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Bipolar Neutrosophic Transportation Problem in Symmetric Graphs

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Abstract The breadth of transportation problems (TP) makes them applicable to real-world scenarios. Real-world issues are often unforeseen, making it impossible to estimate a specific cost. Fuzzy and intuitionistic fuzzy sets resolve uncertainty, but have severe limitations. To solve these challenges, bipolar neutrosophic sets (BNS) generalize fuzzy sets, crisp sets, and intuitionistic fuzzy sets, effectively handling ambiguous, unpredictable, and insufficient information in real-world scenarios. Using BNS provides a more dependable, precise, and trustworthy procedure than conventional methods. In this paper, we use a symmetric graph network to find the shortest path for a bipolar neutrosophic transit problem. The approach is utilized to address bipolar neutrosophic transportation network issues with a single-valued neutrosophic network problems. This integration improves transportation problem-solving skills, providing more precision and reliability. The novel technique serves a variety of businesses, including logistics and supply chain management. By delivering accurate solutions, BNS assists decision-makers in optimizing transportation networks, reducing costs, and increasing efficiency. Our findings show that BNS has the ability to address real-world transportation difficulties by providing a helpful tool for managing uncertainty and complexity, hence contributing to more dependable systems. This study helps design more effective transportation systems. This research leads to the creation of more efficient transportation networks, hence increasing operational effectiveness.

Keywords Symmetric Graph, Optimum Solution, Bipolar Single-valued Neutrosophic Network, Shortest Path

MSC: 05C72, 00A79, 49Q22, 05C85, 05C90

1. Introduction

To cope with ambiguity, Zadeh [1] developed the terms "degree of membership" and "fuzzy set" and described them. In order to define the idea of an intuitionistic fuzzy set, Atanassov [2] added the non-membership degree as a separate factor to the notion of fuzzy set. From a metaphysical perspective, Zhang [3] described neutrosophic set as a generalization of intuitionistic fuzzy set theory, the concept of a neutrosophic set to deal with incomplete, hazy, and erratic data prevalent in the real ecosphere. The phrase "degree of indeterminacy" was also recognized by him as an independent component. Truth, falsity, and indeterminacy membership degrees that fall in a range of the real standard or non-standard units are the three membership degrees for each element in a neutrosophic set. If the range of the neutrosophic set was restricted to the genuine standard unit interval, then it might be easily applied to engineering problems. To find solutions to unforeseen difficulties, Wang et al. [4] created (single-valued neutrosophic set) SVNS as a subset of the neutrosophic set. Deli et al. [5] described bipolar neutrosophic as a generalization of bipolar fuzzy graph and SVNS while looking at a few of their related aspects. For more details regarding the expansions and uses of neutrosophic set theory, we referred to [6].

When the connections between the nodes (or vertices) in an issue are unknown, the idea of fuzzy graphs [7] and expansions such as intuitionistic fuzzy graphs [8, 9], N-graphs [10], bipolar fuzzy graphs [11-13**]**, and bipolar intuitionistic fuzzy graphs [14] are not appropriate to calculate the exactness so neutrosophic sets are introduced.

Jacobe [15] introduced Neutrosophic B-algebras, a field extension of neutrosophic sets that entails algebraic structure. Aspects of indeterminacy in algebraic sets are included, and the basic properties and fresh viewpoint of neutrosophic b-algebras were explored.

Wang and Li [16] offer generalized single-valued neutrosophic hesitant fuzzy prioritized aggregation operators, which are tools for dealing with uncertain, inconsistent, and hesitant decision-making situations. These operators are used to multiple criterion decision-making, demonstrating their usefulness with real applications**.**

The four main categories of neutrosophic graphs, two of which rely on concrete indeterminacy and are referred to as I-edge neutrosophic graph and I-vertex neutrosophic graph. These concepts have been thoroughly explored and have grown in popularity among some researchers [17-18] due to their significance and their application to ecological problems in the real world. The other two graphs were known as ($\mathcal{T}$, $\mathcal{I}$, $\mathcal{F}$)-edge neutrosophic graphs and ($\mathcal{T}$, $\mathcal{I}$, $\mathcal{F}$)-vertex neutrosophic graphs, respectively, and they were based on ($\mathcal{T}$, $\mathcal{I}$, $\mathcal{F}$)-components.

Broumi [19] Isolated SVNS are discussed, and practical instances are solved, providing the researchers with a theoretical framework for discussing real-world situations. Fuzzy graphs and intuitionistic fuzzy graphs were both generalized in the single-valued neutrosophic graph. The same authors [20] also presented a vertex's neighbourhood degree as well as a generalization of the neighbourhood and closed neighbourhood degree of a vertex in the single-valued neutrosophic graph. The associated paper and research requirements for the researchers are provided in [6].

The authors in [21] have provided an outline of the bipolar single-valued neutrosophic graph theory that covers all aspects of fuzzy, intuitionistic fuzzy, and bipolar fuzzy graphs. The comparative study of algorithms used in the field of uncertainty environment has been discussed, and conclusions were made about the best algorithm by the authors **[22]**. A case study held by the author declared that the project scheduling problem, where decision-making is uncertain; the algorithm discussed provided the result for such decision-making in project scheduling problems [23]. For an interval-valued neutrosophic fuzzy number, a novel strategy to solve the SP problem in road surface networking was developed and validated [24].

Day by day, nature challenges human life in all aspects, especially in travelling, transporting goods, foods from country to another. Many new approaches are arriving to sort out these issues. A novel approach and new algorithm termed as Incident Edge Path Algorithm for finding the shortest path have been introduced and implemented, also compared with the existing Dijkstra's algorithm. Where the Dijkstra algorithm fails in finding SP with negative weights, the proposed method gives SP for the taken example problem [25].

In this article, the algorithm proposed in [25] was extended and applied in the area of Bipolar Neutrosophic Network Problems (BNNP). The algorithm has been applied and solved to show its efficacy by applying it in the Neutrosophic Transportation Problem (NTP) whose weights are defined as bipolar single-valued neutrosophic numbers (BNN) and the network diagram satisfies the symmetric property—that is, all edges satisfy the arc symmetric property. The results are tabulated and contrasted with the existing algorithm; the conclusion section discusses the optimal result from the numerical example problem. This article is an application of the algorithm developed and applied in the field of neutrosophic set theory.

In Section 2, preliminaries, the basic definitions and ideas related to neutrosophic set have been given. In Section 3, the algorithm proposed is elaborated and implemented using the numerical example problem. In Section 4, the comparative learning of the numerical example resulting with the existing algorithm was discussed and provided a bar chart. In Section 5, the conclusion was developed in such a way that how the paper has been carried out and future work is declared.

2. Preliminaries

The basic definitions used are referred and cited from the reference part, few definitions are listed below. The definitions can be referred from [6, 7, 10, 11, 13, 15, 18, 24].

Definition 1 [23]: A graph $\mathcal{N}$ is symmetric graph if any two pair of adjacent nodes $v_1 - u_1$ and $v_2 - u_2$ of $\mathcal{N}$, there is an automorphism $f: \mathcal{V}(\mathcal{N}) \longrightarrow \mathcal{V}(\mathcal{N})$ such that $f(u_1) = u_2$ and $f(v_1) = v_2$.

Definition 2 [6]: U is the space of elements. The set $\bar{\mathcal{A}}$ is Neutrosophic set which has an element defined as $\bar{\mathcal{A}} = \{< x: \mathcal{T}_{\bar{\mathcal{A}}}, \mathcal{I}_{\bar{\mathcal{A}}}, \mathcal{F}_{\bar{\mathcal{A}}} >, x \in U$ where $\mathcal{T}, \mathcal{I}, \mathcal{F}$: $\bar{\mathcal{A}} \longrightarrow]^{-}0, 1, [^{+}$ was defined as the functions of degree of Truth, indeterminacy and falsity membership functions, respectively, of $\forall x \in U$ to the set $\bar{\mathcal{A}}$ with the condition $-0 \leq \mathcal{T}_{\bar{\mathcal{A}}} + \mathcal{I}_{\bar{\mathcal{A}}} + \mathcal{F}_{\bar{\mathcal{A}}} \leq 3^{+}$.

Definition 3 [10]: U is a space of discourse by global elements. A set SVNS is defined as $\bar{\mathcal{A}}$, the global elements of a set $\bar{\mathcal{A}}$, $\forall x \in U, \mathcal{T}_{\bar{\mathcal{A}}}, \mathcal{I}_{\bar{\mathcal{A}}}, \mathcal{F}_{\bar{\mathcal{A}}} \in [0,1]$. The SVN set $\bar{\mathcal{A}} = \{x, < \mathcal{T}_{\bar{\mathcal{A}}}, \mathcal{I}_{\bar{\mathcal{A}}}, \mathcal{F}_{\bar{\mathcal{A}}} > | x \in U\}$.

2.1. Bipolar Single-Valued Neutrosophic Set

The following definitions are referred for understanding the concept of Bipolar in SVNS throughout the paper.

Definition 4 [25]: BSVNS

A bipolar neutrosophic set $\bar{\mathcal{A}}$ in X is defined as an element of the form $\bar{\mathcal{A}} = \{\langle x, \mathcal{T}^+_{\bar{\mathcal{A}}}, \mathcal{I}^+_{\bar{\mathcal{A}}}, \mathcal{F}^+_{\bar{\mathcal{A}}}, \mathcal{T}^-_{\bar{\mathcal{A}}}, \mathcal{I}^-_{\bar{\mathcal{A}}}, \mathcal{F}^-_{\bar{\mathcal{A}}}\rangle : x \epsilon X\}$, where $\mathcal{T}^+_{\bar{\mathcal{A}}}, \mathcal{I}^+_{\bar{\mathcal{A}}}, \mathcal{F}^+_{\bar{\mathcal{A}}} : X \to [1,0]$ and $\mathcal{T}^-_{\bar{\mathcal{A}}}, \mathcal{I}^-_{\bar{\mathcal{A}}}, \mathcal{F}^-_{\bar{\mathcal{A}}} : X \to [-1,0]$. $\mathcal{T}^+_{\bar{\mathcal{A}}}, \mathcal{I}^+_{\bar{\mathcal{A}}}, \mathcal{F}^+_{\bar{\mathcal{A}}}$ are the positive truth, indeterminacy and falsity membership degrees of an element belongs to X corresponds to $\bar{\mathcal{A}}$, and $\mathcal{T}^-_{\bar{\mathcal{A}}}, \mathcal{I}^-_{\bar{\mathcal{A}}}, \mathcal{F}^-_{\bar{\mathcal{A}}}$ are the negative truth, indeterminacy and falsity membership degrees of an element belongs to X to few implicit counter-property corresponds to a neutrosophic set $\bar{\mathcal{A}}$.

Definition 5 [26]: BNS

Let $\bar{\mathcal{A}}$ be a bipolar neutrosophic set and defined as $\bar{\mathcal{A}} = \langle \mathcal{T}^+_{\bar{\mathcal{A}}}, \mathcal{I}^+_{\bar{\mathcal{A}}}, \mathcal{F}^+_{\bar{\mathcal{A}}}, \mathcal{T}^-_{\bar{\mathcal{A}}}, \mathcal{I}^-_{\bar{\mathcal{A}}}, \mathcal{F}^-_{\bar{\mathcal{A}}}\rangle$ is empty if its $T^+_{\bar{\mathcal{A}}} = 0$, $\mathcal{I}^+_{\bar{\mathcal{A}}} = 0$, $\mathcal{F}^+_{\bar{\mathcal{A}}} = 1$, and $\mathcal{T}^-_{\bar{\mathcal{A}}} = -1$, $\mathcal{I}^-_{\bar{\mathcal{A}}} = 0$, $\mathcal{F}^-_{\bar{\mathcal{A}}} = 0$.

Definition 6[25]: BNN

Consider $\bar{\mathcal{A}}_1 = \langle \mathcal{T}^+_{\bar{\mathcal{A}}_1}, \mathcal{I}^+_{\bar{\mathcal{A}}_1}, \mathcal{F}^+_{\bar{\mathcal{A}}_1}, \mathcal{T}^-_{\bar{\mathcal{A}}_1}, \mathcal{I}^-_{\bar{\mathcal{A}}_1}, \mathcal{F}^-_{\bar{\mathcal{A}}_1}\rangle$ and $\bar{\mathcal{A}}_2 = \langle \mathcal{T}^+_{\bar{\mathcal{A}}_2}, \mathcal{I}^+_{\bar{\mathcal{A}}_2}, \mathcal{F}^+_{\bar{\mathcal{A}}_2}, \mathcal{T}^-_{\bar{\mathcal{A}}_2}, \mathcal{I}^-_{\bar{\mathcal{A}}_2}, \mathcal{F}^-_{\bar{\mathcal{A}}_2}\rangle$ as two bipolar neutrosophic numbers and $\gamma > 0$. Then, the mathematical operations are defined as follows,

$$\bar{\mathcal{A}}_1 \oplus \bar{\mathcal{A}}_2 = \langle \mathcal{T}^+_{\bar{\mathcal{A}}_1} + \mathcal{T}^+_{\bar{\mathcal{A}}_2} - \left(\mathcal{T}^+_{\bar{\mathcal{A}}_1}\mathcal{T}^+_{\bar{\mathcal{A}}_2}\right), \mathcal{I}^+_{\bar{\mathcal{A}}_1}\mathcal{I}^+_{\bar{\mathcal{A}}_2}, \mathcal{F}^+_{\bar{\mathcal{A}}_1}\mathcal{F}^+_{\bar{\mathcal{A}}_2}, -\mathcal{T}^-_{\bar{\mathcal{A}}_1}\mathcal{T}^-_{\bar{\mathcal{A}}_2}, -\left(-\mathcal{I}^-_{\bar{\mathcal{A}}_1} - \mathcal{I}^-_{\bar{\mathcal{A}}_2} - \left(\mathcal{I}^-_{\bar{\mathcal{A}}_1}\mathcal{I}^-_{\bar{\mathcal{A}}_2}\right)\right), -\left(-\mathcal{F}^-_{\bar{\mathcal{A}}_1} - \mathcal{F}^-_{\bar{\mathcal{A}}_2} - \mathcal{F}^-_{\bar{\mathcal{A}}_1}\mathcal{F}^-_{\bar{\mathcal{A}}_2}\right)\rangle$$

$$\bar{\mathcal{A}}_1 \otimes \bar{\mathcal{A}}_2 = \langle \mathcal{T}^+_{\bar{\mathcal{A}}_1}\mathcal{T}^+_{\bar{\mathcal{A}}_2}, \left(\mathcal{I}^+_{\bar{\mathcal{A}}_1} + \mathcal{I}^+_{\bar{\mathcal{A}}_2} - \mathcal{I}^+_{\bar{\mathcal{A}}_1}\mathcal{I}^+_{\bar{\mathcal{A}}_2}\right), \left(\mathcal{F}^+_{\bar{\mathcal{A}}_1} + \mathcal{F}^+_{\bar{\mathcal{A}}_2} - \mathcal{F}^+_{\bar{\mathcal{A}}_1}\mathrm{F}^+_{\bar{\mathcal{A}}_2}\right), -\left(\mathcal{T}^+_{\bar{\mathcal{A}}_1} - \mathcal{T}^+_{\bar{\mathcal{A}}_2} - \left(\mathcal{T}^+_{\bar{\mathcal{A}}_1}\mathcal{T}^+_{\bar{\mathcal{A}}_2}\right)\right), -\mathcal{I}^-_{\bar{\mathcal{A}}_1}\mathcal{I}^-_{\bar{\mathcal{A}}_2}, -\mathcal{F}^-_{\bar{\mathcal{A}}_1}\mathcal{F}^-_{\bar{\mathcal{A}}_2}\rangle$$

$$\gamma\bar{\mathcal{A}}_1 = \langle 1 - \left(1 - \mathcal{T}^+_{\bar{\mathcal{A}}_1}\right)^\gamma, {\mathcal{I}^+_{\bar{\mathcal{A}}_1}}^\gamma, {\mathcal{F}^+_{\bar{\mathcal{A}}_1}}^\gamma, -\left(\mathcal{T}^-_{\bar{\mathcal{A}}_1}\right)^\gamma, -\left(\mathcal{I}^-_{\bar{\mathcal{A}}_1}\right)^\gamma, -\left(1 - \left(1 - \left(-\mathcal{F}^-_{\bar{\mathcal{A}}_1}\right)\right)^\gamma\right)\rangle$$

$${\bar{\mathcal{A}}_1}^\gamma = \langle \left(\mathcal{T}^+_{\bar{\mathcal{A}}_1}\right)^\gamma, 1 - \left(1 - \mathcal{I}^+_{\bar{\mathcal{A}}_1}\right)^\gamma, 1 - \left(1 - \mathcal{F}^+_{\bar{\mathcal{A}}_1}\right)^\gamma, -\left(1 - \left(1 - \left(-\mathcal{T}^-_{\bar{\mathcal{A}}_1}\right)\right)^\gamma\right), -\left(-\mathcal{I}^-_{\bar{\mathcal{A}}_1}\right)^\gamma, \left(-\left(-\mathcal{F}^-_{\bar{\mathcal{A}}_1}\right)\right)^\gamma\rangle$$

where $\gamma > 0$.

Definition 7 [25]: Score function:

Any two BNS can be compared by means of the score function defined by Deli et al [7]. By this comparison we can easily identify the paths by ranking it. Consider $\bar{\mathcal{A}} = \langle \mathcal{T}^+_{\bar{\mathcal{A}}}, \mathcal{I}^+_{\bar{\mathcal{A}}}, \mathcal{F}^+_{\bar{\mathcal{A}}}, \mathcal{T}^-_{\bar{\mathcal{A}}}, \mathcal{I}^-_{\bar{\mathcal{A}}}, \mathcal{F}^-_{\bar{\mathcal{A}}}\rangle$ as a BNN. Then the score function $\mathcal{S}(\bar{\mathcal{A}})$, accuracy function $a(\bar{\mathcal{A}})$ and certainty function $c(\bar{\mathcal{A}})$ is defined as follows,

$$\mathcal{S}(\bar{\mathcal{A}}) = \left(\frac{1}{6}\right) \times \left[3 + \mathcal{T}^+_{\bar{\mathcal{A}}} - \mathcal{I}^+_{\bar{\mathcal{A}}} - \mathcal{F}^+_{\bar{\mathcal{A}}} + \mathcal{T}^-_{\bar{\mathcal{A}}} - \mathcal{I}^-_{\bar{\mathcal{A}}} - \mathcal{F}^-_{\bar{\mathcal{A}}}\right]$$

$$a(\bar{\mathcal{A}}) = \mathcal{T}^+_{\bar{\mathcal{A}}} - \mathcal{F}^+_{\bar{\mathcal{A}}} + \mathcal{T}^-_{\bar{\mathcal{A}}} - \mathcal{F}^-_{\bar{\mathcal{A}}}$$

$$c(\bar{\mathcal{A}}) = \mathcal{T}^+_{\bar{\mathcal{A}}} - \mathcal{F}^-_{\bar{\mathcal{A}}}$$

Ranking of Bipolar Neutrosophic Number (BNN),

Consider $\bar{\mathcal{A}}_1 = \langle \mathcal{T}^+_{\bar{\mathcal{B}}_1}, \mathcal{I}^+_{\bar{\mathcal{B}}_1}, \mathcal{F}^+_{\bar{\mathcal{B}}_1}, \mathcal{T}^-_{\bar{\mathcal{B}}_1}, \mathcal{I}^-_{\bar{\mathcal{B}}_1}, \mathcal{F}^-_{\bar{\mathcal{B}}_1}\rangle$ and $\bar{\mathcal{A}}_2 = \langle \mathcal{T}^+_{\bar{\mathcal{B}}_2}, \mathcal{I}^+_{\bar{\mathcal{B}}_2}, \mathcal{F}^+_{\bar{\mathcal{B}}_2}, \mathcal{T}^-_{\bar{\mathcal{B}}_2}, \mathcal{I}^-_{\bar{\mathcal{B}}_2}, \mathcal{F}^-_{\bar{\mathcal{B}}_2}\rangle$ as two BNNs then,

i) If $\mathcal{S}(\bar{\mathcal{B}}_1) > \mathcal{S}(\bar{\mathcal{B}}_2)$, then $\bar{\mathcal{B}}_1 > \bar{\mathcal{B}}_2$
ii) If $\mathcal{S}(\bar{\mathcal{B}}_1) = \mathcal{S}(\bar{\mathcal{B}}_2)$, and $a(\bar{\mathcal{B}}_1) > a(\bar{\mathcal{B}}_2)$ then $\bar{\mathcal{B}}_1 > \bar{\mathcal{B}}_2$
iii) If $\mathcal{S}(\bar{\mathcal{B}}_1) = \mathcal{S}(\bar{\mathcal{B}}_2)$, $a(\bar{\mathcal{B}}_1) = a(\mathcal{B}_2)$ and $c(\bar{\mathcal{B}}_1) > c(\bar{\mathcal{B}}_2)$ then $\bar{\mathcal{B}}_1 > \bar{\mathcal{B}}_2$
iv) If $\mathcal{S}(\bar{\mathcal{B}}_1) = \mathcal{S}(\bar{\mathcal{B}}_2)$, $a(\bar{\mathcal{B}}_1) = a(\bar{\mathcal{B}}_2)$ and $c(\bar{\mathcal{B}}_1) = c(\bar{\mathcal{B}}_2)$ then $\bar{\mathcal{B}}_1 = \bar{\mathcal{B}}_2$

Definition 8: Incident edge

A network $\mathcal{G}(\mathcal{V}, \mathcal{E})$ with vertex set $\mathcal{V}$ and arc set $\mathcal{E}$ and we set $P(v_i)$ weight of the vertex v_i and $\mathcal{S}_j$ be the set of arcs incident with v_i of $\mathcal{G}$ where $0 \le i \le n; 1 \le j \le n$.

3. Algorithm: Incident Bipolar Neutrosophic Edge Path Algorithm (IBNEPA)

Step 1: Collect $\mathcal{S}_j$, j takes the values from $1,2,\dots,n$ the set of all Bipolar Neutrosophic Edges incident with origin node. If $\mathcal{S}_j$ is empty proceed to step 5 or proceed step 2.

Step 2: Evaluate the path weight using $P(v_j) = P(v_i) \oplus d(v_i, v_j)$ where i varies from $1,2,\dots m$ and j varies form $1,2,\dots,n$ and $i \ne j$. Bipolar Single-valued Neutrosophic Minimum cost is calculated for selected $P(v_j)$

Step 3: Evaluate using score function defined in definition 7, for Bipolar Neutrosophic Edge to compare and select the minimum path weight by $\mathcal{S}_j < \mathcal{S}_i$ or $\mathcal{S}_j > \mathcal{S}_i$.

Step 4: If $P(v_j)$ has multiple paths then choose the path with minimum of $P(v_j)$.

Step 5: Repeat the steps 1 to 4 until $\mathcal{S}_n = \{\}$, also the destination reached to get shortest path.

Numerical example:

The following numerical example NTP has been taken to verify the algorithm procedure. In this network, the weights are bipolar single valued neutrosophic number and the considered NTP's network is symmetric graph. The symmetric neutrosophic network of transportation problem is shown in Fig 1, and BSVNEW were tabulated in Table 1. Table 1 provides edge distance and crisp number of Fig 1.

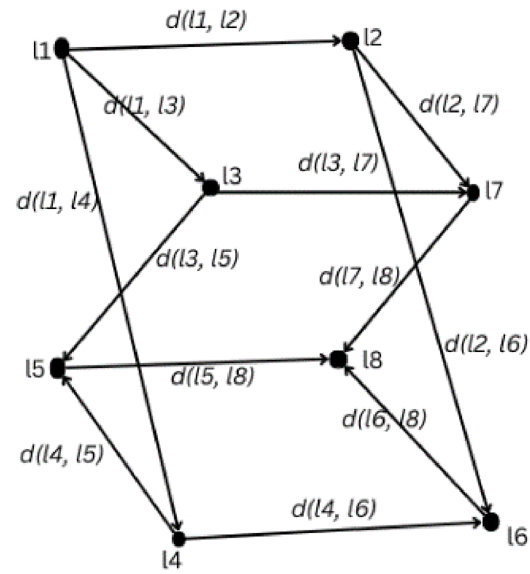

Figure 1. A symmetric bipolar neutrosophic transportation network

Table 1. Edge distance and its crisp number of BSVNTP

Edges $d(l_i, l_j)$	BSVNEW	Crisp weight ($\mathcal{S}(d(l_i, l_j))$)
$d(l_1, l_2)$	$\langle 0.8, 0.4, 0.5, -0.6, -0.2, -0.5\rangle$	0.5
$d(l_1, l_3)$	$\langle 0.7, 0.4, 0.5, -0.4, -0.2, -0.5\rangle$	0.5167
$d(l_1, l_4)$	$\langle 0.8, 0.4, 0.4, -0.8, -0.2, -0.4\rangle$	0.467
$d(l_2, l_6)$	$\langle 0.8,0.4,0.5, -0.6, -0.3, -0.5\rangle$	0.6
$d(l_2, l_7)$	$\langle 0.6, 0.5,0.5, -0.5, -0.5, -0.6\rangle$	0.55
$d(l_3, l_5)$	$\langle 0.7, 0.5, 0.5, -0.4, -0.6, -0.6\rangle$	0.5833
$d(l_3, l_7)$	$\langle 0.6,0.5,0.5, -0.4, -0.6, -0.6\rangle$	0.567
$d(l_4, l_5)$	$\langle 0.8,0.4,0.4, -0.5, -0.2, -0.6\rangle$	0.55
$d(l_4, l_7)$	$\langle 0.8,0.4,0.4, -0.8, -0.2, -0.4\rangle$	0.467
$d(l_5, l_8)$	$\langle 0.5,0.5,0.6, -0.4, -0.5, -0.8\rangle$	0.55
$d(l_6, l_8)$	$\langle 0.5,0.4,0.6, -0.4, -0.3, -0.8\rangle$	0.533
$d(l_7, l_8)$	$\langle 0.5,0.5,0.6, -0.4, -0.5, -0.8\rangle$	0.55

Solution: the algorithm procedure has been followed and executed as steps. The algorithm has to stop when j = n =number of nodes, was reached.

Step 1:

$\mathcal{S}_1 = \{d(l_1, l_2),\ d(l_1, l_3),\ d(l_1, l_4)\ \}$ Initially, the BSVNPW $P(l_1) = \langle 0,1,1,\text{-}1,0,0\rangle$, i = 1 , j = 2,3,4.

Table 2. Path calculation incident with node 1

$d(l_1, l_j)$	$P(l_j) = \{P(l_1) \oplus d(l_1, l_j)\}$	BSVNMC	Crisp weight	BSVNSP
$d(l_1, l_2)$	$P(l_2) = \{P(l_1) \oplus d(l_1, l_2)\}$	$\langle 0.8,0.4,0.5, -0.6,0.2, -0.5\rangle$	0.5	$l_1 - l_2$
$d(l_1, l_3)$	$P(v_3) = \{P(l_1) \oplus d(l_1, l_3)\}$	$\langle 0.7, 0.4, 0.5, -0.4, -0.2, -0.5\rangle$	0.5167	$l_1 - l_3$
$d(l_1, l_4)$	$P(l_4) = \{P(l_1) \oplus d(l_1, l_4)\}$	$\langle 0.8,0.4,0.4, -0.8, -0.2, -0.4\rangle$	0.4667	$l_1 - l_4$

Step 2:

$\mathcal{S}_2 = \{d(l_2, l_6),\ d(l_2, l_7)\ \}$, the BSVNPW $P(l_2) = \langle 0.8,0.4,0.5,\text{-}0.6,\text{-}0.2,\text{-}0.5\rangle$, i = 2 , j = 6, 7.

Table 3. Path calculation incident with node 2

$d(l_2, l_j)$	$P(l_j) = \{P(l_1) \oplus d(l_1, l_j)\}$	BSVNMC	Crisp weight	BSVNSP
$d(l_2, l_6)$	$P(l_6) = \{P(l_2) \oplus\ d(l_2, l_6)$	$\langle 0.96, 0.16, 0.25, -0.36, -0.44, -0.75 \rangle$	0.75	$l_1 - l_2 - l_6$
$d(l_2, l_7)$	$P(l_7) = \{P(l_2) \oplus d(l_2, l_7)\}$	$\langle 0.92, 0.2, 0.25, -0.3, -0.6, -0.8 \rangle$	0.7617	$l_1 - l_2 - l_7$

Step 3:
$\mathcal{S}_3 = \{\ d(l_3, l_5), d(v_3, v_7)\}$ the BSVNPW $P(l_3) = \langle 0.7, 0.4, 0.5, -0.4, -0.2, -0.5 \rangle$, $i = 3$, $j = 5,\ 7$.

Table 4. Path calculation incident with node 3

$d(l_3, l_j)$	$P(l_j) = \{P(l_3) \oplus d(l_3, l_j)\}$	BSVNMC	Crisp weight	BSVNSP
$d(l_3, l_5)$	$P(l_5) = \{P(l_3) \oplus\ d(l_3, l_5)\}$	$\langle 0.91, 0.2, 0.25, -0.16, -0.68, -0.8 \rangle$	0.7967	$l_1 - l_3 - l_5$
$d(l_3, l_7)$	$P(l_7) = \{P(l_3) \oplus d(l_3, l_7)\}$ $P(l) = \min\{0.7617, 0.7917\}$ $P(l_7) = 0.7617$	$\langle 0.88, 0.2, 0.25, -0.16, -0.68, -0.8 \rangle$	0.7917	$l_1 - l_2 - l_7$ path is shortest than $l_1 - l_3 - l_7$

Step 4:
$\mathcal{S}_4 = \{\ d(l_4, l), d(l_4, l_6)\}$, the BSVNPW $P(l_4) = \langle 0.8, 0.4, 0.4, -0.8, -0.2, -0.4 \rangle$, $i = 4$, $j = 5,\ 6$

Table 5. Path calculation incident with node 4

$d(l_4, l_j)$	$P(l_j) = \{P(l_4) \oplus d(l_4, l_j)\}$	BSVNMC	Crisp weight	BSVNSP
$d(l_4, l_5)$	$P(l_5) = \{P(l_4) \oplus\ d(l_4, l_5)\}$ $P(v_5) = \min\{0.7967, 0.7267\} = 0.7267$	$\langle 0.96, 0.16, 0.16, -0.4, -0.36, -0.76 \rangle$	0.7267	$l_1 - l_4 - l_5$ is shortest than $l_1 - l_3 - l_5$
$d(l_4, l_6)$	$P(l_6) = \{P(l_4) \oplus d(l_4, l_7)\}$ $P(l_6) = \min\{0.75, 0.667\}$ $P(l_6) = 0.667$	$\langle 0.96, 0.16, 0.16, -0.64, -0.36, -0.64 \rangle$	0.667	$l_1 - l_4 - l_6$ path is shortest than $l_1 - l_2 - l_6$

Step 5:
$\mathcal{S}_5 = \{\ d(l_5, l_8)\}$, the BSVNPW $P(l_5) = \langle 0.96, 0.16, 0.16, -0.4, -0.36, -0.76 \rangle$, $i = 5$, $j = 8$

Table 6. Path calculation incident with node 5

$d(l_5, l_j)$	$P(l_j) = \{P(l_5) \oplus d(l_5, l_j)\}$	BSVNMC	Crisp weight	BSVNSP
$d(l_5, l_8)$	$P(l_5) = \{P(l_5) \oplus\ d(l_5, l_8)\}$	$\langle 0.98, 0.08, 0.096, -0.16, -0.68, -0.952 \rangle$	0.866	$l_1 - l_4 - l_5 - l_8$

Step 6:
$\mathcal{S}_5 = \{\ d(l_6, l_8)\}$, the BSVNPW $P(l_6) = \langle 0.96, 0.16, 0.16, -0.64, -0.36, -0.64 \rangle$, $i = 6$, $j = 8$

Table 7. Path calculation incident with node 6

$d(l_6, l_j)$	$P(l_j) = \{P(l_6) \oplus d(l_6, l_j)\}$	BSVNMC	Crisp weight	BSVNSP
$d(l_6, l_8)$	$P(l_8) = \{P(l_6) \oplus\ d(l_6, l_8)\}$	$\langle 0.98, 0.064, 0.096, -0.256, -0.552, -0.928 \rangle$	0.841	$l_1 - l_4 - l_6 - l_8$

Step 7:
$\mathcal{S}_7 = \{\ d(l_7, l_8)\}$, the BSVNPW $P(l_7) = \langle 0.96, 0.2, 0.25, -0.3, -0.6, -0.8 \rangle$, $i = 7$, $j = 8$

Table 8. Path calculation incident with node 7

$d(l_7, l_j)$	$P(l_j) = \{P(l_7) \oplus d(l_7, l_j)\}$	BSVNMC	Crisp weight	BSVNSP
$d(l_7, l_8)$	$P(l_8) = \{P(l_7) \oplus\ d(l_7, l_8)\}$ $P(v_8) = \min\{0.866, 0.841, 08917\}$ =0.841	$\langle 0.96, 0.1, 0.15, -0.12, -0.8, -0.96 \rangle$	0.8917	$l_1 - l_4 - l_6 - l_8$ is shortest path than other two paths.

The process has to stop since, $\mathcal{S}_8 = \{\}$

The following Table 9 represents the SP and BSVNMC of all nodes from origin node and the SP is shown in the Figure 2.

Table 9. Shortest Path from node 1

l_j	BSVNMC	Crisp weight	BSVNSP
l_2	$\langle 0.8,0.4,0.5,-0.6,0.2,-0.5\rangle$	0.5	l_1-l_2
l_3	$\langle 0.7,0.4,0.5,-0.4,-0.2,-0.5\rangle$	0.5167	l_1-l_3
l_4	$\langle 0.8,0.4,0.4,-0.8,-0.2,-0.4\rangle$	0.4667	l_1-l_4
l_5	$\langle 0.96,0.16,0.16,-0.4,-0.36,-0.76\rangle$	0.7267	$l_1-l_4-l_5$
l_6	$\langle 0.96,0.16,0.16,-0.64,-0.36,-0.64\rangle$	0.667	$l_1-l_4-l_6$
l_7	$\langle 0.92,0.2,0.25,-0.3,-0.6,-0.8\rangle$	0.7617	$l_1-l_2-l_7$
l_8	$\langle 0.98,0.064,0.096,-0.256,-0.552,-0.928\rangle$	0.841	$l_1-l_4-l_6-l_8$

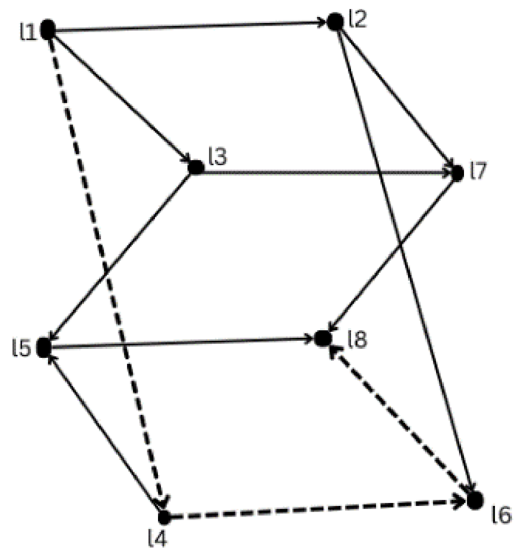

Figure 2. SP of the given BSVNTP of symmetrical graph

4. Comparative Study

The accuracy and certainty of the SP of BSVNTP were checked by plotting its values in bar graph as shown in the following Figure 3.

Figure depicts the 3 possible paths from origin to destination with its score, accuracy and certainty values to do the comparison by bar graphs as P1, P2 and P3. P1 is the minimum among the 3 values, it is obtained by the Incident Bipolar Neutrosophic Edge Path Algorithm

The following Table 11 provides the result brought by the existing Broumi et al., and proposed algorithm with their outcomings. From Figure 3, P1 is the required optimum result obtained by the IBNEPA.

Broumi extended Dijkstra algorithm to find the shortest path in the field of neutrosophic set theory, but Dijkstra algorithm fails to solve with negative weighted edges in TP; so the proposed algorithm is executed and extended to apply in Bipolar Neutrosophic set by comparing its results in Table 11.

Table 10. Score, Accuracy and certainty values of 3 SP's to find the finest one.

SP	Score value	Accuracy value	Certainty value
$l_1 - l_4 - l_6 - l_8$	0.841	1.556	1.908
$l_1 - l_2 - l_7 - l_8$	0.8917	1.65	1.92
$l_1 - l_4 - l_5 - l_8$	0.866	1.676	1.932

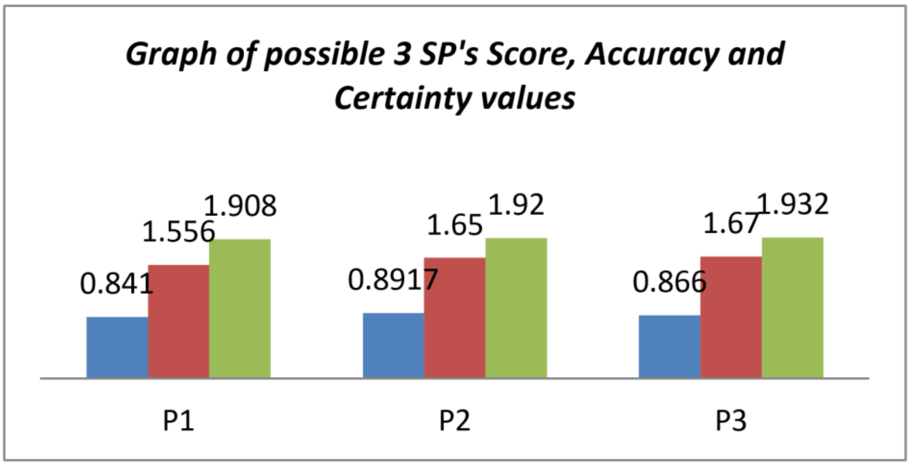

Figure 3. 3 Possible SP obtained from Proposed IBNEPA

Table 11. Comparison table

Methods	BSVNMC	Crisp weight	BSVNSP
Broumi et al.	$\langle 0.98, 0.064, 0.096, -0.256, -0.552, -0.928 \rangle$	0.841	$l_1 - l_4 - l_6 - l_8$
Proposed IBNEPA	$\langle 0.98, 0.064, 0.096, -0.256, -0.552, -0.928 \rangle$	0.841	$l_1 - l_4 - l_6 - l_8$

5. Conclusions

In this article, algorithm established by Kanchana et al., [26] has been extended and applied in the field of BSVNNTP to get the shortest path that too for specifically for symmetrical graphs. To find the optimality and comparison between the BNNs the ranking method is very much useful. The obtained result is optimum and compared with the existing algorithm by Broumi et al. This algorithm not only provides SP from origin to destination node also it provides SP all other nodes as destination from origin without any backward process in the method. This algorithm is applicable in all fields of transportation related network problems to clarify the SP from start to destination places.

Appendix

SP – Shortest Path
NTP – Neutrosophic Transportaion Problem
BNS – Bipolar Neutrosophic Set
BNN – Bipolar Neutrosophic Number
BNNP - Bipolar Neutrosophic Network Problems
BNEW – Bipolar Neutrosophic Edge Weight
BSVNTP – Bipolar Single-Valued Neutosophic Transportation Problem
IBNEPA – Incident Bipolar Neutrosophic Edge Path Algorithm
BSVNEW - Bipolar Single-valued Neutrosophic Edge Weight
BSVNPW - Bipolar Singel-valued Neutrosophic Path Weight
BSVNMC - Bipolar Single-valued Neutrosophic Minimum cost
BSVNSP - Bipolar Single-valued Neutrosophic Shortest Path

Future Work

This algorithm can be applied and developed in the area of spherical, plithogenic or other fuzzy related environments.

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Data Accessibility Statement

There is no data accessibility in this paper.

Conflicts of Interest

The writers announce that there is no conflict of interest.

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