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An Extension of The Hesitant Fuzzy Weight Averaging Operator-VIKOR Method under Hesitant Fuzzy Sets

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Abstract The hesitant fuzzy set (HFS) is an innovative approach to decision-making under uncertainty. This study addresses the aggregated operation of the HFS decision matrix. The introduction of induced VIKOR procedures, various extensions of HFSs aggregation operator, and essential approaches for multi-criteria decision-making (MCDM) are presented. This technique uses the aggregation operator, HFWA operator, to rank alternatives and identify the compromise solution that comes closest to the ideal solution. We developed the hesitant fuzzy weight averaging VIKOR (HFWA-VIKOR) model as a novel technique to achieve this. By combining the hesitant fuzzy elements, the HFWA aggregation operator creates aggregated values that are expressed as a single value. The primary advantage of the HFWA-VIKOR model lies in its initial step of aggregating the hesitant fuzzy element. This results in an initial hesitant fuzzy decision matrix, which provides much more detailed information for decision-making and, through the use of the inducing HFWA operator, represents the complex attitudinal nature of the decision-makers. The multi-criteria location selection problem is then solved using the combined HFWA-VIKOR technique, and the outcomes are presented in an easy-to-understand way owing to aggregation operators. A numerical example is also applied in this new method which gives out the best alternative result. As per the scope of our research work, MCDM under hesitant fuzzy sets with HFWA-VIKOR method have been used and their result revealed the best alternative is to find out. These results indicate good potential for objectives. This technique may also be used for other studies or applications. Further research in this area may provide a more developed technique for this application.

Keywords MCDM, VIKOR Method under Hesitant Fuzzy Data, Decision Making, HFWA Operator, Hesitant Fuzzy Set

1 Introduction

One of the most prevalent processes in human society is MCDM. Selecting an outstanding option out of a range of accessible options based on predetermined criteria or attributes entails this process. Numerous MCDM approaches are suggested in the studies to tackle the MCDM problem [1, 2] effectively. The VIKOR method [3,4], which was founded on concepts of compromise programming [5,6], is a highly helpful tool for decision-making including numerous criteria. The VIKOR approach's key benefits are its ability to handle discrete choice issues with competing and non-commensurable criteria [7] and its ability to solve as close to the ideal as is practicable. The VIKOR technique emphasizes grading and choosing from a variety of possibilities and provides compromise solutions to an issue having opposing parameters in order to aid decision-makers in reaching a final decision [8,9].

The VIKOR approach has recently been researched and used to solve a variety of issues, including [10]. In general, the weighted average and maximum weighted methods are used to derive the independent greatest and worst values as assessed when employing VIKOR in decision-making. The HFWA operator developed by Xia and Xu [11] can be applied in this situation. As a result, it is possible to take into consideration the decision maker's complex attitudinal character by applying order-inducing parameters in the aggregation procedure. Numerous publications have recently examined the HFWA operator; for instance, see [12].

This paper's goal is to introduce the HFWA-VIKOR technique for decision-making under multiple criteria that take into account the complicated attitudinal structure of a decision-maker and to demonstrate the application of induced aggregation operators in the VIKOR technique with hesitant fuzzy data. By utilizing the HFWA operator in the VIKOR technique, it is possible to deal with complex attitudinal factors of the decision-makers and present a much more complete view of the decision-making process. As a result, we are able to handle more challenging issues that are more representative of actual situations, and the decision-maker can choose the solution that best serves his or her interests. We examine several of the HFWA operator's fundamental characteristics as well as numerous special scenarios. Each instance is appropriate for a particular circumstance in accordance with the decision-maker's goals. It gives a more detailed illustration of the particular issue under consideration and uses the HFWA operator as an example.

One of the best tools for making decisions is the fuzzy set theory, which Zadeh [13] first developed. It can deal with ambiguous and imprecise information. Whenever working with insufficient and fuzzy data when several or more sources of uncertainty are present simultaneously, the classic fuzzy set has a number of drawbacks [14]. First, Torra and Narukawa [15,16] defined the hesitant fuzzy set (HFS), providing new research areas for the study of decision-making in uncertain circumstances in the future. This generalized type of fuzzy set is founded on the aforementioned extensional forms of fuzzy sets. The HFS illustrates fuzziness by presenting all possible values when calculating an element's membership degree in a certain set despite the fact that it does not offer an accurate membership function.

The benefit of employing HFS is clear. Two different types of instances can demonstrate the benefits of HFS in decision-making. On the one hand, employing HFS makes it quite similar to how humans think. It should be noted that while determining whether an element belongs to a specific set, the decision-maker, the elicitation of imprecise information via the aforementioned extended forms depends on single or interval values that should include and convey the provided information. However, in some instances, the problem-solving decision-makers may have a range of potential values. Therefore, they cannot supply a single phrase or an expression. Interval values convey their choices or evaluations when they are considering many potential values simultaneously. As a result, the HFS, whose membership includes is expressed by a range of numbers, when used, can perfectly solve this problem, while the extensions listed above are not valid. A different kind of instance occurs frequently in our daily lives because of the rising. Due to the complexity of today's socioeconomic environments, solitary people are becoming less and less likely when assessing the taken into account items; a decision-maker must take into account all pertinent parts of a situation. As mentioned by Yu [17], a group of people who want to strengthen their overall negotiating power may decide to form a union or a corporation with themselves as the shareholders. Typically they have a few differences in opinions. The differences stem from the disparity in their subjective assessments of the emerging decision-making issues. Because the decision-makers may get varying viewpoints on the options being considered due to their distinct knowledge bases or advantages, it might be challenging to arrive at a consistent result because they cannot simply persuade one another. However, some factors can help. Because of possible values, it is more powerful, and the HFS is suited to solve this problem. After that, any more expanded fuzzy sets. As an illustration, assume the decision-making body or constitution is requested to provide the levels at which substitute decision-makers prefer to represent one alternative over another using values between 0 and 1. Consequently, it is more effective and appropriate to use HFS to describe the uncertain evaluation information.

The novelty of this paper is that it uses an aggregation operator namely a hesitant fuzzy weight averaging operator applied on the hesitant fuzzy element and induces these aggregated values in the VIKOR method under hesitant fuzzy information.

The article's remaining sections are organized as follows. The HFWA operator and a few other aggregation operators are presented in Section 2, along with a quick overview of some fundamental ideas that are used throughout the article. We present the enhanced VIKOR approach of HFSs for MCDM issues in Section 3. We describe the creation of the HFWA-VIKOR approach in Section 4 for MCDM with the HFWA operator in HFSs information. In Section 5, an example is given to explain the proposed strategy. The paper's principal results are then briefly discussed.

2 Preliminaries

In this section, we briefly define fuzzy sets, HFSs, HFWA operator and HFWG operator.

2.1 Definition [18].

Suppose X be the universal set. Fuzzy set in X is defined by membership function
$B_1 \subseteq X$, $\mu_{B_1}(\tilde{x}) : X \to [0,1]$ as given as:

$$B_1 = \{(\tilde{x}, \mu_{B_1}(\tilde{x})), \tilde{x} \in X\},$$

where $\mu_{B_1}(\tilde{x})$ is the degree of membership of $\tilde{x}$ in B_1 and each pair $(\tilde{x}, \mu_{B_1}(\tilde{x}))$ is singleton.

Example 1. Suppose $X = \{\tilde{a}, \tilde{b}, \tilde{c}\}$ and $\mu_{B_1}(\tilde{x}) : X \to [0,1]$ defined by $\mu_{B_1}(\tilde{a}) = 0.3, \mu_{B_1}(\tilde{b}) = 0.6, \mu_{B_1}(\tilde{c}) = 0.9$. mathematically as given below:

$$B_1 = \{(\tilde{a}, 0.3), (\tilde{b}, 0.6), (\tilde{c}, 0.9)\}.$$

2.2 Definition[19,20].

Let X be the universal set, then a HFS as B on X is defined by function $\rho_B(\tilde{x})$ that X returns to subset of [0,1]. HFSs can be written asgiven below:

$$B = \{< \tilde{x}, \rho_B(\tilde{x}) > | \tilde{x} \in X\},$$

where $\rho_B(\tilde{x})$ is defined as a set of membership degree for an element under a subset of [0,1], indicating the membership degree of an element $\tilde{x} \in X$.

Example 2. Suppose $X = \{\tilde{a}_1, \tilde{b}_1, \tilde{c}_1\}$ be a universal set, $\rho_B(\tilde{a}_1) = \{0.1, 0.9, 0.6\}, \rho_B(\tilde{b}_1) = \{0.6, 0.7\}, \rho_B(\tilde{c}_1) = \{0.7, 0.3\}$ then HFSs can be written as:

$B =$
$\{< \tilde{a}_1, \{0.1, 0.9, 0.6\} >, < \tilde{b}_1, \{0.6, 0.7\} >, < \tilde{c}_1, \{0.7, 0.3\} >\}$

2.3 Definition[19].

An HFWA operator is defined by $H^n \rightarrow H$ such that HFWA$(h_1, h_2, h_3, ..., h_n)$=$\cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \gamma_3 \in h_3, ..., \gamma_n \in h_n}$ $\{1 - \prod_{j=1}^{n}(1-\gamma_j)^{w_j}\}$, where $w_j \in [0, 1]$ is the weight vector of h_j and $\sum_{j=1}^{n} w_j = 1$.

2.4 Definition[19]

An HFWG operator is defined by $H^n \rightarrow H$ such that HFWG$(h_1, h_2, h_3, ..., h_n)$=$\cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \gamma_3 \in h_3, ..., \gamma_n \in h_n}$ $\{\prod_{j=1}^{n} \gamma_j^{w_j}\}$, where $w_j \in [0, 1]$ is the weight vector of h_j and $\sum_{j=1}^{n} w_j = 1$.

3 Extended VIKOR method on HFSs

VIKOR method was discovered by Opricovic in 1998 [21] for the control and optimization of the multi-criteria system. It is a soothing technique that minimises remorse and benefits the group while allowing rankings of a finite decision to fluctuate. This method is very effective in MCDM, and it can deal with and solve the following problems.
(i) When decision-makers fail to ascertain their priorities.
(ii) When there is an opposing clash and incommensurability in evaluation.
(iii) A decision-maker who encounters confusion realistically accepts the compromise.
Finding the productive outcome, or positive ideal solution (PIS) and negative ideal solution (NIS), and then selecting an optional solution based on the evaluation value and position degree are the key concepts that VIKOR represents. Therefore, the PIS was designated with constraints on benefits that were acceptable and consistent with the procedure.
The VIKOR approach yields solutions that are both close to and a compromise over the ideal solution. By learning from the side of the benefits and removing itself from personal losses, the VIKOR algorithm arrives at a compromise option that is typically preferred by decision-makers. The TOPSIS and VIKOR are only idealized compromise methods, but the VIKOR method does not answer the question that the best solution lines near to the best problem and far from the negative prospects. As a result, it can vilify the solutions. The optimal solution is gained as a result of the VIKOR ideal scheme but TOPSIS is different for the synthesis method.
VIKOR adopts a normal function modeled by the L_p metric. L_i^p is for the measurement of the distance between best ideal and alternatives A_i that is suggested by Duckstein and Opricovic in 1980 [22]. The values obtained by ith alternatives under jth criteria are directly read as F_{ij}. The VIKOR method takes its connection the following L_p metric:
$L_i^p = \left\{\sum_{j=1}^{n}[W_j(Fj^* - Fij)/(Fj^* - Fj^-)]^p\right\}^{1/p}$,
Where $Fj^* = \max_i(F_{ij})$ and $Fj^- = \min_i(F_{ij})$.
The Opricovic [23] adopted the fuzzy VIKOR method to solve problems in an uncertain way where the criteria and weight characterize fuzzy sets [24] which is shown in Figure. 1.
Step 1. Determine PIS and NIS:
$A^* = [h_1^*, ..., h_n^*]$,
where $h_j^* = \cup_i^m h_{ij}$
$A^- = [h_1^-, ..., h_n^-]$,
where $h_j^- = \cap_i^m h_{ij}$ where j=1,2,3,..,m
Step 2. Compute S_i and R_i below
$S_i = \sum_{j=1}^{n} W_j ||h_j^* - h_{ij}||/||h_j^* - h_j^-||$,
Calculate regret measure
$R_i = \max_j(S_i) = W_j||h_j^* - h_{ij}||/||h_j^* - h_j^-||$, where i=1,2,...,$m$.
Step 3. Evaluate Q_i, $i = 1, 2, ..., m$, by the relation given below
$Q_i = V(\frac{S_i - S^-}{S^* - S^-}) + (1 - V)(\frac{R_i - R^-}{R^* - R^-})$ and V is introduced weight, where $S^* = \min_i(S_i), S^- = \max_i(S_i), R^- = \max_i(R_i), R^* = \min_i(R_i)$.
Step 4. Classify the alternatives, categorizing through S, R, and Q values, from largest to smallest. The outcome is in three grade.
Step 5. Provide the alternative (A_1), which is graded top by smallest values by Q, as a compromise solution if the following two factors persist:
C1. Acceptable advantage: $Q(A_2) - Q(A_1) \geq (DQ)$ where A_2 is the alternative with 2nd position in the grading list by Q, DQ=$\frac{1}{m-1}$ where m denotes the possible alternatives.
C2."Acceptable stability in decision making": An alternative A_1 must also possess the highest position from S or R. As suggested the following compromise solution is if one of the requirements is not fulfilled.
• Alternative A_1 and A_2 if only C2 is not satisfied,or
• Alternatives A_i, where $i = 1, 2, 3..., m$ if C1 is not satisfied; A_m is determined by the relation $Q(A_m) - Q(A_1) < (DQ)$ by maximum m.

4 Extension of hesitant fuzzy weight averaging operator-VIKOR method under hesitant fuzzy sets

Each criterion can be seen as an HFS defined in terms of the opinions of the decision-makers [17] as well as helps the membership to choose from a set of possible values, which is an extremely beneficial tool for dealing with ambiguity in avoiding such issues when evaluating the correct numbers of the criterion is difficult or impossible. Since the values of the criteria should more appropriately be considered as hesitant fuzzy elements, benefit criteria are elements. Because of this, in the current research, we expand the HFWA operator-extended VIKOR approach to addressing the MCDM problem using the tentative fuzzy set data and aggregate tentative fuzzy sets in the VIKOR approach, which is depicted in Fig. 2. Consider a decision matrix as shown in Table 1 and Table 2, represented by the symbol, the following structure is used for hesitant fuzzy ele-

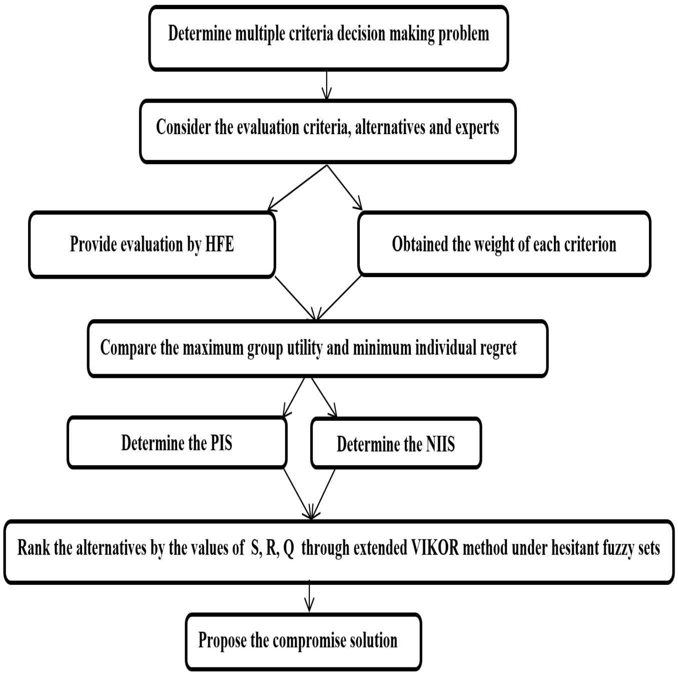

Figure 1. Mathematical steps of extended VIKOR method under hesitant fuzzy information.

ments:

Table 1. Decision matrix of hesitant fuzzy elements.

Alternatives	κ_1	κ_2	κ_3	....	κ_n
A_1	h_{11}	h_{12}	h_{13}	....	h_{1n}
A_2	h_{21}	h_{22}	h_{23}	....	h_{2n}
A_3	h_{31}	h_{32}	h_{33}	....	h_{3n}
...	...	...	...	...	...
A_m	h_{m1}	h_{m2}	h_{m3}	....	h_{mn}

where A_i are alternatives, κ_i are criteria , h_{ij} are the aggregated values and w_j is the weight of criteria.

Step 1. Evaluate the given data through hesitant fuzzy weight averaging operator (HFWA).

Step 2. Determine PIS and NIS:

$A^* = [h_1^*, ..., h_n^*]$,

where $h_j^* = \cup_i^m h_{ij}$

$A^- = [h_1^-, ..., h_n^-]$,

where $h_j^- = \cap_i^m h_{ij}$ where j=1,2,3,..,*m*

Step 3. Compute S_i and R_i below

$S_i = \sum_{j=1}^{n} W_j||h_j^* - h_{ij}||/||h_j^* - h_j^-||$,

Evaluate regret measure

$R_i = \max_j(S_i) = W_j||h_j^* - h_{ij}||/||h_j^* - h_j^-||$, where i=1,2,...,*m*.

Step 4. Evaluate Q_i by the relation given below

$Q_i = V(\frac{S_i - S^-}{S^* - S^-}) + (1 - V)(\frac{R_i - R^-}{R^* - R^-})$ where$V$ is introduced weight, and $S^* = \min_i(S_i), S^- = \max_i(S_i), R^- = \max_i(R_i), R^* = \min_i(R_i), i = 1, 2, ..., m$.

Step 5. Classify the alternatives, categorizing through *S*, *R*, and *Q* values, from largest to smallest. The outcome is in three grade.

Step 6. Provide the alternative (A_1), which is graded top by smallest values by *Q*, as a compromise solution if the following two factors persist:

C1. Acceptable advantage: $Q(A_2) - Q(A_1) \geq (DQ)$ where A_2 is the alternative with 2nd position in the grading list by *Q*, $DQ = \frac{1}{m-1}$ where *m* denotes the possible alternatives.

C2."Acceptable stability in decision making": An alternative A_1 must also possess the highest position from *S* or *R*. As suggested the following compromise solution is if one of the requirements is not fulfilled.

- Alternative A_1 and A_2 if only C2 is not satisfied,or
- Alternatives A_i, where $i = 1, 2, 3..., m$ if C1 is not satisfied; A_m is determine by the relation $Q(A_m) - Q(A_1) < (DQ)$ by maximum *m*.

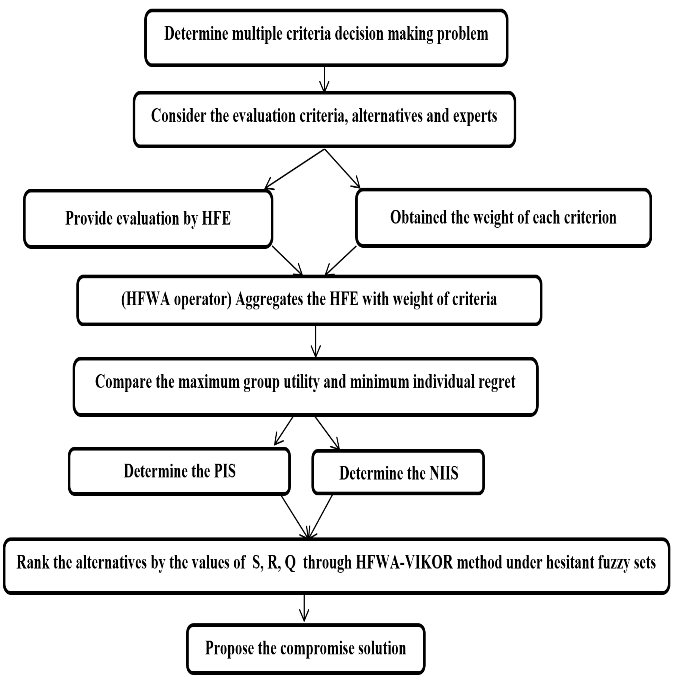

Figure 2. Mathematical steps of HFWA- VIKOR method under hesitant fuzzy information.

5 Numerical example

Suppose the government announces to construct a airport then the engineer decides the best location A_1, A_2, A_3, A_4 for the construction site airport on the basis of criteria with their weight $W = (0.11, 0.21, 0.23, 0.45)$.
κ_1: regional plan
κ_2: airport use
κ_3: proximity to other airport
κ_4: ground accessibility

5.1 Extended VIKOR method under hesitant fuzzy sets
Step 1. Determine PIS and NIS:
$A^* = [0.9, 0.9, 0.8, 0.7]$
$A^- = [0.3, 0.3, 0.2, 0.1]$,
Step 2. Compute S_i and R_i below
$S_i = \sum_{j=1}^{n} W_j ||h_j^* - h_{ij}||/||h_j^* - h_j^-||$,
$S_1 = 0.46455, S_2 = 0.41201, S_3 = 0.31208, S_4 = 0.475.$
Calculate regret measure
$R_i = \max_j(S_i) = W_j||h_j^* - h_{ij}||/||h_j^* - h_j^-||, R_1 = 0.14375, R_2 = 0.13125, R_3 = 0.09625, R_4 = 0.275.$
Step 3. Evaluate Q_i where $i = 1, 2, 3, 4.$ by the relation given below
$Q_1 = V(\frac{S_l - S^-}{S^* - S^-}) + (1 - V)(\frac{R_1 - R^-}{R^* - R^-}) = 0.0416, Q_2 = 0.1115, Q_3 = 1.000, Q_4 = 0.3895.$
where $S^* = 0.31208, S^- = 0.475, R^* = 0.09625$ and $R^- = 0.275.$
Step 4. Classify the alternatives, categorizing through *S*, *R*, and *Q* values, from largest to smallest. The outcome is in three graded as listed in Table 3.
Step 5. Provide the alternative (A_1), which is graded top by smallest values by *Q*, as a compromise solution if the given factor persists:
C1. $Q(A_2) - Q(A_1) \geq (\frac{1}{3})$ is satisfied.

Table 3. Ranking the alternatives.

Alternatives	*S*	*Q*	*R*	Ranking
A_1	0.46455	0.3992	0.14375	2
A_2	0.41201	0.5954	0.13125	3
A_3	0.31208	1.000	0.09625	4
A_4	0.4750	0.000	0.2750	1

5.2 Extension of hesitant fuzzy weight averaging operator-VIKOR method under hesitant fuzzy sets
Step 1. Evaluate the hesitant fuzzy decision matrix through hesitant fuzzy weight averaging operator (HFWA)
$h_{11} = 0.5367, h_{12} = 0.650, h_{13} = 0.505, h_{14} = 0.6149, h_{21} = 0.4608, h_{22} = 0.6736, h_{23} = 0.7085, h_{24} = 0.5900, h_{31} = 0.5140, h_{32} = 0.7846, h_{33} = 0.6149, h_{34} = 0.6903, h_{41} = 0.7763, h_{42} = 0.7191, h_{43} = 0.7191, h_{44} = 0.4457.$
Step 2. Determine PIS and NIS:
$A^* = [0.7763, 0.7846, 0.7191, 0.6903]$
$A^- = [0.4608, 0.650, 0.505, 0.4457]$,
Step 3. Compute S_i and R_i below
$S_i = \sum_{j=1}^{n} W_j ||h_j^* - h_{ij}||/||h_j^* - h_j^-||$,
$S_1 = 0.6621, S_2 = 0.4796, S_3 = 0.2033, S_4 = 0.5522.$
Calculate regret measure
$R_i = \max_j(S_i) = W_j||h_j^* - h_{ij}||/||h_j^* - h_j^-||, R_1 = 0.23, R_2 = 0.1845, R_3 = 0.119, R_4 = 0.45.$
Step 4. Evaluate Q_i where $i = 1, 2, 3, 4.$ by the relation given below
$Q_1 = V(\frac{S_l - S^-}{S^* - S^-}) + (1 - V)(\frac{R_1 - R^-}{R^* - R^-}) = 0.3323, Q_2 = 0.6935, Q_3 = 1.000, Q_4 = 0.1197.$
Where $S^* = 0.2033, S^- = 0.6621, R^* = 0.1119$ and $R^- = 0.45.$
Step 5. Classify the alternatives, categorizing through *S*, *R*, and *Q* values, from largest to smallest. The outcome is in three graded as listed in Table 4.
Step 6. Provide the alternative (A_1), which is graded top by smallest values by *Q*, as a compromise solution if the following two factors persist:
C1. $Q(A_2) - Q(A_1) \geq (\frac{1}{3})$ is not satisfied.
C2. An alternative A_3 has the best position in *S* and *R* so C2 is satified.

Table 4. Ranking the alternatives.

Alternatives	*S*	*Q*	*R*	Ranking
A_1	0.6621	0.3323	0.23	2
A_2	0.4796	0.6935	0.1845	3
A_3	0.2033	1.000	0.1119	4
A_4	0.5522	0.1197	0.45	1

5.3 Discussion and Analysis.
In this section, we discuss solving the numerical analysis on the extension of the VIKOR method under HFSs and the hesitant fuzzy weight averaging operator- VIKOR method. In an extension of the VIKOR method under HFSs, we first find the PIS and NIS and also use hesitant normalized Hamming distance during finding S_i, but if we solve the numerical example on hesitant fuzzy weight averaging operator- VIKOR method, there is no need to apply these steps just applying aggregation operator which aggregates the multiple values to single and the method under hesitant fuzzy sets looks like a general form of VIKOR method and solves the hesitant fuzzy data very easily and soon.

6 Conclusions

When there are competing criteria for a set of options, the VIKOR technique was created as an MCDM model to establish the order of choice. The decision-makers may accept the negotiated agreement because it increases the majority's overall profit and reduces the opponent's personal regret. In this study, we evaluated the use of induced aggregation operators in the VIKOR method under hesitant fuzzy information and

Table 2. Hesitant fuzzy decision matrix

Alternatives	κ_1	κ_2	κ_3	κ_4
A_1	$(0.3, 0.4, 0.6)$	$(0.3, 0.6, 0.7)$	$(0.2, 0.4, 0.5, 0.6)$	$(0.4, 0.5, 0.6, 0.7)$
A_2	$(0.3, 0.4, 0.5)$	$(0.3, 0.5, 0.6, 0.8)$	$(0.3, 0.6, 0.7, 0.8)$	$(0.5, 0.6)$
A_3	$(0.4, 0.5)$	$(0.3, 0.6, 0.7, 0.9)$	$(0.4, 0.5, 0.6, 0.7)$	$(0.6, 0.7)$
A_4	$(0.3, 0.5, 0.7, 0.9)$	$(0.5, 0.6, 0.7, 0.8)$	$(0.5, 0.6, 0.7, 0.8)$	$(0.1, 0.4, 0.5)$

formed an integrated HFWA-VIKOR model to solve multi-criteria problems with differing and non-commensurable criteria, specifically taking into account the complicated subjective character of the decision maker. This operator is particularly helpful because it provides a variety of specified aggregation operators that aggregate hesitant fuzzy sets and a sample method to calculate the separation between the aggregated values and maximum standardized distance. The key benefit of the suggested VIKOR technique is its capacity to model specific measurements between the highest and lowest values on the foundation of a reordering procedure that establishes the amount of orness required for the aggregation. Additionally, it is possible to use rank-producing factors to construct a more complicated retraining approach in decision-making that reflects the complex attitudinal nature of the decision-maker. In a decision-making situation involving the choice of a new integrated method, we have examined an application of airport construction zones where we can observe the various findings made using various distance aggregation operators. A more full image of the decision-making process is provided by the HFWA-VIKOR approach, enabling the choice the decision-maker to select the option that best suits his or her interests.

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