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Study of the Non-Linear Static Elastic Stress-Strain State of Rod Elements

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Abstract Rod elements have wide application in the engineering structures including bridges, buildings, drilling platforms and other types of infrastructure. The ability to predict accurately the behaviour of these elements under various loads is crucial to the safety and reliability of above-mentioned structures. The aim of this work is to improve the reliability prediction of the offshore platform. One important aspect of this prediction is the study of the non-linear static elastic stress state of the rod elements. To solve these problems, designers and engineers use a number of techniques including the finite element analysis to ensure that the platform can withstand the expected loads and perform its intended function safely and reliably. The design of drilling platforms in the Caspian Sea is a complex and non-trivial task that requires careful consideration of various factors including environment, safety regulations and operational requirements. Severe environmental conditions such as strong winds, waves and currents can have a significant impact on the structural integrity of the platform, while the presence of hydrocarbons in the area poses a risk of explosions and fires. This study presents the calculation of amplitude-frequency characteristics of free oscillations of the offshore drilling platforms on an anisotropic layered base using the finite element method. The findings will help to understand better the design of the "offshore drilling platform – seabed" when operating in the Caspian Sea, notably to determine the size and shape of the elements required to support a given load or to optimise the structure design to minimise stresses in the elements.

Keywords Drilling Platform, Finite Element Method, Caspian Sea, Offshore Platform, Fortran, Vibrations

1. Introduction

Rod elements are an important component of drilling platforms used in the oil and gas industry [1]. They usually have high-strength steel in their base and their design serves for high loads and pressures, so they adhere to strict quality control to ensure their strength and durability [2]. The proper use and maintenance of these elements are crucial for safe and efficient drilling operations [3]. Regular inspections and tests are necessary to ensure that the elements are free from defects and able to withstand the extreme conditions encountered during drilling [4].

One widely used approach to studying the non-linear static elastic stress state of rod elements is the finite element method (FEM) [5]. FEM involves discretizing elements into smaller segments and solving the governing equations for each segment to approximate the overall behaviour of the structure [6]. While numerous studies have demonstrated FEM's efficacy in analysing non-linear stresses across various materials, limited attention has been given to alternative methodologies, such as analytical solutions or hybrid approaches, which could complement FEM for specific scenarios, including stress-strain analysis in rod elements [7]. The stress state of the rod elements has a definition of the relationship between the external loads applied to the element, corresponding internal stresses and strains developed in the element [8]. In a non-linear elastic state, the stress-strain relationship is not proportional and the element does not return to its original shape when the load is removed [9]. The study of the non-linear static elastic stress state of rod elements involves the determination of the stresses that evolve in the element under various loads [10]. This usually requires mathematical modelling and analysis techniques, which help the engineers to predict the behaviour of an element under different conditions [11].

Y.Q. Wang et al. [12] conducted an experimental study of the bending behaviour of aluminium alloy beams under concentrated loads. Experimental data and subsequent analysis have shown that beam thicknesses and initial geometric defects have a significant effect on the local load of beam stability loss. Likewise, X. Luo et al. [13] carried out a finite element analysis of a buried polyethylene pipe to study its behaviour in a seismic landslide using their 3D finite element model. The FEM also helps to predict the mechanical behaviour of rubber materials, known for their non-linear properties. A study by A.H. Muhr [14] developed a three-dimensional FEM model to simulate the stress-strain state of rubber materials sustaining large deformations. The study showed that the stress-strain curve of the rubber is prominently non-linear and depends on the strain rate and loading conditions. Another material extensively researched for non-linear stress with FEM is composite materials. For example, J. Zhou et al. [15] used FEM to study the mechanical behaviour of fibre-reinforced composites under tensile loads. They found that the stress-strain behaviour of the composites was prominently non-linear due to interphase friction between the fibres and the matrix.

The above overview shows that FEM is a powerful tool for the analysis of non-linear stresses in various materials. These studies demonstrate that non-linear stress analysis using FEM is crucial for predicting the mechanical behaviour of materials, which do not meet Hooke’s law. The FEM allows the researchers to study the behaviour of materials under different loading conditions and to predict the tensile strength of materials and fracture modes. Another approach is to use the analytical methods to solve equations that directly describe the behaviour of an element [16]. This methodology is frequently applicable for simple elements where analytical solution of the equations is possible. Regardless of the approach, the study of the non-linear static elastic stress state of the rod elements requires a comprehensive understanding of the material properties of the element, including its stiffness, strength and plasticity. It also demands a thorough knowledge of the loading conditions that the element may experience during its intended application.

This study aims to develop a method for predicting the behaviour of the offshore platforms under active operating conditions. This study presents the results of a study of free oscillations of an offshore platform on an anisotropic layered base.

2. Materials and Methods

When using the finite element method, the first step is to select the right differential equation that correctly describes the model under study. The equation of motion of the system in matrix form is:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{Q(t)\}. \tag{1}$$

The solution is in a homogeneous system of differential equations of motion. When determining the solution, the two cases should be distinguished: oscillations with and without damping. This problem’s solution is the following differential equation:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} = 0. \tag{2}$$

If the vector of generalised displacements in the nodes of the system is as:

$$u = u_0 \sin(\omega t + \theta). \tag{3}$$

Then matrix equation (1) takes the following form:

$$([K] - \omega^2[M])\{\bar{u}\} = 0. \tag{4}$$

Represents the equation of eigenoscillations of the system in question. The system of algebraic equations (4) has a solution other than the trivial one only when the determinant of this system is zero, i.e.:

$$[K] - \omega^2[M] = 0. \tag{5}$$

If the expression (5) will be in expanded form, it represents a characteristic polynomial of *n*-order under ω with n roots. Since $[K]$ and $[M]$ are positively determined matrices, all the roots of the polynomial are positive real numbers and represent eigenfrequencies and eigenforms of the oscillations of the system. For each value of ω, there is one eigenvector (u_i), representing the eigenform of the oscillation. These vectors bear a name of the modes of the system. As is well known, the system (5) is also applicable when solving problems concerning the behaviour of the systems under forced oscillations. When the value of ω approaches eigenfrequency, the response increases and the phenomenon of resonance occurs. All methods generally relate to the standard kind of eigenvalue tasks:

$$[H]\{U\} = \lambda\{U\}, \tag{6}$$

where $[H]$ is a symmetric square matrix.

Equation (5), after converting the $[K]$ matrix and introducing the $\lambda = \frac{1}{\omega^2}$ notation, written as:

$$[K]^{-1}[M]\{u\} = \lambda\{u\}. \tag{7}$$

There is no symmetry in general, if the matrix $[K]$ is in the following form:

$$[K] = [L][L]^T, \tag{8}$$

$$[K]^{-1} = [L]^{T-1}[L]^{-1}, \tag{9}$$

where $[L]$ is a matrix with zero coefficients over the main diagonal, then after multiplying (7) by $[L]^T$ it will be:

$$[L]^{-1}[M]\{u\} = \lambda[L]^T\{u\}. \qquad (10)$$

Assuming that $[L]^T\{u\} = \{U\}$, the following equation is got:

$$[H]\{U\} = \lambda\{U\}. \qquad (11)$$

It is the same as (6), but here the matrix $[H]$ is symmetric and has the following form:

$$[H] = [L]^{-1}[M][L]^{T-1}. \qquad (12)$$

After determining several values $\{\lambda_i\}, (i = 1,2 \ldots n)$ that correspond to the basic tones, the eigenvectors and then the eigenvalues are found. When investigating oscillations by the finite element method, the largest eigenvalue is determined by a simple iterative method, where the following is needed:

- set some value of the vector $\{U\}$, further called $\{U_{g1}\}$. Since the eigenvector characterises some eigenfunction of the system, only the relative values of the $\{U\}$ vector components are requisite. Therefore, it can be assumed that one of the unknowns, e.g. $x1$, always equals one;
- calculate the value AX_{g1};
- produce the multiplication AX_{g1} product, which is a vector written in the following form $\lambda_{g2}X_{g2}$, where λ_{g2} is a multiplier that the component x1 of X_{g2} vector is again equal to one and the other variables take on corresponding values;
- compare X_{g2} and X_{g1} and if they do not differ within a given accuracy from each other, then the resulting set forms an eigenvector and the multiplier represents the largest eigenvalue;
- the other values and their corresponding eigenvectors are determined by iterative method. This method modifies the matrix $[H]$ to reduce the maximum eigenvalue of the system to zero. This results in the next highest eigenvalue λ, after which the iteration process continues.

Figure 1 shows a model of a drilling platform. Using the finite element method, the platform is replaced by a system with a finite number of node points interconnected via non-inertia rods. The masses gather at the nodal points, with external loads applied at the nodes.

The mass matrix is a symmetric and positively distributed matrix of n-order, where n is the number of degrees of freedom of the element. While the damping matrix of the element is defined similarly, using a constant damping factor may oversimplify the dynamic interactions of offshore platforms. Real-world scenarios, such as extreme weather conditions and anisotropic seabed interactions, often introduce non-linearities that are not adequately captured by simplified damping models. Incorporating non-linear damping and adaptive boundary conditions can improve the accuracy of the numerical solutions obtained through the finite element method, making them more representative of actual platform behaviour under operational condition

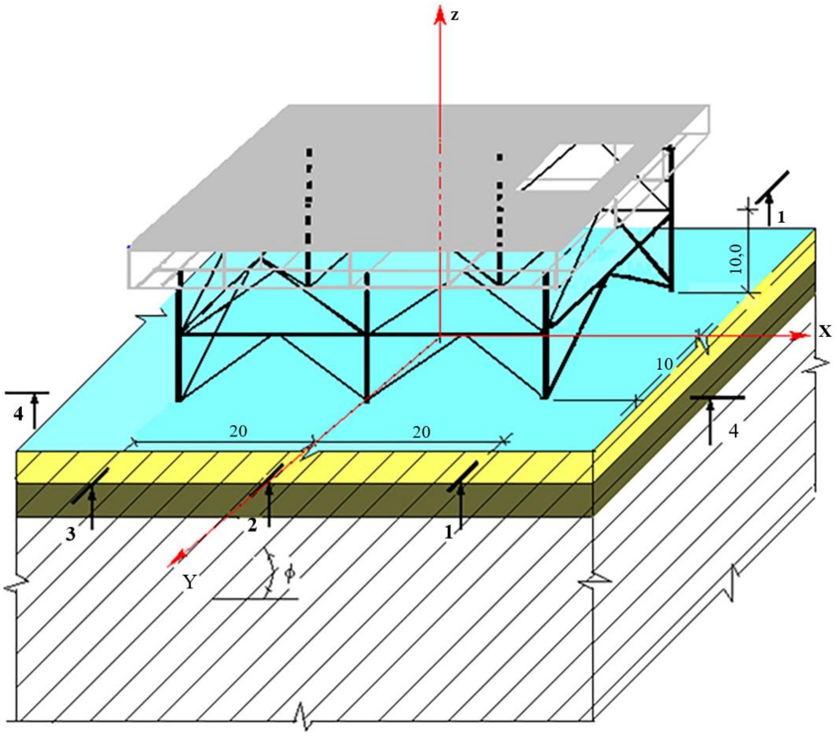


Figure 1. Geometric dimensions of an offshore drilling platform

3. Results

The theoretical basis for offshore platform construction features several areas, including naval architecture, structural engineering, hydrodynamics, and marine ecology. The design and construction of offshore platforms require careful consideration of various factors such as site conditions, wave loads, wind loads, seismic activity, hydrodynamic forces, and environmental impact. Dynamic interactions between seismic activity and hydrodynamic forces introduce additional complexity to the structural design. These interactions involve the coupling of sudden, high-magnitude forces from seismic events with the persistent effects of waves and currents, leading to amplified stresses and potentially resonant vibrations that challenge platform stability. Naval architecture has parallels with the design and construction of marine structures, including ships, boats, and offshore platforms [17]. The principles of naval architecture are essential for the design of sustainable and efficient offshore platforms. The construction design shall take into account the buoyancy, stability, and resistance of the platform to external loads such as waves, currents, and seismic-induced forces. The theoretical basis of naval architecture draws on several disciplines, including physics, mathematics, fluid mechanics and materials science. One of the basic theoretical foundations of naval architecture is the principles of hydromechanics, which relates to the behaviour of fluids in motion. Understanding the behaviour of water and air around the structure is essential for designing platforms that can effectively withstand external forces and remain stable.

Hydrodynamics or fluid dynamics is the study of fluid motion, which is crucial to the design of the offshore platforms [18]. To design a stable and safe platform, it is necessary to understand the behaviour of waves, currents and other hydrodynamic phenomena. The specialists should consider the following factors as wave height, frequency and direction as well as the effect of currents and tides when designing the platform. The basic equations of fluid dynamics are the Navier-Stokes equations, which describe the motion of fluids in terms of speed, pressure and density. Here, the mass conservation equation is of the following form:

$$\frac{\delta\rho}{\delta t} + \nabla + \rho V = 0. \tag{13}$$

The moment of mass conservation with account of the action of external forces is of the following form:

$$\rho\left(\frac{\delta V}{\delta t} + V \cdot \nabla V\right) = -\nabla\rho + \mu\nabla^2 V + f, \tag{14}$$

where ρ is the density of the fluid, v is the velocity vector, p is the pressure, μ is the viscosity of the fluid and f is any external force applied to the fluid. The operator ∇ denotes the gradient operator and the operator denotes the scalar product.

The Navier-Stokes equations are very complex and non-linear and the solutions to these equations are often difficult to obtain. As a result, many simplifications and approximations are present in the practical applications of fluid dynamics, such as the design of aircraft, ships and other vehicles that interact with liquids. As it can be seen, the mathematics plays a significant role in the theoretical basis of hydrodynamics. Mathematical models and simulations help to analyse the behaviour of fluids around objects and predict the behaviour of objects under different conditions. These models and simulations can take into account the factors such as wave height, wave frequency and water viscosity. Physics is also an important component of the theoretical basis of hydrodynamics. Principles of physics, as Newton's laws of motion, help to understand the behaviour of moving fluids and their interaction with objects such as ships and boats. These principles also help to understand the effects of external forces such as waves, wind and currents on the movement of objects.

Another important aspect of the theoretical basis of hydrodynamics is experimental verification. The experiments have goals to test mathematical models and simulations and to collect data on the behaviour of fluids around objects. These experiments help to improve the accuracy of hydrodynamic models and simulations and provide valuable information for the design of ships and other offshore structures. In addition, the theoretical basis for naval architecture is material mechanics, which deals with the behaviour of solids under external loads. The materials used in shipbuilding, such as steel and aluminium alloys, have special properties affecting their strength, stiffness and impact resistance. Naval architects must understand the mechanics of materials in order to design ships that can withstand the stresses and strains of waves, wind and other external loads. Offshore platforms, such as oil derricks, are subject to vibration from waves, wind and currents. The process of offshore platform vibration is the transfer of energy from the external environment to the platform structure.

Mathematics plays a crucial role in construction design, especially when analysing and optimising constructions. The theoretical foundations of mathematics help to calculate the loads and to assess the strength of different materials and design of the structures that can overcome the expected loads. Civil engineers also use mathematical models and simulations to analyse the behaviour of structures in different scenarios. Mathematical analysis helps to calculate the oscillations of the platform. The vibration process can split into two main types: forced vibration and free vibration [19]. Forced vibration occurs when external forces, such as waves and wind, act on the platform and cause it to vibrate. On the other hand, free vibration emerges from the inherent properties of the platform, for example its shape and rigidity. Forced vibration can be split into two categories: deterministic and random. Deterministic vibration occurs when the frequency and amplitude of external forces are known and predictable. For example, a regular wave hitting the platform will cause a predictable vibration. On the other hand, random vibration occurs when external forces are unpredictable like rough seas or strong winds.

Free vibration originates from the structural properties of the platform such as its shape, mass and rigidity. When the platform is a subject to external forces, it will vibrate with its own frequencies, determined by its structural properties. The platform's eigenfrequencies can be calculated using mathematical models, and it is important to understand these frequencies to ensure that the platform's design meets the vibration levels. Offshore platform vibration can have several consequences, including structural damage, equipment failure and discomfort to personnel. To mitigate the effects of vibration, the offshore platforms have vibration-damping systems in their design such as shock absorbers and vibration isolators. These systems absorb or decrease vibration energy, thereby reducing the impact on the platform structure and personnel. In general, the vibration process of offshore platforms is complex and depends on many factors, including external forces, structural properties and damping systems. Understanding the vibration process is crucial to the design and maintenance of the offshore platforms to ensure their safety, stability and performance. Offshore platform vibration has mathematical description through several equations. The equation of motion among these describes the movement of the platform under the influence of external forces. This has the following expression:

$$m\left(\frac{d^2x}{dt^2}\right) + c\left(\frac{dx}{dt}\right) + kx = F, \tag{15}$$

where m is the mass of the platform, c is the damping factor, k is the stiffness of the platform, x is the displacement of the platform and F is the external force acting on the platform.

The calculation of eigenfrequency of the platform uses the following equation:

$$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}, \tag{16}$$

where f is the eigenfrequency of the platform.

The calculation of a damping coefficient uses the following equation:

$$\zeta = \frac{c}{2\sqrt{km}}, \tag{17}$$

where f is the platform-damping factor.

The resonant frequency of the platform occurs when the external force applied to the platform is equal to its eigenfrequency. The resonance frequency of the platform can undergo calculation using the following equation:

$$f_r = \frac{f}{\sqrt{1-\zeta^2}}, \tag{18}$$

where f is the resonance frequency of the platform.

These equations are essential for understanding and predicting the behaviour of the offshore platforms under different conditions. They have application in the design and analysis of the offshore platforms to ensure their safety, stability and performance. Structural engineering provides the theoretical basis for the design of platform support structure. The structural elements of the platform should support the weight of the platform and any additional loads such as equipment and personnel as well as dynamic loads caused by waves and wind.

Structural physics deals with the behaviour of structures under different types of load, such as gravity, wind, earthquakes and temperature changes. Theoretical basis of structural design includes understanding of the laws of physics and the way they apply to structures. This understanding allows construction engineers to design structures that are safe and stable under the expected loads and conditions. This requires a large theoretical basis in load mechanics, which is essential for the design of structures. This area of research deals with the behaviour of materials under different types of loading, such as tension, compression and bending. Construction engineers use the materials mechanics to select suitable materials for different parts of a structure, to define dimensions of the structural elements and to ensure that the structure can withstand the expected loads. The relationship between stress and deformation in construction is described in Hooke's law, which states that the stress in a material is proportional to the strain. Mathematically, this has the following expression:

$$\sigma = E\varepsilon, \tag{19}$$

where σ is stress (force per unit area), E is modulus of elasticity (also known as Young's modulus), and ε is strain (change in length per unit length).

This equation assumes that the material behaves elastically, which means that it will return to its original shape and size after load removal. In reality, the materials can also exhibit plastic deformation or fracture at higher stresses or strains and more complex construction models describe this behaviour. The stress-strain relationship in materials may not always conform to Hooke's law, especially when the material is a subject to high stresses or large deformations. In these cases, a more complex constitutive model helps to describe the behaviour of the material. One such model is the non-linear elasticity model, which assumes that the material is still elastic but that the relationship between stress and deformation is non-linear. The equation for the static non-linear elastic stress-strain curve has the following expression:

$$\sigma = E\varepsilon + k\varepsilon^3, \tag{20}$$

where k is constant representing the non-linearity of the material. The first term on the right side represents the linear elastic behaviour of the material and the second term represents the non-linear behaviour.

It should be noted that the non-linear-elastic model is only valid for small deformations and may not accurately describe the behaviour of the material at large strains or under plastic deformation or fracture. Civil engineers must also consider environmental factors when designing structures. These factors include seismic activity, wind, snow loads and temperature variations. By understanding the physics and mechanics of these environmental factors, they can design structures that will be safe and stable under the expected loads and conditions. Marine ecology deals with these issues. The presence of offshore drilling platforms in the Caspian Sea has profound long-term implications for marine ecology. While the platforms provide opportunities for economic growth and resource extraction, they may also disrupt local ecosystems. The installation and operation of such platforms can lead to habitat fragmentation, seabed disturbance, and alterations in marine biodiversity. Additionally, chronic exposure to hydrocarbons, drilling waste, and noise pollution can significantly impact aquatic life, including migratory species. To mitigate these long-term environmental impacts, advanced design principles have been implemented. These include minimizing the seabed disturbance through optimized structural layouts, employing eco-friendly materials, and integrating noise-reduction systems to protect marine fauna. Furthermore, sediment management techniques are utilized to prevent large-scale displacements, thereby preserving benthic habitats. The integration of monitoring systems is also critical to continuously assess and adapt operations to reduce ecological damage over time. These measures aim to balance operational efficiency with the preservation of the Caspian Sea's ecological integrity, contributing to a sustainable approach to offshore resource development.

Based on the compiled algorithm, the specialists developed software for calculating free oscillations in the Fortran environment. Although Fortran is widely used in scientific and engineering applications due to its efficiency in complex mathematical calculations and support for arrays, matrices, and large datasets, modern alternatives are worth considering. Python-based finite element libraries, such as FEniCS and PyMesh, offer enhanced accessibility, user-friendly syntax, and integration with a wide range of computational tools. These Python libraries provide a high-level interface for finite element algorithms and facilitate visualization and post-processing, broadening the scope of potential applications beyond traditional Fortran environments. Fortran also has several libraries and frameworks available for finite element analysis, for example "FEniCS Project", which provides a high-level interface for implementing finite element algorithms. In addition, there are several commercial and open source software packages written in Fortran, which present advanced finite element analysis capabilities, such as Abaqus and ANSYS. This makes it particularly suitable for numerical modelling and simulations.

Building a finite element model of an offshore platform in a Fortran programming environment can be a complicated task, but it is an important step in the design and analysis process. Fortran is a high-level programming language well suited to scientific and engineering applications due to its powerful numerical capabilities and efficient memory management. The first step in constructing a finite element model of an offshore platform is to gather all necessary information on platform geometry, material properties and loading conditions. This information helps to create a detailed computer-aided design system (CADS), then importable into Fortran-based finite element analysis software. Once the CADS model has been imported, the next step is to create a grid model, which involves splitting the model into smaller elements that can be analysed individually. This process can be time-consuming and requires careful consideration to ensure that the grid accurately represents the geometry of the platform and that the dimensions of the elements meet the desired level of accuracy, as shown in Figure 1.

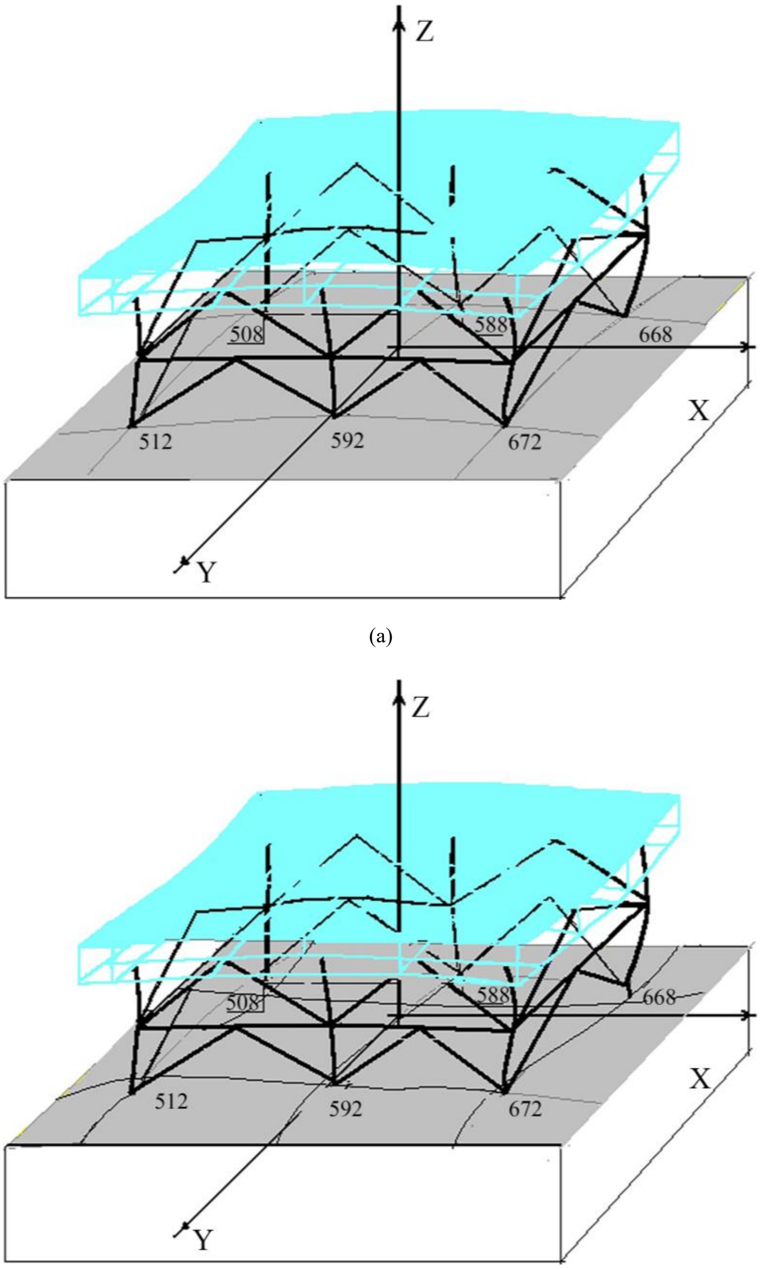
Upon creation of the grid model, material properties and loading conditions are entered for the various elements. This includes determining the stiffness and strength characteristics of the platform structure and the magnitude and direction of any applied loads such as wind, waves and currents. Fully defined model is solvable using a numerical method like the finite element method. This includes solving the set of equations mentioned in the previous section describing the behaviour of the platform under given loading conditions. The results of the analysis can then be applicable to assess the performance of the platform and to optimise its design. In conclusion, building a finite element model of an offshore platform in Fortran requires close attention to detail and a comprehensive understanding of both the geometry of the platform and the basic principles of finite element analysis. However, with the right tools and expertise, a well-built model can provide valuable information on platform behaviour and help ensure its security and reliability.

Developing the software helps to read the input data describing the problem, such as the geometry and material properties of the structure and then to implement the algorithms by finite element method. The software can also include post-processing capabilities for visualisation and analysis of results. With its use, the specialists can study free oscillations of an offshore platform on an anisotropic layered base. The calculation area size helps to exclude the influence of the fixed boundary on the dynamic response. The geometric dimensions of the calculation object and calculation area feature in Figure 1. Figure 2 shows free oscillation patterns of the calculation area and the calculation object.

Table 1 shows the first seven eigenfrequencies of the system, calculated using the developed software.

To understand the dynamic response of the offshore platform under various operational conditions, it is essential to examine the system's vibration characteristics. Figure 3 presents the dynamic response curves of the system, showing how the frequency of oscillations varies with the tilt angle of the seabed. The data, based on the calculated eigenfrequencies of the platform, demonstrate the influence of tilt angle on the frequency response. As the tilt angle increases from 0° to 60°, the system's frequency response decreases, indicating the sensitivity of the platform's behaviour to changes in the seabed conditions. This information is crucial for designing and optimizing offshore platforms, ensuring they can withstand external forces such as waves and wind while maintaining stability.

When examining the results, it becomes clear that tilt angle plays a very important role in determining the values of the oscillation frequency. An effect on the values is significant with angle change. In particular, a tilt angle increase leads to a decrease in the frequency response values. Thus, the authors can conclude that tilt angle has a significant effect on the frequency response of the system and this factor is important in the design and analysis of these systems.



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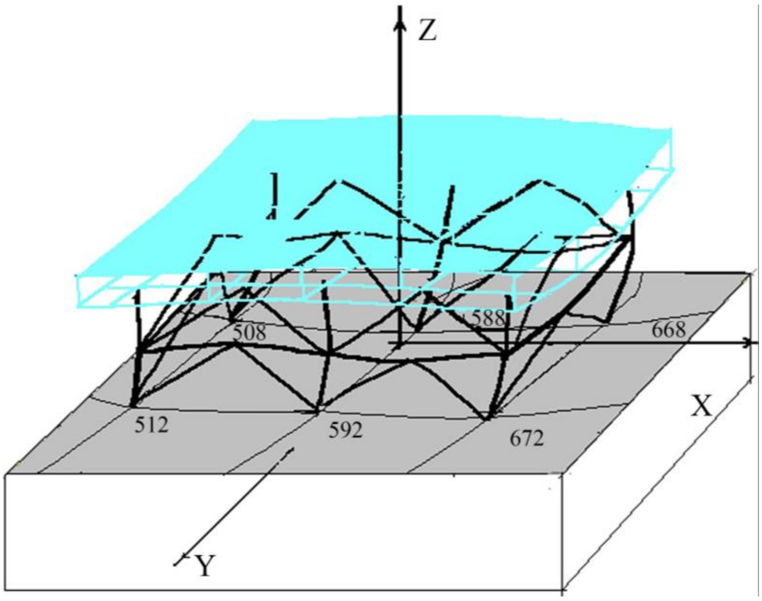

(c)

Figure 2. Forms of eigenoscillations of the "offshore drilling platform – anisotropic base" system, a) first form, b) second form, c) third form

Table 1. Amplitude-frequency characteristics of the free oscillations of the "offshore drilling platform – anisotropic base" system at various tilt angles of the isotropic plane of the seabed

No. frequency	Value ω, sec^{-1}								
	ψ=0 (degrees)			ψ=30 (degrees)			ψ=60 (degrees)		
	φ=0	φ=30	φ=60	φ=0	φ=30	φ=60	φ=0	φ=30	φ=60
1	1.321	1.308	1.001	1.321	1.308	1.001	1.321	1.308	1.001
2	1.332	1.321	1.211	1.332	1.321	1.211	1.332	1.321	1.211
3	1.51	1.481	1.209	1.51	1.481	1.209	1.51	1.481	1.209
4	3.111	2.998	2.565	3.111	2.998	2.565	3.111	2.998	2.565
5	3.674	2.771	2.674	3.674	2.771	2.674	3.674	2.771	2.674
6	3.712	2.412	2.216	3.712	2.412	2.216	3.712	2.412	2.216
7	3.955	3.111	2.989	3.955	3.111	2.989	3.955	3.111	2.989

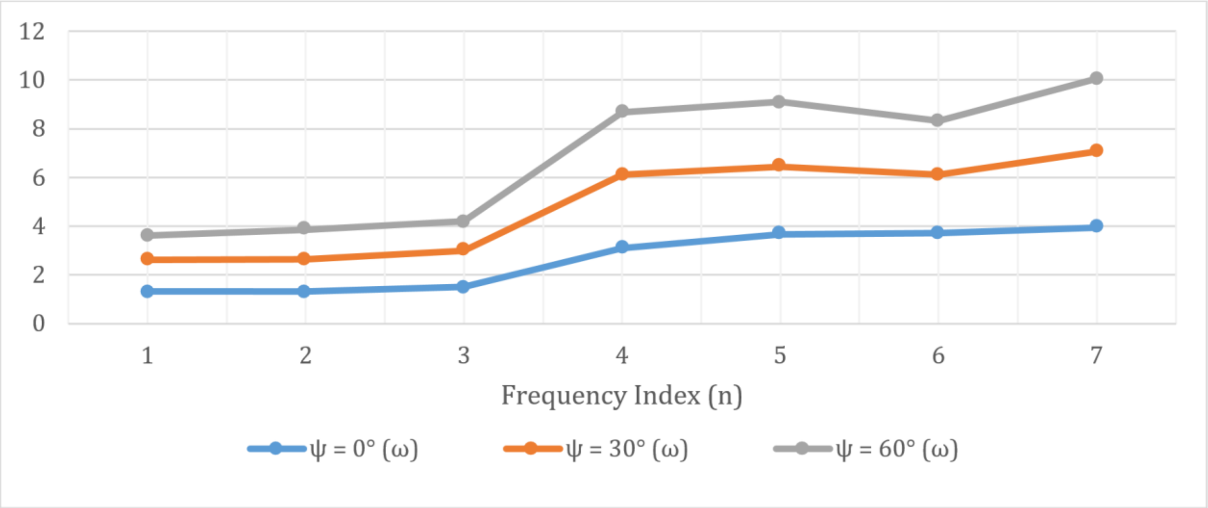

Figure 3. Dynamic response curves for offshore platform under various tilt angles

4. Discussion

The FEM method is active today to study the offshore platform structures. A study by D.E. Brekke et al. presents the findings that included a detailed study of the application of FEM to marine concrete structures [17]. The researchers discuss the various factors to consider in the design of these structures, including wave loads, seabed conditions and structural response. The authors also describe various FEM post-processing techniques capable to analyse the structural response of the offshore concrete structures. The use of these methods will increase the accuracy and reliability of the proposed approach. Post-processing in FEM plays a critical role in analysing simulation data, but several challenges persist. These include the simplification of models, which can introduce inaccuracies, and uncertainties in input data that may lead to erroneous conclusions. Advances in post-processing techniques have significantly improved accuracy and usability. Recent developments, such as adaptive meshing and multi-scale modelling, enhance the precision of simulations by dynamically refining mesh quality and incorporating varied scales within the same model. Additionally, integration of machine learning algorithms for anomaly detection in post-processing has streamlined the identification of discrepancies, reduced manual effort and improved reliability. Studies comparing FEM results with physical tests have highlighted that robust post-processing strategies, when applied with careful parameter optimization, yield results with high accuracy and reliability, offering substantial cost and time efficiencies over conventional testing methods. Overall, it is clear that FEM post-processing techniques are a valuable tool for the design of the offshore structures with account of all limitations.

Another study by Y. Wang et al. [18] applies the finite element method to investigate the bearing capacity of pipes supporting wellheads. The study focuses on the analysis of submarine wellhead stability in deep water conditions. The interface patterns define the area of contact between the pile and the ground, which can have a significant impact on the bearing capacity of the pipe. The authors' conclusions suggest that the use of FEM can provide a more accurate analysis of the bearing capacity of pipes compared to traditional methods. The authors demonstrate that pile-soil contact interface models play a crucial role in determining the bearing capacity of the pipe and suggest a modified interface model that provides a more accurate analysis. In general, using the described experience of the finite element method to analyse the bearing capacity of pipes for wells in deep water can significantly improve the capabilities of the approach presented in this study.

The study by H. Aslaksen et al. [19] demonstrates the application of integrated FEM modelling to optimize drill string systems, achieving greater accuracy and efficiency compared to traditional methods. The modeming accounts for interactions between the drill bit, drill string, and wellbore, enabling the identification of design issues and optimization of operational performance. Similarly, B.B. Sahari et al. [20] utilized FEM to simulate the crash behaviour of natural gas vehicles, specifically analysing the influence of tank mounting designs on impact resistance. These studies emphasize the versatility and adaptability of FEM for addressing complex engineering challenges. The application of FEM in the current study focuses on the vibrational dynamics of offshore platforms supported by anisotropic seabed bases. This approach examines eigenoscillations and assesses the impact of seabed tilt angles on platform stability under operational conditions. The analysis integrates environmental factors and extends the scope of FEM methodologies to address stability optimization in offshore engineering.

The consideration of anisotropic seabed conditions introduces a novel element to FEM-based structural analysis. This refinement provides insights into the vibrational behaviour of offshore platforms, which are critical for ensuring stability and safety in challenging environments such as the Caspian Sea. While H. Aslaksen et al. [19] and B.B. Sahari et al. [20] focused on drill string systems and crash-resistance modelling, respectively, the current study complements these contributions by addressing structural dynamics and stability optimization. These findings enhance FEM's predictive capabilities, offering practical methodologies for advancing engineering solutions in marine and offshore applications.

There are many examples in the literature of analyses of the characteristics of free oscillations and oscillation patterns of different objects. For example, J.W. Ringsberg et al. [21] analyse the vibration behaviour of a semi-submersible platform, widely used in offshore oil and gas exploration and production. This example demonstrates the use of the finite element method to analyse the dynamic behaviour of a similar platform designed in a way to show a stable performance, important in rough sea conditions. J.A. Ruhl [22] conducts a detailed review of the behaviour of the offshore platforms and compares the observed behaviour with theoretical predictions. The author draws on observations of the construction and operation of various offshore platforms to describe their behaviour and discusses the different types of the offshore platforms and their response to wave forces, current forces and wind forces. The findings of this study suggest that theoretical predictions often overestimate the response of the offshore platforms. The future research should focus on developing high accuracy models to predict the behaviour of the sea platforms. The tilt angle study presented in this study can in turn significantly improve a research methodology of the offshore platform by reducing the difference between modelling and practical experimentation.

A specialized software environment is indispensable for performing advanced mathematical calculations and addressing complex problems in mathematical physics. The choice of such an environment should prioritize accuracy, reliability, and user-friendliness to ensure the effective application of computational methods. Although the Fortran software platform is known for its precision and dependability in finite element method (FEM) analyses, exploring alternative platforms could reveal tools with greater flexibility and enhanced functionality. For example, P.R.M. Lindstrom and M. Faraji [23] conducted an in-depth comparison of various FEM software packages tailored for in-service welding analysis in academic and industrial applications. Their findings underscore the importance of evaluating software not only based on technical capabilities but also on criteria such as cost-effectiveness and ease of use. They demonstrated that aligning the platform's features with specific project needs is critical for achieving accurate and efficient outcomes in FEM-based studies.

While Fortran remains a robust option, newer FEM platforms, including Python-based libraries like FEniCS, offer user-friendly interfaces and broader compatibility with modern computational tools. These tools allow for enhanced visualization, seamless integration, and efficient handling of large datasets, which are particularly advantageous for post-processing and iterative modelling. Selecting the appropriate FEM platform is fundamental to the success of advanced engineering projects. An optimal choice enhances the accuracy of simulations, reduces computational overhead, and allows researchers to adapt to evolving project demands. In light of the increasing complexity of engineering challenges, future research should focus on integrating advanced software tools with traditional platforms to maximize the benefits of FEM methodologies while ensuring adaptability and precision.

The analysis of offshore platforms requires a thorough consideration of environmental conditions to ensure reliable and realistic modelling outcomes. In this context, the specific conditions of the Caspian Sea provide a valuable framework for understanding platform behaviour. The unique environmental factors in this region, such as hydrodynamic forces, seabed conditions, and ecological considerations, are critical for developing accurate predictive models. H.R. Ghafari et al. [24] conducted a study examining the hydrodynamic interaction between two floating platforms in the Caspian Sea, with a particular focus on the performance and stability of these structures under varying environmental conditions. Using finite element modelling, they analysed the forces and moments acting on the platforms. Their findings revealed that interactions between platforms significantly impact their dynamic behaviour, with increased hydrodynamic forces and moments caused by wave activity leading to reduced performance and stability. This study underscores the importance of accounting for platform interactions in regions with complex hydrodynamic characteristics, such as the Caspian Sea.

In addition to hydrodynamic factors, the broader environmental challenges of the Caspian Sea are comprehensively analysed by I.S. Zonn [25]. I.S. Zonn's work [25] highlights the environmental issues surrounding the region, including the ecological consequences of intensive oil and gas extraction. The interplay between industrial activity and the marine ecosystem necessitates sustainable design approaches for offshore platforms [26]. These considerations ensure operational efficiency while minimizing the ecological footprint of such structures. Integrating these insights into finite element modelling enhances the accuracy and applicability of platform designs [27].

Overall, the reviewed studies emphasize the critical role of environmental conditions in modelling offshore platforms. This research builds upon these findings by presenting a comprehensive analysis of eigenoscillations for platforms on anisotropic seabed bases, addressing dynamic stability challenges in the Caspian Sea. By bridging environmental considerations with advanced FEM methodologies, the study contributes to the development of safer and more efficient offshore platform designs.

It is clear from the above that integrating the latest post-processing techniques, alongside emerging technologies such as AI-driven optimization and sustainable materials, is essential for further research. This integration will provide a more comprehensive understanding of offshore platform behaviour, enhance the precision of predictive models, and support the development of innovative, environmentally conscious designs. Moreover, the inclusion of these advanced methodologies addresses industry challenges, such as sustainability and operational efficiency, fostering a more robust framework for future offshore engineering solutions. Post-processing techniques play a crucial role in the analysis of data obtained from experiments or simulations. They allow extraction of useful information from raw data and improve identification of patterns and trends. In addition, environmental conditions can have a significant impact on the behaviour of systems. When comparing theoretical and experimental data, the factors of temperature, humidity and atmospheric pressure can affect material properties, sensor performance and measurement accuracy. It is therefore important to monitor and follow these conditions carefully in further experiments or simulations and consider them when building models. The use of modern FEM approaches with all environmental parameters will deepen the understanding of the systems under study and identify new opportunities for innovation and improvement.

5. Conclusions

The authors presented the identification of oscillation forms and calculation of eigenoscillations of the system "offshore drilling platform – anisotropic base". This task involved determining the fundamental characteristics of the structural behaviour of the system required to optimise the design and ensure its stability in service. Accurate determination of the vibration modes and eigenfrequencies of the offshore drilling platforms is crucial to prevent structural damage and to ensure the safety of the personnel working on them. Correct identification of vibration modes and eigenfrequencies can provide valuable information about the dynamic behaviour of the system, enabling the engineers to design more efficient and reliable offshore drilling platforms. The ability to predict system eigenfrequencies with accuracy can also help to optimise platform performance, to reduce operating costs and to minimise the environmental impact of drilling operations.

The authors suggested a methodological basis for calculating and improving accuracy and efficiency based on the finite element method. It comprises the developed algorithm and its implementation by a package of application software using a programming language “Microsoft Fortran”. The proposed methodological framework has to simplify the calculation process and to decrease the computational resources required for analysis. This has significantly reduced the time and resources required for modelling and simulation and increased the accuracy of the results. The proposed methodological framework can also fit to the specific needs of different sectors and applications. This provides a more flexible and versatile approach to finite element analysis, adjustable to the requirements of specific projects or applications.

The multi-variant calculations reveal seven first eigenoscillations and three forms of eigenoscillations. Analysis of the data indicates that the required parameters depend only on the value of the angle φ. A complex picture of the deformation process of the studied object and area includes tension, bending, shift and compression in all directions. The first form of oscillation demonstrates a kind of bulging of the base surface. In the third form of oscillation, one side goes down and the other side goes up. With the fifth form of oscillation, the base surface curves inwards. All these processes are superimposing on the eigenoscillations of the offshore platforms. The eigenfrequencies of the system have an order of magnitude lower than the wave and wind load frequencies. In addition, the authors outlined the approaches for further research, which should consider the latest post-processing techniques and the influence of environmental factors to provide a better understanding of the behaviour of the system under study and to improve the accuracy and reliability of the proposed method.

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