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# Hydraulic Evaluation of Morning Glory Spillways with Semi-Elliptical Notches

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**Abstract** Morning glory spillways (MGSs) or shaft spillways have special design features allowing the excess water to be passed from the dam reservoir to the downstream outlet. In this research, holes within MGS featuring semi-elliptical notch shapes were experimentally investigated for discharge coefficient ($C_d$) and flow pattern compared with the classical MGS in case of without notches. This new and innovative design utilizing the semi-elliptical notches was examined for various approach Froude numbers ($F_o$) and notch numbers of $N$ = 2, 4 and 8 notches. The results indicated that the semi-elliptical notches had the ability to enhance the discharge coefficient and also breakdown the streamlines of flow vortex of simple shaft spillways. However, the semi-elliptical notches were more effective for lower $F_o$, and consequently lower flow discharges. The impact of notches on the percentages of improvement in $C_d$ values decreased as $F_o$ rose compared with the classical MGS. A new discharge coefficient ($C_d$) formula for MGS with semi-elliptical notches was proposed which was valid for $0.01 \leq F_o \leq 0.056$, $2 \leq N \leq 8$, $h/b = 0.86$, $b/D_c = 0.29$ and $h/D_c = 0.25$.

**Keywords** Dam Reservoir, Discharge Coefficient, Morning Glory Spillway, Physical Modeling, Semi-Elliptical Notches, Vortex Breakers

## 1. Introduction

Spillways are designed to manage the intentional release of excess water that exceeds the maximum storage capacity of reservoirs or dams to properly control floods downstream [1]. Ensuring permanent flood discharge might provide a challenge in terms of technical difficulty and consequent cost. In particular, choosing the proper spillway type according to the available options is an essential feature. Most used spillways are identified for a certain characteristic such as ogee, chute, morning glory, side channel, siphon, and stepped spillways [2]. Among the types of spillways, MGS or shaft spillway is a unique kind of spillway that functions separately from the main dam body to pass the excess water. These spillways, which are used in dams, also function to prevent erosion in highway culverts and various other structures. This hydraulic structure consists of a circular inlet (bell-mouth), vertical or sloping shaft, and a horizontal or sloping tunnel [3]. It provides a practical advantage where other spillways are impractical. MGS design is ideal for situations where tight control over the maximum spillway discharge is crucial as it provides the benefit of nearly full operating capacity at comparatively low water heads [4]. Using this type of spillway offers a workable alternative and cost effective for spillways at rockfill and earth dams, as well as at dam locations in narrow canyons with steeply rising abutments. For such narrow dam locations, the space for traditional concrete dam is restricted to provide a straight overflow spillway. For utilizing thin arch dam as a solution, the vibration issues arise from the spillway holes in thin arch dams situated either over or through the dam. It is therefore feasible to implement the alternative of a spillway that is away from the dam [5]. In these situations, MGSs are applied with specific basin types (vortex basins) in which provide the flow with an angular velocity creating a rotating flow within the glory hole.

Nevertheless, the spillway becomes submerged when the water level rises, and its hydraulic system functions as an orifice. In this regard, the flow is under pressure, which is referred to as pipe control [6]. Discharge efficiency is greatly decreased in scenarios involving orifices and pipes control conditions [7]. Vortices prevented flow streamlines from forming directly toward the MGS outlet. As a result, the hydraulic efficiency of MGS declines and its discharge coefficient ($C_d$) decreases [8]. Namely, discharge capacity of this type of spillway is influenced by the crest geometry. Thus, altering the crest design can reduce turbulence and eddy currents and consequently increase the overflow discharge capacity.

### 1.1. Performance of MGS for Different Inlet Shapes

A variety of studies have been conducted to examine the discharge capacity of spillway for different inlet shapes [9-15]. Sayadzadeh *et al*. [16] experimentally modelled MGS to examine how the geometry and number of pyramidal vortex breakers affect the spillway discharge coefficient. The MGS discharge coefficient was evaluated for three, four, and six pyramidal vortex breakers with square and triangle bases. The findings demonstrated a considerable increase of up to 50.97% in the crest control and up to 16.13% in the orifice control when comparing the discharge coefficient performance index with a set of six vortex breakers to the typical morning glory spillway. The MGS discharge coefficient was also calculated empirically using pyramidal vortex breakers. Anani *et al*. [17] proposed triangular and rectangular edged notch for MGS inlet to enhance experimentally the discharge coefficient. The study also tested different numbers of notches for various MGS diameters under different flow conditions. The discharge coefficient $C_d$ was assessed for spillways featuring two, four, and eight notches, as compared to a non-notched spillway. The results indicated that both triangular and rectangular notch shapes increased the discharge coefficient, with increasing the notch number compared to without notches. In addition, for triangular notch, the increment percentages of $C_d \approx 143$ % for notch number $N = 8$, compared to corresponding increment percentages of $C_d \approx 132$ % for rectangular notch shape. Musavi-Jahromi *et al*. [18] examined the impact of inclined vortex breakers on the discharge coefficient in the crest and orifice control conditions using a physical model of MGS. Results indicated that using a set of six vortex breakers with inclination angle of 45° in the orifice control can effectively raise the discharge coefficient and decrease the water level. The location and number of piles on the crest of a morning glory spillway were altered by Fattor and Bacchiega [19]. According to their findings, hydraulic performance of the spillway significantly improved once the modifications had been made. Keihanpour and Kabiri-Samani [20] investigated how the hydraulic properties of swirling flow were affected by marguerite-shaped inlets in shaft spillways. Their investigations proposed different equations to estimate discharge coefficients, critical submergence depths and threshold for both free and orifice flow regimes.

### 1.2. Numerical Modelling

Other studies have applied the numerical modelling to investigate the hydraulic performance of MGS for various configurations [21-26]. Further researchers used ANN approach to analyze the flow behavior within MGS [27,28]. Enjilzadeh and Nohani [29] used FLOW3D model to analyze flow pattern of morning glory spillways in different modes. The experimental data was used to validate the numerical model, and the results showed satisfactory agreement for the spillways' discharge rate results, with a relative error of 6.4%. Kazemipour et al. [26] used marguerite-shaped inlets (MSIs) in vertical shaft spillways to reduce the effect of swirling flow and minimize air entrainment. This study was numerically conducted to evaluate the hydraulic and hydrodynamic characteristics of these inlets under an orifice flow regime, varying the geometrical dimensions. It was found that increasing the height and length of the MSI blades reduced the swirling flow strength and increased water discharge, but can lead to severe flow collision and increased water surface level and swirling flow strength if the blades are too long or too high. Aydin and Ulu [30] numerically investigated hybrid spillway design to improve the hydraulic performance of the spillway. Labyrinth-shaft spillway model for different geometries was proposed using computational fluid dynamics (CFD) to analyze its performances. The results demonstrated that the labyrinth-shaft spillway resulted better discharge capacity and discharge coefficient compared with conventional shaft spillways. In addition, the study developed a new formula to estimate the discharge coefficient of labyrinth-shaft spillway under the study limitations. Aydin and Ulu [31] examined the effect of aeration through holes placed on the hood to mitigate cavitation in high-head siphon-shaft spillways. A three-dimensional computational fluid dynamics (CFD) technique, using the finite-volume method to solve Reynolds-averaged Navier-Stokes (RANS) equations, was employed for the incompressible viscous and turbulent fluid motion. The volume of fluid (VOF) scheme was used to simulate two-phase (water-air) flow. The numerical results demonstrated that aeration was highly effective in reducing siphon sub-pressures and cavitation. An optimal relative aeration diameter of 0.45

was found to provide sufficient air entrainment to protect against cavitation without significantly decreasing discharge performance. Furthermore, Aydin and Ulu [32] addressed cavitation issues in siphon-shaft spillways by proposing a novel, pressure-controlled shaft profile. The study proposed a wider upper section of the shaft, where siphonic pressure was lower, to mitigate cavitation. In addition, developed analytically by Bernoulli's principle and cavitation pressure, the profile's hydraulic performance was validated through computational fluid dynamics (CFD) simulations, benchmark model comparison, and recommended error estimation. The new design significantly reduced cavitation and improved hydraulic performance compared to the traditional Wagner profile.

### 1.3. Objectives

Among the previous studies a lack of knowledge has been observed for utilizing semi-elliptical notch shapes for MGS. The current study presents a new and innovative design based on using semi-elliptical notches in MGS to enhance the discharge coefficient and flow pattern compared with the classical MGS without any notches as a reference case. In order to evaluate the $C_d$ of the spillway, tests were carried out for MGS with two, four, eight semi-elliptical notches and a no notch scenario for different flow rates.

## 2. Materials and Methods

The study employed a precise methodology to comprehensively understand the impact of using semi-elliptical notches in MGS to enhance the discharge coefficient and flow pattern. The detailed methodology is depicted in Figure 1.

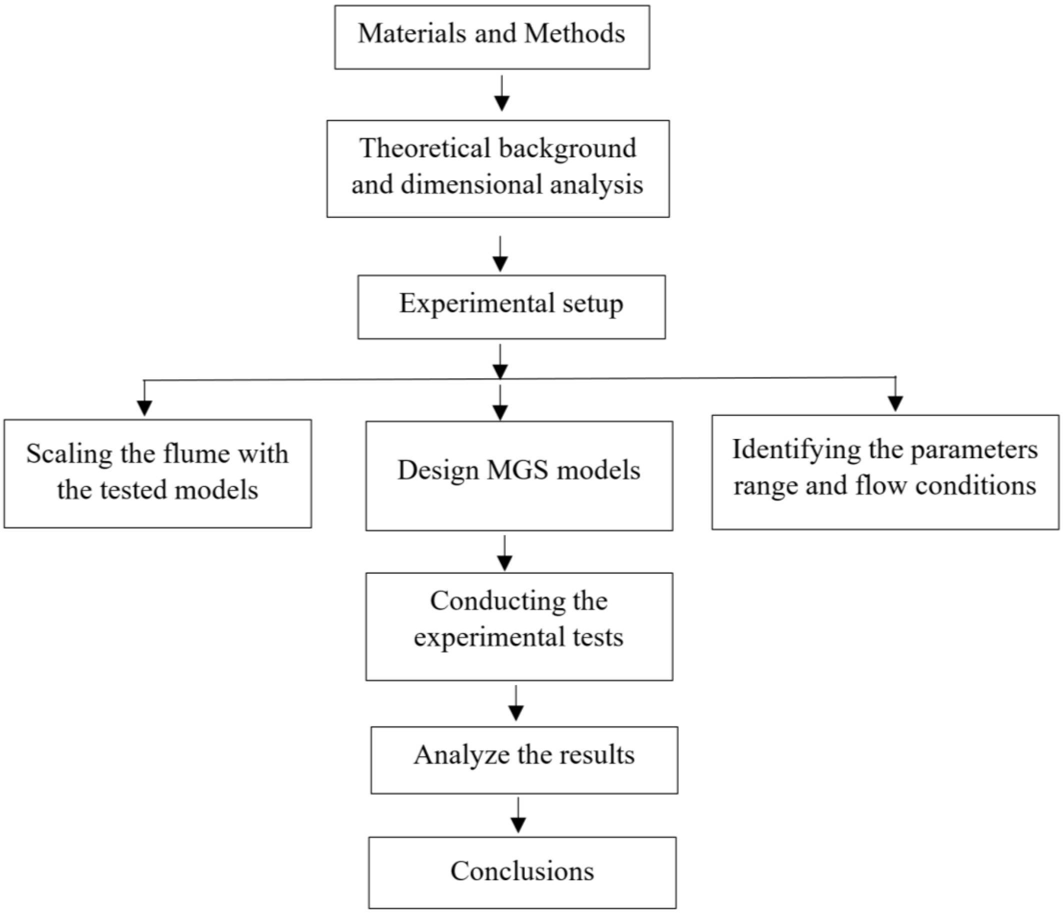

**Figure 1.** Detailed methodology adopted in this paper

### 2.1. Experimental Setup

A physical model was constructed in the Hydraulic Laboratory of the Department of Civil Engineering at Al-Azhar University in Cairo, Egypt. Tests were carried out utilizing semi-elliptical notches in MGS to investigate the $C_d$ of the spillway. The hydraulic model consisted of several parts to provide the experiments of the MGS model for different configurations (Figure 2). The main tank of the model had a capacity of 1000 L. In addition, it included two vents: one for flow volume measurement and calibration, and the other with a piezometer to indicate the water height. Reservoir with dimension of 1 m long × 1 m wide × 0.60 m depth was attached to the model to accommodate the funnel-shaped section of MGS. Two sidewalls of the reservoir with thickness of 8 mm Plexiglas and the other two sides and bottom were made of steel. A pump was used with a maximum discharge of 1.6 L/s to circulate water through feeding net with control valves and flow meter to control the inlet discharges. To avoid volatility and turbulence, a metal screen was placed inside reservoir to counteract the water rush thus preventing the bubbles formation. To fabricate the spillways, spillway consisted of both a fixed 4 mm thick steel main structure and the proposed spillway crest shapes, which were replaceable reinforced plastic with different diameters. To calibrate the flow discharge measurements, discharges were measured and contrasting the result against measurements by utilizing the volumetric approach. Therefore, a piezometer was used to measure the changes in water height during 30 s, from which the measured discharge was obtained. Consequently, the obtained discharges from using the flow-meter reading were regularly compared with those acquired from the volumetric technique to ensure the accuracy.

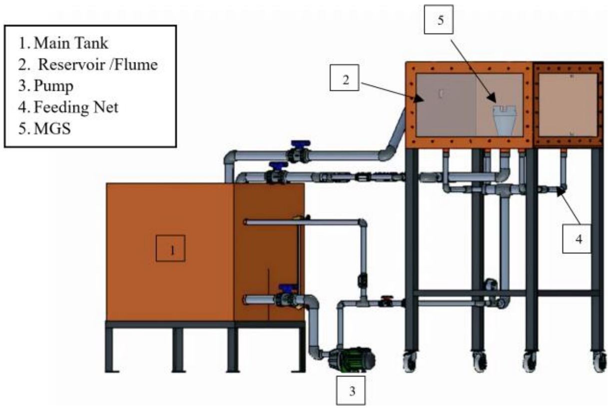

**Figure 2.** 3D view of the physical model with MGS

In the present study, crest diameter of spillway $D_c$ = 203.2 mm and a throat diameter of $d_t$ = 50.8 mm were tested for a number of two, four, eight semi-elliptical notches and no notch scenario under different flow rates; whilst, $D_c$ = 50.8, 101.6 and 152.4 mm were used for test reliability (Table 1). Holes within the wall of crest diameter of MGS featuring the semi-elliptical notches were accurately designed. The design details of semi-elliptical notches are shown in Figure 3 and Table 2. For the semi-elliptical notch, the range of relative height from the center to the top width *h/b* is 0.86 for the tested diameter. Five flow discharges $Q$ = 0.25, 0.50, 0.75, 1.00, and 1.33 L/s were used during the study to attain various flow conditions. In these spillways, the water-conveying tunnel was 0.6 m long with a 90° bend and a vertical shaft. MGS for circular crest and without any notches was considerably tested as the classical and reference spillway case.

**Table 1.** Specifications of MGS model and parameters used during the study

| $F_o$ | $d_t$ (mm) | $D_c$ (mm) for test reliability | N |
|---|---|---|---|
| 0.01 | 50.8 | 50.8;101.6;152.4 | 0 |
| 0.01 | 50.8 | 50.8;101.6;152.4 | 2 |
| 0.01 | 50.8 | 50.8;101.6;152.4 | 4 |
| 0.01 | 50.8 | 50.8;101.6;152.4 | 8 |
| 0.021 | 50.8 | 50.8;101.6;152.4 | 0 |
| 0.021 | 50.8 | 50.8;101.6;152.4 | 2 |
| 0.021 | 50.8 | 50.8;101.6;152.4 | 4 |
| 0.021 | 50.8 | 50.8;101.6;152.4 | 8 |
| 0.031 | 50.8 | 50.8;101.6;152.4 | 0 |
| 0.031 | 50.8 | 50.8;101.6;152.4 | 2 |
| 0.031 | 50.8 | 50.8;101.6;152.4 | 4 |
| 0.031 | 50.8 | 50.8;101.6;152.4 | 8 |
| 0.04 | 50.8 | 50.8;101.6;152.4 | 0 |
| 0.04 | 50.8 | 50.8;101.6;152.4 | 2 |
| 0.04 | 50.8 | 50.8;101.6;152.4 | 4 |
| 0.04 | 50.8 | 50.8;101.6;152.4 | 8 |
| 0.056 | 50.8 | 50.8;101.6;152.4 | 0 |
| 0.056 | 50.8 | 50.8;101.6;152.4 | 2 |
| 0.056 | 50.8 | 50.8;101.6;152.4 | 4 |
| 0.056 | 50.8 | 50.8;101.6;152.4 | 8 |

**Figure 3.** Model of semi-elliptical notches design within MGS crest

**Table 2.** Notch geometric design for the tested cases

| $D_c$ (mm) | h (mm) | Thickness (mm) | b (mm) | h/b | h/$D_c$ |
|---|---|---|---|---|---|
| 50.8 | 14 | 5 | 16.2 | 0.86 | 0.28 |
| 101.6 | 24.9 | 4 | 28.8 | 0.86 | 0.25 |
| 152.4 | 38.3 | 4 | 44.3 | 0.86 | 0.25 |
| 203.2 | 50.6 | 4 | 58.5 | 0.86 | 0.25 |

To check the reliability of the tests, the experiments were repeated for different diameters of $D_c$ = 50.8, 101.6 and 152.4 mm for the same ratio of semi-elliptical notch dimensions with height to width $h/b$ = 0.86 corresponding to each diameter as in Table 2. This concept also means that the results were checked for a smaller scale than the initial $D_c$ = 203.2 mm. Using Eq. (1), the average prediction error was determined for each diameter compared with the initial $D_c$ = 203.2 mm based on $C_d$ values to estimate the errors.

$$\text{Prediction Error} = \frac{|C_d - C_{dr}|}{C_d} \times 100\ (\%) \tag{1}$$

where $C_{dr}$ is the repeated discharge coefficient values for different crest diameters. The average prediction errors were less than 20% for the tested diameters at the tested flow conditions.

### 2.2. Theoretical Background

The discharge for morning glory spillways can be derived based on Eq. (2) [3,6].

$$\mathrm{Q} = \mathrm{C_d}\ .\ (2\pi\mathrm{R}).\ \mathrm{H}^{1.5} \tag{2}$$

where $Q$ is the discharge of MGS, $C_d$ is discharge coefficient (dimensionless), $R$ is the radius of crest, $H$ is the flow head above MGS crest.

To determine the relationships between a physical model and a prototype, dimensionless quantities were identified through dimensional analysis. Application the method of Buckingham's Π-theorem [33], allows MGS discharge coefficient formula using dimensionless terms. In this study, the following parameters affected the MGS discharge coefficient $C_d$:

$$f\ (C_d, V_o, H_o, g, N) = 0 \tag{3}$$

where, $V_o$ is approach average velocity, $H_o$ is flow head above MGS crest in case without notch, $g$ is gravitational acceleration and $N$ is number of semi-elliptical notch (dimensionless). The resulted dimensionless parameters based on Buckingham Π-theorem [33] are resulted as:

$$C_d = f\ (\mathrm{F_o}\ ,\ N) \tag{4}$$

which $\mathrm{F_o}$ is Froude number = $4\mathrm{Q}/[\pi \mathrm{D_c}^2(\mathrm{gH_o})^{0.5}]$. A new formula for predicting the coefficient of discharge using a multiple regression approach was proposed by the authors using the dimensionless terms as in Eq. (5).

$$C_d = m(F_o)^w\ \ (N)^s \tag{5}$$

where $m$, $w$ and $s$ are valued by regression.

## 3. Results and Discussion

### 3.1. Effects of Semi-Elliptical Notches on the Discharge Coefficient and Vortices

The effects of semi-elliptical notches and $\mathrm{F_o}$ on discharge coefficient ($C_d$) in the range of $N$ = 2, 4, 8 and no notches ($N$= 0, reference case) for $\mathrm{F_o}$ = 0.01, 0.021, 0.031, 0.04, and 0.056 were studied. Diagram of $C_d$ versus the number of semi-elliptical notches ($N$) is demonstrated for the tested approach Froude numbers at $d_t/D_c$= 0.25 in Figure 4. The results show that the $C_d$ increases with increasing the number of notches compared with the spillway without notches. The same trend is observed at all tested $\mathrm{F_o}$. For $\mathrm{F_o}$ = 0.031, $C_d$ amounted to 0.66, 0.69 and 0.71 for $N$ = 2, 4 and 8, respectively compared to $C_d$ = 0.44 in case of spillway without notches. In addition, $C_d$ trendline for non-notched spillway has a sharp bend compared to notched ones, especially at lower $\mathrm{F_o}$. For $\mathrm{F_o}$ = 0.01, the values of $C_d$ = 0.34 for non-notched spillway; whilst, $C_d$ = 0.58, 0.60 and 0.62 for notched spillway at $N$ = 2, 4 and 8, respectively. These results mean that the increment percentages of $C_d \approx 70$ % for notched spillway at $N$ = 2 compared to non-notched spillway for $\mathrm{F_o}$ = 0.01; whilst, the increment percentages of $C_d \approx 8$ % for $N$ = 8 compared to $N$ = 2.

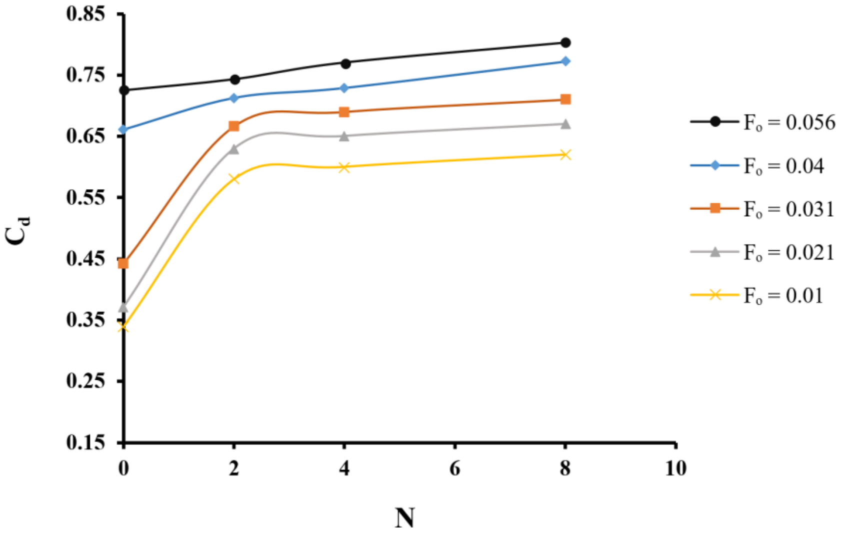

**Figure 4.** Relationships between $C_d$ and $N$ for approach Froude numbers

Providing the circular crest of spillway with opening notch reduces the flow resistance due to higher penetrability of flow through the openings, leading to a reduction in water level and as a result higher $C_d$. This result reveals that the discharge capacity is enhanced with the presence of semi-elliptical notches for MGS.

Figure 5 shows the increment percentages of $C_d$ for the tested $F_o$ and notches relative to case of spillway without notch. The impact of notches on the percentages of improvement in $C_d$ values compared to the reference case diminishes as $F_o$ increases. All tested numbers of notches show this tendency. For $F_o = 0.01$, the increment percentages of $C_d \approx 70$ %, 76 % and 82% for $N = 2$, 4 and 8, respectively compared to corresponding increment percentages of $C_d \approx 3$ %, 6 % and 11%, respectively for $F_o = 0.056$ (Figure 5). At the same spillway area, the shaft capacity is more effective for smaller discharges, but higher discharges cause the flow to be delayed and raise the water level as a result. In particular, more flow rates develop the higher water level, leading to a delay in discharging the water from the spillway opening and resulting a lower increment percentages of $C_d$ for higher $F_o$. This indicates that the notches work better with higher increment percentages of $C_d$ values when the Froude numbers are smaller. The results of the current study match the results obtained by Anani *et al.* [17], who studied triangular and rectangular notches within MGS. Anani et al. [17] concluded that triangular and rectangular notch shapes significantly increased the discharge coefficient in MGS inlets with varying notch numbers, particularly at low discharges.

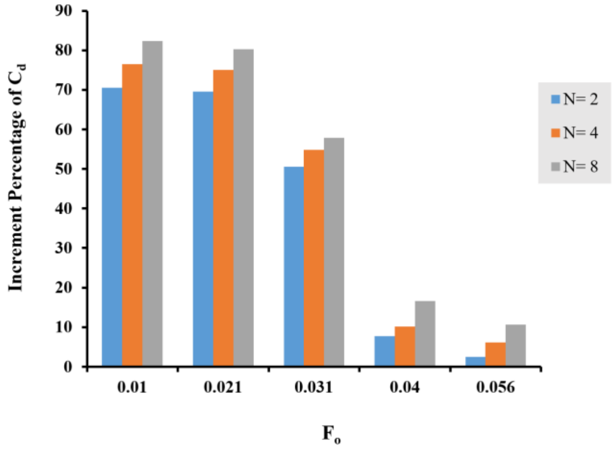

**Figure 5.** Increment percentages of $C_d$ for the tested Fo and notches relative to the spillway without notch

Figure 6 shows the observations of flow pattern and vortex developing through the tested MGS under tested notches. According to Figure 6a, with increasing flow discharges the flow nappe is thicker, forming flow shaft vortices and submergence of the crest in case of MGS without notches. In Figure 6b, 6c, 6d the semi-elliptical notches for MGS reduce the water level with increasing the number of the notches, and consequently delay the shaft vortex. In addition, breakdown of vortex streamlines increases as the numbers of semi-elliptical notches increase, and as a result, the discharge capacity increases. In particular, the notches had the ability to be a breaker for the flow shaft vortex, leading to an increase in discharge coefficient. These result concepts match with the vortex breakers studies such as Christodoulou *et al*. [3] and Sayadzadeh *et al*. [16] who used vortex breakers over the crest or at the inlet of vertical intake to direct the streamlines toward the outlet and diminish the vortex forces. Whereas, the current study designed openings with semi-elliptical shape within the crest itself to increase the discharge coefficient. As a result, the semi-elliptical notches had a significant role in breaking the vortex streamline and improving $C_d$ of MGS.

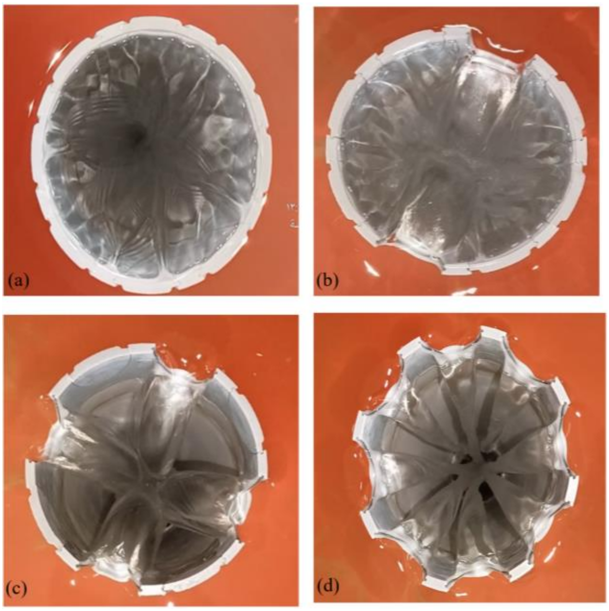

**Figure 6.** Observations of MGS(a) without notch, (b) two notches, (c) four notches, (d) eight notches

### 3.2. Effects of Approach Froude Numbers on the Discharge Coefficient

According to Figure 4, the values of $C_d$ in case of MGS for all arrangements increase with increasing the approach Froude numbers. For $F_o = 0.056$, values of $C_d = 0.72$, 0.74, 0.77 and 0.80 at $N = 0$ (without notch), 2, 4, and 8, respectively compared to corresponding $C_d = 0.34$, 0.58, 0.60 and 0.62, respectively for $F_o = 0.01$ (Figure 4). Specifically, for $F_o = 0.056$, 0.04, 0.031 and 0.021 the average values of $C_d$ for the tested MGS configurations increased by 42%, 34%, 17% and 8.5%, respectively relative to average value of $C_d$ at $F_o = 0.01$. These results are due to the corresponding increase of flow discharges and consequently $F_o$. Regarding the relationship between the approach Froude numbers $F_o$ and the resultant $F_r$ for the tested notch numbers, Figure 7 shows $F_r$ vs $F_o$. The results show an increasing trend between $F_o$ and $F_r$. This trend is similar to all tested numbers of notches. For $F_o = 0.01$, values of $F_r = 0.017$, 0.023, and 0.28 at $N = 2$, 4, and 8, respectively compared to corresponding $F_r = 0.067$, 0.091, and 0.11, respectively for $F_o = 0.056$ (Figure 7). In addition, increasing the number of notches leads to an increase in $F_r$ values. The flow head over the crest decreases with increasing *N*, resulting higher resultant $F_r$. The following expressions represent the relationships between $F_r$ and $F_o$ for the tested *N*.

$$F_r = 1.05\,(F_o)^{0.79} \qquad \text{For } N = 8\ (R^2 = 0.99) \qquad (6)$$

$$F_r = 0.86\,(F_o)^{0.78} \qquad \text{For } N = 4\ (R^2 = 0.99) \qquad (7)$$

$$F_r = 0.63\,(F_o)^{0.78} \qquad \text{For } N = 2\ (R^2 = 0.99) \qquad (8)$$

### 3.3. Effects of Semi-Elliptical Notches on the Rating Curve

Figure 8 presents rating curve for all MGS configurations. The results indicate that the semi-elliptical notches reduce the flow head on the crest for a constant discharge. For $Q = 0.75$ L/s, $H = 0.032$, 0.017 and 0.012 m for $N = 2$, 4 and 8, respectively compared to flow head of 0.057 m for MGS without notches. Specifically, for $N = 2$, 4 and 8 the average values of *H* over the crest for the tested flow rates decreased by 46%, 70% and 80%, respectively relative to average value of flow head for MGS without notches. This result indicates that the discharge coefficient $C_d$ and consequently the flow capacity increases with semi-elliptical notches for MGS. In addition, with increasing flow discharges, the flow head over crest increases as well at constant number of notches.

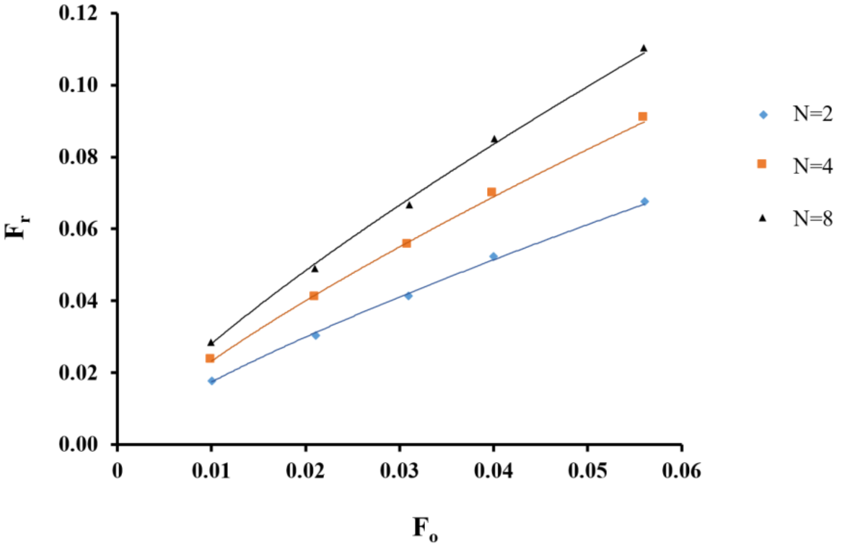

**Figure 7.** Variations of $F_r$ versus $F_o$ for the tested notch numbers

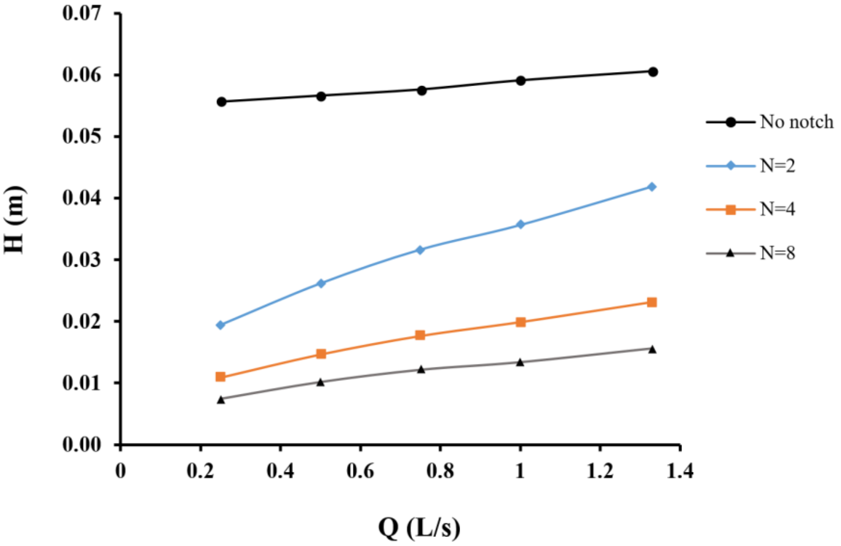

**Figure 8.** Variations of flow head versus discharges for the tested notch arrangements

### 3.4. Developing $C_d$ Equation for MGS with Semi-Elliptical Notches

Multiple nonlinear regression was applied to develop a formula to predict discharge coefficient for MGS with semi-elliptical notches for the tested $\mathrm{F_o}$ and $N$ at a 95% confidence level. Regression approach suggested Eq. (9) with adjusted $R^2 = 0.98$.

$$C_d = \quad 1.17(\mathrm{F}_o)^{0.17} \quad (N)^{0.047} \tag{9}$$

Both the approach Froude number $\mathrm{F_o}$ and semi-elliptical notches $N$ revealed $p$-values $< 0.0001$ with standard errors 0.0068 and 0.007, respectively. Therefore, $\mathrm{F_o}$ and $N$ were significant variables in predicting $C_d$ for MGS with semi-elliptical notches. Eq. (9) was valid for $0.01 \leq \mathrm{F_o} \leq 0.056$, $2 \leq N \leq 8$, $h/b = 0.86$, $b/D_c = 0.29$ and $h/D_c = 0.25$. Figure 9 compares the measured $C_d$ values with the predicted values using Eq. (9). Figure 9 shows that the data points are within ± 5 % prediction range with $R^2 = 0.99$, referring a good fit to the proposed Eq. (9).

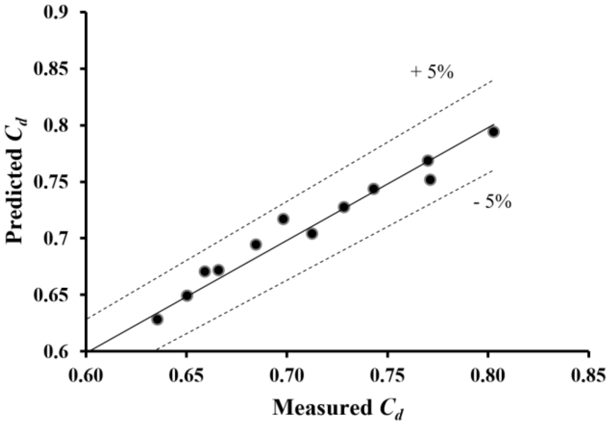

**Figure 9.** Measured $C_d$ and the predicted by Eq. (9) with ±5% predication range

## 4. Conclusions

Morning glory spillways (MGSs), also known as shaft spillways, are characterized by specialized design features that facilitate the passage of excess water from the dam reservoir to the downstream outlet. In this study, we experimentally investigated the effect of incorporating semi-elliptical notch-shaped holes within MGSs on the discharge coefficient ($C_d$) and flow pattern, in comparison to a traditional MGS without notches. The impacts of semi-elliptical openings featuring 2, 4 and 8 notches number within a morning glory spillway were examined for different flow discharges through experimental models. The study revealed the following:

- The incorporation of semi-elliptical notches within the MGS significantly enhanced the discharge coefficients compared to the MGS without notches. The discharge coefficient ($C_d$) increased proportionally with the number of semi-elliptical notches, attributed to the reduction in flow head.
- Semi-elliptical notches were more effective for lower flow discharges. As $\mathrm{F_o}$ increased, the effect of notches on the percentages of improvement in $C_d$ levels decreased relative to the reference case. For $\mathrm{F_o} = 0.01$, the increment percentage of $C_d \approx 82\%$ for $N = 8$ compared to corresponding increment percentage of $C_d \approx 11\%$ for $\mathrm{F_o} = 0.056$.
- For MGS without notches, higher flow discharges caused the flow nappe to thicken, subsequently generating flow shaft vortices and submerging the crest of the MGS.
- The semi-elliptical notches in the MGS reduced the water level proportionally to the increase in the number of notches, thereby delaying the formation of shaft vortices. Furthermore, the disintegration of vortex streamlines intensified with an increase in the number of semi-elliptical notches, subsequently enhancing the discharge capacity. Notably, these notches effectively disrupted the shaft vortex streamlines, resulting in an elevated discharge coefficient.
- The $C_d$ value levels in case of MGS for all configurations increased when the approach Froude numbers increased due to the corresponding rise of flow discharges. Furthermore, $\mathrm{F_r}$ rose as $\mathrm{F_o}$ increased during the test sets.
- The average water level (H) over the crest, at tested flow rates, decreased by 46%, 70%, and 80% for 2, 4, and 8 semi-elliptical notches, respectively, when compared to the average flow head of the MGS without notches.
- A new equation to predict $C_d$ for semi-elliptical notches within MGS was proposed which was valid for $0.01 \leq \mathrm{F_o} \leq 0.056$, $2 \leq N \leq 8$, $h/b = 0.86$, $b/D_c = 0.29$ and $h/D_c = 0.25$.

It is recommended that the current study be repeated on a larger scale to validate the results of the current study scale. Furthermore, it may be necessary to examine both free and orifice flow conditions for different notch shapes within MGS to have a better understanding of the notch flow pattern. Investigating various shapes of notches as vortex breaker countermeasures are also recommended.

## Notations

$B$ = top width of semi-elliptical notch
$C_d$ = discharge coefficient
$D_c$ = crest diameter of spillway
$d_t$ = throat diameter
$\mathrm{F_o}$ = Froude number based on flow head in case of GMS without notch
$\mathrm{F_r}$ = Froude number based on flow head in case of GMS with notch

$G$ = gravitational acceleration
$H$ = flow head above MGS crest
$H_o$ = flow head above MGS crest in case without notch
$H$ = height of semi-elliptical notch from the center
$N$ = number of semi-elliptical notch
$Q$ = flow discharge
$R^2$ = coefficient of determination
$V_o$ = approach average velocity

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## Conflict of Interest

The authors declare that there is no conflict of interest with other parties

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