

# A Comparative Study of Adomian–Kamal Decomposition and Euler-Based Methods for Solving the Fractional Abel Differential Equation

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**Abstract** This study presents the development and application of two semi-analytical methods—namely the Adomian Laplace Theorem and the Adomian–Kamal Theorem—for solving the Fractional Abel Differential Equation (FADE). Both approaches integrate the Adomian Decomposition Method (ADM) with distinct integral transforms to enhance accuracy and computational efficiency. The Adomian–Laplace method combines ADM with the Laplace Transform (LT), while the Adomian–Kamal method incorporates the Kamal Integral Transform (KIT), enabling improved handling of the non-local and long-memory characteristics inherent in fractional-order systems. Additionally, a fractional extension of the classical Euler method is implemented for comparative purposes. The methods are evaluated through two case studies, where approximate solutions are compared to exact solutions for various fractional orders  $\alpha$ . Graphical analyses demonstrate that both semi-analytical methods yield results that perfectly overlap with exact solutions, indicating high accuracy and convergence. In contrast, the fractional Euler method exhibits reduced accuracy at lower fractional orders due to its limited capability of capturing memory effects. The findings

highlight the superior performance and reliability of the Adomian–Kamal and Adomian–Laplace approaches for solving nonlinear FADEs, offering a robust framework for analytical and semi-analytical modeling in physics, engineering, and applied sciences.

**Keywords** FADEs, ADM, KIT, Caputo Derivative, Semi-Analytical Method, Fractional Euler Method

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## 1. Introduction

Fractional calculus has emerged as a powerful mathematical tool for modeling complex systems in science and engineering, particularly those exhibiting memory effects and hereditary properties that cannot be adequately captured by classical integer-order models. Among its applications, the FADE plays a central role due to its ability to describe non-local and long-memory dynamics in diverse fields such as physics, biology, control theory, finance, and engineering. Unlike conventional differential equations, FADE incorporates

fractional-order derivatives that account for the system's past states, making it more suitable for accurately representing anomalous diffusion, viscoelastic behavior, and other phenomena where history-dependent processes are significant.

The study of FADEs is significant due to their broad applications in natural sciences and engineering, which require the development of efficient and accurate solution methods [1]. The Abel differential equation, particularly in its fractional form, plays a key role in modeling complex systems in physics, astrophysics, and applied sciences, where traditional methods often fail due to the lack of exact solutions [2]. Several innovative approaches have been proposed to address these challenges.

For instance, the optimal  $q$ -homotopy analysis method (q-HAM) has demonstrated improved convergence for approximate solutions of Abel-type equations, providing a powerful analytical tool for investigating FADEs [3]. Similarly, the use of generalized Bessel functions (GBFs) in conjunction with spectral and Newton–Krylov subspace methods has yielded reliable numerical algorithms for solving nonlinear FADEs, effectively transforming them into solvable algebraic systems [4]. Beyond theoretical studies, FADEs have practical applications in image processing, particularly in edge detection, where they enhance accuracy and computational speed in identifying structural features [5]. Furthermore, reproducing kernel algorithms employing the Caputo–Fabrizio fractional derivative offer an attractive numerical approach with high accuracy and efficiency, making them valuable for solving nonlinear time-fractional differential equations across diverse scientific domains [6]. The homotopy analysis method has also proven to be a stable and convergent technique for solving FADEs, delivering both accuracy and ease of implementation [7].

The growing advancement of science and technology underscores the critical need for continued research and development of numerical methods that can effectively address the complexity of FADEs, thereby expanding their applicability across various scientific and engineering disciplines [8]. The ADM is a well-established semi-analytical technique widely used to solve both linear and nonlinear differential equations, including fractional differential equations (FDEs) [9]. ADM is particularly effective for handling nonlinear initial value problems (IVPs) and boundary value problems (BVPs) by decomposing nonlinear operators into a series of functions, with each term generated from Adomian polynomials via analytic function expansions [10].

ADM has been successfully applied to numerous engineering problems such as electrical networks and fluid flow, where it provides analytical solutions in the form of infinite series. The convergence and error analysis of these series solutions is essential, and ADM has consistently produced results in agreement with exact and numerical methods [11]. Its versatility has been further

demonstrated in solving multi-term FDEs, where ADM has been used alongside numerical approaches such as the iterative vibration method to tackle complex systems [12]. In the context of non-Newtonian fluid dynamics, ADM has shown superior performance compared to spectral Chebyshev methods when solving nonlinear differential equations [13]. Moreover, ADM has been adapted to solve nonlinear FDEs involving the Riemann–Liouville fractional derivative, offering simplicity and effectiveness in implementation [14]. Its adaptability is also evident in applications to fuzzy Hilfer fractional differential equations, where it provides interpolation between Riemann–Liouville and Caputo derivatives, offering greater flexibility along with high accuracy and efficiency [15].

Despite its effectiveness, the ADM–Laplace method exhibits limitations when solving FADEs, particularly in obtaining approximate solutions, due to difficulties in handling algebraic constraints inherent in such systems [16]. Moreover, classical numerical methods like Euler's method are inadequate for capturing the intrinsic memory effects of FADEs, as they are generally designed for integer-order systems without nonlocal characteristics [17], [18]. Euler's method is only valid for  $\alpha$  that denotes the derivative order, and thus fails to generalize fractional-order equations in which memory effects are significant [19]. The long-memory principle has been suggested as a potential solution for FADEs, but it requires careful implementation to maintain solution validity [20]. These limitations highlight the need for more robust analytical and numerical methods capable of incorporating memory and nonlocal effects—capabilities that traditional ADM–Laplace and classical Euler methods do not adequately address [21].

To overcome these challenges, the combination of ADM with the KIT, referred to as the ADM Kamal method represents a significant innovation in solving fractional-order differential equations. This approach extends ADM Laplace by incorporating the Kamal transform, enhancing both solution accuracy and computational efficiency. The ADM–Kamal method has been shown to provide highly accurate approximate solutions, as reported in various studies [22]. It has been successfully applied to nonlinear fractional differential equations, producing analytical, approximate, and numerical results comparable to those obtained via ADM–Laplace [23]. Its iterative formulation, supported by graphical and numerical validation, demonstrates its efficiency and simplicity, particularly in solving complex equations such as nonlinear wave-like equations with variable coefficients [24].

An important component of these methods is the Mittag–Leffler function and its generalizations, which naturally arise in fractional calculus and are essential for modeling non-exponential relaxation and anomalous diffusion processes [25]–[27]. When combined with

integral transforms, these functions form a powerful framework for solving FDEs and related integral equations [28]-[30]. The computational implementation using Adomian polynomials and iterative schemes further enhances numerical efficiency and accuracy [31].

This research makes a significant contribution to fractional calculus and the modeling of complex systems by presenting an efficient framework for solving FADEs, validated through both theoretical formulation and numerical experimentation. The proposed methods have cross-disciplinary relevance, with potential applications in engineering, biomedical signal processing, finance, and other scientific fields where long-memory and nonlinear characteristics are prevalent [32], [33].

The main contribution and novelty of this work are as follows:

- a. This work introduces the application of the Adomian–Kamal decomposition method—combining the ADM with the KIT—to the solution of nonlinear FADEs.
- b. The study provides a systematic comparison between two semi-analytical approaches (ADM–Laplace and ADM–Kamal) and an extended fractional Euler method, showing that the proposed semi-analytical methods produce results that match exact solutions.
- c. By incorporating Mittag–Leffler functions and leveraging integral transforms within the Adomian-based framework, the paper offers a computationally efficient and highly accurate methodology that can be extended to solve other classes of nonlinear fractional-order differential equations.

The remaining part of the paper is organized as follows: Section 2 presents the mathematical formulation of the FADE along with the descriptions of the Adomian–Laplace, Adomian–Kamal, and fractional Euler methods. Section 3 provides the numerical results, including illustrative examples, graphical comparisons, and error analyses for different fractional orders. Section 4 concludes the study by highlighting the main findings and suggesting possible directions for future research.

## 2. Methodology

This study employs a hybrid approach that combines semi-analytical methods—the Adomian–Laplace theorem and the Adomian–Kamal theorem—with a numerical fractional Euler method to solve the FADE.

To ensure a fair and systematic comparison, the performance of the Adomian–Laplace, Adomian–Kamal, and fractional Euler methods is evaluated based on their accuracy, convergence behavior, and computational efficiency [34], [35]. Accuracy is assessed through comparisons with exact solutions for different fractional

orders, while convergence is examined by analyzing the stability of recursive iterations [36], [37]. Computational aspects, such as the number of iterations and ease of implementation, are also taken into account. All symbolic manipulations and numerical computations were performed using MATLAB, ensuring reproducibility of the results. This framework provides a comprehensive basis for evaluating the strengths and limitations of the proposed methods in solving nonlinear FADEs [38]-[40].

### 2.1. Problem Definition: Fractional Abel Differential Equation

The general form of the FADE considered in this study is:

$${}^c D_t^\alpha y(t) + f(t, y(t)) = 0, \quad y(0) = y_0 \quad (1)$$

where  ${}^c D_t^\alpha$  denotes the Caputo fractional derivative of order  $(0 < \alpha \leq 1)$ , and  $f(t, y(t))$  is a nonlinear function. The fractional order introduces non-local and long-memory effects that cannot be effectively captured by standard integer-order methods.

### 2.2. Adomian–Laplace Theorem

The Adomian–Laplace theorem combines the ADM with the LT to solve the FADE.

Consider the fractional Abel differential equation in the Caputo sense:

$$D_t^\alpha y(t) = g(t, y(t)), \quad 0 < \alpha \leq 1, \quad (2)$$

with the initial condition

$$y(0) = y_0. \quad (3)$$

Applying the LT to both sides and using the property of the Caputo derivative:

$$\mathcal{L}\{D_t^\alpha y(t)\} = s^\alpha Y(s) - s^{\alpha-1}y(0). \quad (4)$$

Applying this to the FADE and rearranging yields:

$$Y(s) = \frac{y_0}{s} + \frac{1}{s^\alpha} \mathcal{L}\{g(t, y(t))\}. \quad (5)$$

Applying the inverse LT gives:

$$y(t) = y_0 + \mathcal{L}^{-1}\left\{\frac{1}{s^\alpha} \mathcal{L}\{g(t, y(t))\}\right\}. \quad (6)$$

Using the ADM:

$$y(t) = \sum_{n=0}^{\infty} y_n(t), \quad g(t, y(t)) = \sum_{n=0}^{\infty} A_n, \quad (7)$$

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} g(t, \sum_{k=0}^{\infty} \lambda^k y_k(t)) \Big|_{\lambda=0}$$

The recursive relation is:

$$\begin{cases} y_0(t) = y_0, \\ y_{n+1}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^\alpha} \mathcal{L}\{A_n\}\right\} \end{cases} \quad n \geq 0 \quad (8)$$

### 2.3. Adomian–Kamal Theorem

The Adomian–Kamal theorem extends the ADM by employing the KIT instead of the LT. Replacing the LT with the Kamal transform  $\mathcal{K}\{\cdot\}$ , we have:

$$\begin{aligned}\mathcal{K}\{D_t^\alpha y(t)\} &= p^\alpha Y(p) - p^{\alpha-1}y(0), \\ Y(p) &= \frac{y_0}{p} + \frac{1}{p^\alpha} \mathcal{K}\{g(t, y(t))\}.\end{aligned}\quad (9)$$

Inverse Kamal transforms:

$$y(t) = y_0 + \mathcal{K}^{-1}\left\{\frac{1}{p^\alpha} \mathcal{K}\{g(t, y(t))\}\right\}.\quad (10)$$

Using ADM decomposition and Adomian polynomials as above, the recursive form becomes:

$$\begin{cases} y_0(t) = y_0, \\ y_{n+1}(t) = \mathcal{K}^{-1}\left\{\frac{1}{p^\alpha} \mathcal{K}\{A_n\}\right\} \quad n \geq 0. \end{cases}\quad (11)$$

### 2.4. Fractional Euler Method

The fractional Euler discretization is:

$$y_{m+1} = y_m + h^\alpha \sum_{k=0}^m w_k g(t_k, y_k),\quad (12)$$

Where  $w_k$  is fractional weights determined from the discretized fractional integral. For  $\alpha=1$ , it reduces to the classical Euler method:

$$y_{m+1} = y_m + hg(t_m, y_m).\quad (13)$$

## 3. Results and Discussion

### 3.1. New Theorem

This section presents the combined theorem of the ADM and the Adomian–Laplace for solving the FADE. The Adomian–Laplace theorem is a well-established method widely used for solving the FADE, whereas the Adomian–Kamal theorem is a combination of the Adomian–Kamal. Both theorems are employed to solve the FADE.

**Adomian–Laplace Theorem.** Given a FADE as follows:

$$D_t^\alpha \phi(t) = A + B\phi(t) + C\phi^2(t) + D\phi^3(t), \quad t > 0, A, B, C, D \in \mathbb{R}\quad (14)$$

with the initial condition  $\phi(0) = k$  and  $D_t^\alpha \phi(t)$  being the Caputo fractional derivative of the function  $\phi(t)$  with respect to  $t$  of order  $\alpha$ , where  $0 < \alpha \leq 1$ , the approximate solution of equation (14) is

$$\phi(t) = \sum_{n=0}^{\infty} \phi_n(t),$$

where;

$$\phi_0(t) = \phi(0) + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[A]\right],$$

and

$$\phi = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[B\phi_{n-1}]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[C\phi_{n-1}^2]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[D\phi_{n-1}^3]\right], n \geq 1 .$$

**Proof.**

Based on equation (14), the FADE is given as follows.

$$D_t^\alpha \phi(t) = A + B\phi(t) + C\phi^2(t) + D\phi^3(t), \quad t > 0, A, B, C, D \in \mathbb{R}$$

where  $\alpha$  is the order of the Caputo fractional derivative,  $n-1 < \alpha \leq n, n \in \mathbb{Z}^+$ , and  $\mathbb{Z}^+$  denotes the set of positive integers.

Applying the LT to equation (14) yields

$$\begin{aligned} \mathcal{L}[D_t^\alpha \phi(t)] &= \mathcal{L}[A] + \mathcal{L}[B\phi(t)] + \mathcal{L}[C\phi^2(t)] + \mathcal{L}[D\phi^3(t)]. \\ s^\alpha \mathcal{L}[\phi(t)] - \sum_{m=0}^{n-1} s^{\alpha-m-1} \phi^{(m)}(0) &= \mathcal{L}[A] + \mathcal{L}[B\phi(t)] + \mathcal{L}[C\phi^2(t)] + \mathcal{L}[D\phi^3(t)], \\ s^\alpha \mathcal{L}[\phi(t)] - s^{\alpha-1} \phi(0) &= \mathcal{L}[A] + \mathcal{L}[B\phi(t)] + \mathcal{L}[C\phi^2(t)] + \mathcal{L}[D\phi^3(t)], \end{aligned} \tag{15}$$

Dividing both sides by  $s^\alpha$ , we obtain

$$\mathcal{L}[\phi(t)] = \frac{\phi(0)}{s} + \frac{1}{s^\alpha} \mathcal{L}[A] + \frac{1}{s^\alpha} \mathcal{L}[B\phi(t)] + \frac{1}{s^\alpha} \mathcal{L}[C\phi^2(t)] + \frac{1}{s^\alpha} \mathcal{L}[D\phi^3(t)].$$

Using the inverse LT to find the solution  $\phi(t)$ , we obtain

$$\begin{aligned} \mathcal{L}^{-1}[\mathcal{L}[\phi(t)]] &= \mathcal{L}^{-1}\left[\frac{\phi(0)}{s}\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[A]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[B\phi(t)]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[C\phi^2(t)]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[D\phi^3(t)]\right], \\ \phi(t) &= \phi(0) + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[A]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[B\phi(t)]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[C\phi^2(t)]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[D\phi^3(t)]\right]. \end{aligned} \tag{16}$$

ADM assumes that the function  $\phi(t)$  can be expressed as an infinite series as follows:

$$\phi(t) = \phi_0 + \phi_1 + \phi_2 + \dots = \sum_{n=0}^{\infty} \phi_n(t), \tag{17}$$

and the nonlinear operators  $\phi^2(t)$  and  $\phi^3(t)$  can be decomposed as:

$$\begin{aligned} \mathcal{F}\phi(t) = \phi^2(t) &= \sum_{n=0}^{\infty} P_n, \\ \phi^3(t) &= \sum_{n=0}^{\infty} Q_n. \end{aligned} \tag{18}$$

where  $P_n = P_n(\phi_0, \phi_1, \phi_2, \phi_3, \dots, \phi_n)$  and  $Q_n = Q_n(\phi_0, \phi_1, \phi_2, \phi_3, \dots, \phi_n)$  are the Adomian polynomials, which can be expanded as follows:

For  $\phi^2(t)$

$$\begin{aligned} P_0 &= \mathcal{F}(\phi_0) = \phi_0^2, \\ P_1 &= \phi_1 \mathcal{F}'(\phi_0) = 2\phi_0 \phi_1, \\ P_2 &= \phi_2 \mathcal{F}'(\phi_0) + \frac{\phi_1^2}{2!} \mathcal{F}''(\phi_0) = 2\phi_0 \phi_2 + \phi_1^2, \\ &\vdots \end{aligned}$$

For  $\phi^3(t)$

$$\begin{aligned} Q_0 &= \mathcal{F}(\phi_0) = \phi_0^3, \\ Q_1 &= \phi \mathcal{F}'(\phi_0) = 3\phi_0^2 \phi_1, \\ Q_2 &= \phi_2 \mathcal{F}'(\phi_0) + \frac{\phi_1^2}{2!} \mathcal{F}''(\phi_0) = 3\phi_0^2 \phi_2 + 3\phi_0 \phi_1^2, \\ &\vdots \end{aligned}$$

Substituting equations (17) and (18) into equation (16) yields

$$\begin{aligned} \sum_{n=0}^{\infty} \phi_n(t) &= \phi(0) + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[A]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}\left[B \sum_{n=0}^{\infty} \phi_n(t)\right]\right] \\ &+ \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}\left[C \sum_{n=0}^{\infty} P_n(t)\right]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}\left[D \sum_{n=0}^{\infty} Q_n(t)\right]\right], \\ \phi(t) &= \phi_0(t) + \phi_1(t) + \phi_2(t) + \phi(t) \dots \\ &= \phi(0) + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[A]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[B(\phi_0 + \phi_1 + \phi_2 + \dots)]\right] \\ &+ \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[C(P_0 + P_1 + P_2 + \dots)]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha} \mathcal{L}[D(Q_0 + Q_1 + Q_2 + \dots)]\right]. \end{aligned} \tag{19}$$

From equation (19), the recursive iteration is obtained as follows:

$$\begin{aligned}\phi_0(t) &= \phi(0) + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[A] \right], \\ \phi_1(t) &= \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[B\phi_0] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[CP_0] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[DQ_0] \right], \\ \phi_2(t) &= \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[B\phi_1] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[CP_1] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[DQ_1] \right], \\ \phi_n(t) &= \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[B\phi_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[CP_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[DQ_{n-1}] \right] .. \\ &\vdots\end{aligned}$$

Thus, the approximate solution of equation (14) is proven to be

$$\phi(t) = \sum_{n=0}^{\infty} \phi_n(t),$$

where  $\phi_0(t) = \phi(0) + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[A] \right]$ , and

$$\phi_n = \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[B\phi_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[CP_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[DQ_{n-1}] \right], n \geq 1 \quad \blacksquare$$

**Adomian–Kamal Theorem.** If the following Abel-type fractional differential equation is given

$$D_t^\alpha \phi(t) = A + B\phi(t) + C\phi^2(t) + D\phi^3(t), \quad t > 0, A, B, C, D \in \mathbb{R} \quad (20)$$

with the initial condition  $\phi(0) = k$  and  $D_t^\alpha \phi(t)$  being the Caputo fractional derivative of the function  $\phi(t)$  with respect to  $t$  of order  $\alpha$ , where  $0 < \alpha \leq 1$ , the approximate solution of equation (20) is

$$\phi(t) = \sum_{n=0}^{\infty} \phi_n(t),$$

where  $\phi_0(t) = \phi(0) + \mathcal{K}^{-1}[v^{2\alpha} \mathcal{K}[A]]$ , and

$$\phi_n(t) = \mathcal{K}^{-1}[v^{2\alpha} \mathcal{K}[B\phi_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha} \mathcal{K}[CP_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha} \mathcal{K}[DQ_{n-1}]], n \geq 1.$$

**Proof.**

From equation (20), the Abel-type FDE is given as follows

$$D_t^\alpha \phi(t) = A + B\phi(t) + C\phi^2(t) + D\phi^3(t), \quad t > 0, A, B, C, D \in \mathbb{R}$$

where  $\alpha$  is the order of the Caputo fractional derivative,  $n - 1 < \alpha \leq n, n \in \mathbb{Z}^+$ , and  $\mathbb{Z}^+$  denotes the set of positive integers.

Applying the kamal transform to equation (20), we obtain

$$\begin{aligned}\mathcal{K}[D_t^\alpha \phi(t)] &= \mathcal{K}[A + B\phi(t) + C\phi^2(t) + D\phi^3(t)], \\ \frac{\phi(v)}{v^{2\alpha}} - \sum_{k=0}^{n-1} \frac{\phi^k(0)}{v^{2(\alpha-k)-1}} &= \mathcal{K}[A] + \mathcal{K}[B\phi(t)] + \mathcal{K}[C\phi^2(t)] + \mathcal{K}[D\phi^3(t)], \\ \frac{\phi(v)}{v^{2\alpha}} - \frac{\phi(0)}{v^{2\alpha-1}} &= \mathcal{K}[A] + \mathcal{K}[B\phi(t)] + \mathcal{K}[C\phi^2(t)] + \mathcal{K}[D\phi^3(t)],\end{aligned}$$

Multiplying both sides by  $v^{2\alpha}$ , we obtain

$$\begin{aligned}\phi(v) - v\phi(0) &= v^{2\alpha} \mathcal{K}[A] + v^{2\alpha} \mathcal{K}[B\phi(t)] + v^{2\alpha} \mathcal{K}[C\phi^2(t)] + v^{2\alpha} \mathcal{K}[D\phi^3(t)], \\ \phi(v) &= v\phi(0) + v^{2\alpha} \mathcal{K}[A] + v^{2\alpha} \mathcal{K}[B\phi(t)] + v^{2\alpha} \mathcal{K}[C\phi^2(t)] + v^{2\alpha} \mathcal{K}[D\phi^3(t)]. \quad (21)\end{aligned}$$

Applying the inverse Kamal transform to determine  $\phi(x)$ , we obtain

$$\mathcal{K}^{-1}[\phi(v)] = \mathcal{K}^{-1}[v\phi(0) + v^{2\alpha}\mathcal{K}[A] + v^{2\alpha}\mathcal{K}[B\phi(t)] + v^{2\alpha}\mathcal{K}[C\phi^2(t)] + v^{2\alpha}\mathcal{K}[D\phi^3(t)]],$$

$$\phi(x) = \phi(0) + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[A]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[B\phi(t)]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[C\phi^2(t)]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[D\phi^3(t)]]. \quad (22)$$

ADM assumes that the function  $\phi(t)$ , can be expressed as an infinite series as follows:

$$\phi(t) = \sum_{n=0}^{\infty} \phi_n(t), \quad (23)$$

and the nonlinear operators  $\phi^2(t)$  and  $\phi^3(t)$  can be decomposed as:

$$\mathcal{F}\phi(t) = \phi^2(t) = \sum_{n=0}^{\infty} P_n,$$

$$\phi^3(t) = \sum_{n=0}^{\infty} Q_n \quad (24)$$

where  $P_n$  is the Adomian polynomials, which can be expanded as follows:

For  $\phi^2(t)$

$$P_0 = \mathcal{F}(\phi_0) = \phi_0^2,$$

$$P_1 = \phi_1 \mathcal{F}'(\phi_0) = 2\phi_0\phi_1,$$

$$P_2 = \phi_2 \mathcal{F}'(\phi_0) + \frac{\phi_1^2}{2!} \mathcal{F}''(\phi_0) = 2\phi_0\phi_2 + \phi_1^2,$$

$$\vdots$$

For  $\phi^3(t)$

$$Q_0 = \mathcal{F}(\phi_0) = \phi_0^3,$$

$$Q_1 = \phi_1 \mathcal{F}'(\phi_0) = 3\phi_0^2\phi_1,$$

$$Q_2 = \phi_2 \mathcal{F}'(\phi_0) + \frac{\phi_1^2}{2!} \mathcal{F}''(\phi_0) = 3\phi_0^2\phi_2 + 3\phi_0\phi_1^2,$$

$$\vdots$$

Substituting equations (23) and (24) into equation (22) yields

$$\sum_{n=0}^{\infty} \phi_n(t) = \phi(0) + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[A]] + \mathcal{K}^{-1} \left[ v^{2\alpha}\mathcal{K} \left[ B \sum_{n=0}^{\infty} \phi_n(t) \right] \right]$$

$$+ \mathcal{K}^{-1} \left[ v^{2\alpha}\mathcal{K} \left[ C \sum_{n=0}^{\infty} P_n \right] \right] + \mathcal{K}^{-1} \left[ v^{2\alpha}\mathcal{K} \left[ D \sum_{n=0}^{\infty} Q_n \right] \right],$$

$$\phi(t) = \phi_0(t) + \phi_1(t) + \phi_2(t) + \phi_3(t) \dots$$

$$= \phi(0) + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[A]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[B(\phi_0 + \phi_1 + \phi_2 + \dots)]]$$

$$+ \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[C(P_0 + P_1 + P_2 + \dots)]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[D(Q_0 + Q_1 + Q_2 + \dots)]]]. \quad (25)$$

From equation (25), the recursive iteration is obtained as follows:

$$\phi_0(t) = \phi(0) + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[A]],$$

$$\phi_1(t) = \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[B\phi_0]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[CP_0]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[DQ_0]],$$

$$\phi_2(t) = \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[B\phi_1]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[CP_1]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[DQ_1]],$$

$$\phi_n(t) = \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[B\phi_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[CP_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[DQ_{n-1}]].$$

$$\vdots$$

Thus, the approximate solution of equation (20) is proven to be characteristics.

$$\phi(t) = \sum_{n=0}^{\infty} \phi_n(t)$$

where  $\phi_0(t) = \phi(0) + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[A]]$ , and

$$\phi_n(t) = \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[B\phi_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[CP_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[DQ_{n-1}]], n \geq 1.$$

### 3.2. Example 1.

Given the FADE:

$$D_t^\alpha \phi(t) = 8 - 12\phi(t) + 6\phi^2(t) - \phi^3(t), \quad (26)$$

with  $0 \leq t \leq 1$ ,  $0 < \alpha \leq 1$ , for  $\phi(0) = 0$

- Determine the exact solution  $\phi(t)$
- Obtain approximate solutions using the Adomian–Laplace method for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$ .
- Obtain approximate solutions using the Adomian–Kamal method for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$ .

#### 3.2.1. Exact Solution Example 1

$D_t^\alpha \phi(t) = 8 - 12\phi(t) + 6\phi^2(t) - \phi^3(t)$ , with  $\alpha = 1$ , we can write

$$D_t^\alpha \phi(t) = -(\phi - 2)^3 \text{ for } \phi(0) = 0$$

$$\frac{d\phi}{dt} = -(\phi - 2)^3$$

$$d\phi = -(\phi - 2)^3 dt$$

$$\frac{dy}{(\phi - 2)^3} = - dt$$

$$\int \frac{dy}{(\phi - 2)^3} = \int -dt$$

$$-\frac{1}{2}(\phi - 2)^{-2} = -t + C_1$$

$$\frac{1}{(\phi - 2)^2} = 2t + C, \text{ given that for } \phi(0) = 0$$

$$\frac{1}{(0 - 2)^2} = 0 + C, \text{ we obtain } C = \frac{1}{4},$$

$$\frac{1}{(\phi - 2)^2} = 2t + \frac{1}{4}$$

Therefore, the exact solution is given by  $(\phi - 2)^2 = \frac{1}{2t + \frac{1}{4}}$

Figure 1 presents the graph of the exact solution  $\phi(t)$  in Example 1 for  $0 < t < 0.2$ . The solution gradually converges to the classical case, reflecting the transition from fractional dynamics with strong memory effects to standard integer-order behavior. For smaller values of  $\alpha$ , the solution curves deviate more significantly, highlighting the influence of fractional order on the system's evolution. This demonstrates the sensitivity of the FADE to changes in  $\alpha$  and emphasizes the role of fractional derivatives in capturing long-memory

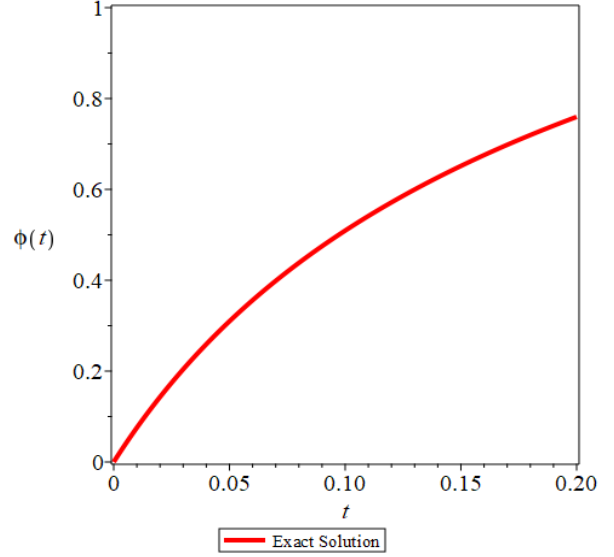


Figure 1. Graph of the exact solution  $\phi(t)$  for  $0 < t < 0.2$

#### 3.2.2. Solutions Using the Adomian–Laplace Method for Example 1

From the Adomian–Laplace Theorem, the approximate solution of the FADE (26) is given by

$$\phi_0(t) = 0 + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[A] \right],$$

$$\phi_0(t) = \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[8] \right].$$

$$\phi_n = \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[B\phi_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[CP_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[DQ_{n-1}] \right], n \geq 1,$$

$$\phi_n(t) = \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[-12\phi_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[6P_{n-1}] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[Q_{n-1}] \right], n \geq 1. \quad (27)$$

where  $P_n$  denotes the Adomian polynomial corresponding to the nonlinear operators  $\phi^2(t)$  and  $\phi^3(t)$ , and can be expressed as follows:

For  $\phi^2(t)$

$$P_0 = \phi_0^2,$$

$$P_1 = 2\phi_0\phi_1,$$

$$P_2 = 2\phi_0\phi_2 + \phi_1^2,$$

⋮

For  $\phi^3(t)$

$$Q_0 = \phi_0^3,$$

$$Q_1 = 3\phi_0^2\phi_1,$$

$$Q_2 = 3\phi_0^2\phi_2 + 3\phi_0\phi_1^2,$$

⋮

The following is a description of equation (27)

$$\begin{aligned}
 \phi_0 &= \phi_0(t) = \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[8] \right] = \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \cdot \frac{8}{s} \right] = \mathcal{L}^{-1} \left[ \frac{8}{s^{\alpha+1}} \right] = \frac{8t^\alpha}{\alpha!} = \frac{8t^\alpha}{\Gamma(\alpha+1)}, \\
 \phi_1 &= \phi_1(t) = -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[12\phi_0] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[6P_0] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[Q_0] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ \frac{12 \cdot 8t^\alpha}{\Gamma(\alpha+1)} \right] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[6\phi_0^2] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[\phi_0^3] \right], \\
 &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ \frac{12 \cdot 8t^\alpha}{\Gamma(\alpha+1)} \right] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ \frac{6 \cdot 8^2 t^{2\alpha}}{\Gamma^2(\alpha+1)} \right] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ \frac{8^3 t^{3\alpha}}{\Gamma^3(\alpha+1)} \right] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{96}{\Gamma(\alpha+1)s^\alpha} \mathcal{L}[t^\alpha] \right] + \mathcal{L}^{-1} \left[ \frac{384}{\Gamma^2(\alpha+1)s^\alpha} \mathcal{L}[t^{2\alpha}] \right] - \mathcal{L}^{-1} \left[ \frac{512}{\Gamma^3(\alpha+1)s^\alpha} \mathcal{L}[t^{3\alpha}] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{96}{\Gamma(\alpha+1)s^\alpha} \cdot \frac{\alpha!}{s^{\alpha+1}} \right] + \mathcal{L}^{-1} \left[ \frac{384}{\Gamma^2(\alpha+1)s^\alpha} \cdot \frac{2\alpha!}{s^{2\alpha+1}} \right] - \mathcal{L}^{-1} \left[ \frac{512}{\Gamma^3(\alpha+1)s^\alpha} \cdot \frac{3\alpha!}{s^{3\alpha+1}} \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{96}{\Gamma(\alpha+1)} \cdot \frac{\Gamma(\alpha+1)}{s^{2\alpha+1}} \right] + \mathcal{L}^{-1} \left[ \frac{384}{\Gamma^2(\alpha+1)} \cdot \frac{\Gamma(2\alpha+1)}{s^{3\alpha+1}} \right] - \mathcal{L}^{-1} \left[ \frac{512}{\Gamma^3(\alpha+1)} \cdot \frac{\Gamma(3\alpha+1)}{s^{4\alpha+1}} \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{96}{s^{2\alpha+1}} \right] + \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)} \mathcal{L}^{-1} \left[ \frac{384}{s^{3\alpha+1}} \right] - \frac{\Gamma(3\alpha+1)}{\Gamma^3(\alpha+1)} \mathcal{L}^{-1} \left[ \frac{512}{s^{4\alpha+1}} \right] \\
 &= -\frac{96t^{2\alpha}}{2\alpha!} + \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)} \cdot \frac{384t^{3\alpha}}{3\alpha!} - \frac{512\Gamma(3\alpha+1)}{\Gamma^3(\alpha+1)} \cdot \frac{t^{4\alpha}}{4\alpha!} \\
 &= -\frac{96t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{384}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{512}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \Gamma(3\alpha+1)t^{4\alpha} \\
 \phi_2 &= \phi_2(t) = -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[12\phi_1] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[6P_1] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[Q_1] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[12\phi_1] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[12\phi_0\phi_1] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[3\phi_0^2\phi_1] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ 12 \left( -\frac{96t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{384}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{512}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \Gamma(3\alpha+1)t^{4\alpha} \right) \right] \right] \\
 &+ \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ 12 \left( \frac{8t^\alpha}{\Gamma(\alpha+1)} \right) \left( -\frac{96t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{384}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{512}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \Gamma(3\alpha+1)t^{4\alpha} \right) \right] \right] \\
 &- \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ 3 \left( \frac{8t^\alpha}{\Gamma(\alpha+1)} \right)^2 \left( -\frac{96t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{384}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{512}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \Gamma(3\alpha+1)t^{4\alpha} \right) \right] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ -\frac{1.152t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{4.608}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{6.144}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \Gamma(3\alpha+1)t^{4\alpha} \right] \right] \\
 &+ \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ -\frac{9.216t^{3\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} + \frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)} - \frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)} \Gamma(3\alpha+1)t^{5\alpha} \right] \right] \\
 &- \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ -\frac{18.432t^{4\alpha}}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)} + \frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)} - \frac{98.304}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)} \Gamma(3\alpha+1)t^{6\alpha} \right] \right] \\
 &= -\mathcal{L}^{-1} \left[ -\frac{1}{s^\alpha} \cdot \frac{1.152}{\Gamma(2\alpha+1)} \cdot \frac{2\alpha!}{s^{2\alpha+1}} + \frac{1}{s^\alpha} \cdot \frac{4.608}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{3\alpha!}{s^{3\alpha+1}} - \frac{1}{s^\alpha} \cdot \frac{6.144}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{4\alpha!}{s^{4\alpha+1}} \right] \\
 &+ \mathcal{L}^{-1} \left[ -\frac{1}{s^\alpha} \cdot \frac{9.216}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} \cdot \frac{3\alpha!}{s^{3\alpha+1}} + \frac{1}{s^\alpha} \cdot \frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{4\alpha!}{s^{4\alpha+1}} - \frac{1}{s^\alpha} \cdot \frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{5\alpha!}{s^{5\alpha+1}} \right]
 \end{aligned}$$

$$\begin{aligned}
& -\mathcal{L}^{-1} \left[ -\frac{1}{s^\alpha} \cdot \frac{18.432}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)} \cdot \frac{4\alpha!}{s^{4\alpha+1}} + \frac{1}{s^\alpha} \cdot \frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{5\alpha!}{s^{5\alpha+1}} - \frac{1}{s^\alpha} \cdot \frac{98.304}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{6\alpha!}{s^{6\alpha+1}} \right] \\
& = -\mathcal{L}^{-1} \left[ -\frac{1.152}{\Gamma(2\alpha+1)} \cdot \frac{\Gamma(2\alpha+1)}{s^{3\alpha+1}} + \frac{4.608}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{\Gamma(3\alpha+1)}{s^{4\alpha+1}} - \frac{6.144}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{\Gamma(4\alpha+1)}{s^{5\alpha+1}} \right] \\
& + \mathcal{L}^{-1} \left[ -\frac{9.216}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} \cdot \frac{\Gamma(3\alpha+1)}{s^{4\alpha+1}} + \frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{\Gamma(4\alpha+1)}{s^{5\alpha+1}} - \frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{\Gamma(5\alpha+1)}{s^{6\alpha+1}} \right] \\
& - \mathcal{L}^{-1} \left[ -\frac{18.432}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)} \cdot \frac{\Gamma(4\alpha+1)}{s^{5\alpha+1}} + \frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{\Gamma(5\alpha+1)}{s^{6\alpha+1}} - \frac{98.304}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{\Gamma(6\alpha+1)}{s^{7\alpha+1}} \right] \\
& = \frac{1.152t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{4.608}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma(2\alpha+1)} + \frac{6.144}{\Gamma^3(\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(3\alpha+1)t^{5\alpha}}{\Gamma(3\alpha+1)} \\
& - \frac{9.216}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(3\alpha+1)} + \frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma(2\alpha+1)\Gamma(4\alpha+1)} \\
& - \frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)} \frac{\Gamma(3\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma(3\alpha+1)\Gamma(5\alpha+1)} + \frac{18.432}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma(4\alpha+1)} \\
& - \frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)\Gamma(6\alpha+1)} \frac{\Gamma(2\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma(2\alpha+1)\Gamma(5\alpha+1)} + \frac{98.304\Gamma(3\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)\Gamma(7\alpha+1)} \\
& \quad \vdots
\end{aligned}$$

By applying the Adomian–Laplace Theorem, the solution to equation (26) is obtained as:

$$\begin{aligned}
\phi(t) & = \phi_0(t) + \phi_1(t) + \phi_2(t) + \dots \\
& = \frac{8t^\alpha}{\Gamma(\alpha+1)} - \frac{96t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{384\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{512\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \\
& + \frac{1.152t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{4.608}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma(2\alpha+1)} + \frac{6.144}{\Gamma^3(\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(3\alpha+1)t^{5\alpha}}{\Gamma(3\alpha+1)} \\
& - \frac{9.216}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(3\alpha+1)} + \frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma(2\alpha+1)\Gamma(4\alpha+1)} \\
& - \frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)} \frac{\Gamma(3\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma(3\alpha+1)\Gamma(5\alpha+1)} + \frac{18.432}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma(4\alpha+1)} \\
& - \frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)\Gamma(6\alpha+1)} \frac{\Gamma(2\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma(2\alpha+1)\Gamma(5\alpha+1)} + \frac{98.304\Gamma(3\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)\Gamma(7\alpha+1)}.
\end{aligned}$$

For  $\alpha = 0.1$

$$\begin{aligned}
\phi(t) & = \phi_0(t) + \phi_1(t) + \phi_2(t) + \dots \\
& = \frac{8t^{0,1}}{\Gamma(0,1+1)} - \frac{96t^{2(0,1)}}{\Gamma(2(0,1)+1)} + \frac{384}{\Gamma^2(0,1+1)\Gamma(3(0,1)+1)} \frac{\Gamma(2(0,1)+1)t^{3(0,1)}}{\Gamma(2(0,1)+1)} - \frac{512}{\Gamma^3(0,1+1)\Gamma(4(0,1)+1)} \frac{\Gamma(3(0,1)+1)t^{4(0,1)}}{\Gamma(3(0,1)+1)} \\
& + \frac{1.152t^{3(0,1)}}{\Gamma(3(0,1)+1)} - \frac{4.608}{\Gamma^2((0,1)+1)\Gamma(4(0,1)+1)} \frac{\Gamma(2(0,1)+1)t^{4(0,1)}}{\Gamma(2(0,1)+1)} + \frac{6.144}{\Gamma^3((0,1)+1)\Gamma(5(0,1)+1)} \frac{\Gamma(3(0,1)+1)t^{5(0,1)}}{\Gamma(3(0,1)+1)} \\
& - \frac{9.216}{\Gamma((0,1)+1)\Gamma(2(0,1)+1)\Gamma(4(0,1)+1)} \frac{\Gamma(3(0,1)+1)t^{4(0,1)}}{\Gamma(3(0,1)+1)} + \frac{36.864}{\Gamma^3(0,1+1)\Gamma(3(0,1)+1)\Gamma(5(0,1)+1)} \frac{\Gamma(2(0,1)+1)\Gamma(4(0,1)+1)t^{5(0,1)}}{\Gamma(2(0,1)+1)\Gamma(4(0,1)+1)} \\
& - \frac{49.152}{\Gamma^4(0,1+1)\Gamma(4(0,1)+1)\Gamma(6(0,1)+1)} \frac{\Gamma(3(0,1)+1)\Gamma(5(0,1)+1)t^{6(0,1)}}{\Gamma(3(0,1)+1)\Gamma(5(0,1)+1)} + \frac{18.432}{\Gamma^2(0,1+1)\Gamma(2(0,1)+1)\Gamma(5(0,1)+1)} \frac{\Gamma(4(0,1)+1)t^{5(0,1)}}{\Gamma(4(0,1)+1)} \\
& - \frac{73.728}{\Gamma^4(0,1+1)\Gamma(3(0,1)+1)\Gamma(6(0,1)+1)} \frac{\Gamma(2(0,1)+1)\Gamma(5(0,1)+1)t^{6(0,1)}}{\Gamma(2(0,1)+1)\Gamma(5(0,1)+1)} + \frac{98.304}{\Gamma^5(0,1+1)\Gamma(4(0,1)+1)\Gamma(7(0,1)+1)} \frac{\Gamma(3(0,1)+1)\Gamma(6(0,1)+1)t^{7(0,1)}}{\Gamma(3(0,1)+1)\Gamma(6(0,1)+1)} + \dots \\
& = \frac{8t^{0,1}}{\Gamma(1,1)} - \frac{96t^{0,2}}{\Gamma(1,2)} + \frac{384\Gamma(1,2)t^{0,3}}{\Gamma^2(1,1)\Gamma(1,3)} - \frac{512}{\Gamma^3(1,1)\Gamma(1,4)} \frac{\Gamma(1,3)t^{0,4}}{\Gamma(1,3)} + \frac{1.152}{\Gamma(1,3)} \frac{t^{0,3}}{\Gamma(1,3)} - \frac{4.608}{\Gamma^2(1,1)\Gamma(1,4)} \frac{\Gamma(1,2)t^{0,4}}{\Gamma(1,2)}
\end{aligned}$$

$$\begin{aligned}
 & + \frac{6.144 \Gamma(1,3)t^{0.5}}{\Gamma^3(1,1)\Gamma(1,5)} - \frac{9.216 \Gamma(1,3)t^{0.4}}{\Gamma(1,1)\Gamma(1,2)\Gamma(1,4)} + \frac{36.864 \Gamma(1,2)\Gamma(1,4)t^{0.5}}{\Gamma^3(1,1)\Gamma(1,3)\Gamma(1,5)} \\
 & - \frac{49.152 \Gamma(1,3)\Gamma(1,5)t^{0.6}}{\Gamma^4(1,1)\Gamma(1,4)\Gamma(1,6)} + \frac{18.432 \Gamma(1,4)t^{0.5}}{\Gamma^2(1,1)\Gamma(1,2)\Gamma(1,5)} - \frac{73.728 \Gamma(1,2)\Gamma(1,5)t^{0.6}}{\Gamma^4(1,1)\Gamma(1,3)\Gamma(1,6)} \\
 & + \frac{98.304 \Gamma(1,3)\Gamma(1,6)t^{0.7}}{\Gamma^5(1,1)\Gamma(1,4)\Gamma(1,7)} + \dots \\
 & = \frac{8t^{0.1}}{0,9514} - \frac{96t^{0.2}}{0,9182} + \frac{384(0,9182)t^{0.3}}{(0,9514)^2(0,8975)} - \frac{512(0,8975)t^{0.4}}{(0,9514)^3(0,8873)} + \frac{1.152t^{0.3}}{0,8975} \\
 & - \frac{4.608(0,9182)t^{0.4}}{(0,9514)^2(0,8873)} + \frac{6.144(0,8975)t^{0.5}}{(0,9514)^3(0,8862)} - \frac{9.216(0,8975)t^{0.4}}{(0,9514)(0,9182)(0,8873)} \\
 & + \frac{36.864(0,9182)(0,8873)t^{0.5}}{(0,9514)^3(0,8975)(0,8862)} - \frac{49.152(0,8975)(0,8862)t^{0.6}}{(0,9514)^4(0,8873)(0,8935)} + \frac{18.432(0,8873)t^{0.5}}{(0,9514)^2(0,9182)(0,8862)} \\
 & - \frac{73.728(0,9182)(0,8862)t^{0.6}}{(0,9514)^4(0,8975)(0,8935)} + \frac{98.304(0,8975)(0,8935)t^{0.7}}{(0,9514)^5(0,8873)(0,9086)} + \dots \\
 & = 8,41t^{0.1} - 104,56t^{0.2} + 434,06t^{0.3} - 601,47t^{0.4} + 1.283,61t^{0.3} - 5.268,67t^{0.4} \\
 & + 7.226,11t^{0.5} - 10.672,02t^{0.4} + 43.852,11t^{0.5} - 60.198,95t^{0.6} + 22.206,31t^{0.5} - 91.330,3t^{0.6} + 125.471,79t^{0.7} \\
 & = 8,41t^{0.1} - 104,56t^{0.2} + 1.717,67t^{0.3} - 16.542,16t^{0.4} + 73.284,53t^{0.5} - 151.529,25t^{0.6} + 125.471,79t^{0.7} + \dots
 \end{aligned}$$

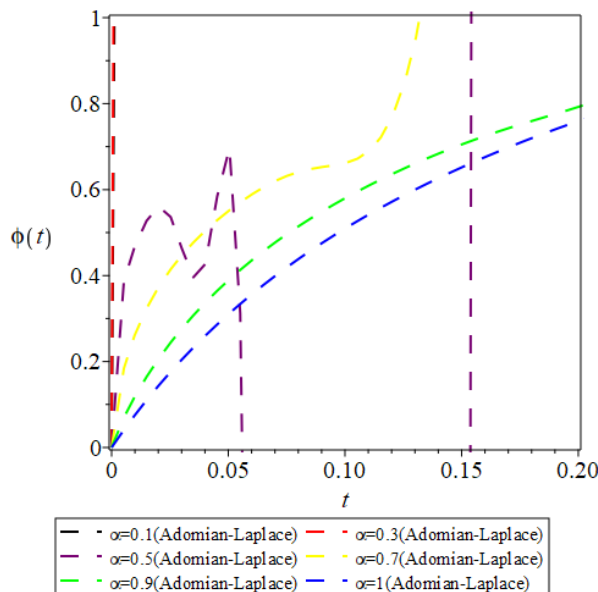
**For  $\alpha = 0.3$**   
 $\phi(t) = 8,91t^{0.3} - 107,44t^{0.6} + 1.640,71t^{0.9} - 15.289,71t^{1.2} + 66.644,98t^{1.5} - 136.157,8t^{1.8} + 112.430,44t^{2.1} + \dots$

**For  $\alpha = 0.5$**   
 $\phi(t) = 9,03t^{0.5} - 95,99t + 1.234,39t^{1.5} - 10.334,47t^2 + 41.630,6t^{2.5} - 79.136,83t^3 + 61.653,77t^{3.5} + \dots$

**For  $\alpha = 0.7$**   
 $\phi(t) = 8,8t^{0.7} - 77,28t^{1.4} + 787,1t^{2.1} - 5.619,06t^{2.8} + 20.009,58t^{3.5} - 33.880,82t^{4.2} + 23.903t^{4.9} + \dots$

**For  $\alpha = 0.9$**   
 $\phi(t) = 8,32t^{0.9} - 57,26t^{1.8} + 443,09t^{2.7} - 2.584,98t^{3.6} + 7.847,16t^{4.5} - 11.419,64t^{5.4} + 7.052,74t^{6.3} + \dots$

**For  $\alpha = 1$**   
 $\phi(t) = 8t - 48t^2 + 320t^3 - 1.664t^4 + 4.608t^5 - 6.144t^6 + 3.510,86t^7 + \dots$



**Figure 2.** Graph of the exact solution  $\phi(t)$  using the Adomian–Laplace method for  $0 < t < 0.2$

Figure 2 shows the graph of  $\phi(t)$ , the solution to Example 1 obtained by the Adomian–Laplace method for  $0 < t < 0.2$ , up to the 10th iteration. Figure 2 presents the graph of the solution of the Abel-type FDE using the Adomian–Laplace Theorem for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$ . Figure 2 shows that as the fractional order  $\alpha$  of the Abel-type FDE approaches 1, the graph gradually converges to the graph of the solution for  $\alpha=1$ .

### 3.2.3. Solutions using the Adomian–Kamal Theorem for Example 1

From the Adomian–Kamal Theorem, the approximate solution of the fractional Abel differential equation (26) is given by

$$\begin{aligned}\phi_0(t) &= 0 + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[A]], \\ \phi_0(t) &= \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[8]], \\ \phi_n(t) &= \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[B\phi_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[CP_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[DQ_{n-1}]], n \geq 1 \\ \phi_n(t) &= \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[-12\phi_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[6P_{n-1}]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[Q_{n-1}]], n \geq 1,\end{aligned}\quad (28)$$

where  $P_n$  denotes the Adomian polynomial corresponding to the nonlinear operators  $\phi^2(t)$  and  $\phi^3(t)$ , and can be expressed as follows:

For  $\phi^2(t)$

$$\begin{aligned}P_0 &= \phi_0^2, \\ P_1 &= 2\phi_0\phi_1, \\ P_2 &= 2\phi_0\phi_2 + \phi_1^2, \\ &\vdots\end{aligned}$$

For  $\phi^3(t)$

$$\begin{aligned}Q_0 &= \phi_0^3, \\ Q_1 &= 3\phi_0^2\phi_1, \\ Q_2 &= 3\phi_0^2\phi_2 + 3\phi_0\phi_1^2, \\ &\vdots\end{aligned}$$

The following is a description of equation (20):

$$\begin{aligned}\phi_0 &= \phi_0(t) = \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[8]] = \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[8t^0]] = \mathcal{K}^{-1}[8v^{2\alpha}\Gamma(0+1)v^{2(0)+1}] \\ &= \mathcal{K}^{-1}[8v^{2\alpha}v] = \mathcal{K}^{-1}[8v^{2\alpha+1}] = \frac{8t^\alpha}{\Gamma(\alpha+1)}, \\ \phi_1 &= \phi_1(t) = -\mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[12\phi_0]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[6P_0]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[Q_0]] \\ &= -\mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[12\phi_0]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[6\phi_0^2]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[\phi_0^3]] \\ &= -\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[\frac{12 \cdot 8t^\alpha}{\Gamma(\alpha+1)}\right]\right] + \mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[\frac{6 \cdot 8^2 t^{2\alpha}}{\Gamma^2(\alpha+1)}\right]\right] - \mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[\frac{8^3 t^{3\alpha}}{\Gamma^3(\alpha+1)}\right]\right] \\ &= -\mathcal{K}^{-1}\left[v^{2\alpha}\frac{96}{\Gamma(\alpha+1)}v^{2\alpha+1}\right] + \mathcal{K}^{-1}\left[v^{2\alpha}\frac{384}{\Gamma^2(\alpha+1)}v^{4\alpha+1}\right] - \mathcal{K}^{-1}\left[v^{2\alpha}\frac{512}{\Gamma^3(\alpha+1)}v^{6\alpha+1}\right] \\ &= -\mathcal{K}^{-1}[96v^{4\alpha+1}] + \mathcal{K}^{-1}\left[\frac{384}{\Gamma^2(\alpha+1)}v^{6\alpha+1}\right] - \mathcal{K}^{-1}\left[\frac{512}{\Gamma^3(\alpha+1)}v^{8\alpha+1}\right] \\ &= -\frac{96t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{384}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha} - \frac{512(3\alpha+1)t^{4\alpha}}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}, \\ \phi_2 &= \phi_2(t) = -\mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[12\phi_1]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[6P_1]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[Q_1]] \\ &= -\mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[12\phi_1]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[12\phi_0\phi_1]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[3\phi_0^2\phi_1]] \\ &= -\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[12\left(-\frac{96t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{384}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha} - \frac{512}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}\right)\right]\right]\end{aligned}$$

$$\begin{aligned}
 & +\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[12\left(\frac{8t^\alpha}{\Gamma(\alpha+1)}\right)\left(-\frac{96t^{2\alpha}}{\Gamma(2\alpha+1)}+\frac{384}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{512}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(4\alpha+1)}\right)\right]\right] \\
 & -\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[3\left(\frac{8t^\alpha}{\Gamma(\alpha+1)}\right)^2\left(-\frac{96t^{2\alpha}}{\Gamma(2\alpha+1)}+\frac{384}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{512}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(4\alpha+1)}\right)\right]\right] \\
 & =-\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[-\frac{1.152t^{2\alpha}}{\Gamma(2\alpha+1)}+\frac{4.608}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{6.144}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(4\alpha+1)}\right]\right] \\
 & +\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[-\frac{9.216t^{3\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}+\frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma(3\alpha+1)}-\frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)t^{5\alpha}}{\Gamma(4\alpha+1)}\right]\right] \\
 & -\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[-\frac{18.432t^{4\alpha}}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)}+\frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)t^{5\alpha}}{\Gamma(3\alpha+1)}-\frac{98.304}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)t^{6\alpha}}{\Gamma(4\alpha+1)}\right]\right] \\
 & =-\mathcal{K}^{-1}\left[v^{2\alpha}\left(-\frac{1.152}{\Gamma(2\alpha+1)}\frac{\Gamma(2\alpha+1)}{\Gamma(2\alpha+1)}v^{4\alpha+1}+\frac{4.608}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)\Gamma(3\alpha+1)}{\Gamma(3\alpha+1)}v^{6\alpha+1}\right)\right. \\
 & \left.-\frac{6.144\Gamma(3\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}v^{8\alpha+1}\right] \\
 & +\mathcal{K}^{-1}\left[v^{2\alpha}\left(-\frac{9.216}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}\frac{\Gamma(3\alpha+1)}{\Gamma(2\alpha+1)}v^{6\alpha+1}+\frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma(3\alpha+1)}v^{8\alpha+1}\right.\right. \\
 & \left.\left.-\frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)\Gamma(5\alpha+1)}{\Gamma(4\alpha+1)}v^{10\alpha+1}\right)\right] \\
 & -\mathcal{K}^{-1}\left[v^{2\alpha}\left(-\frac{18.432}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)}\frac{\Gamma(4\alpha+1)}{\Gamma(2\alpha+1)}v^{8\alpha+1}+\frac{73.728\Gamma(2\alpha+1)\Gamma(5\alpha+1)}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)}v^{10\alpha+1}\right.\right. \\
 & \left.\left.-\frac{98.304}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)\Gamma(6\alpha+1)}{\Gamma(4\alpha+1)}v^{12\alpha+1}\right)\right] \\
 & =-\mathcal{K}^{-1}\left[-1.152v^{6\alpha+1}+\frac{4.608}{\Gamma^2(\alpha+1)}\frac{\Gamma(2\alpha+1)}{\Gamma(2\alpha+1)}v^{8\alpha+1}-\frac{6.144\Gamma(3\alpha+1)}{\Gamma^3(\alpha+1)}v^{10\alpha+1}\right] \\
 & +\left[-\frac{9.216}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}\frac{\Gamma(3\alpha+1)}{\Gamma(2\alpha+1)}v^{8\alpha+1}+\frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma(3\alpha+1)}v^{10\alpha+1}-\frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)\Gamma(5\alpha+1)}{\Gamma(4\alpha+1)}v^{12\alpha+1}\right] \\
 & -\mathcal{K}^{-1}\left[-\frac{18.432}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)}\frac{\Gamma(4\alpha+1)}{\Gamma(2\alpha+1)}v^{10\alpha+1}+\frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)}\frac{\Gamma(2\alpha+1)\Gamma(5\alpha+1)}{\Gamma(3\alpha+1)}v^{12\alpha+1}\right. \\
 & \left.-\frac{98.304}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)\Gamma(6\alpha+1)}{\Gamma(4\alpha+1)}v^{14\alpha+1}\right] \\
 & =\frac{1.152t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{4.608}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma(4\alpha+1)}+\frac{6.144}{\Gamma^3(\alpha+1)\Gamma(5\alpha+1)}\frac{\Gamma(3\alpha+1)t^{5\alpha}}{\Gamma(5\alpha+1)} \\
 & -\frac{9.216}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)}\frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(4\alpha+1)}+\frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}\frac{\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma(5\alpha+1)} \\
 & -\frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)}\frac{\Gamma(3\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma(6\alpha+1)}+\frac{18.432}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)\Gamma(5\alpha+1)}\frac{\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma(5\alpha+1)} \\
 & -\frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)\Gamma(6\alpha+1)}\frac{\Gamma(2\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma(6\alpha+1)}+\frac{98.304\Gamma(3\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)\Gamma(7\alpha+1)} \\
 & \vdots
 \end{aligned}$$

By applying the Adomian–Kamal Theorem, the solution to equation (26) is obtained as:

$$\phi(t) = \phi_0(t) + \phi_1(t) + \phi_2(t) + \dots$$

$$\begin{aligned}
&= \frac{8t^\alpha}{\Gamma(\alpha+1)} - \frac{96t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{384\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{512\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \\
&\quad + \frac{1.152t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{4.608}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)}\Gamma(2\alpha+1)t^{4\alpha} + \frac{6.144}{\Gamma^3(\alpha+1)\Gamma(5\alpha+1)}\Gamma(3\alpha+1)t^{5\alpha} \\
&\quad - \frac{9.216}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)}\Gamma(3\alpha+1)t^{4\alpha} + \frac{36.864}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha} \\
&\quad - \frac{49.152}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)}\Gamma(3\alpha+1)\Gamma(5\alpha+1)t^{6\alpha} + \frac{18.432}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)\Gamma(5\alpha+1)}\Gamma(4\alpha+1)t^{5\alpha} \\
&\quad - \frac{73.728}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)\Gamma(6\alpha+1)}\Gamma(2\alpha+1)\Gamma(5\alpha+1)t^{6\alpha} + \frac{98.304\Gamma(3\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)\Gamma(7\alpha+1)}.
\end{aligned}$$

For  $\alpha = 0.1$

$$\begin{aligned}
\phi(t) &= \phi_0(t) + \phi_1(t) + \phi_2(t) + \dots \\
&= \frac{8t^{0,1}}{\Gamma(0,1+1)} - \frac{96t^{2(0,1)}}{\Gamma(2(0,1)+1)} + \frac{384}{\Gamma^2(0,1+1)\Gamma(3(0,1)+1)}\Gamma(2(0,1)+1)t^{3(0,1)} - \frac{512}{\Gamma^3(0,1+1)\Gamma(4(0,1)+1)}\Gamma(3(0,1)+1)t^{4(0,1)} \\
&\quad + \frac{1.152t^{3(0,1)}}{\Gamma(3(0,1)+1)} - \frac{4.608}{\Gamma^2((0,1)+1)\Gamma(4(0,1)+1)}\Gamma(2(0,1)+1)t^{4(0,1)} + \frac{6.144}{\Gamma^3((0,1)+1)\Gamma(5(0,1)+1)}\Gamma(3(0,1)+1)t^{5(0,1)} \\
&\quad - \frac{9.216}{\Gamma((0,1)+1)\Gamma(2(0,1)+1)\Gamma(4(0,1)+1)}\Gamma(3(0,1)+1)t^{4(0,1)} + \frac{36.864}{\Gamma^3(0,1+1)\Gamma(3(0,1)+1)\Gamma(5(0,1)+1)}\Gamma(2(0,1)+1)\Gamma(4(0,1)+1)t^{5(0,1)} \\
&\quad - \frac{49.152}{\Gamma^4(0,1+1)\Gamma(4(0,1)+1)\Gamma(6(0,1)+1)}\Gamma(3(0,1)+1)\Gamma(5(0,1)+1)t^{6(0,1)} + \frac{18.432}{\Gamma^2(0,1+1)\Gamma(2(0,1)+1)\Gamma(5(0,1)+1)}\Gamma(4(0,1)+1)t^{5(0,1)} \\
&\quad - \frac{73.728}{\Gamma^4(0,1+1)\Gamma(3(0,1)+1)\Gamma(6(0,1)+1)}\Gamma(2(0,1)+1)\Gamma(5(0,1)+1)t^{6(0,1)} + \frac{98.304}{\Gamma^5(0,1+1)\Gamma(4(0,1)+1)\Gamma(7(0,1)+1)}\Gamma(3(0,1)+1)\Gamma(6(0,1)+1)t^{7(0,1)} + \dots \\
&= \frac{8t^{0,1}}{\Gamma(1,1)} - \frac{96t^{0,2}}{\Gamma(1,2)} + \frac{384\Gamma(1,2)t^{0,3}}{\Gamma^2(1,1)\Gamma(1,3)} - \frac{512}{\Gamma^3(1,1)\Gamma(1,4)}\Gamma(1,3)t^{0,4} + \frac{1.152}{\Gamma(1,3)}t^{0,3} - \frac{4.608}{\Gamma^2(1,1)\Gamma(1,4)}\Gamma(1,2)t^{0,4} \\
&\quad + \frac{6.144}{\Gamma^3(1,1)\Gamma(1,5)}\Gamma(1,3)t^{0,5} - \frac{9.216}{\Gamma(1,1)\Gamma(1,2)\Gamma(1,4)}\Gamma(1,3)t^{0,4} + \frac{36.864}{\Gamma^3(1,1)\Gamma(1,3)\Gamma(1,5)}\Gamma(1,2)\Gamma(1,4)t^{0,5} \\
&\quad - \frac{49.152}{\Gamma^4(1,1)\Gamma(1,4)\Gamma(1,6)}\Gamma(1,3)\Gamma(1,5)t^{0,6} + \frac{18.432}{\Gamma^2(1,1)\Gamma(1,2)\Gamma(1,5)}\Gamma(1,4)t^{0,5} - \frac{73.728}{\Gamma^4(1,1)\Gamma(1,3)\Gamma(1,6)}\Gamma(1,2)\Gamma(1,5)t^{0,6} \\
&\quad + \frac{98.304}{\Gamma^5(1,1)\Gamma(1,4)\Gamma(1,7)}\Gamma(1,3)\Gamma(1,6)t^{0,7} + \dots \\
&= \frac{8t^{0,1}}{0,9514} - \frac{96t^{0,2}}{0,9182} + \frac{384(0,9182)}{(0,9514)^2(0,8975)}t^{0,3} - \frac{512(0,8975)t^{0,4}}{(0,9514)^3(0,8873)} + \frac{1.152t^{0,3}}{0,8975} \\
&\quad - \frac{4.608(0,9182)t^{0,4}}{(0,9514)^2(0,8873)} + \frac{6.144(0,8975)t^{0,5}}{(0,9514)^3(0,8862)} - \frac{9.216(0,8975)t^{0,4}}{(0,9514)(0,9182)(0,8873)} \\
&\quad + \frac{36.864(0,9182)(0,8873)t^{0,5}}{(0,9514)^3(0,8975)(0,8862)} - \frac{49.152(0,8975)(0,8862)t^{0,6}}{(0,9514)^4(0,8873)(0,8935)} + \frac{18.432(0,8873)t^{0,5}}{(0,9514)^2(0,9182)(0,8862)} \\
&\quad - \frac{73.728(0,9182)(0,8862)t^{0,6}}{(0,9514)^4(0,8975)(0,8935)} + \frac{98.304(0,8975)(0,8935)t^{0,7}}{(0,9514)^5(0,8873)(0,9086)} + \dots \\
&= 8,41t^{0,1} - 104,56t^{0,2} + 434,06t^{0,3} - 601,47t^{0,4} + 1.283,61t^{0,3} - 5.268,67t^{0,4} \\
&+ 7.226,11t^{0,5} - 10.672,02t^{0,4} + 43.852,11t^{0,5} - 60.198,95t^{0,6} + 22.206,31t^{0,5} - 91.330,3t^{0,6} + 125.471,79t^{0,7} \\
&= 8,41t^{0,1} - 104,56t^{0,2} + 1.717,67t^{0,3} - 16.542,16t^{0,4} + 73.284,53t^{0,5} - 151.529,25t^{0,6} + 125.471,79t^{0,7} + \dots
\end{aligned}$$

For  $\alpha = 0.3$

$$\phi(t) = 8,91t^{0,3} - 107,44t^{0,6} + 1.640,71t^{0,9} - 15.289,71t^{1,2} + 66.644,98t^{1,5} - 136.157,8t^{1,8} + 112.430,44t^{2,1} + \dots$$

**For  $\alpha = 0.5$**

$$\phi(t) = 9,03t^{0.5} - 95,99t + 1.234,39t^{1.5} - 10.334,47t^2 + 41.630,6t^{2.5} - 79.136,83t^3 + 61.653,77t^{3.5} + \dots$$

**For  $\alpha = 0.7$**

$$\phi(t) = 8,8t^{0.7} - 77,28t^{1.4} + 787,1t^{2.1} - 5.619,06t^{2.8} + 20.009,58t^{3.5} - 33.880,82t^{4.2} + 23.903t^{4.9} + \dots$$

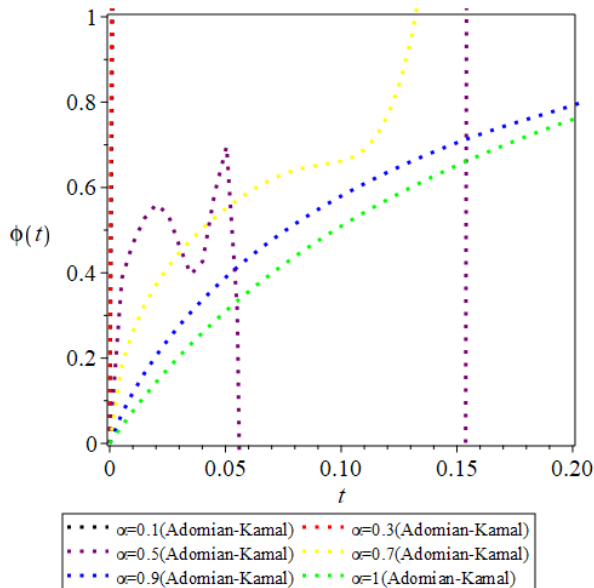
**For  $\alpha = 0.9$**

$$\phi(t) = 8,32t^{0.9} - 57,26t^{1.8} + 443,09t^{2.7} - 2.584,98t^{3.6} + 7.847,16t^{4.5} - 11.419,64t^{5.4} + 7.052,74t^{6.3} + \dots$$

**For  $\alpha = 1$**

$$\phi(t) = 8t - 48t^2 + 320t^3 - 1.664t^4 + 4.608t^5 - 6.144t^6 + 3.510,86t^7 + \dots$$

Figure 3 shows a visual representation of the solution of the Abel-type FDE in Example 1 using the Adomian–Kamal Theorem for  $\alpha = 1$ . The solutions of the Abel-type FDE in the first example obtained using the Adomian–Laplace Theorem and the Adomian–Kamal Theorem are identical, and their graphs coincide for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$  for any  $t > 1$ . This demonstrates that both the Adomian–Laplace Theorem and the Adomian–Kamal Theorem are highly effective in solving the FADE. The solution methods based on these two theorems are highly accurate and exhibit strong consistency in solving the FADE.

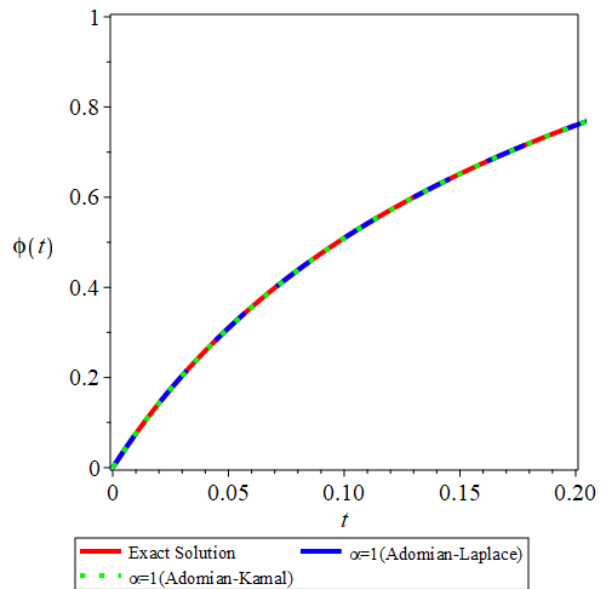


**Figure 3.** Graph of the exact solution  $\phi(t)$  using the Adomian–Kamal method for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$

Figures 1–3 indicate that the solution exhibits strong memory effects when the fractional order  $\alpha$  is small. In this regime, the dynamics evolve more slowly, and the curves display a noticeable deviation from the classical case. As  $\alpha$  increases, the influence of the historical terms weakens, and the trajectories converge smoothly toward the integer-order solution. This behaviour highlights the sensitivity of the FADE to fractional-order variation and confirms that both the Adomian–Laplace and Adomian–

Kamal approaches are capable of accurately capturing these trends.

The comparison in Figure 4 shows that both semi-analytical methods reproduce the exact solution with high precision. This agreement arises from the structure of the integral transforms used. The Laplace transform provides a convolution-based representation that effectively captures the long-memory behaviour, while the Kamal transform simplifies the inversion step and reduces the accumulation of local numerical errors.



**Figure 4.** Comparison of the graphs of the Exact Solution, the Adomian–Laplace Theorem, and the Adomian–Kamal Theorem for  $\alpha=1$

**3.3. Example 2**

Given the FADE:

$$D_t^\alpha(t) = 4 - 3\phi(t) + 2\phi^2(t) - \phi^3(t), \quad (29)$$

With  $0 \leq t \leq 0.2, 0 < \alpha \leq 1,$   
for  $\phi(0) = 0$

- a. Obtain approximate solutions using the Adomian–Laplace method for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1.$
- b. Obtain approximate solutions using the Adomian–Kamal method for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1.$

## 3.3.1. Solutions Using the Adomian–Laplace Method for Example 2

From the Adomian–Laplace Theorem, the approximate solution of the FADE (26) is given by

$$\begin{aligned}\phi_0(t) &= 0 + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[A] \right], \\ \phi_0(t) &= \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[4] \right], \\ \phi_n &= \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[B\phi_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[CP_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[DQ_{n-1}] \right], n \geq 1, \\ \phi_n(t) &= \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[-3\phi_{n-1}] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[2P_{n-1}] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[Q_{n-1}] \right], n \geq 1.\end{aligned}\quad (30)$$

where  $P_n$  denotes the Adomian polynomial corresponding to the nonlinear operators  $\phi^2(t)$  and  $\phi^3(t)$ , and can be expressed as follows:

For  $\phi^2(t)$

$$\begin{aligned}P_0 &= \phi_0^2, \\ P_1 &= 2\phi_0\phi_1, \\ P_2 &= 2\phi_0\phi_2 + \phi_1^2, \\ &\vdots\end{aligned}$$

For  $\phi^3(t)$

$$\begin{aligned}Q_0 &= \phi_0^3, \\ Q_1 &= 3\phi_0^2\phi_1, \\ Q_2 &= 3\phi_0^2\phi_2 + 3\phi_0\phi_1^2, \\ &\vdots\end{aligned}$$

The following is a description of equation (30)

$$\begin{aligned}\phi_0 &= \phi_0(t) = \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[4] \right] = \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \cdot \frac{4}{s} \right] = \mathcal{L}^{-1} \left[ \frac{4}{s^{\alpha+1}} \right] = \frac{4t^\alpha}{\alpha!} = \frac{4t^\alpha}{\Gamma(\alpha+1)}, \\ \phi_1 &= \phi_1(t) = -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[3\phi_0] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[2P_0] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[Q_0] \right] \\ &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ \frac{3 \cdot 4t^\alpha}{\Gamma(\alpha+1)} \right] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[2\phi_0^2] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[\phi_0^3] \right], \\ &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ \frac{12t^\alpha}{\Gamma(\alpha+1)} \right] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ \frac{2 \cdot 4^2 t^{2\alpha}}{\Gamma^2(\alpha+1)} \right] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ \frac{4^3 t^{3\alpha}}{\Gamma^3(\alpha+1)} \right] \right] \\ &= -\mathcal{L}^{-1} \left[ \frac{12}{\Gamma(\alpha+1)s^\alpha} \mathcal{L}[t^\alpha] \right] + \mathcal{L}^{-1} \left[ \frac{32}{\Gamma^2(\alpha+1)s^\alpha} \mathcal{L}[t^{2\alpha}] \right] - \mathcal{L}^{-1} \left[ \frac{64}{\Gamma^3(\alpha+1)s^\alpha} \mathcal{L}[t^{3\alpha}] \right] \\ &= -\mathcal{L}^{-1} \left[ \frac{12}{\Gamma(\alpha+1)s^\alpha} \cdot \frac{\alpha!}{s^{\alpha+1}} \right] + \mathcal{L}^{-1} \left[ \frac{32}{\Gamma^2(\alpha+1)s^\alpha} \cdot \frac{2\alpha!}{s^{2\alpha+1}} \right] - \mathcal{L}^{-1} \left[ \frac{64}{\Gamma^3(\alpha+1)s^\alpha} \cdot \frac{3\alpha!}{s^{3\alpha+1}} \right] \\ &= -\mathcal{L}^{-1} \left[ \frac{12}{\Gamma(\alpha+1)} \cdot \frac{\Gamma(\alpha+1)}{s^{2\alpha+1}} \right] + \mathcal{L}^{-1} \left[ \frac{32}{\Gamma^2(\alpha+1)} \cdot \frac{\Gamma(2\alpha+1)}{s^{3\alpha+1}} \right] - \mathcal{L}^{-1} \left[ \frac{64}{\Gamma^3(\alpha+1)} \cdot \frac{\Gamma(3\alpha+1)}{s^{4\alpha+1}} \right] \\ &= -\mathcal{L}^{-1} \left[ \frac{12}{s^{2\alpha+1}} \right] + \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)} \mathcal{L}^{-1} \left[ \frac{32}{s^{3\alpha+1}} \right] - \frac{\Gamma(3\alpha+1)}{\Gamma^3(\alpha+1)} \mathcal{L}^{-1} \left[ \frac{64}{s^{4\alpha+1}} \right] \\ &= -\frac{12t^{2\alpha}}{2\alpha!} + \frac{32\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)} \cdot \frac{t^{3\alpha}}{3\alpha!} - \frac{64\Gamma(3\alpha+1)}{\Gamma^3(\alpha+1)} \cdot \frac{t^{4\alpha}}{4\alpha!} \\ &= -\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{64\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}\end{aligned}$$

$$\begin{aligned}
 \phi_2 = \phi_2(t) &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[3\phi_1] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[2P_1] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[Q_1] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[3\phi_1] \right] + \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[4\phi_0\phi_1] \right] - \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L}[3\phi_0^2\phi_1] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ 3 \left( -\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(2\alpha+1)} - \frac{64}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(3\alpha+1)} \right) \right] \right] \\
 &+ \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ 4 \left( \frac{4t^\alpha}{\Gamma(\alpha+1)} \right) \left( -\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(2\alpha+1)} - \frac{64}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(3\alpha+1)} \right) \right] \right] \\
 &- \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ 3 \left( \frac{4t^\alpha}{\Gamma(\alpha+1)} \right)^2 \left( -\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(2\alpha+1)} - \frac{64}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(3\alpha+1)} \right) \right] \right] \\
 &= -\mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ -\frac{36t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{96}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma(2\alpha+1)} - \frac{192}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(3\alpha+1)} \right] \right] \\
 &+ \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ -\frac{192t^{3\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} + \frac{512}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)} \frac{\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma(2\alpha+1)} - \frac{1.024\Gamma(3\alpha+1)t^{5\alpha}}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)} \right] \right] \\
 &- \mathcal{L}^{-1} \left[ \frac{1}{s^\alpha} \mathcal{L} \left[ -\frac{576t^{4\alpha}}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)} + \frac{1.536}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)} \frac{\Gamma(2\alpha+1)t^{5\alpha}}{\Gamma(2\alpha+1)} - \frac{3.072}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(3\alpha+1)t^{6\alpha}}{\Gamma(3\alpha+1)} \right] \right] \\
 &= -\mathcal{L}^{-1} \left[ -\frac{1}{s^\alpha} \cdot \frac{36}{\Gamma(2\alpha+1)} \cdot \frac{2\alpha!}{s^{2\alpha+1}} + \frac{1}{s^\alpha} \cdot \frac{96}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{3\alpha!}{s^{3\alpha+1}} - \frac{1}{s^\alpha} \cdot \frac{192}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{4\alpha!}{s^{4\alpha+1}} \right] \\
 &+ \mathcal{L}^{-1} \left[ -\frac{1}{s^\alpha} \cdot \frac{192}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} \cdot \frac{3\alpha!}{s^{3\alpha+1}} + \frac{1}{s^\alpha} \cdot \frac{512}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{4\alpha!}{s^{4\alpha+1}} - \frac{1}{s^\alpha} \cdot \frac{1.024}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{5\alpha!}{s^{5\alpha+1}} \right] \\
 &- \mathcal{L}^{-1} \left[ -\frac{1}{s^\alpha} \cdot \frac{576}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)} \cdot \frac{4\alpha!}{s^{4\alpha+1}} + \frac{1}{s^\alpha} \cdot \frac{1.536}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{5\alpha!}{s^{5\alpha+1}} - \frac{1}{s^\alpha} \cdot \frac{3.072}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{6\alpha!}{s^{6\alpha+1}} \right] \\
 &= -\mathcal{L}^{-1} \left[ -\frac{36}{\Gamma(2\alpha+1)} \cdot \frac{\Gamma(2\alpha+1)}{s^{3\alpha+1}} + \frac{96}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{\Gamma(3\alpha+1)}{s^{4\alpha+1}} - \frac{192}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{\Gamma(4\alpha+1)}{s^{5\alpha+1}} \right] \\
 &+ \mathcal{L}^{-1} \left[ -\frac{192}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} \cdot \frac{\Gamma(3\alpha+1)}{s^{4\alpha+1}} + \frac{512}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{\Gamma(4\alpha+1)}{s^{5\alpha+1}} - \frac{1.024}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{\Gamma(5\alpha+1)}{s^{6\alpha+1}} \right] \\
 &- \mathcal{L}^{-1} \left[ -\frac{576}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)} \cdot \frac{\Gamma(4\alpha+1)}{s^{5\alpha+1}} + \frac{1.536\Gamma(2\alpha+1)}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)} \cdot \frac{\Gamma(5\alpha+1)}{s^{6\alpha+1}} - \frac{3.072\Gamma(3\alpha+1)}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)} \cdot \frac{\Gamma(6\alpha+1)}{s^{7\alpha+1}} \right] \\
 &= \frac{36t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{96}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma(2\alpha+1)} + \frac{192}{\Gamma^3(\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(3\alpha+1)t^{5\alpha}}{\Gamma(3\alpha+1)} \\
 &- \frac{192}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)} \frac{\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(3\alpha+1)} + \frac{512}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma(2\alpha+1)\Gamma(4\alpha+1)} \\
 &- \frac{1.024}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)} \frac{\Gamma(3\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma(3\alpha+1)\Gamma(5\alpha+1)} + \frac{576}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)\Gamma(5\alpha+1)} \frac{\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma(4\alpha+1)} \\
 &- \frac{1.536}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)\Gamma(6\alpha+1)} \frac{\Gamma(2\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma(2\alpha+1)\Gamma(5\alpha+1)} + \frac{3.072}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)\Gamma(7\alpha+1)} \frac{\Gamma(3\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{\Gamma(3\alpha+1)\Gamma(6\alpha+1)} \\
 &\vdots
 \end{aligned}$$

By applying the Adomian–Laplace Theorem, the solution to equation (21) is given by

$$\begin{aligned}
 \phi(t) &= \phi_0(t) + \phi_1(t) + \phi_2(t) + \dots \\
 &= \frac{4t^\alpha}{\Gamma(\alpha+1)} - \frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{64\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}
 \end{aligned}$$

$$\begin{aligned}
& + \frac{36t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{96\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)} + \frac{192\Gamma(3\alpha+1)t^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(5\alpha+1)} \\
& - \frac{192\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)} + \frac{512\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} \\
& - \frac{1.024\Gamma(3\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)} + \frac{576\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)\Gamma(5\alpha+1)} \\
& - \frac{1.536\Gamma(2\alpha+1)\Gamma(5\alpha+1)t^{6\alpha}}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)\Gamma(6\alpha+1)} + \frac{3.072\Gamma(3\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)\Gamma(7\alpha+1)}.
\end{aligned}$$

For  $\alpha = 0.1$

$$\begin{aligned}
\phi(t) &= \phi_0(t) + \phi_1(t) + \phi_2(t) + \dots \\
&= \frac{4t^{0,1}}{\Gamma(0,1+1)} - \frac{12t^{2(0,1)}}{\Gamma(2(0,1)+1)} + \frac{32\Gamma(2(0,1)+1)t^{3(0,1)}}{\Gamma^2(0,1+1)\Gamma(3(0,1)+1)} - \frac{64\Gamma(3(0,1)+1)t^{4(0,1)}}{\Gamma^3(0,1+1)\Gamma(4(0,1)+1)} \\
&+ \frac{36t^{3(0,1)}}{\Gamma(3(0,1)+1)} - \frac{96\Gamma(2(0,1)+1)t^{4(0,1)}}{\Gamma^2((0,1)+1)\Gamma(4(0,1)+1)} + \frac{192\Gamma(3(0,1)+1)t^{5(0,1)}}{\Gamma^3((0,1)+1)\Gamma(5(0,1)+1)} \\
&- \frac{192\Gamma(3(0,1)+1)t^{4(0,1)}}{\Gamma((0,1)+1)\Gamma(2(0,1)+1)\Gamma(4(0,1)+1)} + \frac{512\Gamma(2(0,1)+1)\Gamma(4(0,1)+1)t^{5(0,1)}}{\Gamma^3(0,1+1)\Gamma(3(0,1)+1)\Gamma(5(0,1)+1)} \\
&- \frac{1.024\Gamma(3(0,1)+1)\Gamma(5(0,1)+1)t^{6(0,1)}}{\Gamma^4(0,1+1)\Gamma(4(0,1)+1)\Gamma(6(0,1)+1)} + \frac{576\Gamma(4(0,1)+1)t^{5(0,1)}}{\Gamma^2(0,1+1)\Gamma(2(0,1)+1)\Gamma(5(0,1)+1)} \\
&- \frac{1.536\Gamma(2(0,1)+1)\Gamma(5(0,1)+1)t^{6(0,1)}}{\Gamma^4(0,1+1)\Gamma(3(0,1)+1)\Gamma(6(0,1)+1)} + \frac{3.072\Gamma(3(0,1)+1)\Gamma(6(0,1)+1)t^{7(0,1)}}{\Gamma^5(0,1+1)\Gamma(4(0,1)+1)\Gamma(7(0,1)+1)} + \dots \\
&= \frac{4t^{0,1}}{\Gamma(1,1)} - \frac{12t^{0,2}}{\Gamma(1,2)} + \frac{32\Gamma(1,2)t^{0,3}}{\Gamma^2(1,1)\Gamma(1,3)} - \frac{64\Gamma(1,3)t^{0,4}}{\Gamma^3(1,1)\Gamma(1,4)} + \frac{36t^{0,3}}{\Gamma(1,3)} - \frac{96\Gamma(1,2)t^{0,4}}{\Gamma^2(1,1)\Gamma(1,4)} + \frac{192\Gamma(1,3)t^{0,5}}{\Gamma^3(1,1)\Gamma(1,5)} \\
&- \frac{192\Gamma(1,3)t^{0,4}}{\Gamma(1,1)\Gamma(1,2)\Gamma(1,4)} + \frac{512\Gamma(1,2)\Gamma(1,4)t^{0,5}}{\Gamma^3(1,1)\Gamma(1,3)\Gamma(1,5)} - \frac{1.024\Gamma(1,3)\Gamma(1,5)t^{0,6}}{\Gamma^4(1,1)\Gamma(1,4)\Gamma(1,6)} + \frac{576\Gamma(1,4)t^{0,5}}{\Gamma^2(1,1)\Gamma(1,2)\Gamma(1,5)} \\
&- \frac{1.536\Gamma(1,2)\Gamma(1,5)t^{0,6}}{\Gamma^4(1,1)\Gamma(1,3)\Gamma(1,6)} + \frac{3.072\Gamma(1,3)\Gamma(1,6)t^{0,7}}{\Gamma^5(1,1)\Gamma(1,4)\Gamma(1,7)} + \dots \\
&= \frac{4t^{0,1}}{0,9514} - \frac{12t^{0,2}}{0,9182} + \frac{32(0,9182)t^{0,3}}{(0,9514)^2(0,8975)} - \frac{64(0,8975)t^{0,4}}{(0,9514)^3(0,8873)} + \frac{36t^{0,3}}{0,8975} \\
&- \frac{96(0,9182)t^{0,4}}{(0,9514)^2(0,8873)} + \frac{192(0,8975)t^{0,5}}{(0,9514)^3(0,8862)} - \frac{192(0,8975)t^{0,4}}{(0,9514)(0,9182)(0,8873)} \\
&+ \frac{512(0,9182)(0,8873)t^{0,5}}{(0,9514)^3(0,8975)(0,8862)} - \frac{1.024(0,8975)(0,8862)t^{0,6}}{(0,9514)^4(0,8873)(0,8935)} + \frac{576(0,8873)t^{0,5}}{(0,9514)^2(0,9182)(0,8862)} \\
&- \frac{1.536(0,9182)(0,8862)t^{0,6}}{(0,9514)^4(0,8975)(0,8935)} + \frac{3.072(0,8975)(0,8935)t^{0,7}}{(0,9514)^5(0,8873)(0,9086)} + \dots \\
&= 4,2t^{0,1} - 13,07t^{0,2} + 36,17t^{0,3} - 75,18t^{0,4} + 40,11t^{0,3} - 109,76t^{0,4} + 225,82t^{0,5} \\
&- 222,33t^{0,4} + 609,06t^{0,5} - 1.254,14t^{0,6} + 693,95t^{0,5} - 1.902,71t^{0,6} + 3.920,99t^{0,7} \\
&= 4,2t^{0,1} - 13,07t^{0,2} + 76,28t^{0,3} - 407,28t^{0,4} + 1.528,82t^{0,5} - 3.156,86t^{0,6} + 3.920,99t^{0,7} + \dots
\end{aligned}$$

For  $\alpha = 0.3$

$$\phi(t) = 4,47t^{0,3} - 13,43t^{0,6} + 74,34t^{0,9} - 382,94t^{1,2} + 1.400,91t^{1,5} - 2.836,62t^{1,8} + 3.513,45t^{2,1} + \dots$$

For  $\alpha = 0.5$

$$\phi(t) = 4,51t^{0,5} - 12t + 57,73t^{1,5} - 26623t^2 + 884,69t^{2,5} - 1.648,68t^3 + 1.926,68t^{3,5} + \dots$$

For  $\alpha = 0.7$

$$\phi(t) = 4,4t^{0,7} - 9,668t^{1,4} + 38,29t^{2,1} - 150,35t^{2,8} + 430,69t^{3,5} - 705,85t^{4,2} + 746,97t^{4,9} + \dots$$

For  $\alpha = 0.9$

$$\phi(t) = 4,16t^{0,9} - 7,16t^{1,8} + 22,54t^{2,7} - 72,54t^{3,6} + 171,29t^{4,5} - 237,91t^{5,4} + 220,39t^{6,3} + \dots$$

For  $\alpha = 1$

$$\phi(t) = 4t - 6t^2 + 16,67t^3 - 48t^4 + 101,33t^5 - 128t^6 + 109,71t^7 + \dots$$

Figure 5 shows the graph of  $\phi(t)$ , the solution to Example 2 obtained by the Adomian–Laplace method for  $0 < t < 0.2$ , up to the 10th iteration.

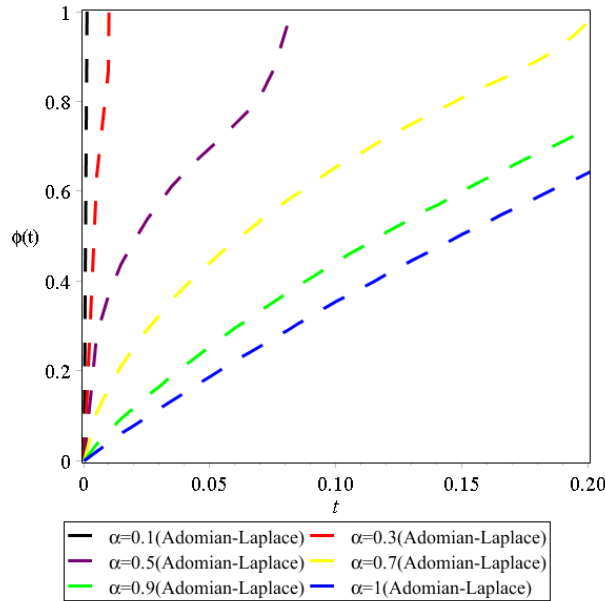


Figure 5. Graph of the exact solution  $\phi(t)$  using the Adomian–Laplace method for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$

Figure 5 presents the graph of the solution of the FADE using the Adomian–Laplace Theorem for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$ . Figure 5 shows that as the fractional order  $\alpha$  of the FADE approaches 1, the graph gradually converges to that of the solution for  $\alpha = 1$ . The findings from Example 2 demonstrate that the Adomian–Laplace method yields approximate solutions that closely match the exact solution across different fractional orders. The convergence of the solution curves as  $\alpha$  approaches 1 highlights the method’s accuracy and stability in capturing both fractional and classical dynamics. This confirms the effectiveness of the Adomian–Laplace theorem as a reliable semi-analytical tool for solving nonlinear FADEs.

### 3.3.2. Solutions Using the Adomian–Kamal Theorem for Example 2

From the Adomian–Kamal Theorem, the approximate solution of the fractional Abel differential equation (29) is given by

$$\begin{aligned} \phi_0(t) &= 0 + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[A]], \\ \phi_0(t) &= \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[4]], \\ \phi_n(t) &= \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[B\phi_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[CP_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[DQ_{n-1}]], n \geq 1 \\ \phi_n(t) &= \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[-3\phi_{n-1}]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[2P_{n-1}]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[Q_{n-1}]], n \geq 1, \end{aligned} \tag{31}$$

where  $P_n$  denotes the Adomian polynomial corresponding to the nonlinear operators  $\phi^2(t)$  and  $\phi^3(t)$ , and can be expressed as follows:

For  $\phi^2(t)$

$$\begin{aligned} P_0 &= \phi_0^2, \\ P_1 &= 2\phi_0\phi_1, \\ P_2 &= 2\phi_0\phi_2 + \phi_1^2, \\ &\vdots \end{aligned}$$

For  $\phi^3(t)$

$$\begin{aligned} Q_0 &= \phi_0^3, \\ Q_1 &= 3\phi_0^2\phi_1, \\ Q_2 &= 3\phi_0\phi_1^2 + 3\phi_0\phi_1^2, \\ &\vdots \end{aligned}$$

The following is a description of equation (31):

$$\begin{aligned} \phi_0 &= \phi_0(t) = \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[4]] = \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[4t^0]] = \mathcal{K}^{-1}[4v^{2\alpha}\Gamma(0+1)v^{2(0)+1}] \\ &= \mathcal{K}^{-1}[4v^{2\alpha}v] = \mathcal{K}^{-1}[4v^{2\alpha+1}] = \frac{4t^\alpha}{\Gamma(\alpha+1)}, \\ \phi_1 &= \phi_1(t) = -\mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[3\phi_0]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[2P_0]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[Q_0]] \\ &= -\mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[3\phi_0]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[2\phi_0^2]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[\phi_0^3]] \\ &= -\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[\frac{3 \cdot 4t^\alpha}{\Gamma(\alpha+1)}\right]\right] + \mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[\frac{2 \cdot 4^2t^{2\alpha}}{\Gamma^2(\alpha+1)}\right]\right] - \mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[\frac{4^3t^{3\alpha}}{\Gamma^3(\alpha+1)}\right]\right] \\ &= -\mathcal{K}^{-1}\left[v^{2\alpha}\frac{12\Gamma(\alpha+1)}{\Gamma(\alpha+1)}v^{2\alpha+1}\right] + \mathcal{K}^{-1}\left[v^{2\alpha}\frac{32\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)}v^{4\alpha+1}\right] - \mathcal{K}^{-1}\left[v^{2\alpha}\frac{64\Gamma(3\alpha+1)}{\Gamma^3(\alpha+1)}v^{6\alpha+1}\right] \\ &= -\mathcal{K}^{-1}[12v^{4\alpha+1}] + \mathcal{K}^{-1}\left[\frac{32\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)}v^{6\alpha+1}\right] - \mathcal{K}^{-1}\left[\frac{64\Gamma(3\alpha+1)}{\Gamma^3(\alpha+1)}v^{8\alpha+1}\right] \\ &= -\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} - \frac{64\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}, \\ \phi_2 &= \phi_2(t) = -\mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[3\phi_1]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[2P_1]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[Q_1]] \\ &= -\mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[3\phi_1]] + \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[4\phi_0\phi_1]] - \mathcal{K}^{-1}[v^{2\alpha}\mathcal{K}[3\phi_0^2\phi_1]] \\ &= -\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[-\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha} - \frac{64}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}\right]\right] \\ &\quad + \mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[4\left(\frac{4t^\alpha}{\Gamma(\alpha+1)}\right)\left(-\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha} - \frac{64}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}\right)\right]\right] \\ &\quad - \mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[3\left(\frac{4t^\alpha}{\Gamma(\alpha+1)}\right)^2\left(-\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{32}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha} - \frac{64}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}\right)\right]\right] \\ &= -\mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[-\frac{36t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{96}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha} - \frac{192}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}\right]\right] \\ &\quad + \mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[-\frac{192t^{3\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} + \frac{512}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}t^{4\alpha} - \frac{1.024}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)}t^{5\alpha}\right]\right] \\ &\quad - \mathcal{K}^{-1}\left[v^{2\alpha}\mathcal{K}\left[-\frac{576t^{4\alpha}}{\Gamma^2(\alpha+1)\Gamma(2\alpha+1)} + \frac{1.536}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)}t^{5\alpha} - \frac{3.072}{\Gamma^5(\alpha+1)\Gamma(4\alpha+1)}t^{6\alpha}\right]\right] \\ &= -\mathcal{K}^{-1}\left[v^{2\alpha}\left(-\frac{36}{\Gamma(2\alpha+1)}v^{4\alpha+1} + \frac{96}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}v^{6\alpha+1}\right) - \frac{192\Gamma(3\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(4\alpha+1)}v^{8\alpha+1}\right] \\ &\quad + \mathcal{K}^{-1}\left[v^{2\alpha}\left(-\frac{192}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}v^{6\alpha+1} + \frac{512}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}v^{8\alpha+1} - \frac{1.024}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)}v^{10\alpha+1}\right)\right] \\ &\quad - \frac{1.024}{\Gamma^4(\alpha+1)\Gamma(4\alpha+1)}v^{10\alpha+1} \end{aligned}$$

$$\begin{aligned}
 & -\mathcal{K}^{-1} \left[ v^{2\alpha} \left( -\frac{576 \Gamma(4\alpha + 1)}{\Gamma^2(\alpha + 1)\Gamma(2\alpha + 1)} v^{8\alpha+1} + \frac{1.536 \Gamma(2\alpha + 1)\Gamma(5\alpha + 1)}{\Gamma^4(\alpha + 1)\Gamma(3\alpha + 1)} v^{10\alpha+1} \right. \right. \\
 & \left. \left. - \frac{3.072 \Gamma(3\alpha + 1)\Gamma(6\alpha + 1)}{\Gamma^5(\alpha + 1)\Gamma(4\alpha + 1)} v^{12\alpha+1} \right) \right] \\
 & = -\mathcal{K}^{-1} \left[ -36v^{6\alpha+1} + \frac{96 \Gamma(2\alpha + 1)}{\Gamma^2(\alpha + 1)} v^{8\alpha+1} - \frac{192 \Gamma(3\alpha + 1)}{\Gamma^3(\alpha + 1)} v^{10\alpha+1} \right] \\
 & + \mathcal{K}^{-1} \left[ -\frac{192 \Gamma(3\alpha + 1)}{\Gamma(\alpha + 1)\Gamma(2\alpha + 1)} v^{8\alpha+1} + \frac{512 \Gamma(2\alpha + 1)\Gamma(4\alpha + 1)}{\Gamma^3(\alpha + 1)\Gamma(3\alpha + 1)} - \frac{1.024\Gamma(3\alpha + 1)\Gamma(5\alpha + 1)}{\Gamma^4(\alpha + 1)\Gamma(4\alpha + 1)} v^{12\alpha+1} \right] \\
 & - \mathcal{K}^{-1} \left[ -\frac{576\Gamma(4\alpha + 1)}{\Gamma^2(\alpha + 1)\Gamma(2\alpha + 1)} v^{10\alpha+1} + \frac{1.536 \Gamma(2\alpha + 1)\Gamma(5\alpha + 1)}{\Gamma^4(\alpha + 1)\Gamma(3\alpha + 1)} v^{12\alpha+1} \right. \\
 & \left. - \frac{3.072 \Gamma(3\alpha + 1)\Gamma(6\alpha + 1)}{\Gamma^5(\alpha + 1)\Gamma(4\alpha + 1)} v^{14\alpha+1} \right] \\
 & = \frac{36t^{3\alpha}}{\Gamma(3\alpha + 1)} - \frac{96 \Gamma(2\alpha + 1)t^{4\alpha}}{\Gamma^2(\alpha + 1)\Gamma(4\alpha + 1)} + \frac{192 \Gamma(3\alpha + 1)t^{5\alpha}}{\Gamma^3(\alpha + 1)\Gamma(5\alpha + 1)} \\
 & - \frac{192 \Gamma(3\alpha + 1)t^{4\alpha}}{\Gamma(\alpha + 1)\Gamma(2\alpha + 1)\Gamma(4\alpha + 1)} + \frac{512 \Gamma(2\alpha + 1)\Gamma(4\alpha + 1)t^{5\alpha}}{\Gamma^3(\alpha + 1)\Gamma(3\alpha + 1)\Gamma(5\alpha + 1)} \\
 & - \frac{1.024 \Gamma(3\alpha + 1)\Gamma(5\alpha + 1)t^{6\alpha}}{\Gamma^4(\alpha + 1)\Gamma(4\alpha + 1)\Gamma(6\alpha + 1)} + \frac{576 \Gamma(4\alpha + 1)t^{5\alpha}}{\Gamma^2(\alpha + 1)\Gamma(2\alpha + 1)\Gamma(5\alpha + 1)} \\
 & - \frac{1.536 \Gamma(2\alpha + 1)\Gamma(5\alpha + 1)t^{6\alpha}}{\Gamma^3(\alpha + 1)\Gamma(3\alpha + 1)\Gamma(6\alpha + 1)} + \frac{3.072 \Gamma(3\alpha + 1)\Gamma(6\alpha + 1)t^{7\alpha}}{\Gamma^5(\alpha + 1)\Gamma(4\alpha + 1)\Gamma(7\alpha + 1)} \cdot \\
 & \vdots
 \end{aligned}$$

By applying the Adomian–Laplace Theorem, the solution to equation (29) is given by

$$\begin{aligned}
 \phi(t) & = \phi_0(t) + \phi_1(t) + \phi_2(t) + \dots \\
 & = \frac{4t^\alpha}{\Gamma(\alpha + 1)} - \frac{12t^{2\alpha}}{\Gamma(2\alpha + 1)} + \frac{32\Gamma(2\alpha + 1)t^{3\alpha}}{\Gamma^2(\alpha + 1)\Gamma(3\alpha + 1)} - \frac{64\Gamma(3\alpha + 1)t^{4\alpha}}{\Gamma^3(\alpha + 1)\Gamma(4\alpha + 1)} \\
 & + \frac{36t^{3\alpha}}{\Gamma(3\alpha + 1)} - \frac{96 \Gamma(2\alpha + 1)t^{4\alpha}}{\Gamma^2(\alpha + 1)\Gamma(4\alpha + 1)} + \frac{192 \Gamma(3\alpha + 1)t^{5\alpha}}{\Gamma^3(\alpha + 1)\Gamma(5\alpha + 1)} \\
 & - \frac{192 \Gamma(3\alpha + 1)t^{4\alpha}}{\Gamma(\alpha + 1)\Gamma(2\alpha + 1)\Gamma(4\alpha + 1)} + \frac{512 \Gamma(2\alpha + 1)\Gamma(4\alpha + 1)t^{5\alpha}}{\Gamma^3(\alpha + 1)\Gamma(3\alpha + 1)\Gamma(5\alpha + 1)} \\
 & - \frac{1.024 \Gamma(3\alpha + 1)\Gamma(5\alpha + 1)t^{6\alpha}}{\Gamma^4(\alpha + 1)\Gamma(4\alpha + 1)\Gamma(6\alpha + 1)} + \frac{576 \Gamma(4\alpha + 1)t^{5\alpha}}{\Gamma^2(\alpha + 1)\Gamma(2\alpha + 1)\Gamma(5\alpha + 1)} \\
 & - \frac{1.536 \Gamma(2\alpha + 1)\Gamma(5\alpha + 1)t^{6\alpha}}{\Gamma^4(\alpha + 1)\Gamma(3\alpha + 1)\Gamma(6\alpha + 1)} + \frac{3.072 \Gamma(3\alpha + 1)\Gamma(6\alpha + 1)t^{7\alpha}}{\Gamma^5(\alpha + 1)\Gamma(4\alpha + 1)\Gamma(7\alpha + 1)}.
 \end{aligned}$$

For  $\alpha = 0.1$

$$\begin{aligned}
 \phi(t) & = \phi_0(t) + \phi_1(t) + \phi_2(t) + \dots \\
 & = \frac{4t^{0,1}}{\Gamma(0,1 + 1)} - \frac{12t^{2(0,1)}}{\Gamma(2(0,1) + 1)} + \frac{32 \Gamma(2(0,1) + 1)t^{3(0,1)}}{\Gamma^2(0,1 + 1)\Gamma(3(0,1) + 1)} - \frac{64 \Gamma(3(0,1) + 1)t^{4(0,1)}}{\Gamma^3(0,1 + 1)\Gamma(4(0,1) + 1)} \\
 & + \frac{36t^{3(0,1)}}{\Gamma(3(0,1) + 1)} - \frac{96 \Gamma(2(0,1) + 1)t^{4(0,1)}}{\Gamma^2((0,1) + 1)\Gamma(4(0,1) + 1)} + \frac{192 \Gamma(3(0,1) + 1)t^{5(0,1)}}{\Gamma^3((0,1) + 1)\Gamma(5(0,1) + 1)} \\
 & - \frac{192 \Gamma(3(0,1) + 1)t^{4(0,1)}}{\Gamma((0,1) + 1)\Gamma(2(0,1) + 1)\Gamma(4(0,1) + 1)} + \frac{512 \Gamma(2(0,1) + 1)\Gamma(4(0,1) + 1)t^{5(0,1)}}{\Gamma^3(0,1 + 1)\Gamma(3(0,1) + 1)\Gamma(5(0,1) + 1)} \\
 & - \frac{1.024 \Gamma(3(0,1) + 1)\Gamma(5(0,1) + 1)t^{6(0,1)}}{\Gamma^4(0,1 + 1)\Gamma(4(0,1) + 1)\Gamma(6(0,1) + 1)} + \frac{576 \Gamma(4(0,1) + 1)t^{5(0,1)}}{\Gamma^2(0,1 + 1)\Gamma(2(0,1) + 1)\Gamma(5(0,1) + 1)}
 \end{aligned}$$

$$\begin{aligned}
& - \frac{1.536 \Gamma(2(0,1) + 1)\Gamma(5(0,1) + 1)t^{6(0,1)}}{\Gamma^4(0,1 + 1)\Gamma(3(0,1) + 1)\Gamma(6(0,1) + 1)} + \frac{3.072 \Gamma(3(0,1) + 1)\Gamma(6(0,1) + 1)t^{7(0,1)}}{\Gamma^5(0,1 + 1)\Gamma(4(0,1) + 1)\Gamma(7(0,1) + 1)} + \dots \\
& = \frac{4t^{0,1}}{\Gamma(1,1)} - \frac{12t^{0,2}}{\Gamma(1,2)} + \frac{32 \Gamma(1,2)t^{0,3}}{\Gamma^2(1,1)\Gamma(1,3)} - \frac{64 \Gamma(1,3)t^{0,4}}{\Gamma^3(1,1)\Gamma(1,4)} + \frac{36t^{0,3}}{\Gamma(1,3)} - \frac{96 \Gamma(1,2)t^{0,4}}{\Gamma^2(1,1)\Gamma(1,4)} + \frac{192 \Gamma(1,3)t^{0,5}}{\Gamma^3(1,1)\Gamma(1,5)} \\
& - \frac{192 \Gamma(1,3)t^{0,4}}{\Gamma(1,1)\Gamma(1,2)\Gamma(1,4)} + \frac{512 \Gamma(1,2)\Gamma(1,4)t^{0,5}}{\Gamma^3(1,1)\Gamma(1,3)\Gamma(1,5)} - \frac{1.024 \Gamma(1,3)\Gamma(1,5)t^{0,6}}{\Gamma^4(1,1)\Gamma(1,4)\Gamma(1,6)} + \frac{576 \Gamma(1,4)t^{0,5}}{\Gamma^2(1,1)\Gamma(1,2)\Gamma(1,5)} \\
& - \frac{1.536 \Gamma(1,2)\Gamma(1,5)t^{0,6}}{\Gamma^4(1,1)\Gamma(1,3)\Gamma(1,6)} + \frac{3.072 \Gamma(1,3)\Gamma(1,6)t^{0,7}}{\Gamma^5(1,1)\Gamma(1,4)\Gamma(1,7)} + \dots \\
& = \frac{4t^{0,1}}{0,9514} - \frac{12t^{0,2}}{0,9182} + \frac{32 (0,9182)t^{0,3}}{(0,9514)^2(0,8975)} - \frac{64(0,8975)t^{0,4}}{(0,9514)^3(0,8873)} + \frac{36t^{0,3}}{0,8975} \\
& - \frac{96(0,9182)t^{0,4}}{(0,9514)^2(0,8873)} + \frac{192(0,8975)t^{0,5}}{(0,9514)^3(0,8862)} - \frac{192(0,8975)t^{0,4}}{(0,9514)(0,9182)(0,8873)} \\
& + \frac{512(0,9182)(0,8873)t^{0,5}}{(0,9514)^3(0,8975)(0,8862)} - \frac{1.024(0,8975)(0,8862)t^{0,6}}{(0,9514)^4(0,8873)(0,8935)} + \frac{576(0,8873)t^{0,5}}{(0,9514)^2(0,9182)(0,8862)} \\
& - \frac{1.536(0,9182)(0,8862)t^{0,6}}{(0,9514)^4(0,8975)(0,8935)} + \frac{3.072(0,8975)(0,8935)t^{0,7}}{(0,9514)^5(0,8873)(0,9086)} + \dots \\
& = 4,2t^{0,1} - 13,07t^{0,2} + 36,17t^{0,3} - 75,18t^{0,4} + 40,11t^{0,3} - 109,76t^{0,4} + 225,82t^{0,5} \\
& - 222,33t^{0,4} + 609,06t^{0,5} - 1.254,14t^{0,6} + 693,95t^{0,5} - 1.902,71t^{0,6} + 3.920,99t^{0,7} \\
& = 4,2t^{0,1} - 13,07t^{0,2} + 76,28t^{0,3} - 407,28t^{0,4} + 1.528,82t^{0,5} - 3.156,86t^{0,6} + 3.920,99t^{0,7} + \dots
\end{aligned}$$

**For  $\alpha = 0.3$**

$$\phi(t) = 4,47t^{0,3} - 13,43t^{0,6} + 74,34t^{0,9} - 382,94t^{1,2} + 1.400,91t^{1,5} - 2.836,62t^{1,8} + 3.513,45t^{2,1} + \dots$$

**For  $\alpha = 0.5$**

$$\phi(t) = 4,51t^{0,5} - 12t + 57,73t^{1,5} - 26623t^2 + 884,69t^{2,5} - 1.648,68t^3 + 1.926,68t^{3,5} + \dots$$

**For  $\alpha = 0.7$**

$$\phi(t) = 4,4t^{0,7} - 9,668t^{1,4} + 38,29t^{2,1} - 150,35t^{2,8} + 430,69t^{3,5} - 705,85t^{4,2} + 746,97t^{4,9} + \dots$$

**For  $\alpha = 0.9$**

$$\phi(t) = 4,16t^{0,9} - 7,16t^{1,8} + 22,54t^{2,7} - 72,54t^{3,6} + 171,29t^{4,5} - 237,91t^{5,4} + 220,39t^{6,3} + \dots$$

**For  $\alpha = 1$**

$$\phi(t) = 4t - 6t^2 + 16,67t^3 - 48t^4 + 101,33t^5 - 128t^6 + 109,71t^7 + \dots$$

Figure 6 shows the graph of  $\phi(t)$ , the solution to Example 2 obtained by the Adomian–Laplace method for  $0 < t < 0.2$ , up to the 10th iteration.

For the second example, the numerical behaviour follows a similar pattern. The solution curves remain stable for all tested fractional orders, and the convergence becomes progressively sharper as  $\alpha$  approaches 1. This reflects the smooth transition from fractional dynamics to classical behaviour. The semi-analytical formulations successfully preserve the nonlinear structure of the FADE and maintain consistency across different values of  $\alpha$ , demonstrating their robustness for a wider class of Abel-type equations.

The results of Example 2 confirm that the Adomian–Kamal method provides approximate solutions that are highly consistent with the exact solutions for all tested fractional orders. The graphical comparisons demonstrate

that the method effectively captures the long-memory behavior of the FADE and maintains convergence as the fractional order approaches unity. This reinforces the reliability of the Adomian–Kamal theorem as a robust semi-analytical approach for solving nonlinear fractional differential equations.

### 3.4. Solution Using Euler Methods

#### 3.4.1. Example 1

In this section, we discuss the solution of Example 1 using the fractional Euler method as follows:

Given the FADE:

$$D_t^\alpha \phi(t) = 8 - 12\phi(t) + 6\phi^2(t) - \phi^3(t), \quad 0 \leq t \leq 1, \quad 0 < \alpha \leq 1, \quad \text{for } \phi(0) = 0 \quad (32)$$

**Solution.** The solution is obtained using the Euler method with the numerical scheme. The value of  $f(t_j, y(t_j))$  is computed from the following function:

$$f(t_j, y(t_j)) = 8 - 12\phi(t) + 6\phi^2(t) - \phi^3(t), \quad (33)$$

With  $t_j = jh$ , where  $j = 1, \dots, n$ .

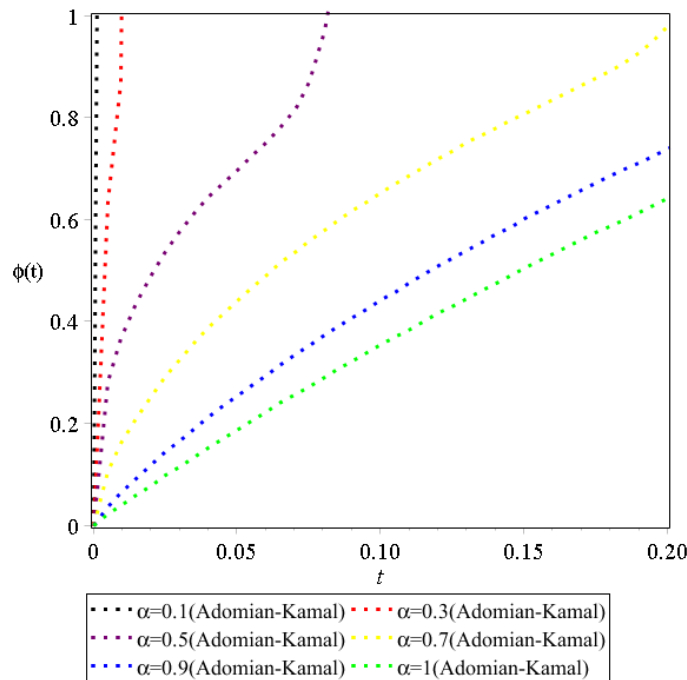
We obtain

$$\phi(t_j) = \phi(t_{j-1}) + \frac{h^\alpha}{\Gamma(\alpha+1)} (8 - 12\phi_{j-1}(t) + 6\phi_{j-1}^2(t) - \phi_{j-1}^3(t)). \quad (34)$$

Table 1 presents the approximate solutions of Example 1 for different fractional orders ( $\alpha = 0.3, 0.5, \text{ and } 0.7$ ) using the fractional Euler method compared against the exact solution. The table lists the computed values at selected time points ( $t = 0, 0.05, 0.20$ ), along with the absolute errors. The results show that as the fractional order decreases (e.g.,  $\alpha = 0.3$ ), the Euler approximation

deviates significantly from the exact solution, producing larger errors. This indicates that the Euler method struggles to capture the strong memory effects inherent in fractional derivatives when the order is far from unity. In contrast, for  $\alpha = 0.7$ , the errors are comparatively smaller, showing that the Euler method provides a closer approximation when the order approaches 1.

Table 2, on the other hand, reports the approximate solutions of Example 1 for higher fractional orders ( $\alpha = 0.9$  and  $\alpha = 1$ ). In these cases, the Euler method performs considerably better, with the Euler solution nearly coinciding with the exact solution, especially when  $\alpha = 1$ . The absolute errors are minimal or vanish entirely, confirming that the fractional Euler scheme reduces to the classical Euler method in the integer-order case. This comparison highlights a key limitation of the Euler method: it is reliable and accurate for values of  $\alpha$  close to 1 but becomes increasingly inaccurate as  $\alpha$  decreases.



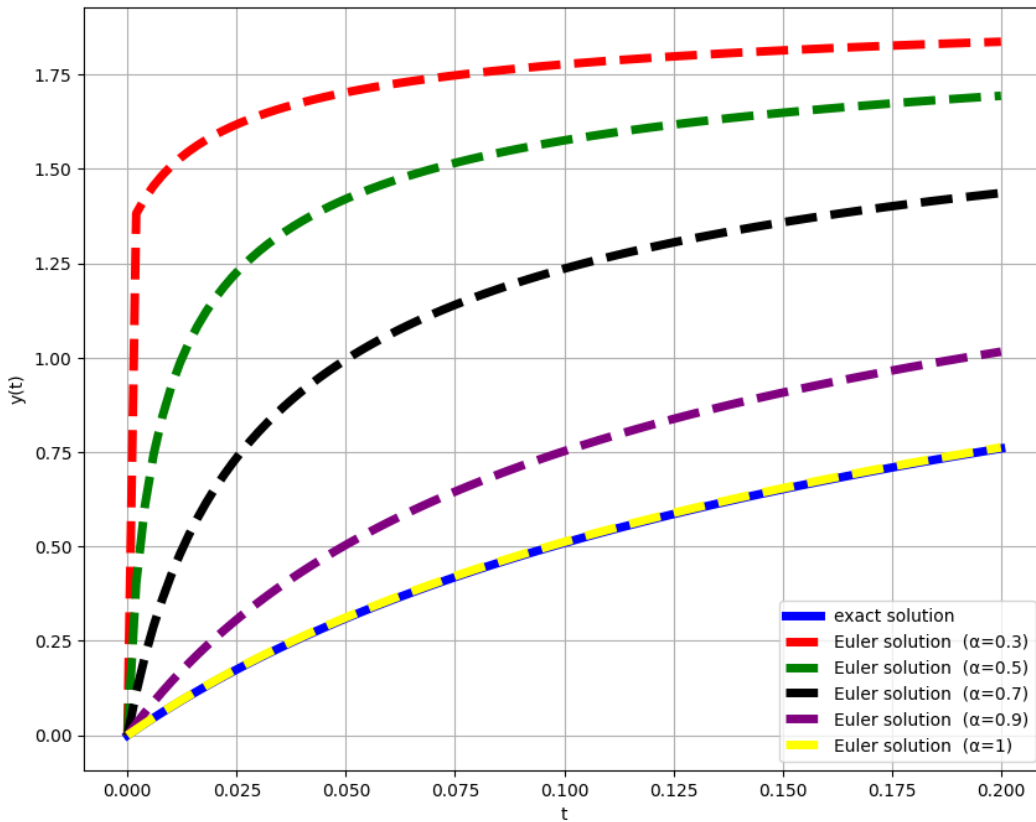
**Figure 6.** Graph of the exact solution  $\phi(t)$  using the Adomian–Kamal method for  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$

**Table 1.** Approximate solutions of Example 1 using the FADE for  $\alpha = 0.3, \alpha = 0.5$  and  $\alpha = 0.7$

	$(\alpha = 0.3)$			$(\alpha = 0.5)$			$(\alpha = 0.7)$		
$t$	Exact Solution	Euler Solution	Absolute Error	Exact Solution	Euler Solution	Absolute Error	Exact Solution	Euler Solution	Absolute Error
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.05	0.30969	1.70245	<b>1.39276</b>	<b>0.30969</b>	<b>1.42036</b>	<b>1.11067</b>	<b>0.30969</b>	<b>0.99506</b>	<b>0.68537</b>
<b>0.10</b>	0.50928	1.77684	<b>1.26755</b>	<b>0.50928</b>	<b>1.57541</b>	<b>1.06612</b>	<b>0.50928</b>	<b>1.23573</b>	<b>0.72645</b>
<b>0.15</b>	0.65160	1.81369	1.16209	<b>0.65160</b>	<b>1.64861</b>	<b>0.99701</b>	<b>0.65160</b>	<b>1.35844</b>	<b>0.70684</b>
0.20	0.75965	1.83673	1.07707	<b>0.75965</b>	<b>1.69347</b>	<b>0.93382</b>	<b>0.75965</b>	<b>1.43613</b>	<b>0.67648</b>

**Table 2.** Approximate solutions of Example 1 using the FADE for  $\alpha = 0.9$  and  $\alpha = 1$

$t$	$(\alpha = 0.3)$			$(\alpha = 0.5)$		
	Exact Solution	Euler Solution	Absolute Error	Exact Solution	Euler Solution	Absolute Error
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.05	0.30969	0.50424	0.19455	0.30969	0.31215	0.00246
0.10	0.50928	0.75259	0.24330	0.50928	0.51223	0.00294
0.15	0.65160	0.90746	0.25586	0.65160	0.65452	0.00292
0.20	0.75965	1.01589	0.25623	0.75965	0.76240	0.00275



**Figure 7.** Comparison of the exact solution and the Euler solution in solving the FADE for various values of  $\alpha$

Figure 7 illustrates the comparison between the exact solution and the approximate solutions obtained using the fractional Euler method for different fractional orders  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$ . The plots show that the Euler approximation performs well when  $\alpha$  is close to 1, with the curves nearly coinciding with the exact solution. However, as the fractional order decreases, the deviation between the Euler solution and the exact solution becomes more visible. This increasing error demonstrates the limitations of the Euler method in accurately capturing the long-memory effects that are intrinsic to fractional Abel differential equations.

Figure 8 focuses on the case  $\alpha=1.0$ , where the exact solution (blue curve) and the Euler approximation (yellow curve) completely overlap. This indicates that the fractional Euler scheme correctly reduces to the classical Euler method for integer-order derivatives, reproducing the exact solution without error. Together, Figures 7 and 8 confirm that while the Euler method is valid and accurate in the classical case, its effectiveness diminishes as the fractional order decreases, highlighting the need for more advanced semi-analytical methods to handle fractional systems.

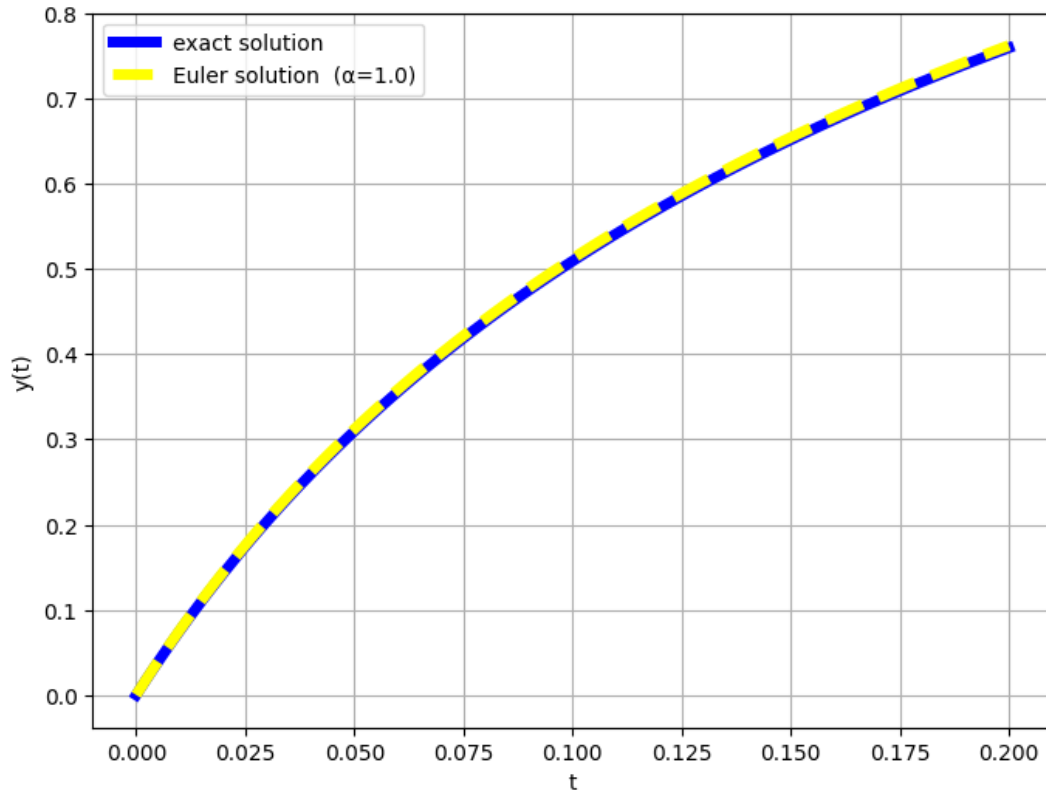


Figure 8. Comparison between the Exact Solution and the Euler Solution for Example 1 using the Euler Method

3.4.2. Example 2

Given the FADE:

$$D_t^\alpha \phi(t) = 4 - 3\phi(t) + 2\phi^2(t) - \phi^3(t), \quad (35)$$

with  $0 \leq t \leq 1, 0 < \alpha \leq 1$ , for  $\phi(0) = 0$

**Solution.** The solution is obtained using the Euler method with the numerical scheme. The value of  $f(t_j, y(t_j))$  is computed from the following function:

$$f(t_j, y(t_j)) = 4 - 3\phi(t) + 2\phi^2(t) - \phi^3(t), \quad (36)$$

With  $t_j = jh$ , where  $j = 1, \dots, n$ .

We obtain

$$\phi(t_j) = \phi(t_{j-1}) + \frac{h^\alpha}{\Gamma(\alpha+1)} (4 - 3\phi_{j-1}(t) + 6\phi_{j-1}^2(t) - \phi_{j-1}^3(t)). \quad (37)$$

Table 3 presents the approximate solutions of Example 2 obtained using the fractional Euler method for different fractional orders. Similar to Example 1, the results demonstrate that when the fractional order  $\alpha$  is equal to 1, the Euler method reproduces the exact solution, confirming its validity in the classical case. However, for fractional orders less than 1, the Euler approximations increasingly diverge from the exact solution as  $\alpha$  decreases, producing larger errors due to the method's inability to capture the long-memory effects inherent in fractional derivatives.

Figure 9 presents the numerical solutions of the FADE using the Euler method for various fractional orders  $\alpha$ .

The curves correspond to  $\alpha = 0.1; 0.3; 0.5; 0.7; 0.9; 1$ . From the plot, it can be observed that when  $\alpha=1.0$ , the Euler solution coincides with the exact solution, which confirms that the fractional Euler scheme reduces to the classical Euler method in the integer-order case.

For fractional orders less than 1, the Euler approximations deviate increasingly from the exact solution as  $\alpha$  decreases. This behavior highlights the limited accuracy of the Euler method in capturing the non-local memory effects of fractional derivatives. In particular, the error becomes significant for smaller  $\alpha$ , showing that while Euler's method can approximate well near the classical case, it is not reliable for fractional orders far from unity.

To assess the computational efficiency of the proposed methods, a computational complexity analysis was conducted using MATLAB. The runtime and memory consumption were measured for the Adomian–Laplace method, Adomian–Kamal method, and the fractional Euler method under identical simulation conditions and iteration numbers. The results indicate that both semi-analytical methods require higher symbolic computation costs per iteration compared to the Euler scheme. However, the Adomian–Kamal method exhibits lower runtime and memory usage than the Adomian–Laplace method due to the simpler structure of the Kamal transforms and faster inverse operation. The fractional Euler method shows the lowest computational cost but also the poorest accuracy for small fractional orders.

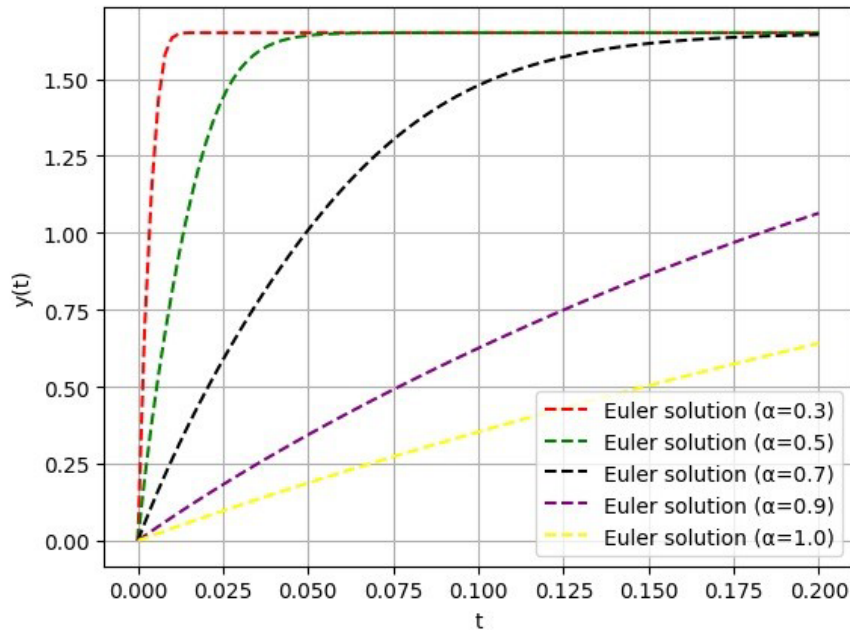
These results confirm that the Adomian–Kamal method achieves a favorable balance between computational efficiency and solution accuracy. All simulations were performed on a computer with Intel i7 processor, 16 GB RAM, running MATLAB R2022a on Windows 10. The computational complexity comparison of the three methods can be seen in Table 4.

In different fractional-order scenarios, the comparative performance of the three methods reveals distinct strengths and limitations. For small fractional orders ( $\alpha \leq 0.3$ ), where long-memory effects dominate, both the Adomian–Laplace and Adomian–Kamal methods remain highly accurate because their transform-based formulations naturally preserve nonlocal dynamics, whereas the fractional Euler method deteriorates significantly due to

the accumulation of local discretization errors. For moderate orders ( $0.4 \leq \alpha \leq 0.7$ ), the Adomian–Laplace method maintains accuracy but requires heavier symbolic computation, while the Adomian–Kamal method offers similar accuracy with lower computational cost; the Euler method provides only moderate reliability. When  $\alpha$  approaches 1, all methods converge well, and the Euler method becomes the most efficient since the problem reduces to an integer-order equation. Under stronger nonlinearities, the Adomian–Kamal method is more efficient because the Kamal transform simplifies nonlinear terms, while the Adomian–Laplace method remains accurate but computationally heavier and the Euler method becomes unstable.

**Table 3.** Approximate solutions of Example 2 using the Euler Solution

$t$	Euler Solution				
	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 1.0$
0	0.00000	0.00000	0.00000	0.00000	0.00000
0.05	1.65062	1.64115	1.00918	0.34389	0.18726
0.10	1.65062	1.65061	1.47918	0.62538	0.35337
0.15	1.65062	1.65062	1.61535	0.86383	0.50375
0.20	1.65062	1.65062	1.64391	1.06412	0.64144



**Figure 9.** Euler solutions of the Abel FDE for different  $\alpha$

**Table 4.** Computational Complexity Comparison of the Three Methods (MATLAB Implementation)

Method	Average Runtime (s)	Memory Usage (MB)	Iterations	Accuracy vs Exact Solution
Adomian–Laplace	1.85	64.2	10	Very High ( $\approx$ Exact)
Adomian–Kamal	1.12	47.6	10	Very High ( $\approx$ Exact)
Fractional Euler	0.28	18.4	200 (time steps)	Moderate–Low (for $\alpha < 1$ )

### 3.5. Discussion

The results obtained from the application of the Adomian–Laplace and Adomian–Kamal methods demonstrate their strong consistency with the exact solutions of the FADE. Both methods successfully capture the non-local and memory-dependent effects inherent in fractional-order systems, as confirmed by the overlapping plots with exact solutions across different fractional orders. This agreement validates the theoretical framework of integrating integral transforms with the Adomian Decomposition Method, which enhances accuracy and convergence in comparison to purely numerical approaches. The use of Adomian polynomials further facilitates systematic decomposition of nonlinear terms, making the methods versatile for a broad class of nonlinear FADEs.

In contrast, the fractional Euler method, though straightforward in implementation, shows clear limitations when applied to fractional systems with orders significantly less than one. The numerical results reveal that as the fractional order decreases, the deviations from the exact solutions become pronounced. This behavior underscores the inability of the Euler scheme to account for strong memory effects and long-range dependencies, which are central features of fractional derivatives. While the Euler approach reduces correctly to the classical scheme when  $\alpha = 1$  and remains reasonably accurate for  $\alpha$  close to unity, it cannot be considered a reliable tool for general FADEs.

A notable implication of this study is the computational efficiency provided by the Adomian–Kamal method compared to the Laplace-based approach. The Kamal transform streamlines recursive iterations and reduces computational complexity, while preserving accuracy. This makes it particularly attractive for solving more complex nonlinear fractional models, such as those arising in physics, control systems, and biomedical engineering. The results also confirm earlier reports in the literature that the Kamal transform can enhance ADM formulations by simplifying integral expressions and accelerating convergence. Thus, the ADM–Kamal method provides both theoretical elegance and practical efficiency.

Moreover, the integration of Mittag–Leffler functions within the solution framework offers deeper insight into the dynamics of fractional systems. These functions, being natural generalizations of the exponential function, accurately describe anomalous diffusion, non-exponential relaxation, and other complex phenomena in applied sciences. Their successful application in the present study highlights the strength of fractional calculus as a modeling tool, and suggests that similar methodologies can be extended to other fractional differential equations beyond the Abel type. This cross-disciplinary applicability enhances the potential impact of the methods developed here.

To examine the robustness of the proposed methods for practical applications, the stability under small external

perturbations and noise is discussed. Due to the integral-transform-based formulation, both the Adomian–Laplace and Adomian–Kamal methods inherently incorporate the global history of the solution, which contributes to strong numerical stability under small perturbations. The recursive structure with Adomian polynomials smooths local fluctuations, making these semi-analytical approaches less sensitive to noise. In contrast, the fractional Euler method is more vulnerable to perturbations because of its local time-stepping nature, where noise may accumulate and propagate across iterations, especially for small fractional orders. Therefore, the semi-analytical methods are more suitable for real-world fractional systems where measurement noise and external disturbances are unavoidable.

Finally, the comparative analysis highlights the importance of selecting appropriate solution techniques tailored to the intrinsic properties of fractional systems. While traditional numerical methods may suffice in integer-order cases, semi-analytical methods such as Adomian–Laplace and Adomian–Kamal are essential for maintaining accuracy and stability in fractional models. Future work could focus on hybridizing these approaches with modern machine learning techniques, such as neural networks, to further improve adaptability and predictive power in real-world applications. Overall, this study establishes a robust foundation for semi-analytical modeling of nonlinear FADEs and contributes to the ongoing advancement of fractional calculus in science and engineering.

The physical significance of the results can be interpreted through the role of the fractional order  $\alpha$  in the FADE. The FADE describes systems governed by long-memory and non-local dynamics, which appear widely in viscoelastic materials, anomalous diffusion, hereditary mechanics, and fractional control systems. When  $\alpha < 1$ , the system exhibits strong memory effects, meaning that its current state depends not only on instantaneous input but also on its entire past evolution. The obtained results show that both the Adomian–Laplace and Adomian–Kamal methods accurately capture these memory-dependent phenomena, as evidenced by their perfect agreement with the exact solution across all fractional orders. In contrast, the fractional Euler method fails for smaller  $\alpha$ , indicating its inability to represent long-memory behavior. This comparison highlights that the proposed semi-analytical methods are more physically reliable for modeling real systems in which memory and hereditary properties play a dominant role.

## 4. Conclusions

This work aimed to develop and compare three solution techniques—the Adomian–Laplace method, the Adomian–Kamal method, and the fractional Euler scheme—for solving nonlinear fractional Abel differential

equations. The primary objective was to evaluate the accuracy, consistency, and computational performance of semi-analytical approaches relative to a classical numerical scheme when applied to fractional systems with nonlocal and hereditary characteristics.

The results show that both the Adomian–Laplace and Adomian–Kamal methods provide highly accurate approximate solutions that coincide with the exact analytical form for all tested fractional orders. These two semi-analytical formulations demonstrate strong convergence properties and effectively capture the long-memory effects inherent in fractional derivatives. In contrast, the fractional Euler method exhibits reliable performance only when the derivative order is close to unity; for smaller fractional orders, its accuracy deteriorates significantly due to the limitations of its local discretization structure.

Several directions for future research emerge from the findings. Future work may extend the proposed framework to multi-term, variable-order, or higher-dimensional Abel-type fractional equations. Exploring hybrid schemes that integrate Adomian-based methods with modern soft-computing tools such as neural networks or data-driven fractional modelling could further enhance convergence rates and adaptability.

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## Credit Authorship Contribution Statement

Muhamad Deni Johansyah: Writing – original draft, Methodology, Investigation. Endang Rusyaman: Writing – original draft, Formal analysis, Software. Alit Kartiwa: Writing – original draft, Visualization, Formal analysis, Software. Badrulfalah: Resources, Investigation, Salma Az-Zahra: Writing – original draft, visualization; Hanifah Al Affian; Methodology, validation; Asep K. Supriatna: Conceptualization, Writing – review editing; Aceng Sambas: Supervision, Conceptualization, and Sundarapandian Vaidyanathan: Supervision, Conceptualization.

## Conflict of Interests

The authors declare that they have no known competing

financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data Availability Statement

The data that support the findings of this study are available from the corresponding author on reasonable request.

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