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Equitable Domination in Fuzzy Digraphs

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Abstract Fuzzy digraphs, an extension of directed graphs enhanced with fuzzy set theory, provide a robust framework for representing uncertainty in networks with relationships of varying strengths. In fuzzy digraphs, nodes represent entities, and directed edges are assigned fuzzy values to measure the degree of connection between them. This approach is particularly useful for modeling uncertainty in network applications like communication networks, transportation systems, and social networks, where relationships may be imprecise, uncertain, or dynamic. Equitable domination in a fuzzy graph involves selecting a dominating set where the membership values between dominated and dominating nodes are balanced. The concept of an equitable dominating set is beneficial in situations when a balanced or equitable distribution is required, as it expands on the idea of a traditional dominating set by including a fairness criteria. By providing a sophisticated tool for assessing and creating networks and systems, this idea closes the gap between dominance and equitability. In this article, we extend equitable concept to weighted graphs and apply it in more complex scenarios where vertices have varying levels of importance or weight. We studied the ideas of fuzzy equitable domination in fuzzy digraphs. We defined the fuzzy equitable domination number and studied the properties of minimal fuzzy equitable dominating set and fuzzy equitable independent sets.

Keywords Fuzzy Digraph, Fuzzy Equitable Domination, Equitable Degree, Minimal Equitable Dominating Set, Fuzzy Independent Set

1 Introduction

The theory of a fuzzy graph developed as a result of uncertainty and ambiguity. Set theory focuses on the conditional statements that use the numbers 0 and 1 to denote whether or not an element is present in the set. The idea of an uncertain situation based on the presence of an element in the closed interval [0,1] is dealt by fuzzy set theory. Lofti. A. Zadeh [1] discovered the value of fuzziness in different situations and put forward the fuzzy logic idea. Additionally, he examined how the idea of a linguistic variable was used [2] and how it related to approximate reasoning. Later, other studies were conducted to evaluate the application of fuzzy logic in real-world scenarios. Fuzzy digraphs first came up in the work of Mordeson and Nair [3] and more recent developments have been noted by Kumar and Lavanya [4]. Numerous applications that are applicable to everyday life are being studied. Fuzzy digraphs are used to represent some of them graphically. Likewise, the properties of fuzzy directed graphs and fuzzy finite state machines are very similar.

The theory of dominance in fuzzy graphs utilizing effective edges was first presented by A. Somasundaram and S. Somasundaram [5]. Dominance in fuzzy graphs using strong arc was studied by A. Nagoorgani and V. T. Chandrasekaran [6]. C. Natarajan and S. K. Ayyaswamy [7] developed the concept of strong (weak) domination in fuzzy graphs and extension of strong (weak) domination in crisp graphs. O. T. Manjusha and M. S. Sunitha [8], developed the idea of 1-strong dominance in fuzzy graphs as an extension of the theory of domination in fuzzy graphs with strong edges. The introduction of a numerical model on ambulance service statistics for certain Indian villages explains why double domination in m-PIVFG(m-Polar Interval-valued Fuzzy Graph) is required for the specific application, this was discussed by Bera Sancharia et.al., [9]. Many authors studied about domination in fuzzy graphs [10, 11, 12].

Domination and its variations are a great tool for network issues. In networks, notably wireless networks and wireless sensor networks, domination in fuzzy graphs plays a significant role. These networks are utilised for a wide range of civil and military purposes, including in the food business, agriculture, disaster recovery, conferences, concerts, environmental detection and health applications. All of these could result in increased social welfare while using fewer resources. Furthermore, domination in fuzzy graphs has been applied to decision-making problems, such as determining optimal strategies in game theory and resource allocation.

In a dominance situation, a vertex can dominate another vertex by being adjacent to it. However, in real-life friendships, equality in status is more likely to lead to friendship. People tend to be friend those who are similar in wealth, education, or social rank. In a democratic organization, the executive body includes representatives from various divisions such as the general public, management, office staff, and labor force. Each delegate represents a specific area, facilitating communication. To model this real-world scenario in a graph, labels such as positive numbers can be assigned to vertices to indicate comparability. Two vertices can be considered equitable if their labels are nearly equal. The idea of equitable domination was proposed by Venkatasubramanian Swaminathan and Kuppusamy Markandan Dharmalingam [13]. The properties of equitable domination in fuzzy graphs was studied by S. Revathi and C. V. R. Harinarayanan [14].

The domain literature has little understanding of the domination in fuzzy digraphs, which is a new idea. Nirmala et al. [15] investigated the idea of domination in fuzzy digraphs. With this novel idea, they suggest a new domination parameter for fuzzy digraphs. This work aims to extend the body of knowledge on domination in both a fuzzy graph and a directed graph. It is inspired by the terms of fuzzy digraphs [3, 4] and the notions of domination of graphs [16].

In this article, we developed theoretical concepts of equitable domination in fuzzy digraphs. In addition, a number of additional concepts such as fuzzy independent and equitable neighborhood in fuzzy digraphs are discussed. Also, we obtained the equivalent condition for a set to be a fuzzy equitable domination set.

2 Preliminaries

This section presents essential definitions and key results to establish the foundation for proving the theorems. It serves as a prerequisite, offering crucial insights and establishing a conceptual framework necessary for the subsequent verification of the theorems under consideration.

Definition 2.1. *[17] A fuzzy graph $\mathscr{Q} = (\mathscr{J}, \varphi, \eta)$ is a triple consisting set $\mathscr{J}(\neq \emptyset)$ together with a pair of functions $\varphi : \mathscr{J} \to [0,1]$ and $\eta : \mathscr{J} \times \mathscr{J} \to [0,1] \ni \eta(sg) \leq \varphi(s) \wedge \varphi(g)$ $\forall\, s, g$ in $\mathscr{J}$, φ is vertex set $\mathscr{Q}$ and η is edge set $\mathscr{Q}$.*

Definition 2.2. *[17] The order p and size q of $\mathscr{Q} = (\varphi, \eta)$ are referred to be $p = \sum\limits_{m \in \mathscr{J}} \varphi(m)$ and $q = \sum\limits_{m,r \in \mathscr{E}} \eta(m, r)$.*

Through out this paper we use fuzzy graph as FG, dominating set as D-set, fuzzy equitable dominating set as FED-set, equitable dominating set as ED-set, fuzzy digraph as FDG, fuzzy independent set as FI-set, fuzzy equitable independent set as FEI-set, equitable independent set as EI-set.

Definition 2.3. *[14] Let $\mathscr{Q} = (\varphi, \eta)$ be a FG on a graph $\mathscr{Q}^* = (\mathscr{J}, \mathscr{E})$. $\mathbb{B} \subset \mathscr{J}$ is defined as D-set in $\mathscr{Q}$ if every vertex f in $\mathscr{J} \backslash \mathbb{B}$, $\exists\, h \in \mathbb{B} \ni hf \in \mathscr{E}$. The domination number of $\mathscr{Q}$ is the minimum cardinality taken over all D-sets in $\mathscr{Q}$ and is indicated by $\gamma(\mathscr{Q})$ or simply γ_f. A fuzzy D-set $\mathbb{B}$ of a FG $\mathscr{Q}$ is said to be minimal D-set of $\mathscr{Q}$, if for all node $f \in \mathbb{B}, \mathbb{B} \backslash \{f\}$ is not a fuzzy D-set of $\mathscr{Q}$.*

Definition 2.4. *[18] Let $\mathscr{Q} = (\varphi, \eta)$ be a FG on a graph $\mathscr{Q}^* = (\mathscr{J}, \mathscr{E})$. $\mathbb{B} \subset \mathscr{J}$ is termed as FED-set if for every $m \in \mathscr{J} \backslash \mathbb{B}$ there exists a vertex $g \in \mathbb{B}$ such that $sm \in \mathscr{E}(\mathscr{Q})$ and $|d_{\mathscr{Q}}(g) - d_{\mathscr{Q}}(m)| \leq 1$. The minimum cardinality of such a D-set is indicated by γ_{fe} and is known as FED number of $\mathscr{Q}$.*

Definition 2.5. *[18] Let $\mathscr{Q} = (\varphi, \eta)$ be a FG on a graph $\mathscr{Q}^* = (\mathscr{J}, \mathscr{E})$. A vertex $m \in \mathscr{J}$ is called degree equitable with a vertex $w \in \mathscr{J}$ if $|d_{\mathscr{Q}}(m) - d_{\mathscr{Q}}(w)| \leq 1$ and $\eta(mw) \leq \varphi(m) \wedge \varphi(w)$.*

Definition 2.6. *[14] Let $\mathscr{Q} = (\varphi, \eta)$ be a FG on a graph $\mathscr{Q}^* = (\mathscr{J}, \mathscr{E})$. Then $\mathscr{Q} = (\varphi, \eta)$ is called fuzzy regular graph, if all nodes have the same degree.*

Definition 2.7. *[14] Let $\mathscr{Q} = (\varphi, \eta)$ be a FG on a graph $\mathscr{Q}^* = (\mathscr{J}, \mathscr{E})$. Then $\mathscr{Q} = (\varphi, \eta)$ is called a biregular fuzzy graph, if all nodes have the degree either h or $h + 1$.*

Definition 2.8. *[19] Let $\mathscr{Q} = (\varphi, \eta)$ be a FG on a graph $\mathscr{Q}^* = (\mathscr{J}, \mathscr{E})$. Then for any two adjacent vertices r, t, we say that r strongly dominates t if $d_{\mathscr{Q}}(r) \geq d_{\mathscr{Q}}(t)$. Similarly t weakly dominates r if $d_{\mathscr{Q}}(r) \geq d_{\mathscr{Q}}(t)$.*

Definition 2.9. *[20] Let $\mathscr{Q} = (\varphi, \eta)$ be a FG on a graph $\mathscr{Q}^* = (\mathscr{J}, \mathscr{E})$. Then $\mathbb{B} \subseteq \mathscr{J}$ is defined as strong (weak) FED-set of $\mathscr{Q}$, if all vertex $f \in \mathscr{J} \backslash \mathbb{B}$ is strongly (weakly) dominated by some vertex t in $\mathbb{B}$. We indicate a strong (weak) FED-set by sefd-set (wefd-set). The minimum scalar cardinality of a sefd-set (wefd-set) is said to be sefd (wefd) number of $\mathscr{Q}$ and it is donated by $\gamma^{sef}(\mathscr{Q})$ $(\gamma^{wef}(\mathscr{Q}))$.*

Definition 2.10. *[21] Let $\mathscr{Q}_D^* = (\mathscr{J}, \mathscr{E})$ be a directed simple graph, where $\mathscr{J}(\neq \emptyset)$ and $\mathscr{E} = \{(s, c) : s, c \in \mathscr{J}, s \neq c\}$. A fuzzy digraph $\mathscr{Q}_D = (\varphi_D, \eta_D)$ is a pair of two functions $\varphi_D : \mathscr{J} \to [0,1]$ and $\eta_D : \mathscr{J} \times \mathscr{J} \to [0,1] \ni \eta_D(s, c) \leq \varphi_D(s) \wedge \varphi_D(c)$ for all $s, c \in \mathscr{J}$.*

A digraph is a directed graph.

Example 2.1. *Consider $\mathscr{Q}_D = (\varphi_D, \eta_D)$, where φ_D, η_D defined as follows in a fuzzy digraph.*

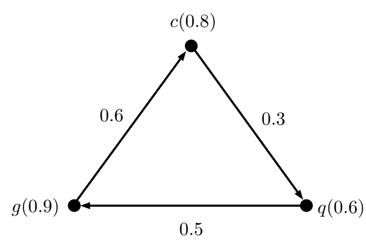


Figure 1. Example of fuzzy digraph.

Definition 2.11. *[4] Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a fuzzy digraph(FDG) on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. An Indegree of a vertex m in a FDG is addition of the η_D values of the edges incident towards the vertex $\varphi_D(m)$. The outdegree of a vertex m in a FDG is addition of the η_D values of the edges incident from the vertex $\varphi_D(m)$ to all the other vertices. We indicate the indegree by $d^-_{\mathscr{Q}_D}(m)$ and the outdegree by $d^+_{\mathscr{Q}_D}(m)$.*

Definition 2.12. *[21] The $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$ is said to be latent directed graph of $\mathscr{Q}_D = (\varphi_D, \eta_D)$.*

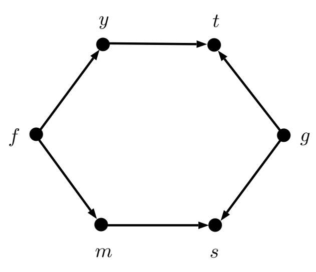
Definition 2.13. *[21] Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. Then an arc $\eta_D(f,u)$ is called as effective arc if*

$$\eta_D(f,u) = \varphi_D(f) \wedge \varphi_D(u) \text{ for } f,u \in \mathscr{J}.$$

Definition 2.14. *[21] Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (R, \mathscr{E})$. Then $\mathbb{B} \subseteq \mathscr{J}$ is a fuzzy out D-set of $\mathscr{Q}_D$ if, for all vertex $s \in \mathscr{J} \backslash \mathbb{B}$, $\exists\, c \in \mathbb{B} \ni \eta_D(s,c) = \varphi_D(s) \wedge \varphi_D(c)$. It is also known as $\varphi_D(s)$ dominates $\varphi_D(c)$.*

Definition 2.15. *[21] Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on directed simple graph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$, where $\mathscr{E}$ is an arc set and $\mathscr{J}$ is a vertex set of a FDG $\mathscr{Q}^*_D$, while φ_D is a vertex set and η_D is an arc set of a FDG $\mathscr{Q}_D$. Then for $\mathbb{B} \subseteq \mathscr{J}$, $t \in \mathscr{J} \backslash \mathbb{B}$ and $s \in \mathbb{B}$, a subset $\varphi_D(\mathbb{B}) \subseteq \varphi_D$ is a D-set of $\mathscr{Q}_D$ if, for every $\varphi_D(t) \in \varphi_D \backslash \varphi_D(\mathbb{B})$, $\exists\, \varphi_D(s) \in \varphi_D(\mathbb{B})$ such that $\varphi_D(s)$ dominates $\varphi_D(t)$. The domination number $\gamma(\mathscr{Q}_D)$ of $\mathscr{Q}_D$ is the smallest cardinality of a D-set $\mathbb{B}$ of $\mathscr{Q}_D$ and it also denoted as γ^+_f.*

Example 2.2. *Consider a FDG $\mathscr{Q}^*_D = (R, \mathscr{E}) \ni R = \{y,t,g,s,m,f\}$ and $\mathscr{E} = \{(y,t),(t,g),(g,s),(s,m),(m,f),(f,y)\}$ and its graphical representation is given below.*



The digraph $\mathscr{Q}^*_D$ is the latent digraphs of $\mathscr{Q}_D$. Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a fuzzy digraph as shown below.

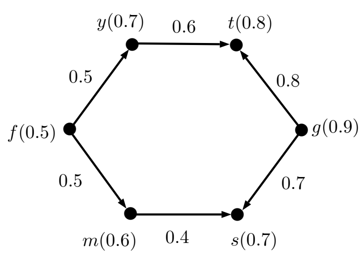


Figure 2. Example of dominating set in fuzzy digraph.

The minimum D-set of $\mathscr{Q}_D$ is $\{f,g\}$ and domination number $\gamma(\mathscr{Q}_D) = 1.4$.

Definition 2.16. *[22] Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$ and t be any vertex in $\mathscr{Q}_D$. The neighbours of t in $\mathscr{Q}$ are all vertices that are adjacent to t, in either direction. The neighbourhood of t in $\mathscr{Q}$ in $\mathscr{Q}$ is the subgraph of $\mathscr{Q}$ induced by t and all its neighbours, indicated by $\mathbb{N}^-(t)$ (inward direction) and $\mathbb{N}^+(t)$ (outward direction).*

Definition 2.17. *[21] A fuzzy digraph $\mathscr{Q}_D$ is said to be a complete FDG if for every pair of directed adjacent vertices $\eta_D(m,r) = \varphi_D(m) \wedge \varphi_D(r)$ for all m,r in φ_D.*

3 Main Results

Definition 3.1. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. Then $\mathbb{B} \subset \mathscr{J}$ is called as FED-set if, for all $r \in \mathscr{J} \backslash \mathbb{B} \exists$ a vertex $m \in \mathbb{B} \ni \varphi_D(m)$ dominates $\varphi_D(r)$ and $|d^+_{\mathscr{Q}_D}(m) - d^+_{\mathscr{Q}_D}(r)| \leq 1$. The minimum cardinality of such a D-set is indicated by γ^+_{fe} and is known as ED number of $\mathscr{Q}_D$.*

Example 3.1. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$ and described as follows.*

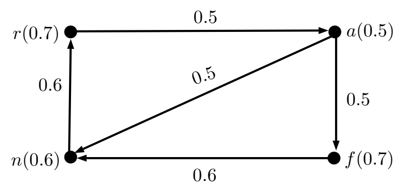


Figure 3. Example of ED-set in fuzzy digraph

Here $\mathbb{B} = \{a,n\}$ is a FED-set as $|d^+_{\mathscr{Q}_D}(a) - d^+_{\mathscr{Q}_D}(f)| = 0.4 < 1$ and $|d^+_{\mathscr{Q}_D}(n) - d^+_{\mathscr{Q}_D}(r)| = 0.1 < 1$ with equitable domination number $\gamma^+_{fe}(\mathscr{Q}_D) = 1.1$.

Example 3.2. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$ and described as follows.*

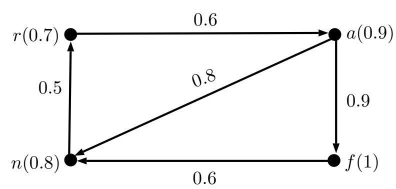


Figure 4. Example of D-set which is not ED-set.

Here $\mathbb{B} = \{a,n\}$ is a D-set but not a ED-set as $|d^+_{\mathscr{Q}_D}(a) - d^+_{\mathscr{Q}_D}(f)| = 1.1 > 1$.

Theorem 3.1. *Consider $\mathscr{Q}_D = (\varphi_D, \eta_D)$ is a fuzzy digraph on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. If $\mathscr{Q}$ is complete fuzzy digraph, then the equitable domination number $\gamma^+_{fe}(\mathscr{Q}_D) = min\{\varphi(g)\}$ for all $g \in \mathscr{J}$, where $\varphi(g)$ is the membership value of g.*

Proof. Consider $\mathscr{Q}_D$ as a complete FG. Then every edge in $\mathscr{Q}_D$ is a strong edge or strong arc and each vertex dominates all other vertices.

Consider $\mathbb{B} = \{t\}$ as an FED-set of $\mathscr{Q}_D$. So for any vertex $l \in \mathscr{J} \backslash \mathbb{B}$, we have $tl \in \mathscr{E}(\mathscr{Q}_D)$. Therefore l is dominated by t. Hence $\gamma^+_{fe}(\mathscr{Q}_D) = min\{\varphi(t)\}$ for all $t \in \mathscr{J}$. □

Theorem 3.2. *For a fuzzy digraph $\mathscr{Q}_D = (\varphi_D, \eta_D)$ on a digraph $\mathscr{Q}_D^* = (\mathscr{J}, \mathscr{E})$ and the order p of $\mathscr{Q}_D$, the below statements are satisfied*

(i) $\gamma(\mathscr{Q}_D) \leq \gamma_{fe}^+(\mathscr{Q}_D) \leq p - \Delta_{\mathbb{N}}^+(\mathscr{Q}_D) \leq p - \Delta_{\mathscr{E}}^+(\mathscr{Q}_D)$,

(ii) $\gamma(\mathscr{Q}_D) \leq \gamma_{fe}^+(\mathscr{Q}_D) \leq p - \delta_{\mathbb{N}}^+(\mathscr{Q}_D) \leq p - \delta_{\mathscr{E}}^+(\mathscr{Q}_D)$.

where $\Delta_{\mathbb{N}}^+(\mathscr{Q}_D), \delta_{\mathbb{N}}^+(\mathscr{Q}_D)$ denote the maximum and minimum neighborhood degree and $\Delta_{\mathscr{E}}^+(\mathscr{Q}_D), \delta_{\mathscr{E}}^+(\mathscr{Q}_D)$ indicate the maximum and minimum effective degree.

Proof. Assume $\mathscr{Q}_D$ is a fuzzy digraph and every ED-set, $\mathbb{B}$ of a fuzzy digraph $\mathscr{Q}_D$ is a D-set in $\mathscr{Q}_D$. So

$$\gamma(\mathscr{Q}_D) \leq \gamma_{fe}^+(\mathscr{Q}_D) \tag{1}$$

Let $r, t \in \mathscr{J}$ of $\mathscr{Q}_D$. Then $p - \Delta_{\mathbb{N}}^+(\mathscr{Q}_D)$ is the addition of the membership values of vertices excluding the maximum neighborhood degree of a vertex. Hence

$$\gamma_{fe}^+(\mathscr{Q}_D) \leq p - \Delta_{\mathbb{N}}^+(\mathscr{Q}_D) \tag{2}$$

$p - \delta_{\mathbb{N}}^+(\mathscr{Q}_D)$ is the addition of the membership values of vertices excluding minimum neighborhood degree of a vertex. it is clear that

$$\gamma_{fe}^+(\mathscr{Q}_D) \leq p - \delta_{\mathbb{N}}^+(\mathscr{Q}_D) \tag{3}$$

Let $r, t \in R, p - \Delta_{\mathscr{E}}^+(\mathscr{Q}_D)$ is the addition of the membership values of the vertices excluding the maximum effective edges of $\mathscr{Q}_D$. $\Delta_{\mathscr{E}}^+(\mathscr{Q}_D) \leq \Delta_{\mathbb{N}}^+(\mathscr{Q}_D)$ and $\delta_{\mathscr{E}}^+(\mathscr{Q}_D) \leq \delta_{\mathbb{N}}^+(\mathscr{Q}_D)$ this imply that

$$p - \Delta_{\mathscr{E}}^+(\mathscr{Q}_D) \geq p - \Delta_{\mathbb{N}}^+(\mathscr{Q}_D) \tag{4}$$

$$p - \delta_{\mathscr{E}}^+(\mathscr{Q}_D) \geq p - \delta_{\mathbb{N}}^+(\mathscr{Q}_D) \tag{5}$$

Hence From the equations (1), (2) and (4)

$$\gamma(\mathscr{Q}_D) \leq \gamma_{fe}^+(\mathscr{Q}_D) \leq p - \Delta_{\mathbb{N}}^+(\mathscr{Q}_D) \leq p - \Delta_{\mathscr{E}}^+(\mathscr{Q}_D).$$

From (1), (3) and (5),

$$\gamma(\mathscr{Q}_D) \leq \gamma_{fe}^+(\mathscr{Q}_D) \leq p - \delta_{\mathbb{N}}^+(\mathscr{Q}_D) \leq p - \delta_{\mathscr{E}}^+(\mathscr{Q}_D).$$

□

Theorem 3.3. *Consider $\mathscr{Q}_D = (\varphi_D, \eta_D)$ as a FDG on a digraph $\mathscr{Q}_D^* = (\mathscr{J}, \mathscr{E})$. A FED-set, $\mathbb{B}$ is minimal iff for all vertex $t \in \mathbb{B}$ one of the below statement holds.*

(i) either $\mathbb{N}^+(t) \cap \mathbb{B} = \emptyset$,

(ii) there exists a vertex $h \in \mathscr{J} \backslash \mathbb{B} \ni \mathbb{N}(h) \cap \mathbb{B} = \{t\}$ and $|d_{\mathscr{Q}_D}^+(h) - d_{\mathscr{Q}_D}^+(t)| \leq 1$.

Proof. Assume that $\mathbb{B}$ is a minimal FED-set. Consider (i) and (ii) does not hold. Then for some $t \in \mathbb{B}$, $\exists\, h \in \mathbb{N}^+(t) \cap \mathbb{B}$ such that $|d_{\mathscr{Q}_D}^+(h) - d_{\mathscr{Q}_D}^+(t)| \leq 1$ and for every $h \in \mathscr{J} \backslash \mathbb{B}$ either $\mathbb{N}^+(h) \cap \mathbb{B} \neq \{t\}$ or $|d_{\mathscr{Q}_D}^+(h) - d_{\mathscr{Q}_D}^+(t)| \geq 2$ or both. Hence $\mathbb{B} \backslash \{t\}$ is a FED-set, which contradicts the minimality $\mathbb{B}$ of B. Thus, both (i) and (ii) hold.

Conversely, assume that for all $t \in \mathbb{B}$, one of the suggestions (i) and (ii) holds. Consider $\mathbb{B}$ is not minimal. Then there is $t \in \mathbb{B}$ such that $\mathbb{B} \backslash \{t\}$ is a FED-set. So there exists $h \in \mathbb{B} \backslash \{t\}$ such that h fuzzy equitably dominates t and $h \in \mathbb{N}^+(t)$ and $|d_{\mathscr{Q}_D}^+(h) - d_{\mathscr{Q}_D}^+(t)| \leq 1$. Thus, t does not satisfy (i). So t must satisfy (ii). Then there is $h \in \mathscr{J} \backslash \mathbb{B}$ such that $\mathbb{N}^+(h) \cap \mathbb{B} = \{t\}$ and $|d_{\mathscr{Q}_D}^+(h) - d_{\mathscr{Q}_D}^+(m)| \leq 1$. Since $\mathbb{B} \backslash \{t\}$ is a FED-set, there exists $s \in \mathbb{B} \backslash \{t\} \ni s$ is adjacent to h and s is degree equitable with h. So, $s \in \mathbb{N}^+(h) \cap \mathbb{B}, |d_{\mathscr{Q}_D}^+(s) - d_{\mathscr{Q}_D}^+(h)| \leq 1$ and $s \neq t$, which is a contradiction to $\mathbb{N}^+(h) \cap \mathbb{B} = \{t\}$. Hence $\mathbb{B}$ is a minimal FED-set of $\mathscr{Q}_D$. □

Definition 3.2. *A FDG $\mathscr{Q}_D = (\varphi_D, \eta_D)$ on a digraph $\mathscr{Q}_D^* = (\mathscr{J}, \mathscr{E})$ is called fuzzy regular digraph, if all nodes have the same outdegree.*

Example 3.3. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}_D^* = (\mathscr{J}, \mathscr{E})$ and described as follows. Here $\mathscr{Q}_D$ is a*

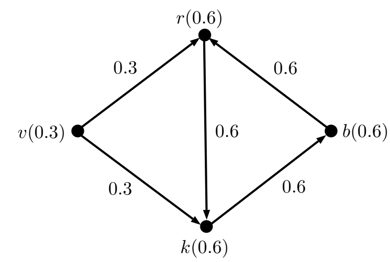


Figure 5. Example of regular fuzzy digraph.

regular FDG, out degree of $\mathscr{Q}_D$ is $d_{\mathscr{Q}_D}^+(v, r, b, k) = 0.6$. $\mathbb{B} = \{v, k\}$ is a dominating set and its each elements has the same degree.

Definition 3.3. *A FDG $\mathscr{Q}_D = (\varphi_D, \eta_D)$ on a digraph $\mathscr{Q}_D^* = (\mathscr{J}, \mathscr{E})$ is called a biregular fuzzy graph, if the outdegree of any vertex is either h or $h + 1$.*

Example 3.4. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}_D^* = (\mathscr{J}, \mathscr{E})$ and described as follows.*

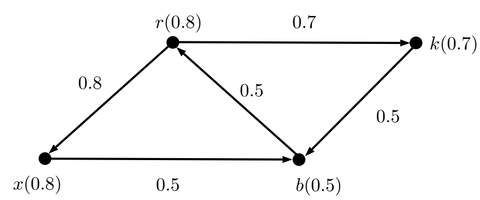


Figure 6. Example of biregular fuzzy digraph.

Here $\mathscr{Q}_D$ is a biregular FDG, degree of $\mathscr{Q}_D$ is $d_{\mathscr{Q}_D}^+(x, b, k) = 0.5$ and $d_{\mathscr{Q}_D}^+(r) = 1.5$. $\mathbb{B} = \{r, b\}$ is a dominating set and its members has outdegree as either 0.5 or 1.5.

Theorem 3.4. *If $\mathscr{Q}_D$ is a regular FDG or $(b, b+1)$ biregular FDG, for some b, then $\gamma_{fe}^+ = \gamma_f^+$.*

Proof. Assume $\mathscr{Q}_D$ is a regular FDG. Then every vertex of $\mathscr{Q}_D$ has the same outdegree say b. Consider $\mathbb{B}$ is a minimum D-set of $\mathscr{Q}_D$. Then $|\mathbb{B}| = \gamma(\mathscr{Q}_D) = \gamma_f^+$.

As $\mathbb{B}$ is a D-set for $t \in \mathscr{J} \backslash \mathbb{B}$ there exists $h \in \mathbb{B}$ and $th \in \mathscr{E}(\mathscr{Q}_D)$ such that $d_{\mathscr{Q}_D}^+(t) = d_{\mathscr{Q}_D}^+(h) = b$ and $|d_{\mathscr{Q}_D}^+(t) - d_{\mathscr{Q}_D}^+(h)| = 0 \leq 1$ imply that $\mathbb{B}$ is a FED-set of $\mathscr{Q}_D$, so that $\gamma_{fe}^+(\mathscr{Q}) \leq |\mathbb{B}| = \gamma_f^+$. But $\gamma_f^+ \leq \gamma_{fe}^+$. Hence $\gamma_f^+ = \gamma_{fe}^+$.

Suppose $\mathscr{Q}_D$ is a biregular fuzzy digraph and for every vertex of $\mathscr{Q}_D$ has outdegree either b (or) $b + 1$. Let $\mathbb{B}$ be a minimum D-set of $\mathscr{Q}_D$. Then $|\mathbb{B}| = \gamma_f^+$. As $\mathbb{B}$ is a D-set, for $t \in R \backslash \mathbb{B}$ there exists $h \in \mathbb{B}$ and $th \in \mathscr{E}(\mathscr{Q})$ such that $d^+_{\mathscr{Q}_D}(t) = b$ (or) $b+1$ and $d^+_{\mathscr{Q}_D}(h) = b$ (or) $b+1$. So $|d^+_{\mathscr{Q}_D}(t) - d^+_{\mathscr{Q}_D}(h)| \leq 1$ which implies $\mathbb{B}$ is a FED-set of $\mathscr{Q}_D$ such that $\gamma^+_{fe} \leq |\mathbb{B}| = \gamma^+_f$. But $\gamma^+_f \leq \gamma^+_{fe}$. Hence $\gamma^+_f = \gamma^+_{fe}$. □

Example 3.5. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$ and described as follows. Here $\mathscr{Q}_D$ is a biregular fuzzy digraph and $d^+_{\mathscr{Q}_D}(m, r, b) = 0.2$ and $d^+_{\mathscr{Q}_D}(s) = 1.2$. $\mathbb{B}_D = \{s\}$ is a FED-set with $\gamma^+_{fe} = \gamma^+_f = 0.5$.*

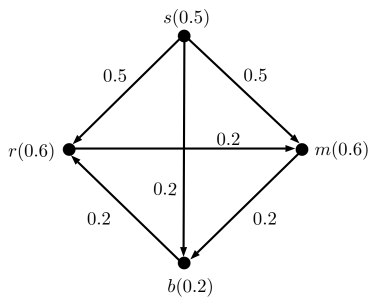


Figure 7. Example of biregular fuzzy digraph.

Definition 3.4. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. If a vertex $t \in \mathscr{J} \ni |d^+_{\mathscr{Q}_D}(t) - d^+_{\mathscr{Q}_D}(h)| \geq 2$ for every $h \in \mathbb{N}^+(t)$, then t is in every ED-set and the nodes are said to be equitable isolates. Set of all equitable isolates are indicated by I^+_{fe}.*

Theorem 3.5. *A FDG $\mathscr{Q}_D = (\varphi_D, \eta_D)$ on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$ has a unique minimal FED-set if and only if the set of all fuzzy equitable isolates forms an ED-set.*

Proof. Sufficient condition is obvious. Consider $\mathscr{Q}_D$ have a unique minimal FED-set $\mathbb{B}$.

Suppose $D = \{t \in \mathscr{J} : t$ is an equitable isolate$\}$. Then $D \subseteq \mathbb{B}$. We will prove that $D = \mathbb{B}$. Consider $\mathbb{B} \backslash D \neq \phi$. Then for $h \in \mathbb{B} \backslash D$, h is not a fuzzy equitable isolate and $\mathscr{J} \backslash \{h\}$ is a FED-set. So there exists a minimal FED-set, $\mathbb{B}_1 \subseteq \mathscr{J} \backslash \{h\}$ and $\mathbb{B}_1 \neq \mathbb{B}$, a contradicts to the fact that $\mathscr{Q}_D$ has a unique FED-set. Hence $D = \mathbb{B}$. □

Definition 3.5. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. For $t \in \mathscr{J}$, the equitable neighborhood of t in digraph indicated by $\mathbb{N}^+_{fe}(t)$ and it is described as $\mathbb{N}^+_{fe}(t) = \{h \in \mathscr{J} : h \in \mathbb{N}^+(t), th$ is a strong arc and $|d^+_{\mathscr{Q}_D}(t) - d^+_{\mathscr{Q}_D}(h)| \leq 1\}$ and $t \in I^+_{fe} \Leftrightarrow \mathbb{N}^+_{fe}(t) = \emptyset$. The cardinality of $\mathbb{N}^+_{fe}(t)$ is known as fuzzy equitable degree of t and it is indicated by $d^{fe^+}_{\mathscr{Q}_D}(t)$. The maximum and minimum equitable degree of a point in $\mathscr{Q}_D$ are indicated by $\Delta^+_{fe}(\mathscr{Q}_D)$ and $\delta^+_{fe}(\mathscr{Q}_D)$. That is $\Delta^+_{fe}(\mathscr{Q}_D) = \max_{t \in R(\mathscr{Q})} \left|\mathbb{N}^+_{fe}(t)\right|$ and $\delta^+_{fe}(\mathscr{Q}_D) = \min_{t \in R(\mathscr{Q})} \left|\mathbb{N}^+_{fe}(t)\right|$.*

Theorem 3.6. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. Then a D-set, $\mathbb{B}$ of $\mathscr{Q}_D$ is a minimal ED-set iff for each $s \in \mathbb{B}$ one of the below two statements satisfies:*

(i) $\mathbb{N}^+_{fe}(s) \cap \mathbb{B} = \emptyset$,

(ii) $\exists$ vertex $b \in \mathscr{J} \backslash \mathbb{B}$ such that $\mathbb{N}^+_{fe}(b) \cap \mathbb{B} = \{s\}$.

Proof. $(i) \implies (ii)$ Consider $\mathbb{B}$ is a minimal ED-set and $s \in \mathbb{B}$. Then $\mathbb{B}_s = \mathbb{B} \backslash \{s\}$ is not a ED-set and so $\exists\, l \in \mathscr{J} \backslash \mathbb{B}_s \ni l$ is not dominated by any element of $\mathbb{B}_s$.
Case (1): If $l = s$ we have (i),
Case (2): If $l \neq s$ we have (ii).

Conversely, for each $s \in \mathbb{B}$ one of the below two conditions satisfies:

(i) $\mathbb{N}^+_{fe}(s) \cap \mathbb{B} = \emptyset$,

(ii) there exists vertex $b \in \mathscr{J} \backslash \mathbb{B}$ such that $\mathbb{N}^+_{fe}(b) \cap \mathbb{B} = \{s\}$.

Suppose $\mathbb{B}$ is not minimal. Then there is $k \in \mathbb{B}$ such that $\mathbb{B} \backslash \{k\}$ is an ED-set. So, there exists $h \in \mathbb{B} \backslash \{k\}$ such that h is equitable dominates k, hence $h \in \mathbb{N}^+(k)$ and $|d^+_{\mathscr{Q}_D}(h) - d^+_{\mathscr{Q}_D}(k)| \leq 1$. So, k does not satisfy (i). Then k must satisfy (ii) $\exists\, h \in \mathscr{J} \backslash \mathbb{B}$ such that $\mathbb{N}^+_{fe}(h) \cap \mathbb{B} = \{k\}$ and $|d^+_{\mathscr{Q}_D}(h) - d^+_{\mathscr{Q}_D}(k)| \leq 1$. As $\mathbb{B} \backslash \{k\}$ is a FED-set, there exists $l \in \mathbb{B} \backslash \{k\} \ni l$ is adjacent to h and l is degree fuzzy equitable with h. So $l \in \mathbb{N}^+_{fe}(h) \cap \mathbb{B}, |d^+_{\mathscr{Q}_D}(l) - d^+_{\mathscr{Q}_D}(h)| \leq 1$ which imply $l = k$, a contradiction to (ii). Therefore $\mathbb{B}$ is a minimal FED-set. □

Definition 3.6. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. Two nodes of a FDG are defined as fuzzy independent(FI) if, there is no strong arc between them. $\mathbb{B} \subset \mathscr{J}$ is defined as a FI-set of $\mathscr{Q}$ if any two vertices of $\mathbb{B}$ are FI.*

Definition 3.7. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. $\mathbb{B} \subset \mathscr{J}$ is called EI-set, if for any $t \in \mathbb{B}, h \notin \mathbb{N}^+_{fe}(t)$ for all $h \in \mathbb{B} \backslash \{t\}$.*

Theorem 3.7. *Let $\mathbb{B}$ be a maximal EI-set. Then $\mathbb{B}$ is a minimal FED-set.*

Proof. Assume $\mathbb{B}$ is a maximal fuzzy EI-set and $t \in \mathscr{J} \backslash \mathbb{B}$. If $t \notin \mathbb{N}^+_{fe}(h)$ for every $h \in \mathbb{B}$, then $\mathbb{B} \cup \{t\}$ is a FEI-set, a contradiction to the maximality of $\mathbb{B}$. Thus $t \in \mathbb{N}^+_{fe}(h)$ for some $h \in \mathbb{B}$, which implies $\mathbb{B}$ is an ED-set. As for any $t \in \mathbb{B}, t \notin \mathbb{N}^+_{fe}(h)$ for all $h \in \mathbb{B} \backslash \{t\}$, either $\mathbb{N}^+(t) \cap \mathbb{B} = \emptyset$ or $|d^+_{\mathscr{Q}_D}(h) - d^+_{\mathscr{Q}_D}(t)| \geq 2$ for every $h \in \mathbb{N}^+(t) \cap \mathbb{B}$. Hence $\mathbb{B}$ is a minimal FED-set. □

Definition 3.8. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. For any $t, h \in \mathbb{B}$, t strongly dominates h if $\eta(th) = \varphi(t) \wedge \varphi(h)$ and $d^+_{\mathscr{Q}D}(t) \geq d^+_{\mathscr{Q}D}(h)$. Similarly t weakly dominates h if $\eta(ht) = \varphi(h) \wedge \varphi(t)$ and $d^+_{\mathscr{Q}D}(h) \geq d^+_{\mathscr{Q}D}(t)$.*

Definition 3.9. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$. Then $\mathbb{B} \subseteq \mathscr{J}$ is defined as strong ED-set of $\mathscr{Q}_D$, if all vertex in $\mathscr{J} \backslash \mathbb{B}$ is strongly dominated by at least one vertex in $\mathbb{B}$. A set $\mathbb{B} \subseteq \mathscr{J}$ is a weak ED-set of $\mathscr{Q}_D$, if all vertex in $\mathscr{J} \backslash \mathbb{B}$ is weakly dominated by at least one vertex in $\mathbb{B}$.*

The minimum fuzzy cardinality of a strong ED-set is said to be strong equitable domination number and is indicated by $\gamma^+_{fse}(\mathscr{Q}_D)$. The minimum fuzzy cardinality of a weak D-set is known as weak equitable domination number and is indicated by $\gamma^+_{fwe}(\mathscr{Q}_D)$.

Theorem 3.8. *In any complete fuzzy digraph $\mathscr{Q}_D = (\varphi_D, \eta_D)$ on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$, the conditions inequality holds $\gamma^+_{fse}(\mathscr{Q}_D) \geq \gamma^+_{fwe}(\mathscr{Q})$.*

Proof. Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a complete fuzzy digraph. Then

Case (i): Assume for every $r_i \in \mathscr{J}, \varphi(r_i)$ are equal. As $\mathscr{Q}_D$ is complete fuzzy digraph. $\eta(r,h) = \varphi(r) \wedge \varphi(h)$ for all $rh \in \mathscr{E}$. Hence $\eta(r,h) = \varphi(r_i)$, for all $r_i \in \mathscr{J}$. Thus

$$\gamma^+_{fwe}(\mathscr{Q}_D) = \gamma^+_{fse}(\mathscr{Q}_D). \tag{6}$$

Case (ii): Consider for every $r_i \in \mathscr{J}, \varphi(r_i)$ are not equal. In a complete FDG, one node dominates all others. If it is the least among them, the D-set with that vertex is said to be weak D-set. Hence the fuzzy cardinality of the set is the weak domination number. That is $\gamma^+_{fwe}(\mathscr{Q}_D) = \wedge\varphi(r_i)$ for all $r_i \in \mathscr{J}$. Obviously the strong D-set includes a vertex that is not the smallest in the vertex set, so the strong domination number is greater than the weak domination number.

$$\gamma^+_{fse}(\mathscr{Q}_D) > \gamma^+_{fwe}(\mathscr{Q}_D) \tag{7}$$

From (6) and (7), we have,

$$\gamma^+_{fse}(\mathscr{Q}_D) \geq \gamma^+_{fwe}(\mathscr{Q}_D).$$

□

Theorem 3.9. *Let $\mathscr{Q}_D = (\varphi_D, \eta_D)$ be a FDG on a digraph $\mathscr{Q}^*_D = (\mathscr{J}, \mathscr{E})$ of order p, then:*
(i) $\gamma^+_{fe}(\mathscr{Q}_D) \leq \gamma^+_{sfe}(\mathscr{Q}_D) \leq p - \Delta^+_{fe}(\mathscr{Q}_D)$,
(ii) $\gamma^+_{fe}(\mathscr{Q}_D) \leq \gamma^+_{wfe}(\mathscr{Q}_D) \leq p - \delta^+_{fe}(\mathscr{Q}_D)$.

Proof. Every strong ED-set is an ED-set of $\mathscr{Q}_D, \gamma^+_{fe}(\mathscr{Q}_D) \leq \gamma^+_{sfe}(\mathscr{Q}_D)$ and all weak ED-set is an ED-set of $\mathscr{Q}_D, \gamma^+_{fe}(\mathscr{Q}_D) \leq \gamma^+_{wfe}(\mathscr{Q}_D)$. For $t, h \in \mathscr{J}$, if $d^{fe^+}_{\mathscr{Q}_D}(t) = \Delta^+_{fe}(\mathscr{Q}_D)$ and $d^{fe^+}_{\mathscr{Q}_D}(h) = \delta^+_{fe}(\mathscr{Q}_D)$, then $\mathscr{J} \backslash \mathbb{N}^+_{fe}(t)$ is a strong ED-set and $\mathscr{J} \backslash \mathbb{N}^+_{fe}(h)$ is a weak ED-set. Hence $\gamma^+_{sfe}(\mathscr{Q}_D) \leq \left|\mathscr{J} \backslash \mathbb{N}^+_{fe}(t)\right|$ and $\gamma^+_{wfe}(\mathscr{Q}_D) \leq \left|\mathscr{J} \backslash \mathbb{N}^+_{fe}(h)\right|$ which imply that $\gamma^+_{sfe}(\mathscr{Q}_D) \leq p - \Delta^+_{fe}(\mathscr{Q}_D)$ and $\gamma^+_{wfe}(\mathscr{Q}_D) \leq p - \delta^+_{fe}(\mathscr{Q}_D)$. □

4 Application of Equitable Domination in Fuzzy Directed Graphs for Network Resource Allocation

In network resource allocation, equitable distribution of resources across a network is crucial, especially in situations where some nodes have more or less access to resources due to factors like bandwidth, capacity, or communication quality. A fuzzy directed graph (fuzzy digraph) can model these networks, where the directed edges represent the directional flow of resources between nodes, and the fuzzy weights represent the quality or capacity of the connections.

By applying the concept of equitable domination in fuzzy digraphs, resources (such as bandwidth, computational power, or data storage) can be allocated equitably across the network nodes, ensuring that no single node is overwhelmed or underserved.

Illustration:

Select a balanced set of dominating nodes to efficiently manage resources and distribute services to other nodes. This ensures that the workload is evenly distributed among the dominating nodes, preventing any single node from becoming overloaded. The objective is to distribute resources fairly and efficiently among all nodes in the network to ensure equitable allocation.

Fuzzy Directed Graph: Consider fuzzy directed graph $\mathscr{Q} = (\mathscr{J}, \mathscr{E}, \mu)$, where
$\mathscr{J}$: Set of nodes in the network.
$\mathscr{E}$: Directed edges representing connections between nodes.
$\mu(w, g)$: Fuzzy membership value representing the strength or quality of the connection between node w and node g.

Resource Allocation Context: Each node $r \in \mathscr{J}$ has a resource demand (or load) that it needs to satisfy. Some nodes may act as dominators that allocate resources to other nodes (dominated nodes) in the network.

Dominating Set in Fuzzy Digraph: A dominating set consists of nodes that can provide resources to their neighboring nodes. Each node not in the dominating set must be connected to at least one dominating node through a directed edge with a non-zero fuzzy membership value.

Equitable Domination:

The load handled by each dominating node must be distributed equitably among all dominating nodes, meaning that no node should be overloaded while others remain underutilized.

Algorithm:

Step 1:

Vertices $\mathscr{J}$ represents the computers, servers and devices.

Edges $\mathscr{E}$ indicates the quality or capacity of computers. Edges takes the membership valve depends upon the quality and capacity(bandwidth).

Step 2:

Calculate the fuzzy degree of each vertex in a graph by summing the fuzzy values of its adjacent vertices. The fuzzy degree $d^+_{\mathscr{Q}}(h)$ of a vertex h is calculated as sum of the neighourhood of h, $\mathbb{N}^+(h)$ is the set of adjacent vertices in outward direction to h.

Step 3: Nodes with higher out-degrees are more effective at dominating due to their stronger connections, allowing them to allocate resources more efficiently. Dominating Set Selection: Nodes with the highest out-degree are chosen as dominators to cover a significant part of the network.

Step 4: To ensure balanced load distribution among dominating nodes, check that the difference in load between any two nodes is less than or equal one, i.e., $|d^+_{\mathscr{Q}}(h) - d^+_{\mathscr{Q}}(g)| \leq 1$. If there is an imbalance where one node is overloaded while others are underutilized, take corrective action. In Figure 8 equitable dominating set represent high load distribution vertices.

Here fuzzy equitable dominating set $\mathbb{B} = \{n_1, n_2, n_6\}$, which has high degree equitable.

This algorithm is commonly used to distribute bandwidth, computational power, or other network resources among servers in a distributed computing system or routers in a communication network. By ensuring fair allocation of resources, it prevents bottlenecks and under utilization, leading to more

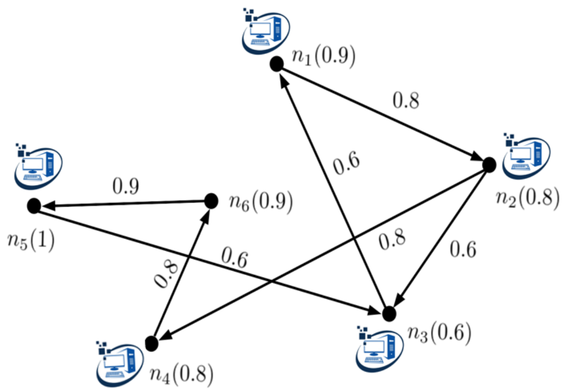


Figure 8. Example of internet fuzzy digraph.

efficient and reliable network operation. This method is especially valuable in cloud computing, data centers, and telecommunications networks where equitable resource distribution is essential for optimal performance and service reliability.

5 Conclusion

Digraphs are useful in many applications such as analysing electrical circuits, developing project schedules and to finding shortest routes. Domination in fuzzy digraphs has proven to be a versatile and valuable concept with numerous applications in various fields such as transportation systems, healthcare, image processing, pattern recognition, ecological networks, decision-making problems, social networks, and network security. In this article, we have concluded the idea of equitable domination in fuzzy digraphs and basic features and a few intriguing findings have been established. Also we covered the fuzzy independent and equitable neighborhood in fuzzy digraphs. Moreover, the characterizations of minimal equitable domination sets are obtained.

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