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Study on Stability for Compressive Elastic-Fixed Strip

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Abstract Studies on the influence of various types of imperfections in elastic systems in the case of several external forces were analyzed. The analysis showed that in most studies, a loading function was introduced at a certain stage of research, and the loads ceased to be independent. The two most frequently used static stability criteria in the mechanics of deformable solids are the bifurcation criterion and criterion of the method of initial imperfections. It is shown that these are methods for finding singular points of the implicit function theorem (only those pairs of spaces with corresponding norms in which the derivative of the Fréchet map is an isomorphism are considered). Using this result, we analyzed the continuous dependence of the function describing the state of the compressed elastic strip on the characteristics of the initial imperfections. A condition is obtained that is imposed on the external influence parameter and base stiffness coefficient; if violated, the cross-sectional shape of the strip will no longer be close to a rectangle; that is, the strip loses its shape. Moreover, these parameters remained independent throughout the study. The new results were compared with known classical results.

Keywords Structural Nonlinearity, Bifurcation Analysis, Imperfection Approach, Boundary Condition, Finite Element Analysis, Stress-Strain State, Compressive Elastic- Fixed Strip

1. Introduction

In many works on both engineering and theoretical features, it has been noted that the idealized structure designed by an engineer differs from the one that is made according to the project. This difference was due to numerous minor deviations, defects, and imperfections. Despite these deviations, engineers must be confident that the actual design will perform approximately according to the corresponding idealized design. When operating a physical design, the equilibrium of an idealized solution should be stable with respect to perturbations that distinguish it from the physical solution. In [1], it was highlighted that any research using an idealized solution makes sense only when operating a physical object, and its changes in behavior compared to that of the idealized design depend on characteristics such as imperfections and defects. One of the conditions for the correctness of the formulation of the problem according to Hadamard is the continuous dependence of the solution on the parameters [2-4].

Many works [1-4] note that “stability is a heavily overloaded term with an unclear definition”. The variety of formulations, criteria, and methods for solving stability problems requires a more detailed analysis. There are many research works on the formulated problem, carried out based on various stability criteria [1, 2, 4-12].

In [2], the authors considered the stability criteria for elastic bodies under uniform compression. In addition, the study [5] contains problems with the theory of stability and the theory of oscillations, as well as “instructive mistakes” that were made in solving them. The first part of the book deals with issues of stability of elastic and elastoplastic systems, and the second part deals with issues of vibrations (systems with a non-integer number of degrees of freedom, the destabilizing effect of viscous friction forces, vibrations of pipelines with flowing liquid, the action of a moving load, aeroelastic vibrations, and nonlinear vibrations of some special systems).

The paper [6] attempts to give a broad overview of the vast field of structural stability, including elastic and inelastic structures, static and dynamic response, linear and nonlinear behavior, energy approach, thermodynamic aspects, creep resistance, and instability due to failure or damage. The importance of stability theory for various fields of engineering and applied science is indicated, and the history of this discipline is briefly outlined. The main achievements are briefly reviewed, and recent trends are emphasized, in particular, the analysis of damage localization stability and fractures. A review of the main classical approaches for studying stability in various fields of technology and applied science is provided in [7-9]. Key achievements are briefly reviewed and important recent trends are highlighted.

It is noted that the results obtained in [10] considered the influence of imperfections on the stability of elastic systems, discrete or continuous, with almost simultaneous regimes. After a general analysis of the stability of arbitrary elastic systems with almost simultaneous bifurcation eigenmodes in the presence of imperfections, conditions were provided to determine the worst form of imperfection that minimizes the first local maximum of the load.

In [11], the quantitative influence of the nonlinearity of the material on the theory of Euler-Bernoulli beams was investigated. It was established that the governing equation for the deviation is a nonlinear integral-differential equation, and the equations are solved numerically using a variant of the spectral collocation method. The deflection and spatial stress distribution in the beam were calculated for two sets of models, namely, the standard linearized model and some recent nonlinear models used in the literature to match the experimental data. The main objects studied in the elastic region were rods, rod systems, plates, and shells. Therefore, it is important to develop research methods for an arbitrary elastic body.

Studies of elastic systems under combined loading were carried out in [2, 12-20]. In general, for a system with n degrees of freedom, the stability problem under combined loading was first solved by Papkovich [13]. It was proven that in the space of loads, the boundary line for the stability zone cannot be convex to the stability region. However, in [13] it was shown that if the subcritical stress-strain state of an elastic system is determined by nonlinear theory, then the boundary of the region will not necessarily be convex. In [14,15], a method for assessing the stability of thin-walled shell structures and elastic cylindrical shell under various load types was proposed. As a criterion for the stability of the ground state of a conservative system, the minimum value of the potential energy of the system was selected in relation to the energy value of all adjacent states of the system. Known analytical solutions to linearized problems of the stability of rectangular hinged plates under combined loading in [16] are generalized to similar problems for oblique plates, the mechanics of deformation of which are described by equations of classical theory formed in oblique Cartesian coordinates.

In [17], a numerical method was developed to study the stability of a supported cylindrical shell by considering geometric imperfections in the form of deformation under operational loading. The problem of the nonlinear stability of an imperfectly supported cylindrical shell under combined loading was solved.

In [18], an approach using a single disturbance load was reproduced and modified. The simulation with three disturbance loads showed a reduced (i.e., more critical) buckling value compared with the single disturbance load approach. Global and local dynamic perturbation approaches exhibit behavior that is suitable for estimating the lower bounds of structures with arbitrary geometries. A specific model for the global perturbation approach was developed, and a new procedure was proposed based on this model.

The works [19-20] consider the methodology of numerical analysis of the stress-strain state of structures subjected to combined loading, using the finite element method (FEM). The main aspects of calculating structures under various types of loads are analyzed.

An experimental study of the stability of steel plates under combined loading using the approach proposed by Brown was presented in [21]. The combination of shear stress loading (partial load) and bending stress was discussed. A modification of the interaction equation was proposed.

As an analysis of the literature has shown that, in most studies on the influence of various types of imperfections and inhomogeneities, a loading function is introduced at a certain stage of the research, and the loads cease to be independent.

Improving methods that do not require the introduction of a loading function and studying the continuous dependence of the solution describing the equilibrium state of an arbitrary elastic body on the initial data remains an urgent task.

In the study, it is considered the two most commonly used static stability criteria and implicit function theorem.

2. Bifurcation Analysis

It is assumed that the behavior of a deformable body under static conditions is described by the following equation:

$$H_1(u, \lambda, P)=0 \quad (1)$$

where *u* is the characteristic of the body's stress-strain state, λ characterizes the body's geometry and physical properties, P is the external force or "parameter of several forces". We assume that u_0 is solution of Equation (1) when $\lambda=\lambda_0$, $P=P_0$, i.e. $H_1(u_0, \lambda_0, P_0)\equiv 0$.

In the static approach to solving the stability problem, the conditions under which new equilibrium forms arise along with the initial one are investigated. As a result, a critical point is determined, corresponding to the branching point (bifurcation) of equilibrium states. When conducting stability studies of a deformable body corresponding to u_0 the following equation: $H_1(u_0+u', \lambda_0, P_0)=0$ and the corresponding linear problem for u' are used.

$$L(u', \lambda_0, P_0)=0 \quad (2)$$

The smallest value of P_0 for which Equation (2) admits a nontrivial solution is called critical.

In this case, some ideal system is considered, for example, the cross-section shape is canonical, the material is homogeneous, etc. In real structures, this is not the case. Near critical points, the influence of imperfections increases. This fact served as the basis for the criterion of initial imperfections (another stability criterion for an ideal system).

2.1. The Criterion of Initial Imperfections

This criterion is called imperfection approach as mentioned in [2, 4, 23]

This method consists of creating a mathematical model of a deformable body taking into account imperfections, i.e.

$$H_1(u, \lambda_0+\lambda_1, P)=0 \quad (3)$$

A problem solution is considered for Eq. (3):

$$u=F(\lambda_0+\lambda_1,P) \quad (4)$$

and the behavior of *F* and $\frac{\partial F}{\partial P}$ is analyzed for $\lambda_1\to 0$. If $\lim_{\lambda_1\to 0} F(\lambda_0+\lambda_1, P^*)=\infty$ or $\lim_{\lambda_1\to 0}\frac{\partial F(\lambda_0+\lambda_1,P^*)}{\partial P}=\infty$, then the value of $P=P^*$ is called critical.

The value of P^* may depend on the tendency $\lambda_1\to 0$. For example, in [4], when conducting studies with the initial imperfections approach to the rod stability by the longitudinal bending various results were obtained for function λ_1 tending to zero in different ways.

It should also be noted that in [4, 23] studies were carried out for $\lambda_1\to +0$. In this care, the results were obtained that for the equations under consideration.

$\lim_{\lambda_1\to -0} F(\lambda_0+\lambda_1, P^*)=-\infty$ and $\lim_{\lambda_1\to -0}\frac{\partial F(\lambda_0+\lambda_1,P^*)}{\partial P}=-\infty$. So, it is obvious that the point (λ_0, P^*) is a bifurcation one. Thus, the initial imperfections approach is a methodology that can be used to study the surface behavior (4) in the neighborhood and to show that at the point (λ_0, P^*), not only bifurcation occurs but also $\lim_{\lambda_1\to 0}\frac{\partial F}{\partial P}=\infty$.

For a conservative system, all these criteria give the same result.

2.2. The Implicit Function Theorem

The mathematical model of the object under study can be presented as follows:

$$H(u, \lambda, P_1\dots P_n)=0 \quad (5)$$

Where *u* is the characteristic describing the object behaviour under study, λ is the characteristic describing the object, P_i is environment.

Assume that for Eq. (5), the solution is u_0 for $\lambda=\lambda_0;\ P_i=P_{i0}\ (i=1,2,\dots,n)$, then for the auxiliary variable ζ we have:

$$H(u_0+\zeta, \lambda_0, P_{10}\dots P_{n0})=0 \quad (6)$$

If we linearize Eq. (6) using ζ, we obtain the following linear equation:

$$L(\zeta, u_0, \lambda_0, P_{10}\dots P_{n0})=0 \quad (7)$$

We introduce some limits for the spaces under consideration. Let H in Eq. (5) be a differential operator. We assume that H: Y→Z, where Y is a subset of the Banach space U (U, Z are Banach spaces), is a nonlinear Fredholm map of index zero, and conditions [22] are simultaneously fulfilled:

a) $U\subset Z\subset H$ triple of continuously embedded spaces (*H* is Hilbert space).
b) U is dense in H.

As a rule, when solving static problems of deformable solid mechanics, they consider only the panirs of U, Z spaces with corresponding norms, where the Frechet derivative of the mapping H is an isomorphism. Then the implicit functions theorem can be formulated as follows [23]: Let consider that:

1) On some neighborhood of point $(u_0, \lambda_0, P_{10}\dots P_{n0})$ the derivatives:

$$\frac{\partial H}{\partial\lambda}, \frac{\partial H}{\partial P_1},\dots,\frac{\partial H}{\partial P_n} \quad (8)$$

are defined and limited;

2) Equation (7) admits only a trivial solution.

Then, there is a neighborhood of point $(\lambda_0, P_{10}\dots P_{n0})$ with an unambiguous dependence $u=F(\lambda, P_1,\dots,P_n)$ so that in this neighborhood

+ $H(u, \lambda, P_1 \ldots P_n)=0$

+ $F(\lambda, P_1,\ldots,P_n)$ – is unambiguous and continuous (9)

+ $F, \dfrac{\partial F}{\partial \lambda}, \dfrac{\partial F}{\partial P_1},\ldots,\dfrac{\partial F}{\partial P_n}$ – is continuous and limited (10)

In problems of deformable solid mechanics, condition (8) of the implicit function theorem is usually satisfied. Then, the condition of existence of only a trivial solution to problem (7) is necessary and sufficient for the point $(\lambda_0, P_{10} \ldots P_{no})$ under consideration not to be singular.

Since for n = 1, Eq. (5) exactly coincides with Eq. (1), the following conclusions can be drawn from the above:

Bifurcation analysis searches for a singular point at which condition (9) for the problem solution under consideration isn't satisfied.

The initial imperfections criterion is used to define a specific point at which condition (10) for the problem solution under consideration isn't satisfied.

Since, for n=1, Eqs. (7) and (2) coincide up to the symbols, then, according to the implicit function theorem, the existence of only a trivial solution to (2) is equivalent to the fulfillment of conditions (9) and (10) that coincide with the criteria requirements of bifurcation analysis and the initial imperfections approach.

Thus, determining the conditions for the implicit functions theorem not to be satisfied, the results are obtained equal to those when applying the stability criteria mentioned above.

If the elastic system is under the action of a combined load, then at a certain stage of the research based on static criteria, the loading parameter is set. If the research is based on the energy criterion, then to determine the critical values in this case, these values are found under separate action. Then, approximate critical values of the efforts are obtained based on the principle of minimum potential energy.

When conducting studies based on the implicit function theorem, in contrast to the well-known classical stability criteria given, the load remains independent throughout the entire study.

Using these considerations, we are going to study the state of a compressed elastic strip.

3. The Study of the Stress-Strain State of a Compressed Elastic Strip

3.1. Problem Statement

In this study, let us consider the deformation of an elastic strip whose cross-section is close to a rectangle. The forces on the lateral sides of the cross section are reduced to the main vector equal *2ph* modulus and a zero principal moment applied in the middle of the lateral sides. The strip is elastically supported with modulus of foundation k. The single-parameter Winkler model is chosen as foundation model.

Let the strip edges before deformation be characterized by functions:

$$y = \pm h \pm f(x) \qquad x \in [-\ell; \ell]$$
$$x = \pm \ell$$

and after deformation: $y=g_i(x)$; $x=q_i(y)$ $(i=1,2)$

In case of plane deformation, the stress-strain state of the strip can be described as a solution to the following Equation [25]:

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau}{\partial y}=0; \qquad \frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau}{\partial x}=0;$$
$$\sigma_x=\lambda\theta+2\mu\frac{\partial u}{\partial x}; \quad \sigma_y=\lambda\theta+2\mu\frac{\partial v}{\partial y}; \tag{11}$$
$$\tau=\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right); \quad \theta=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}.$$

We formulate the boundary conditions as follows:

$$P_\tau\Big|_{y=g_i}=0;\ P_n\Big|_{y=g_i}=k(g_i(x)-y_i(x))\ (i=1,2) \tag{12}$$

$$\begin{gathered}\int_{\eta_1}^{\eta_2} P_x dy=\int_{\eta_3}^{\eta_4} P_x dy=-2ph;\\ \int_{\eta_1}^{\eta_2} P_x dy=\int_{\eta_3}^{\eta_4} P_x dy=\int_{\xi_1}^{\xi_2} v dy=\int_{\xi_3}^{\xi_4} v dy=0\end{gathered} \tag{13}$$

In which,

$$\begin{gathered}\xi_1=-h-f(1);\ \xi_2=h+f(1);\xi_3=-h-f(-1);\ \xi_4=h+f(-1)\\ \eta_1=-h-f(1)+v(1,\xi_1);\eta_2=h+f(1)+v(1,\xi_2);\\ \eta_3=-h-f(-1)+v(-1,\xi_3);\eta_4=h+f(-1)+v(-1,\xi_4)\\ y_1=h+f(x);\ y_2=-h-f(x)\end{gathered} \tag{14}$$

For $f(x)\equiv 0$, the conditions (11)-(14) admits the solution

$$\begin{gathered}v=v^{(0)}=\varepsilon_y^0 y; \quad u=u^{(0)}=-\frac{kh+2\mu+\lambda}{\lambda}\varepsilon_y^0 x\\ \sigma_x=\sigma_x^{(0)}=-\frac{\lambda^2-(\lambda+2\mu+kh)(2\mu+\lambda)}{\lambda}\varepsilon_y^0\\ \sigma_y=\sigma_y^{(0)}=-kh\varepsilon_y^0; \quad \tau=\tau^{(0)}=0\end{gathered} \tag{15}$$

In which,

$$\varepsilon_y^0=-\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\lambda}{\lambda^2-(\lambda+2\mu+kh)(2\mu+\lambda)}p}. \tag{16}$$

For $f(x)<<h$ the cross-section shape of the strip in the unstressed state is close to rectangular. If the problem solution (11)-(14) continuously depends on $f(x)$ for $f(x)\equiv 0$, then the cross-section shape remains close to rectangular when strained.

3.2. Continuous Dependence Study

Let us define the conditions for the problem solution (11)-(14) to be continuously dependent on f(x) when f(x)≡0. As follows from [23], for this purpose, it is necessary to represent the problem for auxiliary functions marked by a prime in the following form:

$$\frac{\partial\sigma'_x}{\partial x}+\frac{\partial\tau'}{\partial y}=0;\frac{\partial\sigma'_y}{\partial y}+\frac{\partial\tau'}{\partial x}=0;\sigma'_x=\lambda\theta'+2\mu\frac{\partial u'}{\partial x};$$
$$\sigma'_y=\lambda\theta'+2\mu\frac{\partial v'}{\partial y};\tau'=\mu\left(\frac{\partial v'}{\partial x}+\frac{\partial u'}{\partial y}\right);\theta'=\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y} \quad (17)$$

$$\left(P_\tau^0+P'_\tau\right)\Big|_{y=g_i^0+g'_i}=0;$$
$$\left(P_n^0+P'_n\right)\Big|_{y=g_i^0+g'_i}=k(v^0(\bar{\varphi}_i^0+\bar{\varphi}'_i,h)+v'(\bar{\varphi}_i^0+\bar{\varphi}'_i,h))\ (i=1,2). \quad (18)$$

$$\int_{\eta_1^0+\eta'_1}^{\eta_2^0+\eta'_2}\left(P_x^0+P'_x\right)dy=\int_{\eta_3^0+\eta'_3}^{\eta_4^0+\eta'_4}\left(P_x^0+P'_x\right)dy=-2ph;$$
$$\int_{\eta_1^0+\eta'_1}^{\eta_2^0+\eta'_2}\left(P_x^0+P'_x\right)ydy=\int_{\eta_3^0+\eta'_3}^{\eta_4^0+\eta'_4}\left(P_x^0+P'_x\right)ydy=0; \quad (19)$$
$$\int_{\xi_1^0}^{\xi_2^0}(v^0+v')dy=\int_{\xi_3^0}^{\xi_4^0}(v^0+v')dy=0.$$

In which,

$$g_1^0=\left(1+\varepsilon_y^0\right)h;\ g_2^0=-\left(1+\varepsilon_y^0\right)h;$$
$$\bar{\varphi}_i^0=\frac{x}{1+\varepsilon_x^0};\ \bar{\varphi}'_i=u'\left(\frac{x}{1+\varepsilon_x^0},\pm h\right),\ i=1,2$$
$$g'_1=v'\left(\frac{x}{1+\varepsilon_x^0},h\right);\ g'_2=v'\left(\frac{x}{1+\varepsilon_x^0},-h\right); \quad (20)$$
$$q'_1=u'\left(1,\frac{y}{1+\varepsilon_y^0}\right);\ q'_2=u'\left(-1,\frac{y}{1+\varepsilon_y^0}\right);$$
$$\xi^0{}_1=-h;\ \xi^0{}_2=h;\xi_3{}^0=-h;\ \xi_4{}^0=h$$
$$\eta_1{}^0=-h+v^0(1,-h);\ \eta_2{}^0=h+v^0(1,h);$$
$$\eta_3{}^0=-h+v^0(-1,-h);\eta_4{}^0=h+v^0(-1,h)$$
$$\eta'_1=v'(1,-h);\eta'_2=v'(1,h);\ \eta'_3=v'(-1,-h);$$
$$\eta'_4=v'(-1,h)y_1=h+f(x);\quad y_2=-h-f(x)$$

Let the homogeneous problem corresponding to the linearized equations (17)-(20) have only a trivial solution [22, 23]. This is a condition necessary for the solution to the initial problem to be continuously dependent on f(x) for f(x) ≡0. In general, it is quite difficult to carry out research, so next we are going to consider the case that is usual for $p/\mu<<1$. In this case, the resulting strain will also be small $\varepsilon_y{}^0<<1$; $\varepsilon_x{}^0<<1$

This enables replacing linearized boundary conditions [26, 27] corresponding to (18) and (19) by the following approximated ones (here, it is already taken into account that (15), (16) is the problem solution to (11)-(14) for $f(x)\equiv0$) where $y=\pm h$;

$$\tau'-(\sigma_x^{(0)}+\sigma_y^{(0)})\frac{\partial v'}{\partial x}=\sigma'_y+kv'=0; \quad (21)$$

$$\text{where } x=\pm1;\ \sigma_x^{(0)}(v'(x,h)-v'(x,-h))+\int_{-h}^{h}\sigma'_x(x,y)dy=0;$$
$$\sigma_x^{(0)}h(v'(x,h)+v'(x,-h))+\int_{-h}^{h}\sigma'_x(x,y)ydy=0; \quad (22)$$
$$\int_{-h}^{h}v'(x,y)ydy=0$$

The general solution to the system of equations from (17) is obtained such as in the form of [26]:

$$u'=g(\eta)\cos\alpha x;\quad v'=q(\eta)\sin\alpha x;\ \eta=ay \quad (23)$$

Substituting (23) into (17), we obtain a system for defining unknown functions g(η), q(η):

$$\mu\frac{d^2g}{d\eta^2}-(\lambda+2\mu)g+(\lambda+\mu)\frac{dq}{d\eta}=0;$$
$$(\lambda+2\mu)\frac{d^2q}{d\eta^2}-\mu q-(\lambda+\mu)\frac{dg}{d\eta}=0$$

It follows that

$$g(\eta)=C_1ch\eta+C_2sh\eta+C_3\eta ch\eta+C_4\eta sh\eta,$$
$$q(\eta)=(C_2-a_1C_3)ch\eta+(C_1-a_1C_4)sh\eta+C_1\eta ch\eta+C_3\eta sh\eta;$$
$$a_1=\frac{\lambda+3\mu}{\lambda+\mu}$$

It follows from (17) and (23) that $\sigma'_x=a\left(\lambda\frac{dq}{d\eta}-(\lambda+2\mu)g\right)\sin\alpha x$

Then the boundary conditions (22) are satisfied for $a=n\cdot(\pi/l)$. For other unknowns we get

$$\sigma'_y=a\left((\lambda+2\mu)\frac{dq}{d\eta}-\lambda g\right)\sin\alpha x;\ \tau'=a\mu\left(\frac{dg}{d\eta}+q\right)\cos ax \quad (24)$$

Substituting (23), (24) into bounding conditions (21), we obtain a system of equations

$$\begin{aligned}\alpha_{11}C_1+\alpha_{12}C_2+\alpha_{13}C_3+\alpha_{14}C_4&=0\\-\alpha_{11}C_1+\alpha_{12}C_2+\alpha_{13}C_3-\alpha_{14}C_4&=0\\\alpha_{31}C_1+\alpha_{32}C_2+\alpha_{33}C_3+\alpha_{34}C_4&=0\\\alpha_{41}C_1+\alpha_{42}C_2+\alpha_{43}C_3+\alpha_{44}C_4&=0\end{aligned} \quad (25)$$

$$\alpha_{11}=(2\mu-\sigma_x^0-\sigma_y^0)sh\beta;\ \alpha_{12}=(2\mu-\sigma_x^0-\sigma_y^0)ch\beta;$$
$$\alpha_{13}=(1+a_1(1+\sigma_x^0+\sigma_y^0))ch\beta+\beta(1-\sigma_x^0-\sigma_y^0)sh\beta;$$
$$\alpha_{14}=(1-\sigma_x^0-\sigma_y^0)ch\beta+(1-a_1(\sigma_x^0+\sigma_y^0))sh\beta;$$

$$\alpha_{31}=2\mu ch\beta+\frac{k}{a}sh\beta;\quad \alpha_{32}=2\mu sh\beta+\frac{k}{a}ch\beta;$$
$$\alpha_{33}=\left((\lambda+2\mu)(1-a_1)+\beta\frac{k}{a}\right)sh\beta+\left(2\mu\beta-a_1\frac{k}{a}\right)ch\beta;$$
$$\alpha_{34}=\left((\lambda+2\mu)(1-a_1)+\beta\frac{k}{a}\right)ch\beta+\left(2\mu\beta-a_1\frac{k}{a}\right)sh\beta;$$
$$\alpha_{41}=2\mu ch\beta-\frac{k}{a}sh\beta;\ \alpha_{42}=-2\mu sh\beta+\frac{k}{a}ch\beta; \qquad (26)$$
$$\alpha_{43}=\left(-(\lambda+2\mu)(1-a_1)+\beta\frac{k}{a}\right)sh\beta-\left(2\mu\beta+a_1\frac{k}{a}\right)ch\beta;$$
$$\alpha_{44}=\left((\lambda+2\mu)(1-a_1)-\beta\frac{k}{a}\right)ch\beta+\left(2\mu\beta+a_1\frac{k}{a}\right)sh\beta;$$
$$\beta=ah$$

From Eqs. (25), (26) we have the ratio, at which problem Eqs. (17), (21) and (22) admits a non-trivial solution:

$$\begin{vmatrix}\alpha_{13} & \alpha_{33}\\ \alpha_{12} & \alpha_{32}\end{vmatrix}\begin{vmatrix}\alpha_{11} & \alpha_{14}\\ \alpha_{41} & \alpha_{44}\end{vmatrix}+\begin{vmatrix}\alpha_{12} & \alpha_{42}\\ \alpha_{13} & \alpha_{43}\end{vmatrix}\begin{vmatrix}\alpha_{11} & \alpha_{14}\\ \alpha_{31} & \alpha_{34}\end{vmatrix}=0 \qquad (27)$$

Thus, if the initial data allow (27) to be fulfilled, then the continuous dependence of the problem solution Eqs. (11)-(13) on f(x) for f(x)≡0 is violated. In this case, the homogeneous state Eq. (15) ceases to be stable. Below, there are statically specific curves for a substance with various cross-section sizes and physical constants $\lambda=1.2\cdot10^6\ kg/cm^2$ and $\mu=0.8\cdot10^6\ kg/cm^2$. Fig.1 shows a graph corresponding to Eq. (27) for geometric parameters *h/l*=0.5 in the parameter space of environment and modulus of foundation.

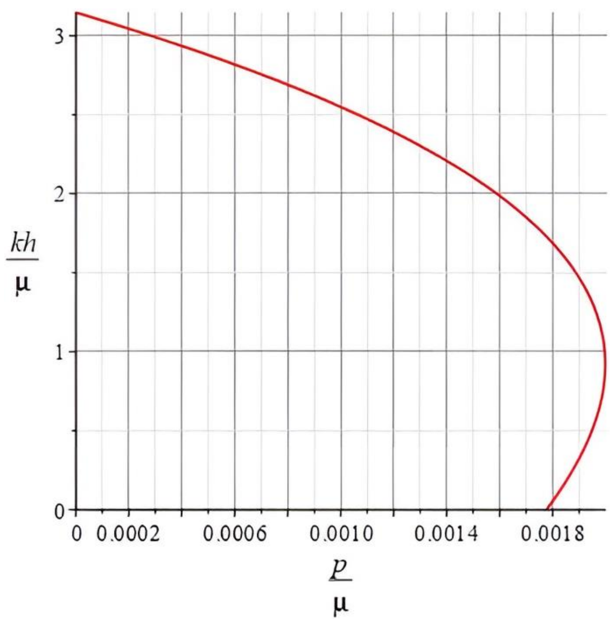

Figure 1. The boundary line of the stability region

Fig. 2 presents statically specific curves for strips with a cross-section elongated in width (*h/l=0.1*). Let us compare the results obtained with those already known, given in the work of Ershov et al. [25] and Ishlinsky [24], and the classical work on the stability of elastic bodies by Volmir [2].

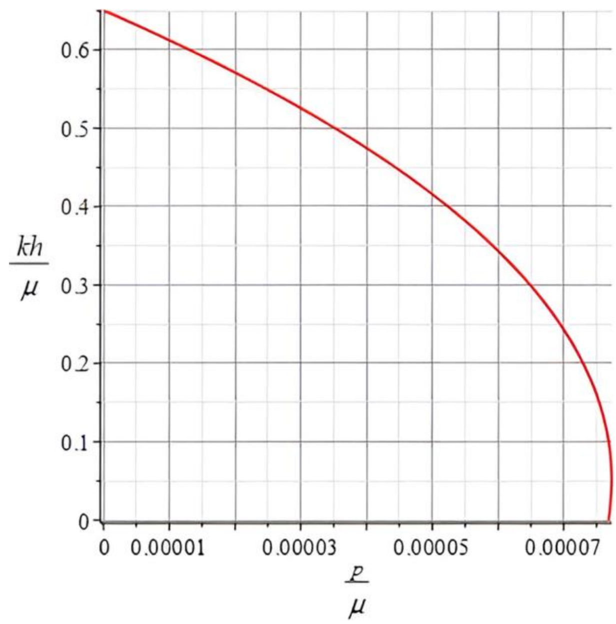

Figure 2. The boundary line of the stability region corresponding to Eq. (7)

Fig. 3 shows statically special curves in the space of the parameters of the compressive force and parameters of the linear dimensions of the section. The solid line shows the curve obtained based on (27), and the dashed line shows the curve obtained based on the equation for determining the critical pressure value obtained by Ershov and Ivlev [25]. Note that these lines coincide practically.

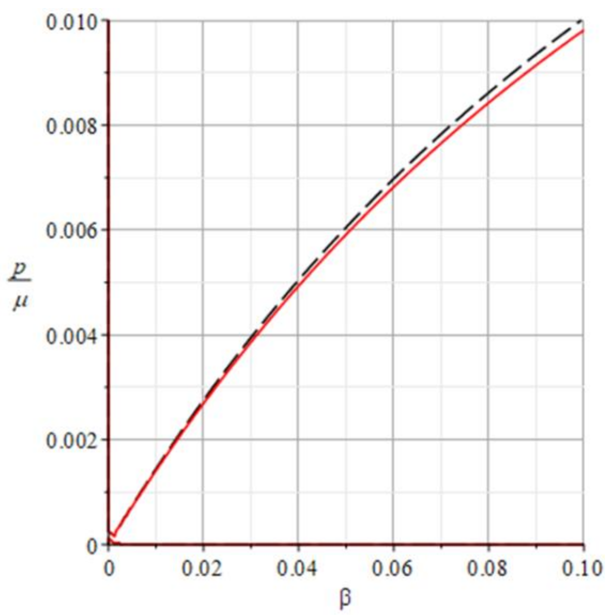

Figure 3. The boundary line of the stability region corresponding to Eq.(7) for *k*=0

If we compare it with the critical load during buckling of a compressed rectangular plate, considered in Volmir's monograph [2], as well as Ishlinsky's work [24] at a value of β=0.04, we obtain minor discrepancies p^*=1219.*067 kg/cm²*, and p_{Eul}=1219.*048kg/cm²*. The condition p/μ<<1 for the values β>0.1 is met, but the stresses will exceed the elastic limit of real materials.

Fig.4 shows a statically specific curve in the parameter space of foundation rigidity and cross-section linear dimensions when p/μ<<1.

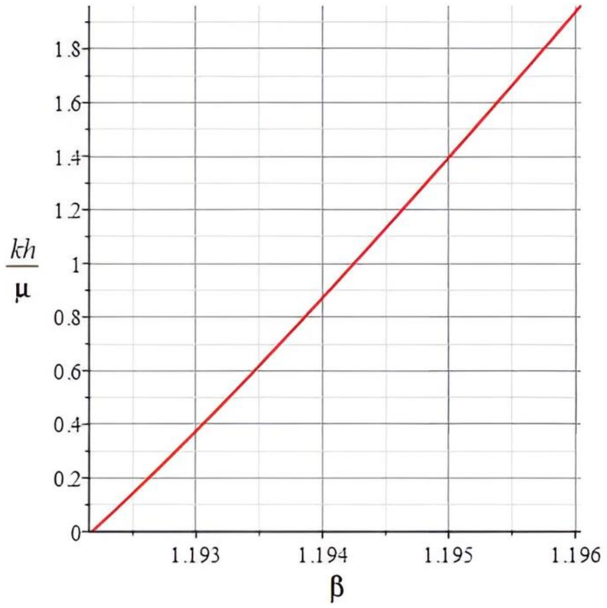

Figure 4. The boundary line of the stability region corresponding to Eq. (7) for ρ/μ=0.005

4. Conclusions

In this study, it is shown that the existence of only a trivial solution for an auxiliary linearized homogeneous problem is equivalent to the fulfillment of the conditions from the criteria of the bifurcation method and the method of initial imperfections. Thus, when conducting research based on the implicit function theorem, it is possible to determine the restrictions imposed on the parameters of the elastic system, under which the continuous dependence (stability) is violated. In this case, there is no need to enter a loading parameter, i.e., the loads remain independent throughout the study.

For the considered elastically reinforced compressed strip, a condition was obtained that determines the critical values of the external influence parameter p and the base stiffness parameter k. In particular, on the plane of these parameters it specifies a statically special curve, when crossed by the loading trajectory, the strip loses stability (its cross-sectional shape will no longer be close to rectangular).

As a result, we find that the existence of only a trivial solution for the auxiliary linearized homogeneous problem and the requirement of definiteness and boundedness $\frac{\partial H}{\partial \lambda}, \frac{\partial H}{\partial P_1}, ..., \frac{\partial H}{\partial P_n}$ in some neighborhood of a point (u_0, λ_0, P_{10}...P_{n0}) is equivalent to the fulfillment of the conditions from the criteria of the bifurcation method and the method of initial imperfections. Thus, when conducting research based on the theorem on implicit functions, it is possible to determine the restrictions imposed on the parameters of the elastic system, upon fulfillment of which the continuous dependence (stability) is violated. The resulting condition can be considered as an implicitly specified function limiting the region of continuous dependence of the solution of the problem on the input data in the space of interest to the researcher. In this case, there is no need to introduce a loading parameter, i.e., the loads remain independent throughout the study.

For the elastically reinforced compressed strip under consideration, a condition is obtained that determines the critical values of the external action parameter *p* and the base rigidity parameter *k*. In particular, on the plane of these parameters, it specifies a statically special curve, upon intersection of which by the loading trajectory, the strip loses stability (the shape of its cross-section will no longer be close to rectangular). For particular cases of the geometric dimensions of the cross-section of the strip, regions are constructed in the space of the base rigidity parameters and the compressive force, within which the state of the strip remains close to uniform during the entire deformation process. A comparison of the obtained results with the known classical ones is carried out.

Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

Author Contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Thuy Van Tran Thi, N. V. Minayeva, S. Yu. Gridnev, Yu. I. Skalko and Vuong Pham Ngoc. The first draft of the manuscript was written by N. V. Minayeva and Thuy Van Tran Thi and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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