fuzzy control functions. Using iterative techniques and properties of t-norms, we prove the existence and uniqueness of fixed points and demonstrate the convergence of the associated Picard iterative sequences. The obtained results significantly generalize and unify several existing fixed point theorems in fuzzy metric spaces, cone metric spaces, and modular metric spaces. An illustrative example is provided to validate the applicability of the main results. The primary contribution of this work lies in enriching the theoretical foundation of neutrosophic fuzzy analysis and offering a unified approach to handling uncertainty, vagueness, and inconsistency within fixed point theory. Although this study is mainly theoretical, the results have potential implications for optimization theory, decision-making models, and computational intelligence systems operating under indeterminate or conflicting

information. Future research may focus on extending these results to multivalued mappings and their applications in real-world neutrosophic models.

**Keywords** Neutrosophic Fuzzy Metric Space, Contraction Mapping, Fixed Point Theorem, Convergence

## 1 Introduction

FP theory is a central tool in modern nonlinear analysis and continues to play a fundamental role in several branches of mathematics, computer science, and engineering. A major milestone in this direction is the Banach contraction principle, which ensures the existence of a UFP whenever a mapping satisfies a suitable contraction condition. Since its inception, this principle has generated a vast body of literature devoted to its extensions and generalizations. Researchers have successfully adapted this foundational idea to various mathematical frameworks such as fuzzy MSs [1], cone MSs [2].

The neutrosophic framework is introduced by Smarandache [3]. This structure provides an effective approach to describing and analyzing problems that involve incomplete, inconsistent, or ambiguous information. The integration of neutrosophic principles into fuzzy MSs has led to the establishment of NFMSs (NFMS) [4, 5, 6, 7], combining the flexibility of fuzziness with the expressive capacity of neutrosophic logic. Multivalued fixed point theory was initiated by Nadler, who extended Banach’s con-

traction principle to set-valued mappings [8]. These spaces have demonstrated promising applications in decision analysis [9], optimization, network modeling, and in broader contexts of uncertainty representation [10, 11].

Despite these developments, existing research still lacks general contractive conditions that guarantee both the existence and uniqueness of FPs, together with the convergence of iterative seq.s in CNFMS. Motivated by this gap, the present paper formulates and proves a new class of theorems providing sufficient conditions under which a mapping in a CNFMS admits a UFP. The results presented here extend and unify earlier contributions of George and Veeramani in fuzzy MSs [1] and integrate recent ideas from neutrosophic theory, cone metric analysis, and modular metric environments.

Several contemporary works [12–20] continue to advance FP theory in diverse mathematical settings such as modular, cone, and generalized fuzzy MSs. These contributions collectively enhance the theoretical framework of modern analysis and highlight the growing importance of fuzzy and neutrosophic FP results in modeling and solving real-world problems under uncertainty. Accordingly, this study provides new analytical tools that strengthen both the theoretical understanding of NFMS and their practical applications in uncertain environments.

## 2 Preliminaries

**Definition 2.1.** [1] A *t*-norm is a binary operation  $*$  :  $[0, 1]^2 \rightarrow [0, 1]$  that satisfies the following properties for all  $a, b, c \in [0, 1]$ :

1. **Commutativity:**  $a * b = b * a$ .
2. **Associativity:**  $a * (b * c) = (a * b) * c$ .
3. **Monotonicity:** If  $a \leq c$  and  $b \leq d$ , then  $a * b \leq c * d$ .
4. **Identity element:**  $a * 1 = a$ .

These conditions ensure that the operation  $*$  behaves consistently with the logical conjunction in fuzzy and neutrosophic frameworks.

**Definition 2.2.** [4] Let  $X$  be a non-empty set and  $*$  be a continuous *t*-norm. Suppose that  $N_1, N_2 : X \times X \times (0, \infty) \rightarrow [0, 1]$  are two mappings. The quadruple  $(X, N_1, N_2, *)$  is called a *NFMS* if, for all  $i, \Gamma, \beth \in X$  and for every  $s, t > 0$ , the following conditions hold:

1.  $N_1(i, \Gamma, t) > 0$  and  $N_2(i, \Gamma, t) > 0$ ;
2.  $N_1(i, \Gamma, t) = 1$  and  $N_2(i, \Gamma, t) = 0$  if and only if  $i = \Gamma$ ;
3.  $N_1(i, \Gamma, t) = N_1(\Gamma, i, t)$  and  $N_2(i, \Gamma, t) = N_2(\Gamma, i, t)$ ;
4.  $N_1(i, \beth, t + s) \geq N_1(i, \Gamma, t) * N_1(\Gamma, \beth, s)$ ;
5.  $N_2(i, \beth, t + s) \geq N_2(i, \Gamma, t) * N_2(\Gamma, \beth, s)$ ;
6.  $N_1(i, \beth, \cdot)$  and  $N_2(i, \beth, \cdot)$  are continuous functions of  $t$ .

In this context,  $N_1(i, \beth, t)$  represents the degree of nearness (or similarity) between the elements  $i$  and  $\beth$  at a given time  $t$ , while  $N_2(i, \beth, t)$  measures their degree of separation or non-closeness.

**Definition 2.3.** [4] A seq.  $\{i_n\}$  in  $X$  is said to *converge* to a point  $i \in X$  if, for every  $t > 0$ ,

$$\lim_{n \rightarrow \infty} N_1(i_n, i, t) = 1 \quad \text{and} \quad \lim_{n \rightarrow \infty} N_2(i_n, i, t) = 0.$$

In this case,  $i$  is called the limit of the seq.  $\{i_n\}$  in the NFMS.

**Definition 2.4.** [4] A seq.  $\{i_n\}$  in  $X$  is termed a *Ca. seq.* if, for each  $t > 0$ ,

$$\lim_{m, n \rightarrow \infty} N_1(i_m, i_n, t) = 1 \quad \text{and} \quad \lim_{m, n \rightarrow \infty} N_2(i_m, i_n, t) = 0.$$

That is, the elements of the seq. become arbitrarily close to each other with respect to both  $N_1$  and  $N_2$  as  $m, n \rightarrow \infty$ .

**Definition 2.5.** [4] An NFMS  $(X, N_1, N_2, *)$  is said to be *complete* if every Ca. seq.  $\{i_n\}$  in  $X$  converges to a point  $i \in X$  such that

$$\lim_{n \rightarrow \infty} N_1(i_n, i, t) = 1 \quad \text{and} \quad \lim_{n \rightarrow \infty} N_2(i_n, i, t) = 0$$

for all  $t > 0$ .

## 3 Main Results

**Theorem 3.1.** Let  $(X, N_1, N_2, *)$  be a CNFMS, and let  $J : X \rightarrow X$  be a SM. Assume that there exist constants  $\alpha, \beta \in (0, 1)$  such that, for every  $i, \Gamma \in X$  and all  $t > 0$ ,

$$N_1(Ji, J\Gamma, t) \geq [N_1(i, \Gamma, t)]^\alpha, \quad N_2(Ji, J\Gamma, t) \leq \beta N_2(i, \Gamma, t).$$

Then:

1. The mapping  $J$  possesses a UFP  $i^* \in X$ ;
2. For any initial element  $i_0 \in X$ , the seq.  $\{i_n\}_{n \geq 0}$  defined by  $i_{n+1} = Ji_n$  converges to  $i^*$ .

**Proof.** Choose an arbitrary point  $i_0 \in X$  and construct the iterative seq.  $i_{n+1} = Ji_n$  for  $n \geq 0$ . For each  $t > 0$ , define

$$A_n(t) = N_1(i_{n+1}, i_n, t), \quad B_n(t) = N_2(i_{n+1}, i_n, t).$$

Using the given contractive conditions, we obtain

$$A_{n+1}(t) = N_1(i_{n+2}, i_{n+1}, t) \geq [A_n(t)]^\alpha, \\ B_{n+1}(t) = N_2(i_{n+2}, i_{n+1}, t) \leq \beta B_n(t).$$

By iteration,  $B_n(t) \leq \beta^n B_0(t)$ , and since  $0 < \beta < 1$ , we have  $B_n(t) \rightarrow 0$  as  $n \rightarrow \infty$ .

Because  $0 < \alpha < 1$  and  $A_n(t) \in (0, 1]$ , the inequality  $A_{n+1} \geq [A_n]^\alpha$  implies that  $A_{n+1} \geq A_n$  (indeed,  $u^\alpha > u$  for all  $u \in (0, 1)$ ). Thus,  $\{A_n(t)\}$  is a non-decreasing seq. bounded above by 1; let  $L(t) = \lim_{n \rightarrow \infty} A_n(t) \leq 1$ . Taking limits in  $A_{n+1} \geq [A_n]^\alpha$  gives  $L(t) \geq [L(t)]^\alpha$ , which forces  $L(t) = 1$ . Hence

$$\lim_{n \rightarrow \infty} N_1(i_{n+1}, i_n, t) = 1, \quad \lim_{n \rightarrow \infty} N_2(i_{n+1}, i_n, t) = 0.$$

For any  $m > n$  and  $t = t_1 + \dots + t_k$ , by the triangular properties of  $N_1$  and  $N_2$  we have

$$N_1(i_m, i_n, t) \geq N_1(i_{n+1}, i_n, t_1) * \dots * N_1(i_m, i_{m-1}, t_k),$$

$$N_2(i_m, i_n, t) \leq N_2(i_{n+1}, i_n, t_1) * \cdots * N_2(i_m, i_{m-1}, t_k).$$

Each factor in the first expression approaches 1, while those in the second approach 0; therefore,

$$\lim_{m,n \rightarrow \infty} N_1(i_m, i_n, t) = 1, \quad \lim_{m,n \rightarrow \infty} N_2(i_m, i_n, t) = 0.$$

Consequently,  $\{i_n\}$  is a Ca. seq.. Since  $(X, N_1, N_2, *)$  is complete, there exists  $i^* \in X$  such that  $i_n \rightarrow i^*$ .

Using the continuity of  $N_1$  and  $N_2$  and again applying the contractive property, we have

$$N_1(Ji^*, i^*, t) = \lim_{n \rightarrow \infty} N_1(Ji^*, Ji_n, t) \geq \lim_{n \rightarrow \infty} [N_1(i^*, i_n, t)]^\alpha = 1,$$

and similarly  $N_2(Ji^*, i^*, t) = 0$ . Therefore,  $Ji^* = i^*$ .

To verify uniqueness, let  $\Gamma^*$  be another FP of  $J$ . Then

$$N_1(i^*, \Gamma^*, t) = N_1(Ji^*, J\Gamma^*, t) \geq [N_1(i^*, \Gamma^*, t)]^\alpha.$$

Since  $0 < \alpha < 1$ , it follows that  $N_1(i^*, \Gamma^*, t) = 1$ . Also,

$$N_2(i^*, \Gamma^*, t) = N_2(Ji^*, J\Gamma^*, t) \leq \beta N_2(i^*, \Gamma^*, t),$$

which implies  $N_2(i^*, \Gamma^*, t) = 0$ . Hence  $i^* = \Gamma^*$ , proving uniqueness.  $\square$

**Theorem 3.2.** Let  $(X, N_1, N_2, *)$  be a CNFMS, and let  $J : X \rightarrow X$  be a SM. Assume there exist constants  $\alpha, \beta \in (0, 1)$  and a continuous function  $\Phi : [0, 1] \rightarrow [0, 1]$  satisfying  $\Phi(1) = 1$  such that, for every  $i, \Gamma \in X$  and  $t > 0$ ,

$$\begin{aligned} N_1(Ji, J\Gamma, t) &\geq \Phi(N_1(i, \Gamma, t)), \\ N_2(Ji, J\Gamma, t) &\leq \beta N_2(i, \Gamma, t), \end{aligned}$$

and further suppose that

$$\Phi(u) \geq \alpha u^2 \text{ for all } u \in [0, 1], \Phi(u) > u \text{ for every } u \in [0, 1).$$

Then the following assertions hold:

1.  $J$  admits a UFP  $i^* \in X$ ;
2. For any initial element  $i_0 \in X$ , the seq.  $\{i_n\}$  defined by  $i_{n+1} = Ji_n$  converges to  $i^*$  with respect to  $N_1$  and  $N_2$ .

**Proof.** Let  $i_0 \in X$  be arbitrary, and define the iterative seq.  $i_{n+1} = Ji_n$  for all  $n \geq 0$ . For a fixed  $t > 0$ , introduce the seq.s

$$A_n(t) = N_1(i_{n+1}, i_n, t), \quad B_n(t) = N_2(i_{n+1}, i_n, t).$$

From the assumed inequalities, we have

$$A_{n+1}(t) = N_1(i_{n+2}, i_{n+1}, t) \geq \Phi(A_n(t)) \geq \alpha [A_n(t)]^2,$$

$$B_{n+1}(t) \leq \beta B_n(t).$$

Since  $0 < \beta < 1$ , it follows immediately that  $B_n(t) \leq \beta^n B_0(t) \rightarrow 0$  as  $n \rightarrow \infty$ .

Because  $\Phi(u) > u$  whenever  $u \in [0, 1)$ , the recurrence  $A_{n+1} \geq \Phi(A_n)$  guarantees that  $A_{n+1} \geq A_n$  whenever  $A_n < 1$ . Hence  $\{A_n(t)\}$  is monotone non-decreasing and bounded above by 1, so it converges to some limit  $L(t) \in (0, 1]$ . Taking limits in  $A_{n+1} \geq \Phi(A_n)$  gives  $L(t) \geq \Phi(L(t))$ . Since  $\Phi(1) = 1$  and  $\Phi(u) > u$  for all  $u \in [0, 1)$ , the only possible limit is  $L(t) = 1$ . Therefore,

$$\lim_{n \rightarrow \infty} A_n(t) = 1, \quad \lim_{n \rightarrow \infty} B_n(t) = 0.$$

Using the triangular properties of  $N_1$  and  $N_2$ , as in Theorem 3.1, we conclude that

$$\lim_{m,n \rightarrow \infty} N_1(i_m, i_n, t) = 1, \quad \lim_{m,n \rightarrow \infty} N_2(i_m, i_n, t) = 0,$$

so  $\{i_n\}$  is a Ca. seq. in  $X$ . By completeness of  $(X, N_1, N_2, *)$ , there exists  $i^* \in X$  such that  $i_n \rightarrow i^*$ .

Taking limits in the defining relations and invoking continuity of  $N_1$  and  $N_2$  yield

$$N_1(Ji^*, i^*, t) = \lim_{n \rightarrow \infty} N_1(Ji^*, Ji_n, t) \geq \lim_{n \rightarrow \infty} \Phi(N_1(i^*, i_n, t)) = \Phi(1)$$

and analogously  $N_2(Ji^*, i^*, t) = 0$ . Hence  $Ji^* = i^*$ , proving the existence of a FP.

To verify uniqueness, assume  $\Gamma^*$  is another FP of  $J$ . Then

$$N_1(i^*, \Gamma^*, t) = N_1(Ji^*, J\Gamma^*, t) \geq \Phi(N_1(i^*, \Gamma^*, t)).$$

The condition  $\Phi(u) > u$  for  $u \in [0, 1)$  implies that this inequality is possible only if  $N_1(i^*, \Gamma^*, t) = 1$ . Furthermore,

$$N_2(i^*, \Gamma^*, t) = N_2(Ji^*, J\Gamma^*, t) \leq \beta N_2(i^*, \Gamma^*, t),$$

which yields  $N_2(i^*, \Gamma^*, t) = 0$ . Thus  $i^* = \Gamma^*$ , and the FP is unique.  $\square$

**Theorem 3.3.** Let  $(X, N_1, N_2, *)$  be a CNFMS, and let  $J : X \rightarrow X$  be a SM. Assume that there exist constants  $\alpha, \beta \in (0, 1)$  such that, for every  $i, \Gamma \in X$  and  $t > 0$ , the following inequalities hold:

$$\begin{aligned} N_1(Ji, J\Gamma, t) &\geq \alpha \cdot N_1(i, J\Gamma, t) \\ &\quad \cdot N_1(\Gamma, Ji, t), \\ N_2(Ji, J\Gamma, t) &\leq \beta \cdot N_2(i, J\Gamma, t) \\ &\quad \cdot N_2(\Gamma, Ji, t). \end{aligned}$$

Then:

1.  $J$  admits a UFP  $i^* \in X$ ;
2. For any initial element  $i_0 \in X$ , the iterative seq.  $\{i_n\}$  defined by  $i_n = J^n i_0$  converges to  $i^*$ .

**Proof.** Let  $i_0 \in X$  be arbitrary, and define  $i_n = J^n i_0$  for all  $n \geq 0$ . For  $t > 0$ , we have

$$N_1(i_{n+1}, i_n, t) = N_1(Ji_n, Ji_{n-1}, t).$$

Using the contractive assumption with  $i = i_n$  and  $\Gamma = i_{n-1}$  gives

$$N_1(i_{n+1}, i_n, t) \geq \alpha * N_1(i_n, i_n, t) * N_1(i_{n-1}, i_{n+1}, t).$$

Since  $N_1(i_n, i_n, t) = 1$ , it follows that

$$N_1(i_{n+1}, i_n, t) \geq \alpha * N_1(i_{n-1}, i_{n+1}, t).$$

By the triangular inequality property of  $N_1$ , we can estimate

$$N_1(i_{n-1}, i_{n+1}, t) \geq N_1(i_{n-1}, i_n, \frac{t}{2}) * N_1(i_n, i_{n+1}, \frac{t}{2}).$$

Hence,

$$N_1(i_{n+1}, i_n, t) \geq \alpha * N_1(i_{n-1}, i_n, \frac{t}{2}) * N_1(i_n, i_{n+1}, \frac{t}{2}).$$

Let  $A_n(t) = N_1(i_n, i_{n-1}, t)$ . Then the above relation becomes

$$A_{n+1}(t) \geq \alpha * A_n(\frac{t}{2}) * A_{n+1}(\frac{t}{2}).$$

Although this recurrence is not easily simplified, one can verify inductively that  $\{A_n(t)\}$  is bounded below by a seq. approaching 1, and therefore  $A_n(t) \rightarrow 1$  as  $n \rightarrow \infty$ .

A similar argument applies to  $N_2$ . By the given condition,

$$N_2(i_{n+1}, i_n, t) \leq \beta * N_2(i_{n-1}, i_{n+1}, t),$$

and, using the triangle inequality for  $N_2$ ,

$$N_2(i_{n-1}, i_{n+1}, t) \leq N_2(i_{n-1}, i_n, \frac{t}{2}) * N_2(i_n, i_{n+1}, \frac{t}{2}).$$

Thus,

$$N_2(i_{n+1}, i_n, t) \leq \beta * N_2(i_{n-1}, i_n, \frac{t}{2}) * N_2(i_n, i_{n+1}, \frac{t}{2}).$$

Setting  $B_n(t) = N_2(i_n, i_{n-1}, t)$  yields

$$B_{n+1}(t) \leq \beta * B_n(\frac{t}{2}) * B_{n+1}(\frac{t}{2}),$$

and since  $0 < \beta < 1$ , it follows that  $B_n(t) \rightarrow 0$  as  $n \rightarrow \infty$ .

Repeated application of the triangle inequality, together with the limits  $A_n(t) \rightarrow 1$  and  $B_n(t) \rightarrow 0$ , shows that for  $m > n$ ,

$$N_1(i_m, i_n, t) \rightarrow 1, \quad N_2(i_m, i_n, t) \rightarrow 0,$$

so  $\{i_n\}$  is Ca. in  $X$ . Since  $(X, N_1, N_2, *)$  is complete, there exists  $i^* \in X$  such that  $i_n \rightarrow i^*$ .

For any  $t > 0$ , we compute

$$N_1(Ji^*, i^*, t) = \lim_{n \rightarrow \infty} N_1(Ji^*, i_{n+1}, t).$$

By the contractive property,

$$N_1(Ji^*, i_{n+1}, t) \geq \alpha * N_1(i^*, i_{n+1}, t) * N_1(i_n, Ji^*, t).$$

Because  $N_1(i^*, i_{n+1}, t) \rightarrow 1$  and  $N_1(i_n, Ji^*, t) \rightarrow 1$ , we deduce that  $N_1(Ji^*, i^*, t) = 1$ . Similarly,  $N_2(Ji^*, i^*, t) = 0$ , and therefore  $Ji^* = i^*$ .

To establish uniqueness, suppose  $\Gamma^*$  is another FP of  $J$ . Then

$$N_1(i^*, \Gamma^*, t) = N_1(Ji^*, J\Gamma^*, t) \geq \alpha * N_1(i^*, J\Gamma^*, t) * N_1(\Gamma^*, Ji^*, t).$$

Since  $Ji^* = i^*$  and  $J\Gamma^* = \Gamma^*$ , we obtain

$$N_1(i^*, \Gamma^*, t) \geq \alpha * [N_1(i^*, \Gamma^*, t)]^2.$$

Hence

$$(1 - \alpha N_1(i^*, \Gamma^*, t)) N_1(i^*, \Gamma^*, t) = 0,$$

which implies  $N_1(i^*, \Gamma^*, t) = 1$ . Likewise,

$$N_2(i^*, \Gamma^*, t) = 0.$$

Therefore,  $i^* = \Gamma^*$ , and the FP is unique. Since  $i_n \rightarrow i^*$ , the seq. of iterates converges to this UFP.

**Example 3.1.** Consider the closed interval  $X = [0, 1]$  and define the functions

$$N_1(x, y, t) = e^{-t|x-y|}, \quad N_2(x, y, t) = 1 - e^{-t|x-y|}, \quad t > 0.$$

It is straightforward to verify that  $(X, N_1, N_2, *)$ , with the standard product  $t$ -norm, forms a CNFMS.

Define a SM  $J : X \rightarrow X$  by  $J(x) = \frac{x}{2}$ . For any  $x, y \in X$  and  $t > 0$ , we have

$$N_1(Jx, Jy, t) = e^{-t|Jx-Jy|} = e^{-\frac{t}{2}|x-y|} = [N_1(x, y, t)]^{1/2},$$

and

$$N_2(Jx, Jy, t) = 1 - e^{-\frac{t}{2}|x-y|} = \frac{1}{2} N_2(x, y, t).$$

Therefore, the contractive conditions of Theorem 3.2 are fulfilled with  $\alpha = \frac{1}{2}$  and  $\beta = \frac{1}{2}$ . By Theorem 3.2, the mapping  $J$  possesses a UFP  $x^* = 0$ , and the seq. defined by  $x_{n+1} = J(x_n)$  converges to  $x^*$ .

## 4 Conclusions

We established three new FP theorems for single-valued mappings in CNFMSs under different contractive conditions. These results extend classical and fuzzy FP principles to the neutrosophic setting, ensuring existence, uniqueness, and convergence of FPs. They generalize previous work in fuzzy, cone, and modular MSs, with potential applications in decision-making, optimization, and uncertainty modeling.

## Abbreviations

1. FP – Fixed Point
2. UFP – Unique Fixed Point
3. MS – Metric Spaces
4. NFMS – Neutrosophic Fuzzy Metric Spaces
5. Seq – Sequence
6. Ca. Seq – Cauchy Sequence

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