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Homogeneous Spaces and Induced Transformation Groups of $\mathcal{S}$-Topological Transformation Group

C. Rajapandiyan, V. Visalakshi*

Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur- 603 203, Tamil Nadu, India

*Corresponding Author: visalakv@srmist.edu.in

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Abstract This paper explores the homogeneous spaces and induced transformation groups of $\mathcal{S}$-topological transformation group. $\mathcal{S}$-topological transformation group is a structure constructed by concatenating a topological group with a topological space through a semi totally continuous action. It is shown that any map from a topological group to the quotient group of a finite Hausdorff topological group by the isotropy group is surjective, continuous, open and it has been proven that any map from the quotient group of a finite Hausdorff topological group by the isotropy group to the homogenous space is both H-isomorphism and semi totally continuous. Furthermore, an equivariant map has been established between homogeneous spaces and discussed the partial order relation on the family of all Hausdorff homogeneous spaces for a compact Hausdorff topological group. Subsequently, an induced $\mathcal{S}$-topological transformation group is constructed by an induced H-action. For any compact subgroup K of a topological group H, it is verified that any map from the topological spcae Y to the orbit space of K-action is continuous and a K-map. For any H-space, K-map and an induced $\mathcal{S}$-topological transformation group; it is proved that there is a unique semi totally continuous H-map. Additionally, it is shown that for a topological group, a subgroup K of topological group and a K-space, there is a unique H-space and a unique injective K-map and also it is established that for a H-space and a semi totally continuous K-map, there exists a unique semi totally continuous H-map. Finally, it is demonstrated that for a finite Hausdorff topological group, finite Frechet space and a M-space, any map from the orbit space of M-action to $H \times_N (N \times_M Y)$ is semi totally continuous, for the subgroups M and N of topological group.

Keywords Topological Transformation Group(TTG), $\mathcal{S}$-topological Transformation Group($\mathcal{S}$-TTG), Isotropy Group, Homogeneous Spaces, Induced Transformation Groups

1 Introduction

Topology and algebra are pivotal branches of mathematics that mutually complement each other. Topology focuses on concepts like continuity and convergence, while algebra scrutinizes various operations. Despite often evolving autonomously, they naturally intertwine with other mathematical domains like functional analysis, representation theory, and dynamical systems. Notably, many significant mathematical entities embody a fusion of topological and algebraic attributes. TTGs exemplify this synergy, embodying both topological and algebraic structures. Montogomery and Zippin's study of Hilbert's fifth problem established the framework for examining the TTG [1]. Understanding the concept of a TTG is crucial in mathematics, especially in the realms of topology and group theory. TTG is a structure that combines the concepts of Topological group(TG), topological space(TS) and continuous group actions to study symmetries and transformations of geometric objects. In 1966, William J. Gray [2] discussed a TTG having an end (fixed) point. J. De Vries [3] described a universal TTG in terms of the actions of any infinite locally compact group. Dimension of a TTG was given by Hsu-Tung Ku et al. [4]. In 1990, David B. Ellis [5] analyzed the suspensions of TTG.

Homogeneous space is a space in which any point can be transformed into any other point by an element of the transformation group acting on the space. Homogeneous space is a mathematical structure that demonstrates particular kinds of symmetry under the group action. Homogeneous spaces can be observed in several fields of mathematics such as group theory, differential geometry and algebraic geometry. Homogeneous spaces enable an approach to examining the symmetries and structure-preserving transformations of a space. The homogeneous sub-Riemannian manifolds whose normal extremals are the orbits of one-parameter subgroups of the group of smooth isometries have been examined by Z. Yan et al. [6] in 2024. The classification of all G-geodesic orbit metrics has been carried out by Z. Chen et al. [7] on a Riemannian homogeneous space that is compact and simply connected in 2023. The existence of strongly tame sets in affine algebraic homogeneous spaces of linear algebraic Lie groups was proven by R. B. Andrist and R. Ugolini [8] in 2022. In 2021, H. Chen et al. [9] proved that every geodesic orbit metric on homogeneous spaces, obtained from two strongly isotropy irreducible spaces, is naturally reductive. In 2020, B. V. Sorin [10] introduced the notion of a strongly homogeneous G- space. In 2017, E. Celakoska et al. [11] introduced a homogenous space of complex constrained vectors in complex velocities and M. Xu, S. Deng [12] studied normal homogeneous Finsler spaces. V. V. Gorbatsevich [13] studied the automorphism group of the homogeneous space in 2016. In 2013, L. K. Kozlov [14] examined the spaces with various forms of homogeneity using the topological property of d-openness of actions of transformation groups. In 2002, D. Guan [15] solved the problem of the classification of compact complex homogeneous spaces with invariant volumes.

The primary emphasis of this paper is the investigation of the homogeneous spaces and an induced transformation groups of $\mathcal{S}$-TTG. It is established that a map from a TG to $H\big/H_y$ is surjective, continuous, open and a map from $H\big/H_y$ to the homogeneous space is determined to be H-isomorphism and semi totally continuous(STC) for a finite Frechet space and a finite Hausdorff topological group(HTG). Later, it is shown that there is a partial order relation on the family of all Hausdorff homogeneous spaces, for a compact HTG. Subsequently, an induced $\mathcal{S}$-TTG is constructed by an induced H-action. For any H-space, K-map and an induced $\mathcal{S}$-TTG, it is proven that a unique STC H-map exists. Also, it is shown that for a TG, a subgroup K of TG and a K-space, there is a unique H-space and a unique injective K-map and further, a unique STC H-map is constructed for a H-space and a STC K-map. Finally, it is shown that for a finite HTG, a finite Frechet space and a M-space, the map $f : H \underset{M}{\times} Y \to H \underset{N}{\times} (N \underset{M}{\times} Y)$ is STC, for the subgroups M and N of TG.

2 Preliminaries

Basic definitions were discussed in this section.

Definition 2.1. [16] A nonempty set H is called a TG, if
(1) H is a group.
(2) H is a TS.
(3) Both multiplication $\sigma : H \times H \to H$ and inversion $\gamma : H \to H$ maps are continuous.

Definition 2.2. [17] Let H be a TG, Y is a TS and the map $\varsigma : H \times Y \to Y$ is continuous. Then the triplet (H, Y, ς) forms a TTG such that the following conditions holds,
(1) $\varsigma(e, y) = y$, for every $y \in Y$, where e is the identity element of H.
(2) $\varsigma(h_2, \varsigma(h_1, y)) = \varsigma(h_2h_1, y)$, for all $h_1, h_2 \in H$ and $y \in Y$.

Definition 2.3. [18] Given a TS Y and its subset B, if $B \subseteq$ cl(int$B$), then B is called semi open.

Definition 2.4. [19] Given two TS, Y and Z, a function $f : Y \to Z$ is STC, if all semi open subsets of Z have an inverse image that is clopen in Y.

The concept of $\mathcal{S}$-TTGs is defined and examined by C. Rajapandiyan et al. [20] in 2024. $\mathcal{S}$-TTGs is formed when a TTG is weakened by STC instead of continuity. $\mathcal{S}$-TTGs imply a TTG but the converse is not necessarily true. The collection of all STC functions of X onto itself, denoted by $STC_G(X)$ constitutes a paratopological group under composition. The extremally disconnectedness property of $STC_G(X)$ creates a Moscow topological group structure on $STC_G(X)$ and it contains an open boolean subgroup.

Definition 2.5. [20] Let H be a TG, Y a TS and a STC map $\delta : H \times Y \to Y$. The triplet (H, Y, δ) forms a $\mathcal{S}$-TTG such that the following conditions hold,
(1) $\delta(e, y) = y$, for every $y \in Y$, where e is the identity element of H.
(2) $\delta(h_2, \delta(h_1, y)) = \delta(h_2h_1, y)$, for all $h_1, h_2 \in H$ and $y \in Y$.

Theorem 2.6. [20] For $\mathcal{S}$-TTG (H, Y, δ), $h \in H$, let a map $\delta_h : Y \to Y$ be defined by $\delta_h(\mathrm{y}) = \delta(h, \mathrm{y})$. Then δ_h and its inverse are semi totally continuous.

Definition 2.7. [21] Let (H, X, δ) and $(H, Y, \overline{\delta})$ be two S-TTGs, a map $\varphi : X \to Y$ is said to be an equivariant map or a H-map if for any $h \in H$ and $x \in X$, $\varphi(\delta(h, x)) = \overline{\delta}(h, \varphi(x))$.

Definition 2.8. [22] The isotropy group at y for $\mathcal{S}$-TTG $\big(H, Y, \delta\big)$ is the set $H_y = \{h \in H \big| hy = y\}$, for a fixed $y \in Y$.

3 Homogeneous Spaces of $\mathcal{S}$-TTG

In this section, homogeneous spaces of $\mathcal{S}$-TTG are defined and some basic properties of $\mathcal{S}$-TTG are discussed.

Definition 3.1. Let (H, Y, δ) be $\mathcal{S}$-TTG. For any $y \in Y$, the subspace $H(y) = \{hy \in Y \big| h \in H\}$ is known as the orbit of y.

Definition 3.2. $\mathcal{S}$-TTG (H, Y, δ) is called transitive if there is precisely one orbit.

Definition 3.3. $\mathcal{S}$-TTG (H, Y, δ) is called homogeneous if the STC action on Y is transitive.

Example 3.4. Let H be a TG, H/K a left coset space of a TG by a subgroup K and δ a STC action. Then $(H, H/K, \delta)$ is $\mathcal{S}$-TTG and the map $\delta : H \times H/K \to H/K$ defined by $\Psi(h^{'}, hK) = h^{'}hK$ is homogeneous.

Example 3.5. Let $H = \{e, i, j, ij | i^2 = e, j^2 = e, ij = ji\}$ be a group, $K = \{i, j\}$ be a subgroup of H and $H/K = h_1K = \{iK, ijK\}$. Now, $(H, H/K, \delta)$ forms $\mathcal{S}$-TTG and the STC action $\delta : H \times H/K \to H/K$ defined by $\delta(h', h_1K) = h'h_1K$ is homogeneous.

Definition 3.6. A map between any two S-TTGs is said to be H-isomorphism, if the map is bijective and equivariant.

Proposition 3.7. The map $\Pi : H \to H/H_y$ is surjective, continuous and open.

Proof. Let Π denote a map from H to its quotient set H/H_y, given by $h \mapsto hk$. Define a topology τ_{H/H_y} on H/H_y by $\tau_{H/H_y} = \{\Pi(U) | U \in \tau_H\}$, where τ_H is the topology on H. Thus, H/H_y is a quotient space. Since every coset has an inverse image, Π is surjective and clearly Π is continuous. Now, let U be an subset of H, then $\Pi^{-1}(\Pi(U)) = Uk = \bigcup\{Uk, k \in K\}$, which is open. So, $\Pi(U)$ is open in H/H_y. Hence Π is an open map. □

Theorem 3.8. Let (H, Y, δ) be $\mathcal{S}$-TTG. If H is finite and Hausdorff, Y is a finite Frechet homogeneous space of H. Then $\varphi : H/H_y \to Y$ is a H-isomorphism and STC.

Proof. Let $\varphi : H/H_y \to Y$ be a map defined by $\varphi(hH_y) = hy$. For $h, h' \in H, hH_y = h'H_y$. Then $H_y = h^{-1}h'H_y, h^{-1}h' \in H_y$. By the definition of $H_y, (h^{-1}h')y = y$, which implies $h'y = hy$ and therefore we have $\varphi(hH_y) = \varphi(h'H_y)$. Hence φ is well-defined. Now, for any $y \in Y$, there exists $h \in H$ with $y = hy$. Then $\varphi(hH_y) = hy = y$. Hence φ is surjective. Also, let $hy = h'y$. Then $h^{-1}h' \in H_y$ and $H_y = h^{-1}h'H_y$. Hence $hH_y = h'H_y$, So φ is injective. For $h, h' \in H$, we have $\varphi(h'(hH_y)) = \varphi((h'h)H_y) = (h'h)y = h'(hy) = h'\varphi(hH_x)$. Hence φ is a H- map. Therefore φ is H- isomorphism.

Since $\overline{\varphi} : H \to Y$ by $\overline{\varphi}(h) = y, \overline{\varphi}$ be STC. STC of φ follows from the commutative diagram given in Figure 1.

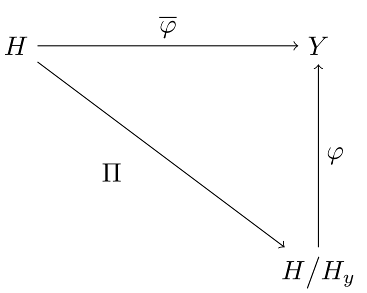

Figure 1. STC of φ

Since Π is continuous and $\overline{\varphi}$ is STC, it follows that φ is STC. □

Definition 3.9. Equivariant map between homogeneous spaces Y and Z is given by the map $f : Y \to Z$ satisfying $f(H(y)) = H(f(y))$ for any $y \in Y$ such that f is locally equivariant map.

Note 3.10. [23] If H is compact and Y is Hausdorff, a homogeneous space usually has the form H/K.

Remark 3.11. Let H be a compact group, K and L be a clopen subgroups of H. Then

1. K is a conjugate subgroup of L if and only if there exists an equivariant map $f : H/K \to H/L$.
2. For $a \in H$ and $aKa^{-1} \subset L$, the map $f_a : H/K \to H/L$ defined by $f_a(hK) = ha^{-1}L$ is a well-defined equivariant map.
3. $ab^{-1} \in L$ if and only if $f_a = f_b$.

Note 3.12. Assume that H be a compact Hausdorff TG. Define the family $\mathcal{F}$ of all Hausdorff homogeneous spaces of H and a relation $\sim$ on $\mathcal{F}$ as follows, for $Y, Z \in \mathcal{F}, Y \sim Z$ if and only if there is a semi totally continuity $f : Y \to Z$. Then $\sim$ is a partial order relation.

4 Induced Transformation Groups of $\mathcal{S}$-TTG

In this section, induced $\mathcal{S}$-TTG is defined and some properties of induced $\mathcal{S}$-TTG are examined.

Let H be a TG and K a subgroup of H. A H-space can be inherently constructed from K-space Y as follows,

Consider a K-action on Y as $\delta : K \times Y \to Y$. Then a K-action on $H \times Y$ is given by $(k, (h, y)) \mapsto (hk^{-1}, \delta(k, y))$ and $H \underset{K}{\times} Y$ indicates the orbit space of K-action, $\Pi : H \times Y \to H \underset{K}{\times} Y$ is the projection and (h, y) represents the K-orbit. A H-action on $H \underset{K}{\times} Y$ defined as $\widetilde{\delta} : H \times H \underset{K}{\times} Y \to H \underset{K}{\times} Y$ by $\widetilde{\delta}(h', (h, y)) = (h'h, y)$ is STC. The H-action $\widetilde{\delta}$ on $H \underset{K}{\times} Y$ that is obtained in this manner is referred to as the induced H-action.

Definition 4.1. The triplet $(H, H \underset{K}{\times} Y, \widetilde{\delta})$ is called the induced $\mathcal{S}$-TTG of (K, Y, δ) and the space $H \underset{K}{\times} Y$ together with the action $\widetilde{\delta}$ is known as the induced H-space.

Lemma 4.2. If K is a compact subgroup of a TG, then the map $j : Y \to H \underset{K}{\times} Y$ is continuous and a K-map.

Proof. For any $k \in K$,

$$\begin{aligned} j(ky) &= (e, ky) \\ &= (k, y) \\ &= k(e, y) \\ &= kj(y) \end{aligned}$$

Hence j is a K-map and from the following commutative diagram in Figure 2, the continuity of j follows.

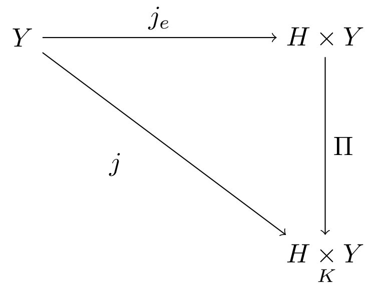

Figure 2. Continuity of j

Since, both j_e and Π are continuous, j is also continuous. □

Remark 4.3. The projection $H \underset{K}{\times} y \to h$ induces a projection $P : H \underset{K}{\times} Y \to H/K$.

Theorem 4.4. For an induced $\mathcal{S}$-TTG $(H, H \underset{K}{\times} Y, \widetilde{\delta})$, any H-space Z and any K-map $f : Y \to Z$, there is a unique H-map $\widetilde{f} : H \underset{K}{\times} Y \to Z$, which is STC such that the following diagram provided in Figure 3 commutes.

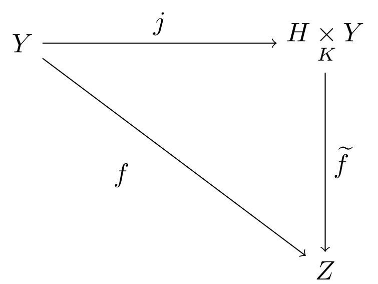

Figure 3. $\widetilde{f} \circ j = f$

Proof. For any H-space Z and any K-map $f : Y \to Z$, let $\widetilde{f} : H \underset{K}{\times} Y \to Z$ be a map defined by $\widetilde{f}(h, y) = hf(y)$, for $h \in H, y \in Y$. If $(h, y) = (h', y')$, there exists $k \in K$ with $h' = hk^{-1}, y' = ky$. Then

$$\begin{aligned} \widetilde{f}(h', y') &= \widetilde{f}(hk^{-1}, ky) \\ &= hk^{-1} f(ky) \\ &= hk^{-1} k f(y) \\ &= hf(y) \\ &= \widetilde{f}(h, y) \end{aligned}$$

$\implies \widetilde{f}$ is well-defined.

$\widetilde{f}$ is a H-map,

$$\begin{aligned} \widetilde{f}(h', (h, y)) &= \widetilde{f}(h'h, y) \\ &= (h'h) f(y) \\ &= h'(hf(y)) \\ &= h' \widetilde{f}(h, y) \end{aligned}$$

Hence $\widetilde{f}$ is H-map. Now, the commutativity of the diagram is given by,

$$\begin{aligned} \widetilde{f} \circ j(y) &= \widetilde{f}(e, y) \\ &= ef(y) \\ &= f(y). \end{aligned}$$

Uniqueness of $\widetilde{f}$,

Let $\widetilde{f}' : H \underset{K}{\times} Y \to Z$ be another $H-$map with $\widetilde{f}' \circ j = f$. Then

$$\begin{aligned} \widetilde{f}'(h, y) &= \widetilde{f}'(h(e, y)) \\ &= h\widetilde{f}'(e, y) \\ &= h\widetilde{f}' \circ j(y) \\ &= hf(y) \\ &= \widetilde{f}(h, y) \end{aligned}$$

$\implies \widetilde{f}' = \widetilde{f}$

Let $\widetilde{\delta}' : H \times Z \to Z$ be the $H-$action on Z. The STC of $\widetilde{f}$ follows from the commutative diagram in Figure 4.

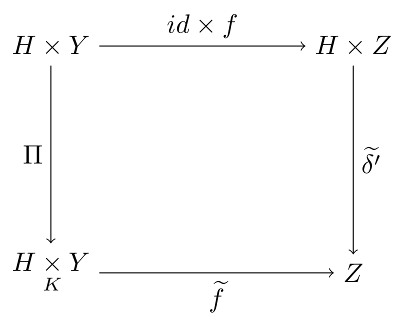

Figure 4. STC of $\widetilde{f}$

Since $id \times f$, Π are continuous, $\widetilde{\delta}'$ is STC and it follows that $\widetilde{f}$ is STC. □

Theorem 4.5. For a TG H, a subgroup K of H and a K-space Y, there is a unique H-space $\widetilde{Y}$ and a unique injective K-map $j : Y \to \widetilde{Y}$. Then for a H-space Z and a STC K-map $f : Y \to Z$, the map $\widetilde{f} : \widetilde{Y} \to Z$ is a unique H-map and STC such that the following diagram given in Figure 5 commutes,

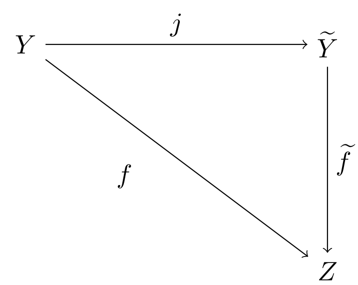

Figure 5. H-map $\widetilde{f}$

Proof. Let $\widetilde{Y} = H \underset{K}{\times} Y, j = j$ in Theorem 4.4 , then $\widetilde{f}$ is a unique H-map, STC and the diagram in Figure 5 commutes. Now, the uniqueness of j is as follows,

Let $j_1 : Y \to \widetilde{Y_1}$ and $j_2 : Y \to \widetilde{Y_2}$ be the maps, then there exists two unique H-maps, $\widetilde{j_2} : \widetilde{Y_1} \to \widetilde{Y_2}$ and $\widetilde{j_1} : \widetilde{Y_2} \to \widetilde{Y_1}$ such that the commutative diagram is provided in Figure 6, Figure 7 commute.

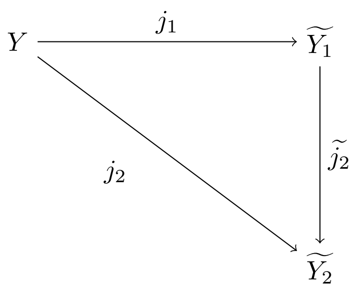

Figure 6. $\widetilde{j_2} \circ j_1 = j_2$

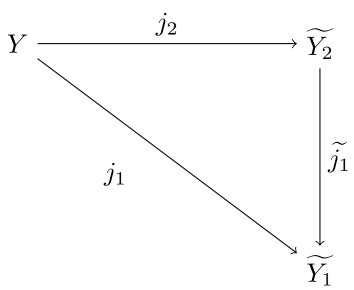

Figure 7. $\widetilde{j_1} \circ j_2 = j_1$

Now, combine the diagrams in Figure 6 and Figure 7, we obtain the commutative diagram which is provided in Figure 8, Figure 9.

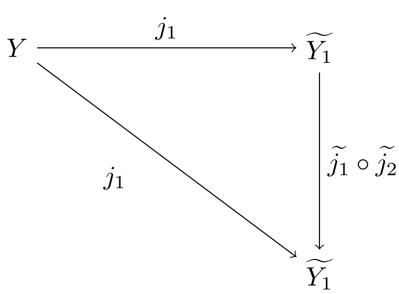

Figure 8. $\widetilde{j_1} \circ \widetilde{j_2}$

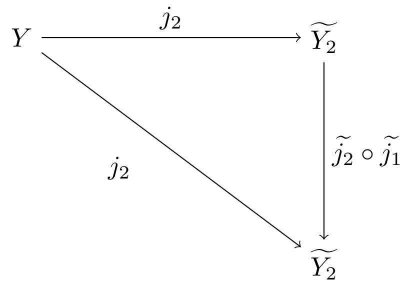

Figure 9. $\widetilde{j_2} \circ \widetilde{j_1}$

Replace both $\widetilde{j_1} \circ \widetilde{j_2}$ and $\widetilde{j_2} \circ \widetilde{j_1}$ with the identity map, then the diagrams in Figure 8, Figure 9 commute. By the uniqueness of $\widetilde{f}$, we have $\widetilde{j_1} \circ \widetilde{j_2} = id, \widetilde{j_2} \circ \widetilde{j_1} = id$ and hence j is unique. □

Theorem 4.6. Let H be a finite HTG and Y a finite Frechet space. Let M and N be subgroups of H with $M \subset N$. Then for a M-space Y, the map $f : H \underset{M}{\times} Y \to H \underset{N}{\times} (N \underset{M}{\times} Y)$ defined by $f(h, y) = (h, (e, y))$ for $h \in H, y \in Y$ is STC.

Proof. For $(h, y) = (h', y')$, there is $m \in M$ with $h' = hm^{-1}, y' = my$. Then

$$\begin{aligned} f(h', y') &= f(hm^{-1}, my) \\ &= (hm^{-1}, (e, my)) \\ &= (hm^{-1}, (m, y)) \\ &= (hm^{-1}, m(e, y)) \\ &= (h, (e, y)) \\ &= f(h, y) \end{aligned}$$

$\Rightarrow f$ is well-defined and the STC of f follows from the commutative diagram provided in Figure 10,

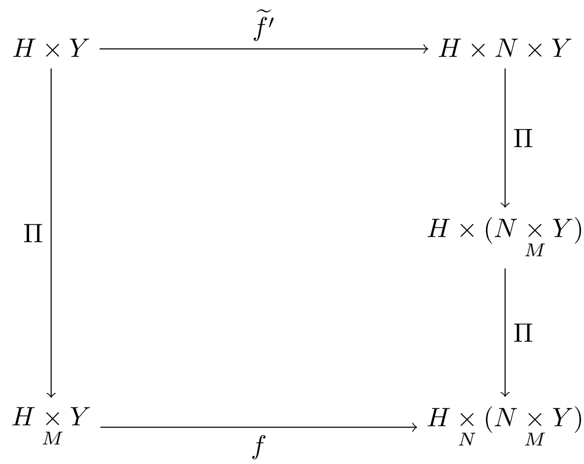

Figure 10. STC of f

In the above, the map $\widetilde{f}' : H \times Y \to H \times N \times Y$ defined by $\widetilde{f}'(h, y) = (h, e, y)$ is STC, for $h \in H, y \in Y$. Since all the Π are continuous and it follows that f is STC. □

Corollary 4.7. Let H be a finite HTG and Y be a finite Frechet space. Let $M_1, ..., M_n$ be subgroups of H with $M_1 \subset M_2 \subset ... \subset M_n$. Then for a M_1-space Y. The map $f : H \underset{M_1}{\times} Y \to H \underset{M_n}{\times} \Big(M_n \underset{M_{n-1}}{\times} ... \Big(M_3 \underset{M_2}{\times} \Big(M_2 \underset{M_1}{\times} Y\Big)\Big)...\Big)$ defined by $f(h, y) = (h, (m_n, (m_{n-1}, (...(m_2, y.))..)))$, for $h \in H, m_i \in M_i, y \in Y$ is STC.

Proof. By induction in Theorem 4.6, the proof follows. □

5 Conclusions

Some essential attributes of homogeneous space and induced transformation groups of $\mathcal{S}$-TTG have been explored and presented. We proved that the map from H/H_y to the homogeneous space is both H-isomorphism and STC for a finite HTG and a finite Frechet space. Subsequently, it is proved that a STC H-map exists, for any given H-space, K-map and induced $\mathcal{S}$-TTG. Furthermore, for a given H-space and a STC K-map, it is established that there is a unique STC H-map. In a future study, we investigate more properties of the homogeneous space and induced transformation groups of $\mathcal{S}$-TTG.

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Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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