

Decoding the Architectural Design Formula of the Hue Imperial Palace, Nguyen Dynasty (1802-1945), Vietnam

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Abstract The Hue Imperial Palace is widely recognized as a rich and significant source of scientific material for academic research. This paper briefly introduces the proportional relationship between the floor plan and the wooden frame of the Twin-Ridge Beam Buildings (abbreviated as Twin Buildings), alongside their dimensional origins and parametric design principles. Furthermore, this paper examines the methods of column creation, timber shaping, and their proportional relationship, aiming to formulate their design methods. With research methods of the Nguyen Dynasty's historical documents study, surveys of the remaining Twin Buildings' timbers, dimensional analyses, and investigations into traditional design methods, this study identifies a calculation system rooted in ancient mathematics. Specifically, it defines the decimal system corresponding to "One Ten" (applied to 10-unit Twin Buildings) and the duodecimal system corresponding to "One Dozen" (applied to 12-unit Twin Buildings). Accordingly, the proposed Design Formula incorporates key elements such as variables, constants, parameters, and dependent quantities that are clearly defined. The Design Formula ensures uniformity in architectural patterns and forms while offering flexibility in adapting usage functions and building scales. It also enables easy parameter adjustments for determining timber dimensions, simplifies wooden material preparation and cost estimation, and facilitates replication and rehabilitation of heritage buildings.

Keywords Design Formula, Twin Buildings, Hue Imperial Palace, Nguyen Dynasty, Vietnam, World Cultural Heritage

1. Introduction

Architectural design methods in many Asian countries have historically existed in the form of traditional formulas used in the design and construction of wooden structures. These methods establish quantifiable relationships that determine the classification, layout, and scale of various building types, particularly within the realm of heritage architecture. Often considered the proprietary knowledge of craft guilds or recorded in construction manuals, such formulas were historically viewed as secondary to the creative process of architectural design, which tends to emphasize individual expression [1], [2], [3]. Nonetheless, from a scientific and methodological standpoint, these traditional design systems played a central role in shaping the architectural identity of successive feudal dynasties and expressing distinct cultural values (Figure 1).

Within a relatively short period of 45 years—from the reign of Gia Long to Thieu Tri (1802–1847)—the Nguyen Dynasty succeeded in constructing an extensive and coherent complex of architectural works [4], [5], [6]. These fulfilled a wide range of functional requirements, from civil and religious to military infrastructure. This raises several critical research questions: What specific design methods were employed during the Nguyen Dynasty? Were these methods governed by analytical design formulas? If so, what parameters and quantities constituted these formulas? Which contextual or cultural factors influenced their development and application, thereby ensuring construction efficiency and consistency?



Figure 1. Front view of the Thai Hoa Dien Palace – a typical feature of the Hue Imperial Palace, Vietnam

By applying the research methodologies including historical documents study, surveys of the remaining Twin Buildings' timbers, dimensional analyses, and investigations into traditional design methods, this study aims to investigate and clarify these questions by examining historical documents, analyzing architectural remains, and exploring traditional mathematical systems. The broader goal is to assess whether these historically influential design principles can be preserved, transmitted, and adapted to contemporary practice. By doing so, this research seeks not only to contribute to the authentic conservation of heritage buildings but also to provide a valuable foundation for emerging architects, enabling them to inherit and creatively reinterpret traditional design methodologies.

2. Materials and Methodology

2.1. Historical Documents Collection

This study draws upon a range of historical sources from the Nguyen Dynasty, particularly official historical texts written in classical Chinese and published during the dynasty's reign (Nguyen, 1802-1945b, 1960, 2004). These primary documents are of significant scholarly value, as they contain detailed records concerning the construction and renovation of imperial architecture, including information on building names, dimensions, functions, structural forms, and underlying architectural principles.

Of particular importance is the text which provides a specialized vocabulary related to timber components (Nguyen, 1802-1945c). This terminology has proven essential for accurately identifying the position and role of individual elements within traditional wooden structures. Furthermore, the treatise offers critical data on the measurement systems used in Nguyen-era construction (Nguyen, 1902). The documentation of ruler lengths within

this source has enabled the conversion of metric measurements (mm) into traditional units, thus allowing for a more precise reconstruction and interpretation of the architectural proportions found in the Hue Imperial Palaces.

2.2. Literature Review

The doctoral dissertation of Takeshi Nakagawa, titled *A Study on the Kiwari Technique of Japan* [1], is a work of significant academic merit and has become a foundational reference in architectural education in Japan. Through an in-depth analysis of traditional Japanese carpentry manuals on *kiwari*—a system of timber proportioning—the study establishes a coherent architectural proportional framework. It further offers practical applications for the design and conservation of traditional Japanese wooden architecture (Figure 2).

Nakagawa's research is particularly notable for its methodological rigor and the development of functional design diagrams that translate traditional knowledge into a systematic and reproducible form. The approach and analytical framework presented in this dissertation have served as a key methodological reference for the present study, inspiring the application of similar principles in the investigation of architectural proportions within the Hue Imperial Palaces.

The master's thesis of Tetsuo Kogure, titled *Analysis on the Planning Method of the Double and Triple Ridge Buildings – Studies on the Imperial Palace of Hue, Vietnam, in the Nguyen Dynasty* [7] can be considered as a foundational study on the spatial organization of the Twin-Ridge Beam and Single-Ridge Beam buildings of the Nguyen Dynasty. While the research offers valuable insights into the spatial planning and typological classification of these structures, it does not engage with the concept of architectural proportion—an essential principle underlying traditional architectural design.

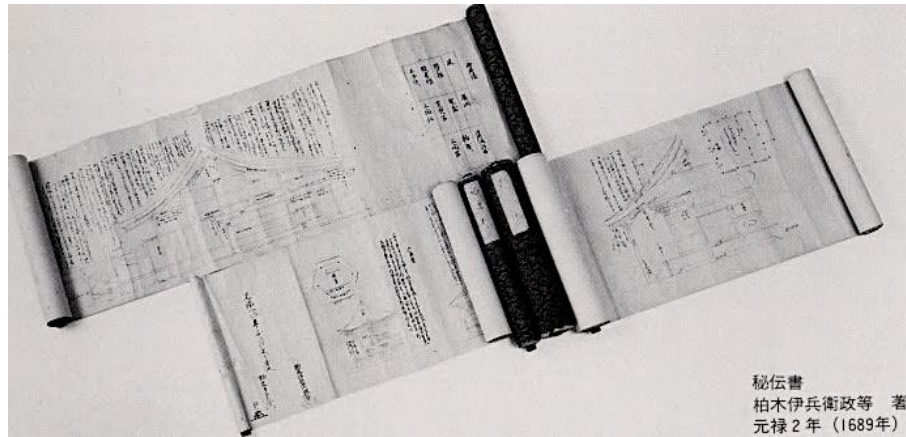


Figure 2. The Japanese “Kiwari” traditional carpentry manuals, 1689 (Source: Nabunken, 2004)

Similarly, the article by Vinh An Le and Ngoc Quynh Chau Truong, entitled “*Practicing on the Re-Construction Study of ‘Can Chanh Dien’ Palace, Hue Imperial City, Vietnam – World Cultural Heritage*” [8], examines the design methodology employed in the reconstruction of the Can Chanh Dien Palace. This structure, once the principal administrative hall for the emperors and their court, was largely destroyed during the Indochina War in 1947, leaving only the stone platform, foundation, and partial components. The article effectively outlines principles for reconstruction based on historical documentation and remaining physical evidence. However, it focuses primarily on a case-specific restoration strategy and does not delve deeply into the broader architectural proportion systems or formalized design formulas of Nguyen Dynasty architecture—gaps that this current study aims to address.

In reality, there has been a limited number of international scholarly studies on this subject. On the one hand, this may be attributed to the perception that the architecture of the Hue Imperial Palace does not constitute a major architectural achievement in the broader context of Asia, especially when compared to the wooden architecture of the Forbidden City in Beijing (China) or historical monuments such as Horyuji Temple, Todaiji Temple, and the Nara Palace Site in Japan. On the other hand, rigorous research in this field requires access to a substantial body of architectural drawings with precise dimensional data essential for proportion analysis—materials that are often difficult for foreign researchers to obtain. Furthermore, conducting interviews with Vietnamese traditional carpenters on design methods and construction techniques presents significant challenges due to language barriers and, more critically, the inherently conservative and secretive nature of Vietnam’s traditional architectural practices. Within this cultural framework, the transmission of esoteric craftsmanship knowledge to external scholars is often considered taboo.

It should also be noted that while various studies on the architecture of the Hue Imperial Palace have been published in the form of reference books, handbooks, or

guild manuals, these sources are largely tangential to the focus of this research and have therefore been excluded from the current study.

2.3. Doctrinal Study of Historical Documents

The historical documents of the Nguyen Dynasty, as referenced above, are composed in classical Chinese script—a logographic writing system characterized by its semantic density and linguistic conciseness. As part of this study, a comprehensive process of data extraction and philological interpretation was undertaken to analyze noun phrases contained within these texts [9]. This includes imperial era names, classifications of functional building complexes, names and construction periods of individual structures, as well as compound and single terms related to architectural spatial concepts and structural components.

Moreover, the examination of these historical records has yielded critical information regarding the measurement systems employed during the Nguyen Dynasty, including the nomenclature and standard lengths of rulers used in architectural construction, along with their corresponding values when converted to the modern metric system (millimeters) [10]. This corpus of data is particularly significant for enabling the analysis of architectural proportions and for uncovering key principles of passive design that were integral to construction practices during the Nguyen Dynasty period.

2.4. Restoration Site Investigation and Oral Interviews with Traditional Master Carpenters

This study represents the culmination of over three decades of archival research and technical engagement, initiated following the inscription of the Complex of Hue Monuments as a UNESCO World Cultural Heritage site. Since that recognition, conservation and restoration efforts have significantly intensified, with a primary focus on the Hue Imperial City, the imperial mausoleums, and other prominent heritage structures within the ancient capital.

These sustained preservation activities have offered a unique and extended opportunity to observe and document traditional construction techniques, architectural design principles, and the structural logic underlying the assembly of timber elements.

Furthermore, these efforts have catalyzed the mobilization of highly skilled traditional carpenters from various regions of Vietnam to participate in conservation projects in Hue. This concentration of craftsmanship has not only contributed directly to restoration outcomes but also fostered the preservation and intergenerational transmission of traditional carpentry knowledge—an essential cultural asset for the ongoing and future safeguarding of Vietnam’s architectural heritage. The interviews with traditional carpenters covered topics such as traditional architectural design methods, criteria for selecting natural timber for construction, wood sawing and processing techniques, and the fabrication of wooden components that have been and are currently applied in the conservation and restoration of wooden architectural complexes in Hue. The results of these interviews provided insights into the architectural thinking of traditional carpenters in design, calculation methods in wooden construction, and the permissible tolerances of timber during the fabrication of components.

2.5. Timber Dimensional Analysis

In addition to the practical insights gained through direct involvement in architectural restoration projects, a collaborative conservation research initiative was undertaken between 1994 and 2000 by Waseda University (Tokyo, Japan) and the Hue Monuments Conservation Center (Hue City, Vietnam). This international program brought together Japanese researchers and Vietnamese technical specialists to conduct extensive architectural surveys and precise measurements across key sites within the Complex of Hue Monuments. The resulting dataset was subsequently digitized, forming a comprehensive and reliable digital archive of architectural dimensions [11], [12].

Utilizing this robust empirical foundation, the study conducted detailed dimensional analyses of timber components, examining their interrelationships and proportional systems. When integrated with historical documentation and qualitative insights gathered through interviews with master Vietnamese carpenters, these analyses revealed fundamental technical principles underpinning the architectural design methodology of the Nguyen Dynasty. This multifaceted approach is instrumental in identifying the critical aspects of traditional design logic, construction techniques, and material usage that are essential to the heritage’s authenticity and preservation.

2.6. Arguments and Diagramming

The scope of this study encompasses a diverse range of

heritage structures, each designed and constructed during distinct historical phases. As a result, variations in design principles, construction techniques, and architectural forms are to be expected across different periods. In order to systematize the collected data and synthesize the findings of the analysis, it is essential to formulate a set of working hypotheses and represent them through diagrammatic means.

This approach involves the use of visual and analytical tools—including photographs, architectural drawings, quantitative tables, and schematic representations of design formulas—to effectively organize and present the research outcomes. By integrating these visual and statistical elements, the study achieves a coherent synthesis of complex data, facilitating a clearer and more scientifically rigorous presentation of the architectural logic underlying the heritage structures under investigation.

3. Results

3.1. Categories of the Survey-Targeted Twin Building's Platforms

Table 1 and Figure 3 present consolidated data on the surveyed platforms (abbreviated as P) of the Twin-Ridge Beam Buildings (hereafter referred to as Twin Buildings), as documented in historical sources [4]. Particular emphasis is placed on key architectural parameters, including platform height, the number of staircases and steps, and the number of structural compartments.

These specific elements warrant focused attention due to their potential significance within the broader framework of traditional design methodology. Their consistent appearance across multiple structures suggests that they may serve as fundamental variables in the formulation of architectural design principles. Accordingly, this raises critical research questions: What is the underlying importance of these parameters? Do they function as integral components within the traditional design formulas, and if so, in what manner do they contribute to the proportional logic and spatial organization of the Twin Buildings?

In general, the height of the platform establishes a proportional aesthetic relationship with the overall elevation of the building, contributing to its visual harmony and perceived monumentality. The number of staircases and steps serves as an indicator of the building’s hierarchical status within the broader architectural complex, while the number of compartment spans reflects the scale and functional importance of the structure.

Notably, a recurrent spatial configuration—referred to as the “5-3-2 Platform,” consisting of five compartments in the front building, three in the main building, and two auxiliary wings at the outermost left and right—appears frequently in standard temples and small shrines. Similarly,

other typologies follow a consistent logic: the “7-5-2 Platform” is commonly associated with standard palaces and large temples; the “9-7-2 Platform” with exceptionally large palaces; the “11-9-2 Platform” with standard shrines; and the “15-13-2 Platform” with monumental or special shrines.

These recurring configurations suggest that the platform composition functions as a constant parameter within the broader design formula. Its standardized application across building types and hierarchical levels highlights its foundational role in shaping architectural typology and will be examined in greater detail in the following sections.

3.2. Dimensional Origin of the Platforms

The floor plan of traditional Vietnamese buildings is organized into a system of spatial modules comprising central compartments, referred to as *Gian* (間), and lateral wings, known as *Ha* (廈). These spatial units are subtly delineated through rows of columns or assemblages of wooden partition panels. The number and arrangement of these compartments and wings are key determinants of the building’s overall scale and functional composition.

As established in prior studies, the design process conventionally begins with the identification of the primary dimensional reference, most notably the central compartment (hereafter abbreviated as A) and/or big beam span of the main hall (abbreviated as E) [2], [3], [13]. This

span functions as the foundational parameter upon which the structural grid of the entire building is based. The dimension of compartment A governs the proportional relationships and spatial hierarchy of adjacent spans, including span of E and the flanking sub-compartments (abbreviated as B) (Figure 4). Accordingly, the central compartment serves not only as the geometric core of the design but also as a critical reference point in the formulation of architectural proportions across the building layout.

By establishing a specific value for the central compartment span (A), the overall spatial composition and proportional framework of the floor plan are determined. Furthermore, empirical observations suggest that the total height of the wooden structural frame (abbreviated as *BuH*) is generally constrained within the range $BuH = A \times 2$. This proportional relationship indicates a direct correlation between the horizontal layout and vertical development of the structure, thereby reinforcing the coherence of the architectural composition.

Nevertheless, critical questions remain regarding the dimensional logic of the individual timber components: How are these dimensions determined, and do they follow a systematic proportional relationship? These fundamental issues concerning the structural design logic and the internal consistency of timber dimensions will be explored in detail in the subsequent chapters.

Table 1. Categories of the Platforms

No	Name of Buildings	Types	Height of Platform (unit)	Number of Stair/Step				Number of Compartment span		
				Front	Back	Left	Right	Front Building	Main Building	Wings
1	Thai To Mieu	Shrine	1.50	3/3	0	1/3	1/3	15	13	2
2	The To Mieu	Shrine	3.60	3/5	2/5	1/5	1/5	11	9	2
3	Phung Tien Mieu	Shrine	<i>1.80</i>	<i>3/3</i>	<i>2/3</i>	<i>1/3</i>	<i>1/3</i>	11	9	2
4	Can Thanh Dien	Palace	2.30	3/4	3/4	1/4	1/4	9	7	2
5	Dien Tho Chinh Dien	Palace	1.40	3/1	2/2	1/3	1/3	5	3	2
6	Long An Dien	Palace	3.20	3/5	1/5	3/5	3/5	7	5	2
7	Thai Hoa Dien	Palace	2.30	3/4	3/4	1/4	1/4	7	5	2
8	Can Chanh Dien	Palace	2.30	3/5	2/2	1/2	1/2	7	5	2
9	Trieu To Mieu	Shrine	2.00	3/3	0	1/3	1/3	5	3	2
10	Minh Thanh Dien	Temple	<i>1.80</i>	<i>3/3</i>	<i>2/3</i>	0	0	5	3	2
11	Sung An Dien	Temple	2.20	3/4	3/4	0	0	5	3	2
12	Bieu Duc Dien	Temple	<i>2.10</i>	<i>3/4</i>	<i>3/4</i>	0	0	5	3	2
13	Hoa Khiem Dien	Temple	<i>2.15</i>	<i>3/4</i>	<i>3/4</i>	<i>2/4</i>	<i>2/4</i>	7	5	2
14	Luong Khiem Dien	Temple	<i>2.15</i>	3/4	1/4	1/4	1/4	5	3	2

Note: Regular is according to historical documents; Italic is the results of site measurement.

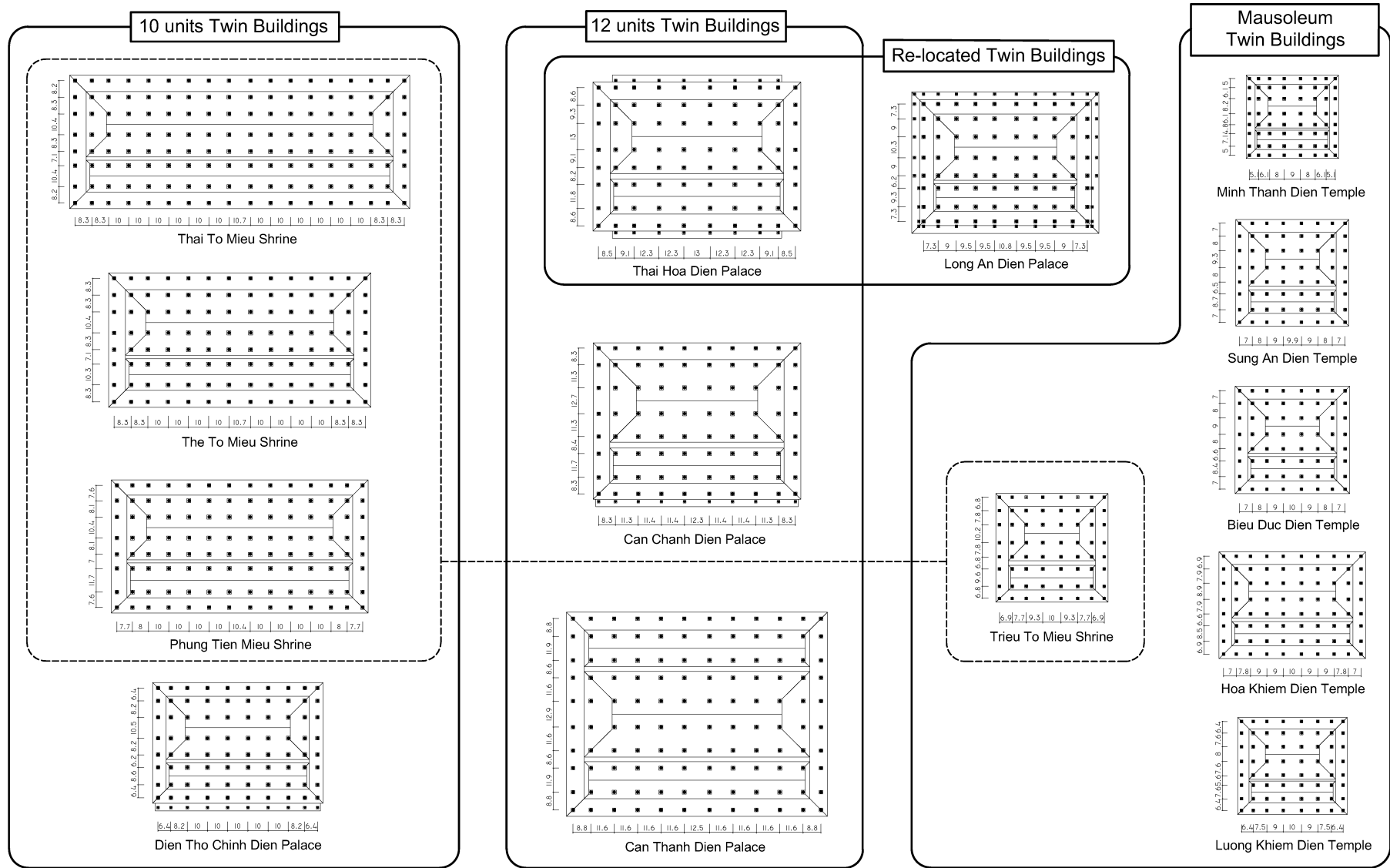


Figure 3. Categorisation of the survey-targeted Twin-Building's Platforms

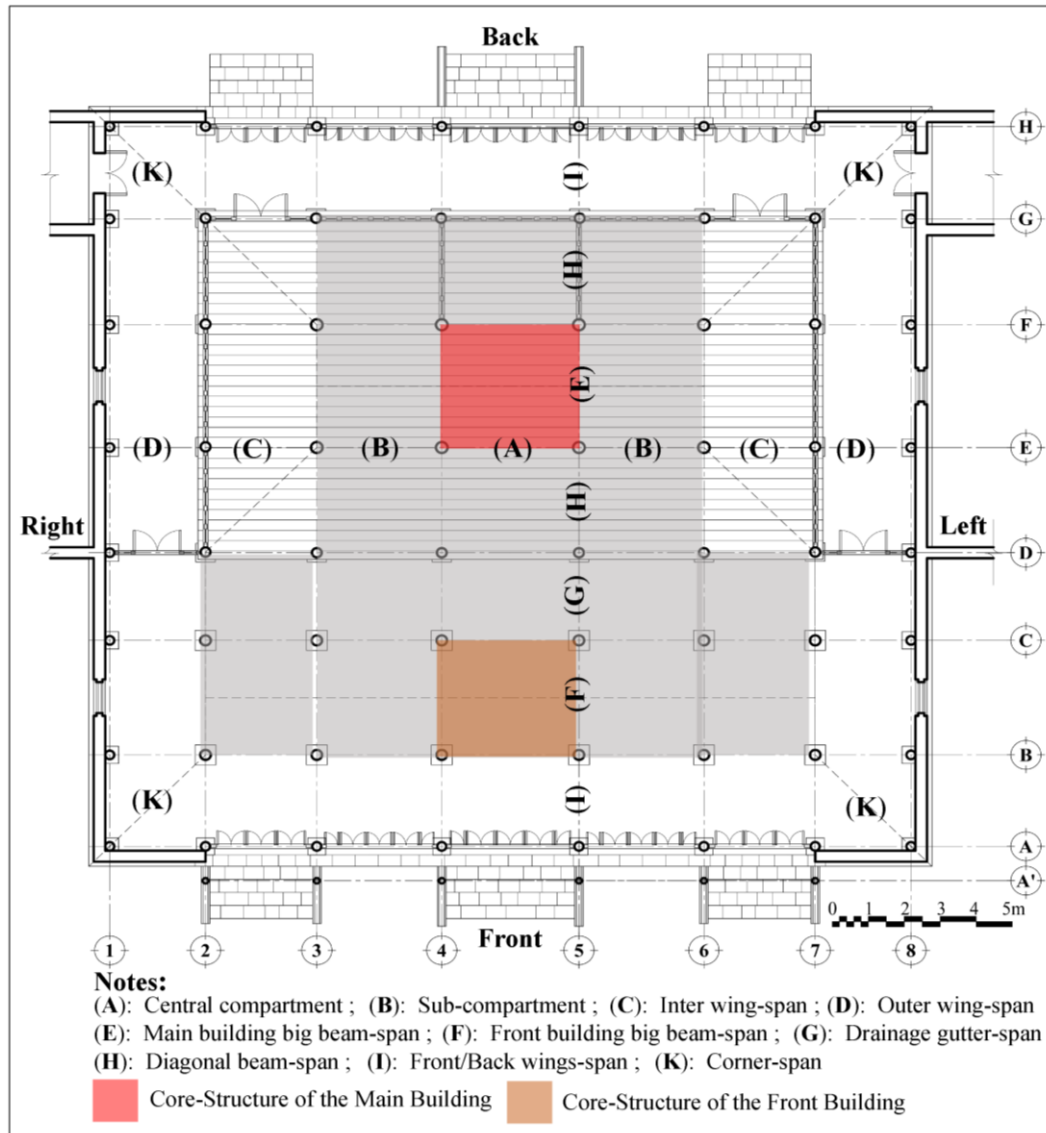


Figure 4. Layout of the floor plan (case of the Sung An Dien Temple, Ming Mang's Mausoleum)

3.3. Timber Components

The timber structure of the Twin-Ridge Beam Buildings (hereafter referred to as Twin Buildings) is composed of a complex assembly of structural elements, including primary components such as large columns, medium columns, and small columns, as well as structural members such as principal beams, main tie beams, head tie beams, necktie beams, lower tie beams, top ridge beams, diagonal braces, purlins, and rafters among others (Figures 5 and 6).

The upper portions of the large columns are structurally linked by principal beams and main tie beams, forming the core horizontal framework. Diagonal beams are strategically installed to provide additional structural stability and are arranged in several configurations:

- Extending from the tops of large and/or medium columns toward the ridge beam of the main hall;
- Extending from the tops of small columns toward the large columns of the front building;

- Extending from the tops of small columns toward the medium columns of the main building.

The superstructure of the front building is supported by its respective principal beams, while the upper structural system of the main building is borne by the main building's principal beams. At the four outermost edges of the Twin Buildings, lower diagonal braces connect the small columns to both the large columns of the front building and the medium columns of the main hall, ensuring lateral stability.

The roof system is composed of rafters and purlins, which are systematically arranged along the slope defined by the diagonal beams, providing a framework for the application of ceramic roof tiles. Additionally, an interior ceiling is typically installed at mid-height within the main hall, vertically dividing the interior space into upper and lower sections and contributing to both spatial definition and environmental control within the building.

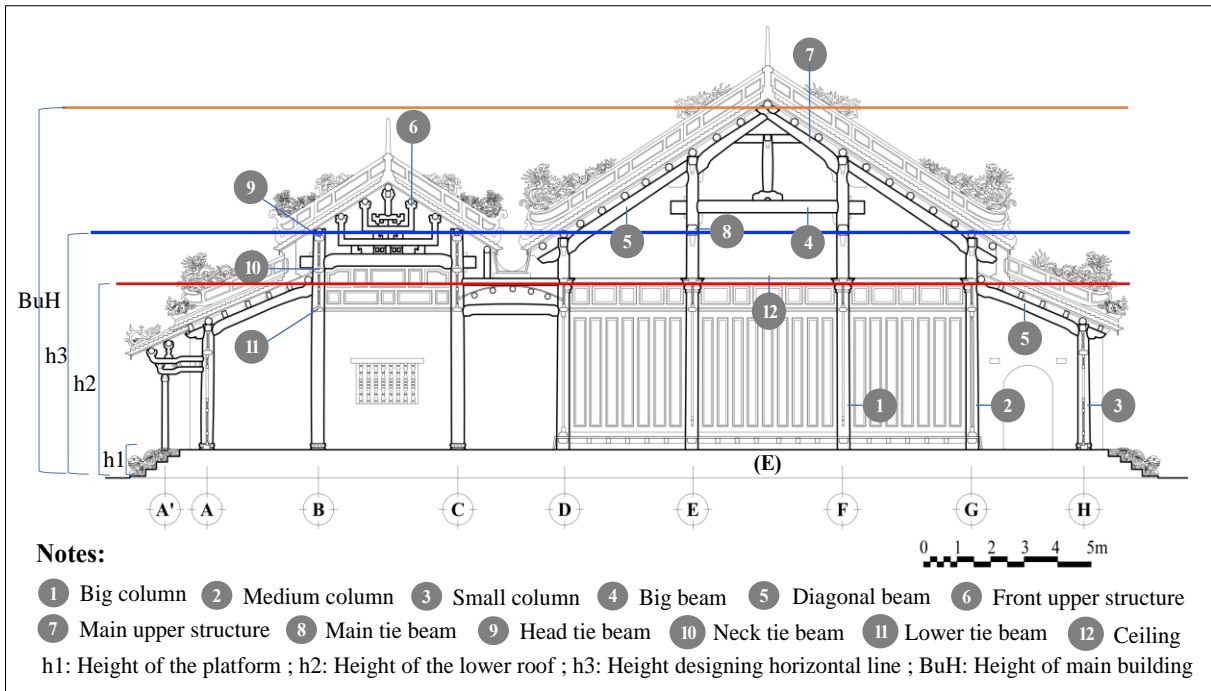


Figure 5. Timber components following the transversal longitude section (Sung An Dien Temple)

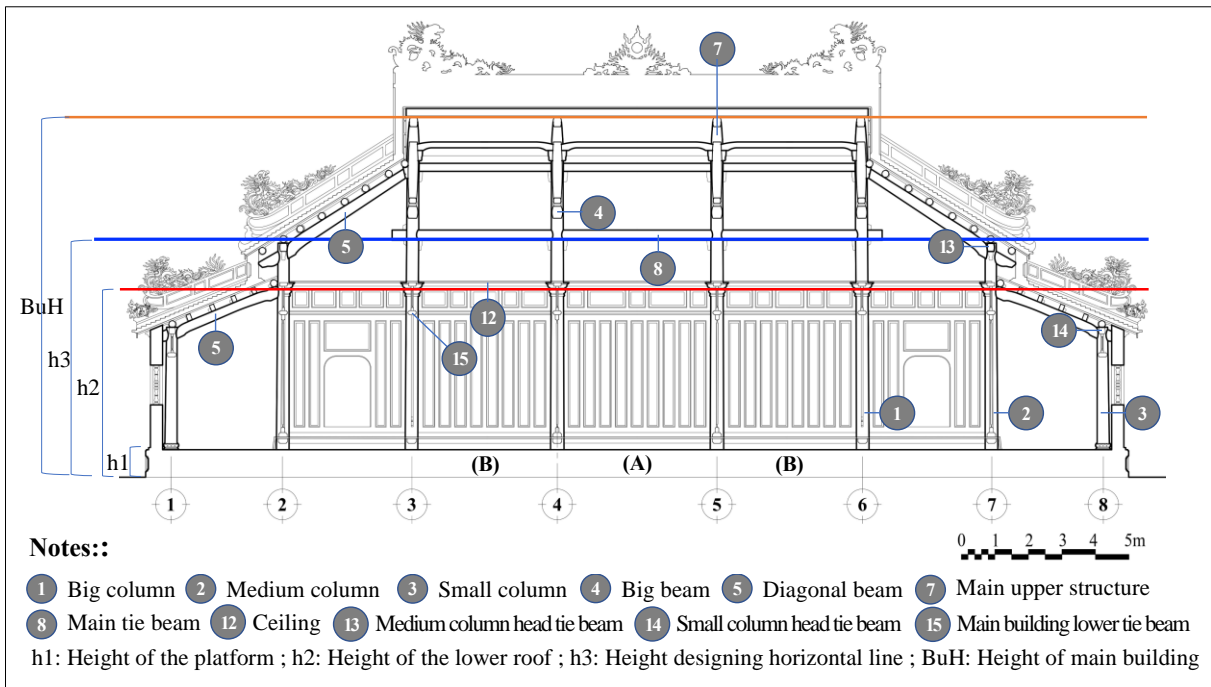


Figure 6. Timber components following the transversal latitude section (Sung An Dien Temple)

3.4. Column Crafting and Timber Shaping

3.4.1. Columns Crafting

Firstly, columns serve a fundamental role in the structural integrity of traditional wooden architecture, functioning as primary load-bearing elements. Based on empirical knowledge derived from traditional construction practices, columns are rarely positioned in perfectly

vertical alignment with the floor plan. Instead, their upper ends are subtly inclined inward from the central vertical axis, a deviation that may reflect long-standing experiential refinements aimed at improving structural performance or visual perception.

In terms of morphology, column diameters vary along their height. The diameter at the base (abbreviated as CoD1) is typically larger than that at the top (CoD3), with both

being smaller than the maximum diameter located at the column's midsection, commonly known as the hip diameter (CoD). The hip is conventionally positioned at approximately one-third of the total column height measured upward from the base, while the neck (CoD2), which engages structurally with the main tie beam, is situated roughly one-third of the total height measured downward from the top (Figure 7).

This graduated tapering of column diameters appears to be the result of accumulated traditional knowledge, developed through iterative trial-and-error practices. It reflects a nuanced understanding of material behavior, structural stability, and aesthetic balance within the architectural composition.

Ideally, a column is fashioned from a naturally long, cylindrical timber, which is meticulously carved and refined to achieve the required form and compatibility with adjoining structural components. In instances where natural round timbers are unavailable or unsuitable, the column is instead constructed from a rectangular or square-sectioned timber, treated as a box-shaped element. Within this configuration, the surface oriented toward the viewer is referred to as the *Face* (面), while the opposite, concealed surface is termed the *Thick* (厚).

The largest cross-sectional dimension of the box-shaped timber establishes the foundational reference for determining the column diameter, and by extension, informs the dimensional relationships with adjacent structural members that connect to the column. This proportional linkage is critical in maintaining coherence throughout the structural system.

Figure 7 illustrates the traditional manual technique employed to transform a box-shaped timber into a circular column. In this process, the diameter of the resulting circular section may be derived either from the perimeter or the diameter, contingent upon the physical characteristics of the timber and the specific design intentions of the carpenter. This practice reflects the adaptability of

traditional craftsmanship to varying material conditions while preserving proportional and structural integrity.

3.4.2. Shaping of the Timber's Cross Section

Secondly, among the various timber elements in traditional wooden architecture, three structural components hold particular significance: the *main tie beam*—called *Chấn Tâm* (振心)—which spans the building transversely along the latitudinal axis; the *principal beam* or *big beam*—called *Lương Tâm* (梁心)—which runs longitudinally along the building's length; and the *diagonal beam*—called *Giao Giá* (交架)—which follows the slope of the roof. These terms and their respective structural roles are formally defined in the historical records of the Nguyen Dynasty [14], [15].

The lengths of these beams are crafted in either straight or gently curved forms, and are designed with symmetry along their central longitudinal and/or latitudinal axes. Structurally, these beams form horizontal and diagonal connections with the columns, thereby contributing to the creation of a three-dimensional wooden framework. The cross-sectional profiles of these beams are typically square or rectangular in shape (as detailed in Table 2), and their dimensions are proportionally derived from the diameter of the column at the hip section. This proportional relationship ensures consistency in structural logic and reflects the underlying geometric coherence characteristic of traditional Nguyen Dynasty timber architecture.

Thirdly, additional structural elements that form integral connections with the columns include the head tie beam (*HeB*), neck-tie beam (*NeB*), lower tie beam (*LoB*), top ridge beam (*d*), purlins (*Pud*), and rafters. These components function within both the longitudinal and latitudinal structural systems, contributing to the overall rigidity and spatial cohesion of the wooden framework. Their integration is further reinforced through the use of infill wooden panels, which help to minimize deformation and enhance structural continuity.

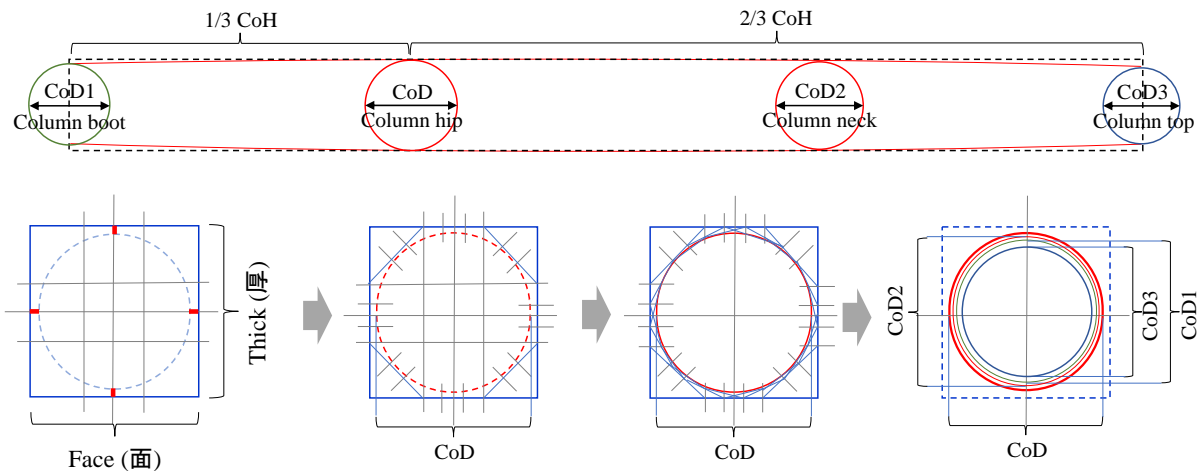
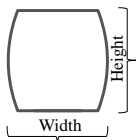
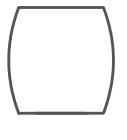
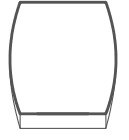
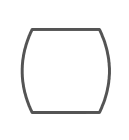
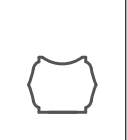
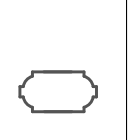
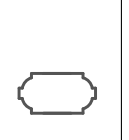
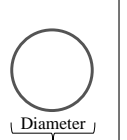
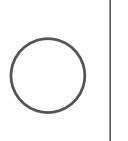


Figure 7. The way of the Columns creation

Table 2. Shape of the cross section of the Timbers

MB Tie-beam (TiB)	MB Big-beam (BiB)	MB Diagonal beam (DiB)	MB Head tie-beam (HeB1)	FB Head tie-beam (HeB2)	FB Neck tie-beam (NeB)	FB Lower tie-beam (LoB)	MB Top-ridge beam (d)	Purlin (Pud)
								

As these elements are directly joined to the columns, the dimensions of their cross-sections are proportionally derived in relation to the column diameter at the hip (CoD), or in some cases, based on the dimension of the head tie beam (HeB). Importantly, the cross-sectional dimensions of these secondary elements are constrained in such a way that they do not exceed those of the primary structural members to which they are connected. This proportional hierarchy reflects a systematic design logic rooted in traditional construction practices, ensuring both structural efficiency and visual harmony across the architectural composition.

3.5. Proportional Analysis

3.5.1. Error and Adjustment in the Analysis

This analysis is grounded in empirical measurements recorded in the metric system, which are subsequently converted into the traditional units of measurement employed during the Nguyen Dynasty. The comparative results reveal discrepancies between the actual recorded values and those derived from the historical unit conversions. However, such deviations are to be expected, as they reflect inherent variations arising from construction tolerances and manual measuring techniques prevalent in traditional craftsmanship.

A fundamental principle in traditional Vietnamese construction practice distinguishes between the treatment of timber and masonry: “*Carpentry is raised, Masonry is reduced.*” This principle is based on the material constraints of each system. In timber construction, the dimensions of natural wood elements cannot be increased once shaped; thus, components are initially cut larger than the intended final size to allow for subsequent trimming and refinement. In contrast, masonry construction requires that allowances be made during design and execution, since excess material is difficult to remove once the structure is built.

In the context of this study, such practical considerations have been taken into account. For instance, material reduction due to finishing is typically estimated at approximately 12 mm for circular columns and 6 mm for square-section timbers. These allowances, although seemingly minor, are essential for achieving dimensional precision and have been incorporated into the analytical

model to more accurately reflect the traditional design and construction process.

3.5.2. The Relation between the Platforms, Central Compartment, and Height of the Twin Buildings

Table 3 presents the proportional relationships between the platform (P) and the vertical dimensions of the associated wooden superstructure. The span of the central compartment (denoted as A) is found to range between 0.48 and 0.53 of the total height of the wooden structure (BuH), thus approximating a 1:2 ratio.

The height of the platform ($h1$) is further analyzed in relation to various vertical references within the architectural composition. Specifically:

- $h1$ ranges from 0.27 to 0.30 of the height of the lower roof ($h2$), corresponding approximately to one-third; in other cases, $h1$ ranges from 0.19 to 0.21 of $h2$, approximating one-fifth;
- $h1$ ranges from 0.23 to 0.24 of the total design reference height ($h3$), which is approximately one-fourth; and in certain configurations, $h1$ ranges from 0.15 to 0.17 of $h3$, roughly equivalent to one-sixth.

These proportional ratios highlight the existence of a consistent dimensional logic underlying the architectural composition of the Twin Buildings. They also suggest the application of traditional mathematical systems—such as decimal and duodecimal divisions—in determining spatial and structural hierarchies within the design formula.

3.5.3. The Relation between the Central Compartment and the Big Column

Table 4 presents the proportional relationships between the central compartment span (A) and the dimensional characteristics of the primary structural column. The diameter of the large column at the hip (CoD) is found to correspond to approximately 0.08, 0.09, or 0.10 of A , which equates to traditional fractional ratios of 1/12, 1/11, and 1/10, respectively.

In terms of girth, the column perimeter (CoP) ranges from 0.25 to 0.31 of A , approximating proportions between 1/4 and 1/3. Additionally, when assessed in relation to the overall height of the column (CoH), the CoD falls within the range of 0.05 to 0.06, corresponding closely to a ratio of 1/20.

Table 3. Proportional relation between the Platform and the height of Wooden structure (calculated in units)

No.	Name of buildings	Central compartment	Big-beam span	Sub-compartment	Total height of Main building	Height of Platform	Height of Lower roof	Height of Designing horizontal line	Algebraic Relation			
		(A)	(E)	(B)	(BuH)	(h1)	(h2)	(h3)	(A)/(BuH)	(E)/(BuH)	(h1)/(h2)	(h1)/(h3)
1	Thai To Mieu	10,70	10,40	10,00	※	1,50	※	※	※	※	※	※
2	The To Mieu	10,70	10,40	10,00	21,50	3,60	11,90	15,20	0,50	0,48	0,30	0,24
3	Phung Tien Mieu	10,40	10,40	10,00	※	1,80	※	※	※	※	※	※
4	Dien Tho Chinh Dien	10,00	10,50	10,50	21,00	1,40	10,80	12,60	0,48	0,50	0,13	0,11
5	Long An Dien	10,80	10,30	9,50	20,50	3,20	11,80	14,00	0,53	0,50	0,27	0,23
6	Can Thanh Dien	12,50	12,90	11,60	※	2,30	※	※	※	※	※	※
7	Thai Hoa Dien	13,00	13,00	12,30	24,40	2,30	12,30	15,20	0,53	0,50	0,19	0,15
8	Can Chanh Dien	12,30	12,70	11,40	※	2,30	※	※	※	※	※	※
9	Trieu To Mieu	10,00	10,20	9,30	20,50	2,00	10,40	12,70	0,49	0,50	0,19	0,16
10	Minh Thanh Dien	9,00	8,20	8,00	17,90	1,80	9,00	11,00	0,50	0,46	0,20	0,16
11	Sung An Dien	9,90	9,30	9,00	19,70	2,20	10,90	12,60	0,50	0,47	0,20	0,17
12	Bieu Duc Dien	10,00	9,00	9,00	19,90	2,10	10,70	12,60	0,50	0,45	0,20	0,17
13	Hoa Khiem Dien	10,00	8,90	9,00	19,70	2,15	10,80	12,50	0,51	0,45	0,20	0,17
14	Luong Khiem Dien	10,00	8,00	9,00	19,90	2,15	10,30	12,50	0,50	0,40	0,21	0,17

Table 4. Proportional relation between A and Big Column (CoD, CoP, and CoH) (calculated in units)

No.	Name of buildings	Central compartment	Big-beam span	Big-column diameter	Big-column perimeter	Big-column height	Height of Designing horizontal line	Algebraic Relation					
		(A)	(E)	(CoD)	(CoP)	(CoH)	(h3)	(CoD)/(A)	(CoP)/(A)	(CoD)/(E)	(CoP)/(E)	(CoD)/(CoH)	(CoD)/(h3)
1	Thai To Mieu	10,70	10,40	1,00	3,14	※	※	0,09	0,29	0,10	0,30	※	※
2	The To Mieu	10,70	10,40	1,00	3,14	20,00	15,20	0,09	0,29	0,10	0,30	0,05	0,07
3	Phung Tien Mieu	10,40	10,40	0,98	3,08	※	※	0,09	0,30	0,09	0,30	※	※
4	Dien Tho Chinh Dien	10,00	10,50	0,84	2,64	17,80	12,60	0,08	0,26	0,08	0,25	0,05	0,07
5	Long An Dien	10,80	10,30	0,85	2,67	17,80	14,00	0,08	0,25	0,08	0,26	0,05	0,06
6	Can Thanh Dien	12,50	12,90	1,00	3,14	※	※	0,08	0,25	0,08	0,24	※	※
7	Thai Hoa Dien	13,00	13,00	1,10	3,45	20,90	15,20	0,08	0,27	0,08	0,27	0,05	0,07
8	Can Chanh Dien	12,30	12,70	1,00	3,14	※	※	0,08	0,26	0,08	0,25	※	※
9	Trieu To Mieu	10,00	10,20	1,00	3,14	15,60	12,70	0,10	0,31	0,10	0,31	0,06	0,08
10	Minh Thanh Dien	9,00	8,20	0,80	2,51	14,80	11,00	0,09	0,28	0,10	0,31	0,05	0,07
11	Sung An Dien	9,90	9,30	0,80	2,51	16,80	12,60	0,08	0,25	0,09	0,27	0,05	0,06
12	Bieu Duc Dien	10,00	9,00	0,90	2,83	17,10	12,60	0,09	0,28	0,10	0,31	0,05	0,07
13	Hoa Khiem Dien	10,00	8,90	0,90	2,83	16,70	12,50	0,09	0,28	0,10	0,32	0,05	0,07
14	Luong Khiem Dien	10,00	8,00	0,85	2,67	16,90	12,50	0,09	0,27	0,11	0,33	0,05	0,07

These proportional relationships underscore a coherent dimensional system embedded within the traditional design logic of the Nguyen Dynasty's timber architecture. Such ratios appear to be governed by established principles of architectural proportion, possibly informed by both structural requirements and long-standing artisanal conventions.

3.5.4. Relation between the Column Diameter and the Main Timbers

Table 5 illustrates the proportional relationships between the column diameter at the hip (*CoD*) and the cross-sectional dimensions of three principal structural members: the tie beam (*TiB*), the big beam (*BiB*), and the diagonal beam (*DiB*).

For the tie beam (*TiB*), the height (*h*) or width (*w*) of the cross-section ranges from 0.60 to 0.75 of *CoD*, corresponding approximately to traditional ratios of 3/5 to 3/4. The big beam (*BiB*) exhibits a broader proportional range, from 0.75 to 1.00 of *CoD*, aligning with fractional

values of 3/4, and 4/5, and in some cases reaching a 1:1 ratio. The diagonal beam (*DiB*) demonstrates proportional values ranging between 0.69 and 0.82 of *CoD*, which approximates standard ratios of 7/10, 3/4, 4/5, and 5/6.

These findings reflect a consistent and deliberate proportional system in the design of timber components, with *CoD* functioning as a key dimensional reference. This system ensures both structural coherence and visual harmony within the architectural framework, in accordance with traditional Nguyen Dynasty construction principles.

3.5.5. Relation between Column Diameter and the Elemental Timbers

Tables 6 and 7 present the proportional relationships between the column diameter at the hip (*CoD*) and the cross-sectional dimensions of several secondary structural components, including the head tie beam (*HeB*), top ridge beam (*d*), neck-tie beam (*NeB*), lower tie beam (*LoB*), and purlins (*Pud*).

Table 5. Proportional relation between Column diameter and the Main Timbers (calculated in units)

Name of buildings	Big-column diameter (CoD)	Across-section of Tie-beam (height & width)		Across-section of Big-beam (height & width)		Across-section of Diagonal-beam (height & width)		Algebraic Relation					
		(TiBh)	(TiBw)	(BiBh)	(BiBw)	(DiBh)	(DiBw)	Tie-beam and Big-column		Big-beam and Big-column		Diagonal beam and Big-column	
								(TiBh)/(CoD)	(TiBw)/(CoD)	(BiBh)/(CoD)	(BiBw)/(CoD)	(DiBh)/(CoD)	(DiBw)/(CoD)
The To Mieu	1,00	0,60	0,60	0,80	0,82	0,81	0,80	0,60	0,60	0,80	0,82	0,81	0,80
Dien Tho Chinh Dien	0,84	0,64	0,60	0,76	0,70	0,63	0,63	0,76	0,71	0,90	0,83	0,75	0,75
Long An Dien	0,85	0,64	0,64	0,71	0,70	0,70	0,64	0,75	0,75	0,84	0,82	0,82	0,75
Thai Hoa Dien	1,10	0,77	0,77	0,91	0,84	0,84	0,76	0,70	0,70	0,83	0,76	0,76	0,69
Trieu To Mieu	1,00	0,60	0,60	0,75	0,75	0,70	0,70	0,60	0,60	0,75	0,75	0,70	0,70
Minh Thanh Dien	0,80	0,53	0,53	0,64	0,60	0,60	0,60	0,66	0,66	0,80	0,75	0,75	0,75
Sung An Dien	0,80	0,60	0,60	0,80	0,72	0,60	0,60	0,75	0,75	1,00	0,90	0,75	0,75
Bieu Duc Dien	0,90	0,63	0,63	0,72	0,64	0,64	0,64	0,70	0,70	0,80	0,71	0,71	0,71
Hoa Khiem Dien	0,90	0,63	0,63	0,81	0,72	0,62	0,62	0,70	0,70	0,90	0,80	0,69	0,69
Luong Khiem Dien	0,85	0,60	0,60	0,68	0,64	0,60	0,60	0,71	0,71	0,80	0,75	0,71	0,71

Table 6. Proportional relation between Column diameter and the elemental Timbers (calculated in units)

No.	Name of buildings	Big-column diameter (CoD)	Across-section of Main Building's Head tie-beam (height & width)		Across-section of Front Building's Head tie-beam (height & width)		Main building's Top-ridge beam diameter (d)	Algebraic Relation					
			(MB-HeBh)	(MB-HeBw)	(FB-HeBh)	(FB-HeBw)		Main building's Head tie-beam and Big-column		Front building's Head tie-beam and Big-column		Main building's Top-ridge beam and Big-coulmn (d)/(CoD)	
								(MB-HeBh)/(CoD)	(MB-HeBw)/(CoD)	(FB-HeBh)/(CoD)	(FB-HeBw)/(CoD)		
1	The To Mieu	1,00	0,50	0,48	0,49	0,51	0,60	0,50	0,48	0,49	0,51	0,60	
2	Dien Tho Chinh Dien	0,84	0,42	0,34	0,42	0,43	0,50	0,50	0,40	0,50	0,51	0,60	
3	Long An Dien	0,90	0,44	0,44	0,43	0,44	0,44	0,49	0,49	0,48	0,49	0,49	
4	Thai Hoa Dien	1,10	0,66	0,66	0,55	0,66	0,66	0,60	0,60	0,50	0,60	0,60	
5	Trieu To Mieu	1,00	0,40	0,40	0,40	0,48	0,60	0,40	0,40	0,40	0,48	0,60	
6	Minh Thanh Dien	0,80	0,40	0,40	0,40	0,40	0,40	0,50	0,50	0,50	0,50	0,50	
7	Sung An Dien	0,80	0,40	0,40	0,40	0,40	0,40	0,50	0,50	0,50	0,50	0,50	
8	Bieu Duc Dien	0,90	0,44	0,44	0,44	0,44	0,45	0,49	0,49	0,49	0,49	0,50	
9	Hoa Khiem Dien	0,90	0,45	0,45	0,44	0,45	0,54	0,50	0,50	0,49	0,50	0,60	
10	Luong Khiem Dien	0,90	0,45	0,45	0,45	0,45	0,54	0,50	0,50	0,50	0,50	0,60	

Table 7. Proportional relation among the Elemental Timbers (calculated in units)

No.	Name of buildings	Front Building's Head tie-beam (FB-HeBw)	Front Building's Neck tie-beam (FB-NeBw)	Front Building's Lower tie-beam (FB-LoBw)	Main Building's Purlin diameter (Pud)	Algebraic Relation		
						Neck tie-beam and Head tie-beam (NeB)/(HeB)	Lower tie-beam and Head tie-beam (LoB)/(HeB)	Purlin diameter and Head tie-beam (Pud)/(HeB)
1	The To Mieu	0,51	0,52	0,50	0,50	1,02	0,98	0,98
2	Dien Tho Chinh Dien	0,43	0,40	0,42	0,42	0,93	0,98	0,98
3	Long An Dien	0,44	0,45	0,45	0,45	1,02	1,02	1,02
4	Thai Hoa Dien	0,66	0,66	0,65	0,65	1,00	0,98	0,98
5	Trieu To Mieu	0,48	0,50	0,50	0,50	1,04	1,04	1,04
6	Minh Thanh Dien	0,40	0,40	0,40	0,40	1,00	1,00	1,00
7	Sung An Dien	0,40	0,40	0,40	0,40	1,00	1,00	1,00
8	Bieu Duc Dien	0,44	0,45	0,45	0,45	1,02	1,02	1,02
9	Hoa Khiem Dien	0,45	0,45	0,45	0,45	1,00	1,00	1,00
10	Luong Khiem Dien	0,45	0,45	0,45	0,45	1,00	1,00	1,00

The height or width of the head tie beam (*HeB*) is found to range from 0.40 to 0.60 of *CoD*, corresponding to conventional proportional ratios of 2/5, 1/2, and 3/5. Similarly, the top ridge beam (*d*) exhibits proportional values between 0.49 and 0.60 of *CoD*, which approximates to 1/2 and 3/5.

The cross-sectional dimensions of the *NeB*, *LoB*, and *Pud* are observed to be equivalent to those of the *HeB*, indicating a consistent application of proportional logic across these interconnected structural elements. This

standardized dimensional system underscores the geometric coherence and rational design methodology characteristic of traditional Nguyen Dynasty timber architecture.

3.5.6. Converting the Arithmetic into the Fractional Relationship

The preceding analysis demonstrates that the dimensional relationships among architectural elements—whether derived from decimal, duodecimal, or

specialized traditional measurement systems—can be systematically translated into fractional ratios to facilitate interpretation and calculation. These fractional expressions, summarized in Table 8, represent commonly employed proportional conventions in traditional timber architecture and reflect a rationalized design framework rooted in empirical construction practices. This approach not only enhances computational accessibility but also underscores the mathematical coherence embedded in the architectural design methodology of the Nguyen Dynasty.

3.6. Assuming the Analysis Results

3.6.1. Relationship between Building Typology and Platform Configuration

Drawing upon the data presented in Tables 1–7, as well as the synthesis of historical documentation and empirical site investigations, several key assumptions can be formulated regarding the proportional organization and dimensional logic of the Twin-Ridge Beam Buildings:

- Firstly, the configuration of the platform (P) is governed primarily by the building typology (T) and its functional requirements, suggesting a typology-function-platform correlation.
- Secondly, the primary dimensional reference, namely the central compartment span (A) and/or the big beam

span of the main building (E), is established based on the length of traditional rulers (R) employed in Nguyen dynasty construction practices.

- Thirdly, a hierarchical derivation of spans is observed: The sub-compartment span (B) is derived from A , the internal wing span (C) is derived from B , and the external wing span (D) is in turn derived from C .
- Fourthly, additional spans—including the big beam span of the front building (F), front and rear diagonal beam spans (H), the drainage gutter span (G), and wing spans at the front and rear elevations (I)—are proportionally determined based on B , C , and D , respectively.

This system of derived proportions reflects a structured approach to determining both the formal composition and the scale of platforms across building types.

3.6.2. Proportional Relationship between Central Compartment Span (A) and Main Building Beam Span (E)

The central compartment span (A) serves as the foundational dimension defining the latitudinal structural module of the timber frame, while the big beam span of the main building (E) defines the corresponding longitudinal module (see Figure 8). The interaction between A and E governs the building's overall spatial framework.

Table 8. Summary table of calculation systems and conversion of analytical values

System	No.	Arithmetic	Faction	Compaction	Factional Relationships
Decimal	1	0,10	1/10		1 CoD = 1/10 A = E
	2	0,20	2/10	1/5	1 h1 = 1/5 h2
	3	0,30	3/10		1 h1 = 3/10 h2; 1 CoP = 3/10 E
	4	0,40	4/10	2/5	1 HeB = 2/5 CoD
	5	0,50	5/10	1/2	1 A or 1 E = 1/2 BuH; 1 HeB = 1/2 CoD; 1 d = 1/2 CoD
	6	0,60	6/10	3/5	1 HeB = 3/5 CoD; 1 d = 3/5 CoD
	7	0,70	7/10		1 TiB = 1 BiB = 1 DiB = 7/10 CoD
	8	0,80	8/10	4/5	1 BiB = 1 DiB = 4/5 CoD
	9	0,90	9/10		1 BiBw = 9/10 CoD
	10	10,0	10/10	1/1	1 BiBh = 1 CoD
Duodecimal	11	0,08	1/12		1 CoD = 1/12 A
	12	0,17	1/6		1 h1 = 1/6 h3
	13	0,25	1/4		1 CoP = 1/4 A; 1 CoP = 1/4 E
	14	0,33	1/3		1 CoP = 1/3 E
	15	0,66	2/3		1 TiB = 2/3 CoD
	16	0,75	3/4		1 TiB = 1 BiB = 1 DiB = 3/4 CoD
	17	0,83	5/6		1 BiB = 5/6 CoD
Special	18	0,05	1/20		1 CoH = 20 CoD
	19	0,09	1/11		1 CoD = 1/11 A

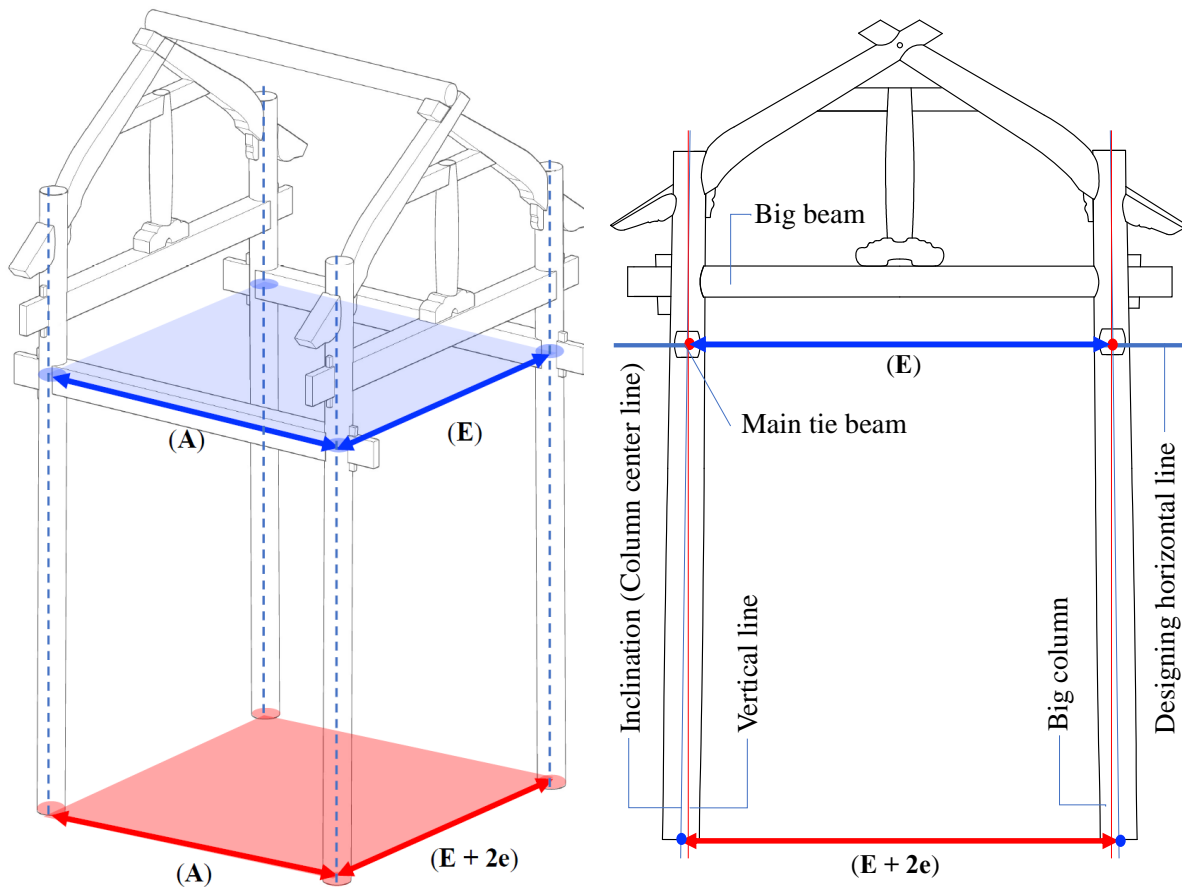


Figure 8. Relation between A and E, and Column inclination ($2e$)

The analysis identifies three principal proportional typologies in the relationship between A and E, which will be elaborated upon in the subsequent subsection. These cases are critical in clarifying the internal structural logic that underpins the architectural design methods of the Nguyen Dynasty. Based on the analysis of structural configurations, three typical relationships can be identified between the central compartment span (A) and the longitudinal big beam span of the main building (E), accounting for the column inclination ($2e$):

- a) $A = (E + 2e) > E$
- b) $A = E < (E + 2e)$
- c) $A > (E + 2e) > E$

In all three cases, the adjusted value ($E + 2e$), which incorporates the inclination of the large columns, consistently exceeds E, defined as the linear distance between the intersection points of the designed horizontal line and the central axis of the columns at the level of the main tie beam.

This indicates that the longitudinal structure, characterized by column inclination, prioritizes technical stability, while the latitudinal structure emphasizes visual regularity through the rhythmic arrangement of compartment spans and column rows.

Consequently, E can be regarded as a derivative

dimension based on A, where E is either increased or reduced by a range typically between 0.2 and 2 units, depending on the specific structural requirements—including the addition of $2e$ to accommodate column inclination. This adjustment forms the basis for a direct proportional relationship between the column diameter (CoD) and A, and/or an indirect proportional relationship with E (see Figures 8, 9, and 10).



Figure 9. Upper wooden structure where the big beam, main tie beams, diagonal beams, and the columns work together



Figure 10. Investigated connecting between big beam, diagonal beam and big column

The column inclination itself remains a largely unresolved technical phenomenon. No empirical studies to date have conclusively explained the structural rationale behind this inclination in terms of load distribution. However, from a mechanical perspective, the column—being the primary load-bearing element—must resist the vertical gravitational load of the roof and superimposed timber framework. A slight inclination may serve as a compensatory mechanism to introduce elastic potential energy, thus offsetting gravitational potential forces and enhancing structural resilience. Alternatively, this practice may reflect an accumulated legacy of empirical trial-and-error rather than a strictly theoretical principle.

Furthermore, as noted in Table 1, historical records place particular emphasis on the number of compartments and platform height as primary design parameters. These values provide the earliest documented quantitative evidence of proportional relationships, suggesting a close correlation between platform configuration and the vertical dimensions of the timber structure. This hypothesis will be empirically investigated in the subsequent section.

3.6.3. Proportional Relationship between the Platforms and the Wooden Structures

Table 8 synthesizes the analytical findings derived from the comparative measurements of architectural components. Two principal numerical systems are identified as governing the design logic of both the platform and the wooden structural framework: the decimal system and the duodecimal system. These systems serve as the foundational basis for proportional calculations within the architectural design of the Nguyen dynasty's Twin Buildings.

Specifically, the height of the lower roof (h_2) and the designed horizontal line (h_3) are expressed as fractional proportions relative to the height of the platform (h_1). The column diameter (CoD) is defined as a fractional ratio in relation to either the central compartment span (A) or the

big beam span of the main building (E), while the column height (CoH) is proportionally derived from its diameter (CoD).

Furthermore, the cross-sectional dimensions of the top ridge beam (d), the main tie beam (TiB), the big beam (BiB), and the diagonal beam (DiB) are all expressed as fractional values of the CoD. Similarly, the dimensions of the neck-tie beam (NeB), the lower tie beam (LoB), and the purlins (Pud) are determined proportionally in relation to the head tie beam (HeB).

In certain cases, the column perimeter (CoP) is shown to maintain a proportional relationship with either A or E, such as:

- $CoP = 3/10 E$, or
- $CoP = 1/4 A = 1/4 E$

Nevertheless, it is important to note that these proportional values serve primarily as referential estimates due to minor discrepancies resulting from the irrational remainder introduced by the mathematical constant π (approximated here as 0.14).

From a methodological perspective, the decimal system, comprising ten divisional options, proves effective for both arithmetic and fractional calculations. In contrast, the duodecimal system, offering seven common divisions, is better suited to fractional calculations, though it is less efficient for arithmetic due to the prevalence of odd divisors and resulting computational complexity.

3.6.4. Decimal and Duodecimal Systems in Construction Practice

The use of decimal and duodecimal systems in traditional architectural construction reflects a deep-rooted influence of ancient mathematical principles widely practiced across East and Southeast Asia. These numerical systems are closely associated with the Stems-and-Branches system (干支, *Ganzhi*), which comprises the Ten Heavenly Stems (十干) and the Twelve Earthly Branches (十二支). Together, they form the Sexagenary Cycle (六十干支) [16]—a sixty-term cyclic framework used extensively in the lunisolar calendar for calculating time units such as hours, days, months, and years, and for applications in traditional astrology, geomancy, and ritual timing.

It is widely held that the Sexagenary Cycle derives its conceptual foundation from the Twenty-Eight Mansions (二十八宿) [17], an ancient system of celestial mapping that played a pivotal role in timekeeping and cosmological observations in East Asian calendrical sciences. These systems not only governed temporal measurement but also informed cultural practices, including architectural planning and construction, where proportional design and symbolic alignment were paramount.

The Ten Heavenly Stems correspond to the decimal system, which includes divisors such as 2 and 5, while the Twelve Earthly Branches align with the duodecimal system, incorporating divisors such as 2, 3, 4, and 6. From

This evidence suggests a codified design logic rooted in traditional numerological principles. Specifically, buildings categorized as “10-unit Twin Buildings” conform to the One Ten system, while those identified as “12-unit Twin Buildings” follow the One Dozen system. These patterns likely reflect the influence of ancient mathematical doctrines and possibly the philosophical or cosmological frameworks derived from astrology and metaphysical traditions.

3.7. Proposed Framework for the Architectural Design Formula

3.7.1. Variables

In the context of this study, a variable is understood as a quantity that may assume multiple values within a defined range, representing different yet equivalent meanings under specific constraints. Among these, the Building Type (T) functions as an independent qualitative variable, classified according to architectural categories such as *Palace*, *Shrine*, and *Temple*. The variable T serves as a primary determinant, guiding the selection of the Ruler (R)—a quantitative variable that defines the fundamental unit of measurement used in building design and construction.

Historically, the Nguyen Dynasty employed two standardized rulers: the Large Ruler, measuring 426 mm, and the Small Ruler, measuring 382 mm [18]. The choice between these rulers (R) is binary and contingent on the determined building type (T), making R a quantitative binary variable, while T is classified as a qualitative nominal variable, with no inherent ordinal relationship among its categorical values. Conversely, the designation of R may reciprocally inform the typology T, thereby establishing a bidirectional interdependence between the two variables. This interplay underscores the integral role of traditional measurement systems in shaping architectural typologies.

3.7.2. Constants

In mathematical terms, a constant refers to a quantity that remains invariant within a given system. Within the traditional architectural design framework of the Nguyen Dynasty, certain elements are established as constants to ensure proportional integrity and typological coherence. One such set of constants is the platform configuration (P), typified by standardized arrangements such as the 5-3-2, 7-5-2, 9-7-2, 11-9-2, and 15-13-2 formats. Each configuration corresponds directly to specific building types (T) as discussed previously in Section 3.1, thereby establishing an immutable foundation for the superstructure.

Additionally, the heights of the lower roof (h_2) and the designed horizontal line (h_3) are fixed in relation to the

height of the platform (h_1), rendering them constants within the vertical dimensioning of the wooden structure. These constants play a foundational role in determining the overall elevation and must remain unaltered once the platform configuration is established.

3.7.3. Parameters

A parameter, while similar in form to a variable, represents a value that governs the behavior of a function or system, allowing for controlled variability within defined limits. In this study, parameters are secondary quantitative variables that define the proportional relationships among key structural components.

Two primary parameters are identified:

- Parameter 1: The span of the central compartment (A),
- Parameter 2: The diameter of the main column (CoD).

These parameters are mutually dependent and function as subordinate quantities to the constant platform configuration (P). From these parameters, critical structural dimensions can be derived, including the total building height (BuH) and column height (CoH). Furthermore, a hierarchical set of proportional relationships emerges, whereby CoD governs the cross-sectional dimensions of major structural timbers—namely the main tie beam (TiB), big beam (BiB), diagonal beam (DiB), head tie beam (HeB), and top ridge beam (d). A subsequent set of dependent proportions can then be determined from HeB to secondary elements, such as the neck tie beam (NeB), lower tie beam (LoB), and purlins (Pud).

3.7.4. Dependent Quantities

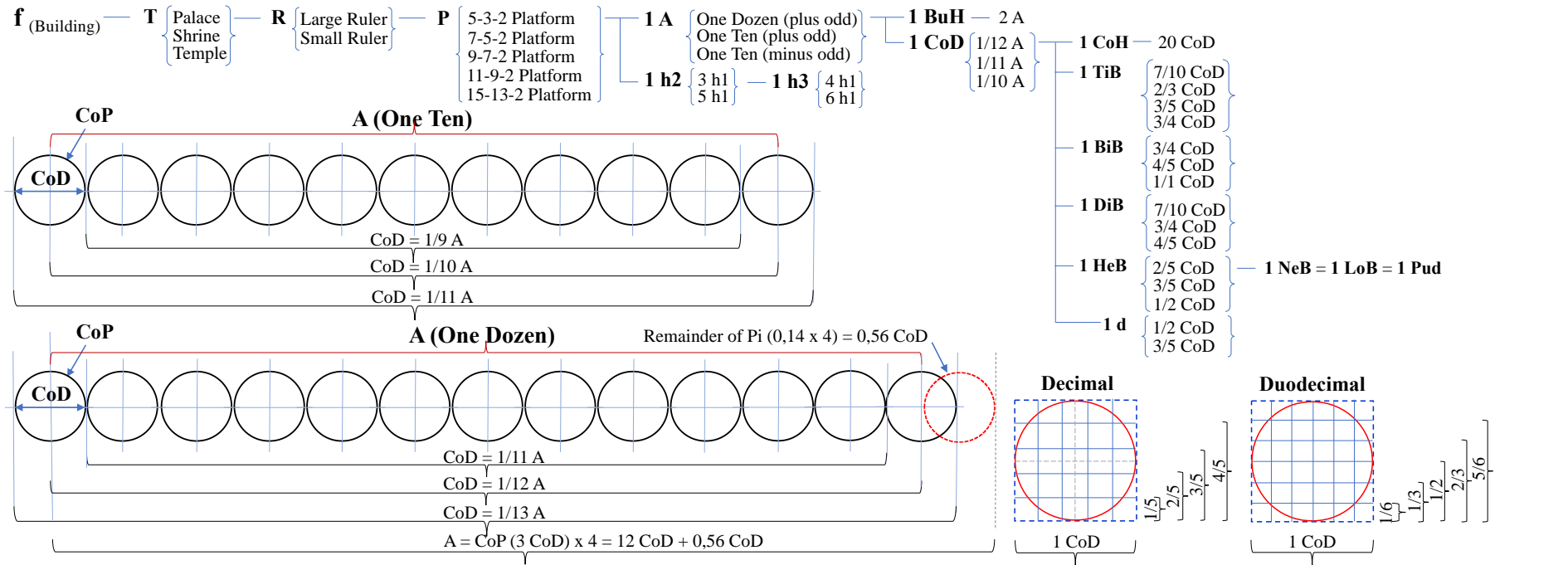
The dependent quantities in this formulaic system are those determined through the interactions among constants and parameters. As illustrated in Table 10 and Figure 12, these quantities include:

- BuH: Dependent on Parameter A,
- CoH, TiB, BiB, DiB, HeB, d: Dependent on Parameter CoD,
- NeB, LoB, Pud: Dependent on HeB.

If Parameter 1 (A) undergoes modification, this change alters its proportional relationship with CoD, thereby triggering a chain reaction that affects the overall structural height and dimensions of all associated timber elements. This dynamic may explain the slight adjustments (ranging from 0.1 to 1.0 unit) observed in empirical data, even when A is set according to canonical units such as *One Ten* or *One Dozen*. Such variations likely reflect adaptations made by traditional carpenters to accommodate the actual dimensions of available timber stock—highlighting a practical synthesis of standardization and flexibility in historical architectural design.

Table 10. Formulated Design Principles

Variables		Constants			Parameters			Dependent Quantities									
Building Types	Rulers	Platforms	Height of Lower Roof	Design Horizontal Line	Center Compartment		Big Column Diameter	Total Height of Building	Big Column Height	Section of Main Timbers (Relate to CoD)					Section of Elemental Timbers (Relate to HeB)		
					(A)	Plus/Minus				(CoD)	(BuH)	(CoH)	(TiB)	(BiB)	(DiB)	(HeB)	(d)
Palace	Large Ruler (426 mm)	7-5-2 Platform 9-7-2 Platform	3 h1 5 h1	4 h1 6 h1	One Dozen One Ten	+ 1 + 0,3 + 0,5 + 0,8	1/12 A	2 A	20 CoD	7/10 CoD 3/4 CoD	4/5 CoD	3/4 CoD 4/5 CoD	1/2 CoD 3/5 CoD	1/2 CoD 3/5 CoD	1 HeB	1 HeB	1 HeB
Shrine	Large Ruler (426 mm)	5-3-2 Platform 11-9-2 Platform 15-13-2 Platform	3 h1	4 h1	One Ten	+ 0,4 + 0,7	1/11 A	2 A	20 CoD	3/5 CoD	3/4 CoD 4/5 CoD	7/10 CoD 4/5 CoD	2/5 CoD 1/2 CoD	3/5 CoD	1 HeB	1 HeB	1 HeB
Temple	Small Ruler (382 mm)	5-3-2 Platform 7-5-2 Platform	5 h1	6 h1	One Ten	± 0 1 - 0,1	1/11 A 1/10 A	2 A	20 CoD	7/10 CoD 2/3 CoD 3/4 CoD	4/5 CoD 1 CoD	7/10 CoD 3/4 CoD	1/2 CoD	1/2 CoD 3/5 CoD	1 HeB	1 HeB	1 HeB



Note: Building Type (T); Ruler (R); Platform (P); Height of lower roof (h2); Design horizontal line (h3); Total height of buildings (BuH); Central compartment (A); Big column diameter (CoD); Big column perimeter (CoP); Big column height (CoH); Box-cross section of Tie-beam (TiB), Big-beam (BiB), Diagonal beam (DiB), Head tie-beam (HeB), Neck-tie beam (NeB), Lower-tie beam (LoB), Top-Ride beam diameter (d), Purlin diameter (Pud); $CoP = CoD \times \pi (3,14)$.

Figure 12. Diagramming of the Formulated Design Principles Corresponding to Decimal and Duodecimal System

3.8. The Comparison between the Design Formula and the Design Methods of Vernacular Architecture

When comparing the design formula of Hue's palaces with the vernacular architectural design methods in Northern and Central Vietnam [19], it becomes evident that this design formula represents a higher level of sophistication, as it has been systematically formulated into a mathematical function with a hierarchical structure, encompassing all the essential components of a functional form, including Variables, Constants, Parameters, and Dependent Quantities. In contrast, the vernacular architectural design methods primarily relied on the traditional measuring unit (1 unit equal about 424 mm), design tools (with two basic forms: the square ruler and the equilateral triangle ruler), and human body dimensions. This indicates an evolution in mathematical thinking and architectural techniques from vernacular architecture to the Hue imperial palaces.

4. Applicability and Significance of Employed Design Formula

4.1. Unity in Architectural Pattern and Form

As previously mentioned [2], [3], [8], the Twin Buildings represent the highest classification within the architectural hierarchy of the Nguyen Dynasty. While these structures may differ in location, scale, and function, they consistently exhibit a unified architectural pattern and form. This uniformity, although potentially leading to aesthetic monotony, simultaneously reinforces distinctions in symbolic rank and intended function, thereby elevating their significance relative to lower-tier constructions. Such homogeneity in spatial organization and formal expression reflects a deliberate application of a standardized design formula, which served as a core mechanism in achieving architectural cohesion across the heritage complex.

Despite the repetition inherent in the Twin Building typology, their placement in spatially and symbolically prominent locations—such as central ceremonial axes or religious precincts—amplifies their visual and cultural impact. It is important to note, however, that differences in roofing materials and decorative elements (both interior and exterior) remain notable markers of differentiation. Nevertheless, these features fall outside the scope of the present study, as they introduce extrinsic variables that do not directly pertain to the analysis of architectural proportions.

4.2. Flexibility in Adapting Functional Usage and Spatial Scale

The variables P (Platform Pattern) and T (Building Type) incorporate both qualitative and quantitative dimensions within the architectural design framework. Functionally, T captures the intended use (e.g., Palace, Shrine, or Temple), while P reflects its scale, determined by the number of compartments and platform dimensions. Notably, buildings classified as Palaces or Shrines are designed using the Large Ruler, while Temples utilize the Small Ruler, thereby embedding scale variability into the design process through unit selection.

This inherent flexibility is exemplified by comparing the Thai Hoa Dien Palace (a civil imperial building) and the Hoa Khiem Dien Temple (a worship structure). Both buildings share an identical architectural configuration, yet differ markedly in physical scale due to the application of different ruler lengths (Figure 13). The Palace, designed using the Large Ruler, possesses a significantly greater span and volume than the Temple, designed with the Small Ruler.

This distinction highlights the design formula's adaptability to functional requirements and site-specific contexts. By modifying the input variables—while preserving the underlying structural and spatial schema—the formula effectively maintains stylistic consistency while accommodating diverse scales and usages. This dual use of qualitative and quantitative variables demonstrates a sophisticated and context-sensitive approach to architectural standardization within the imperial design practice.

4.3. Flexibility in Parameter Adjustment for Efficient Timber Determination

Once the constants within the design formula are defined—typically as irreducible fractional values with fixed denominators—they serve as foundational references for determining all subsequent proportions. The numerators, acting as adjustable parameters, enable precise calibration of component dimensions. While the central compartment span A is assigned one of two primary values—One Ten or One Dozen—these values invariably include an odd number, allowing for nuanced variation.

As illustrated in Table 10 and Figure 12, the simultaneous adjustment of Parameter 1 (A) and Parameter 2 (CoD) directly influences the dimensional characteristics of the column, which in turn governs the sizing of associated timber elements. These adjustments preserve the internal proportional relationships between elements and are informed by empirical knowledge accumulated through traditional trial-and-error practices. The system thereby facilitates efficient timber specification while maintaining structural integrity and visual coherence across diverse architectural contexts.

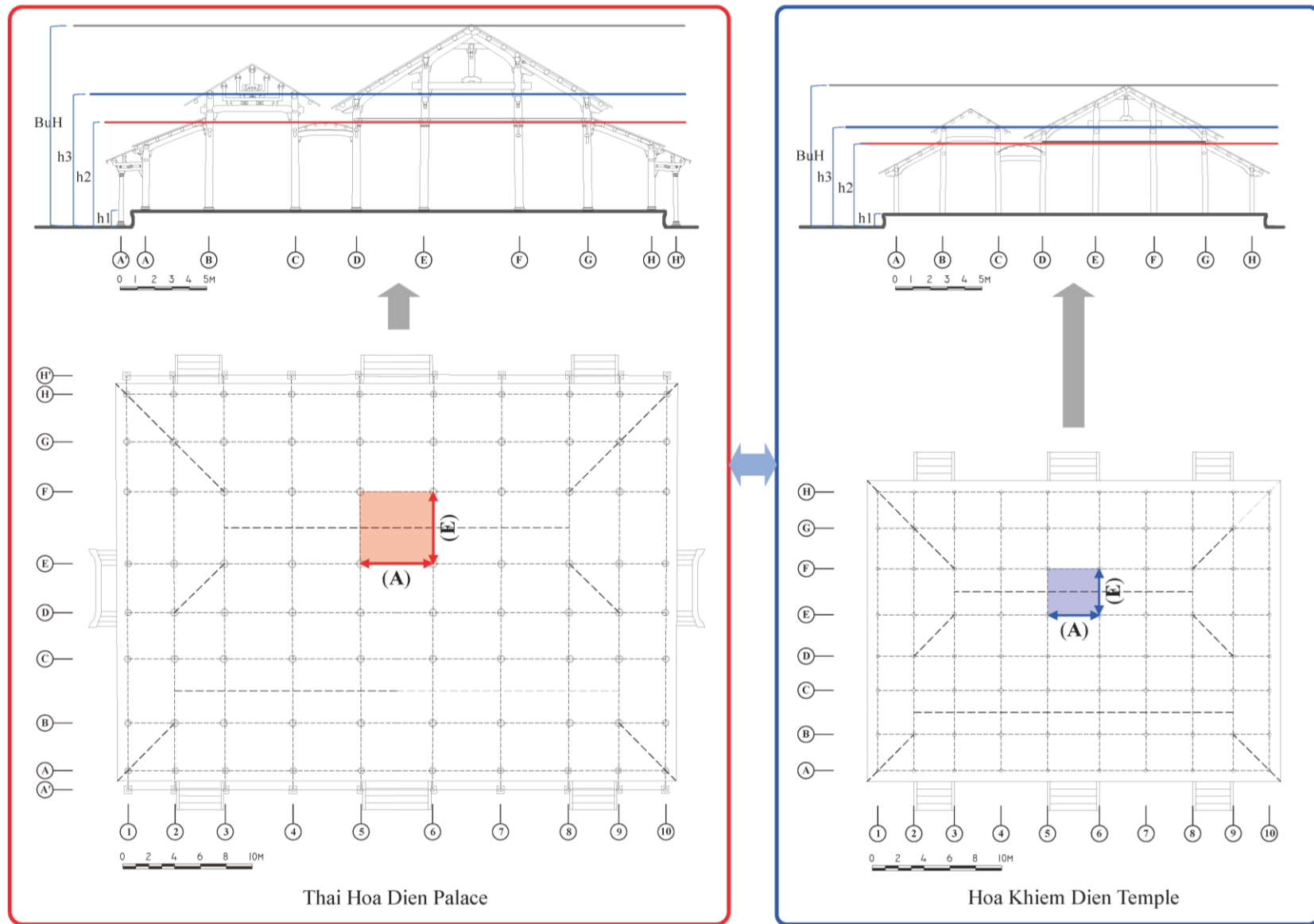


Figure 13. Scale comparison between Thai Hoa Dien Palace (located in the Imperial City) and Hoa Khiem Dien Temple (located in the Mausoleum of Emperor Tu Duc)

Adjustments to the value of A—the central compartment span—directly influence the CoD (column diameter), and consequently, all timber dimensions that are proportionally related to CoD. There are three principal scenarios regarding the value of A: (i) A remains fixed (e.g., assigned as One Ten or One Dozen), (ii) A is increased, and (iii) A is decreased. In the first case, when A is held constant, the proportional relationship between A and CoD is considered ideal. In the second scenario, an increase in A—as commonly observed in larger structures such as Palaces and Shrines—necessitates a decrease in CoD to maintain structural harmony. Conversely, a reduction in A—typically found in smaller structures such as Temples—results in a proportional increase in CoD.

This adjustment process is inherently linked to the availability and physical limitations of natural timber materials. Ideally, CoD is equal to 1/10 of A, a proportion that represents the optimal structural condition. In contrast, when CoD reaches 1/12 of A, the system approaches a critical load-bearing threshold, particularly for box-shaped cross-sections of the main tie beam (TiB) and the big beam (BiB) in relation to A. This threshold signifies the practical limit of structural efficiency in traditional construction techniques.

Thus, the decision to increase or decrease A is primarily determined by the natural size of the available timber stock and the associated structural requirements. Once the core variables—T (Building Type), R (Ruler Type), and A—are established, spatial expansion can be achieved by augmenting the number of sub-compartment spans (B) in the latitudinal direction. This is done by selecting an appropriate platform configuration from the established typologies (e.g., 5-3-2, 7-5-2, etc.), thereby enabling internal spatial modulation without altering the overall proportional logic of the design system.

4.4. Convenience in Timber Preparation and Cost Estimation

Once the platform pattern (P) is established, the central compartment span (A) becomes a dependent parameter, from which both the column diameter (CoD) and column height (CoH) are derived. For traditional timber buildings, the early determination of timber volume and specifications is a critical step in the planning and construction process. The predefined platform pattern serves as the basis for calculating the quantity and dimensions of timber required for the entire wooden structural system.

To ensure appropriate seasoning and structural performance, the timbers must be processed and stored well in advance, particularly to achieve optimal moisture content. Should a specific piece of timber prove inadequate for use as a column due to substandard dimensions or physical properties, it can be reassigned to secondary structural components—such as the main tie beam (TiB), head tie beam (HeB), or other elements with smaller

cross-sectional requirements—by applying either the decimal or duodecimal proportional systems with reference to CoD.

Importantly, cost estimation for such buildings is primarily governed by the total volume of timber required, which directly influences material procurement and labor expenditures. The formulaic approach thus supports both efficient resource planning and budget forecasting, contributing to the economic feasibility of conservation and reconstruction projects.

4.5. Replicability and Rehabilitation Potential

The adoption of a systematic design formula significantly enhances the replicability and rehabilitation capacity of traditional wooden heritage architecture. Once codified, this formula facilitates the rapid and accurate reproduction of architectural components across multiple structures, while also enabling partial replacements without necessitating full dismantlement of the building. In cases of localized structural failure, individual components can be restored in situ based on standardized dimensions and joinery logic.

Moreover, in the event of a total collapse, the remaining platform—as a foundational element encoded with dimensional and spatial information—serves as a reliable basis for reconstructing the original structure. This approach ensures both architectural fidelity and historical authenticity as mentioned in the international conventions [20], [21], [22]. Over time, the deterioration or loss of wooden heritage buildings is inevitable; however, the design formula provides a scientific and replicable framework that safeguards the intangible cultural values embedded in these architectural works.

In this sense, the formula functions as a form of architectural "Heritage DNA" [23], allowing for the informed recovery and transmission of structural knowledge across generations. It offers a methodological foundation not only for academic research but also for practical conservation efforts, ensuring that restoration and reconstruction processes remain faithful to the original design intent and material logic of the Nguyen Dynasty's wooden architecture.

5. Conclusions

The *Twin Buildings* have been constituted the highest tier of architectural typology within the Complex of Hue Monuments, serving as Palaces, Shrines, and Temples. Despite their consistent architectural patterns and forms, these structures are distinguished by their scale and function through the application of traditional measurement systems—specifically, the Large Ruler and Small Ruler—as well as the use of ancient mathematical principles in their design. Two key computational frameworks were identified: the decimal system

(corresponding to “One Ten”) and the duodecimal system (corresponding to “One Dozen”), each applied according to the function and symbolic hierarchy of the buildings.

Central to this architectural system is the design formula, which incorporates a coherent structure of variables, constants, parameters, and dependent quantities. This formula facilitates both the unity and adaptability of the built environment. One of its most significant outcomes is the ability to maintain uniform architectural patterns and forms, thereby reinforcing a sense of visual harmony and cultural identity across imperial structures. Simultaneously, the formula allows for flexibility in both function and scale, ensuring that adjustments can be made without compromising either structural coherence or aesthetic integrity.

The use of well-defined proportional parameters enables efficient planning and precise timber preparation, optimizing the use of available materials while reducing waste and labor. Furthermore, the replicable nature of the design formula provides a valuable foundation for rehabilitation and restoration, offering reliable guidelines for reconstructing buildings based on original proportions and modular logic as the same case of the Daigokuden Imperial Audience Hall in the Nara Palace Site, and the case of the relocated Yakushiji Pagoda (718) in Nara, the reconstruction of the West Pagoda (in 1980) based on its source of genetics referenced the design of the East Pagoda (the original one dates back to the 8th century). This capability is particularly significant for the sustainable management of cultural heritage, where authentic reconstruction must be balanced with technical feasibility.

From a broader perspective, the Nguyen Dynasty's architectural design system reflects an ingenious integration of traditional mathematical knowledge into construction practices. Far from being merely empirical or intuitive, this approach represents a sophisticated architectural logic that addresses both aesthetic and structural requirements. The design formula not only responds to complex technical challenges but also ensures the long-term resilience and sustainability of heritage structures. Its underlying philosophical and mathematical foundations demonstrate the depth of indigenous Vietnamese architectural thought.

While the Imperial Palaces may not feature radical innovation in decorative elements or stylistic experimentation, their harmonious beauty lies in their uniformity and formal rigor, which together underscore their symbolic and architectural significance. From a technological standpoint, the Nguyen Dynasty's systematic use of design formulas constitutes a remarkable intellectual and technical contribution to world heritage, highlighting an early form of standardized architectural practice.

In sum, the architectural principles underpinning the Twin Buildings—anchored in balance, regularity, adaptability, and mathematical reasoning—represent a

timeless design philosophy. They continue to offer practical solutions for contemporary heritage conservation, affirming their relevance in both academic discourse and applied restoration. Preserving and disseminating this knowledge is not only vital for safeguarding Vietnam's cultural legacy but also contributes to the global understanding of sustainable architectural traditions.

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