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Some New Oscillation Criteria for Euler-Bernoulli Beam Equations with Damping Term

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Abstract The main objective of this study is to investigate some new oscillation criteria for Euler-Bernoulli beam equations with damping term by using the integral average method and Riccati technique. Philo introduces the following new integral operator, which is the main tool in this paper. Our plan of action is to reduce the multidimensional problems to ordinary differential problem by using Jenson's inequality, assuming the assumptions and integration by parts with boundary conditions. With hinged, sliding and hinged-sliding end boundary conditions, several new sufficient conditions are established. The results improve and generalize those given in some previous papers, which can be seen by the examples given at the end of this paper. The majority of engineering constructions, ships, support buildings, airplanes, and rotor blades all use beams as structural elements. It is presumed that these elements are only subjected to static loads; yet, dynamic loads induce vibrations, which affect the stress and strain values. These mechanical phenomena also result in noise, instability, and the potential for resonance, which enhances deflections and failure. We analyze the spatial force load $f(x,t)$ the equations of a damped Euler-Bernoulli beam derived from the equation for the velocity or final time displacement that we measured. Usually, internal damping determines the nature of this term.

Keywords Euler-Bernoulli Beam, Oscillation, Damping Term, Hinged-sliding Ends

1. Introduction

Since ancient times, beams have been utilized to reinforce structures such as a certain type of bridges, framed buildings, thin engineering projects, robotic arms, nanotechnology, airplane wings and stabilizers [1-7]. The manuscript of Leonardo de Vinci [8] which accurately determined the stresses and strains in a beam was subject to bending and writings of Galileo who [9] made incorrect assumptions when identified the principle of virtual work as a general law. A mathematical model for second-order spatial derivatives was not provided until the elasticity theory evolution in the last century of the 17th century by Laonhard Euler and Daniel Bernoulli. By including rotational inertia effects and shear deformation, Stephen Timoshenko enhanced the model in 1921 and obtained a fourth-order mathematical model.

In elasticity, the Euler-Bernoulli beam theory is a simplification that provides a technique for predicting the load carrying and deflection characteristic of beams. It is derived from the linear theory of elasticity and is used in beam design. Even though other mathematical models, like as plate theory, have been developed, the simplicity of beam theory has made it an important tool in study, particularly in structural and mechanical engineering.

Engineers and architects usually have to design structure that responds minimally to imposed dynamic loading, so that high stresses, large displacement amplitude, radiated noise decreases and structural fatigue are minimized. In other applications, like musical instruments, the objective is to achieve specific values in the frequencies of natural vibrations.

Several engineering problems can be reasonably approximated by using the Euler-Bernoulli beam model [10, 11]. The natural frequencies are slightly overestimated by the Euler-Bernoulli problem, respectively. The problem is made worse by the higher modes' natural frequencies [12, 13]. Existence and uniqueness of nonlocal solutions, existence results for a model of nonlinear beam and existence of a unique global work solution were studied [14-17].

In 1955, it was initiated the oscillation theory of partial differential equations [18] and have been developed by numerous authors, see [19-22]. Several authors investigated the problem of beam equation oscillation and nonoscillation, like Feireisl and Herrmann [23], Herrmann [24], Kusano and Yoshida [25], Priyadharshini, Chatzarakis and Sadhasivam [26], Timoshenko [27], Yoshida [28-30] and the references therein.

In this article, we initiate the some new oscillation criteria for Euler-Bernoulli beam equations of the form,

$$\frac{\partial}{\partial t}\left(r(t)\frac{\partial w(x,t)}{\partial t}\right)+q(x,t)\frac{\partial w(x,t)}{\partial t}+p\frac{\partial^4 w(x,t)}{\partial x^4} = f(x,t),(x,t)\in \Omega\times\mathbb{R}_+ = G \quad (1)$$

where, Ω = (0, L), $\mathbb{R}_+$ = (0,∞). Then p = EI, E is the modulus of elasticity, I is the moment of inertia and w is beam deflection at the axial location x and time t.

We use the following assumptions,

(A_1) $r\in \mathbb{C}^1(\mathbb{R}_+,\mathbb{R}_+), r'(t)\geq 0, t\in\mathbb{R}_+$.

(A_2) $q\in\mathbb{C}(G,\mathbb{R}_+)$ *and* $q(x,t)\geq \min_{x\in\bar{\Omega}} q(x,t), t\in \mathbb{R}_+$, is damping co-efficient.

(A_3) The load distribution of $f\in\mathbb{C}(G,\mathbb{R}_+)$, $\int_\Omega f(x,t)\psi(x)dx\leq 0$.

The nature of this term q(x,t) $\frac{\partial w(x,t)}{\partial t}$ is usually controlled by external damping systems, while the term's internal damping may also have an effect [31].

In [32], it introduces the following new integral operator, which is the main tool in this paper. Let $\mathbb{J}(t,s)\in\mathbb{C}(\mathbb{D}_*,\mathbb{R}_+), t>s\geq t_0$ and suppose that $\mathbb{J}$ has continuous derivatives such that

$$\frac{\partial\mathbb{J}}{\partial t}=\mu_1\sqrt{\mathbb{J}(t,s)},$$

$$\frac{\partial\mathbb{J}}{\partial s}=-\mu_2\sqrt{\mathbb{J}(t,s)}, \qquad (t,s)\in\mathbb{D}_*,$$

where $\mu_1(t,s),\mu_2(t,s)\in\mathbb{L}_{loc}(\mathbb{D}_*,\mathbb{R}_+)$. Now we define operator

$$A_\tau^t(j(s))=\int_\tau^t \mathbb{J}(t,s)j(s)\rho(s)ds, \qquad t\geq\tau,$$

$$A_t^\tau(j(s))=\int_\tau^t \mathbb{J}(t,s)j(s)\rho(s)ds, t\leq\tau,$$

where $j\in\mathbb{C}([0,\infty),\mathbb{R}_+), \rho\in\mathbb{C}'([0,\infty),\mathbb{R}_+)$.

If $j'\in\mathbb{C}([0,\infty),\mathbb{R})$, then

$$A_\tau^t(j'(s))=-\rho(\tau)\mathbb{J}(t,\tau)j(\tau) + A_\tau^t\left(\left[\mu_2(t,s)-\frac{\rho'(s)}{\rho(s)}\right]j(s)\right), t\geq\tau,$$

$$A_t^\tau(j'(s))=\rho(\tau)\mathbb{J}(t,\tau)j(\tau) - A_t^\tau\left(\left[\mu_2(t,s)-\frac{\rho'(s)}{\rho(s)}\right]j(s)\right), t\leq\tau,$$

A function $w: G\to\mathbb{R}^1$ is said to be oscillatory in G, if it has a zero in $\Omega\times\mathbb{R}_+$ for any t > 0, otherwise it is nonoscillatory.

2. Oscillation of Euler-Bernoulli Beam with Hinged

We derive the oscillation of (1) with hinged ends in this section. Our plan of action is to reduce the multidimensional problems to ordinary differential problem.

We treat the beam's hinged ends and satisfy the requirement

$$\frac{\partial^2 w(0,t)}{\partial x^2}=\frac{\partial^2 w(L,t)}{\partial x^2}=w(0,t) = w(L,t)=0. \quad (B_1)$$

We apply the integral operator and Riccati techniques to study some new oscillations in the following theorem.

Theorem 1

Consider that $\boldsymbol{\psi^4(x)\geq \in\psi(x), x}$ in $\boldsymbol{\Omega}$ for some $\in\ \geq \boldsymbol{0}$ and $\boldsymbol{\psi''(0)=\psi''(L)= \psi(0)=\psi(L)=0}$. If

$$\boldsymbol{(rW'(t))'+qW'(t)+p\in W(t)\leq 0}, \quad (2)$$

has no solution which is nonnegative, then each solution of (1) and (B_1) tends to zero.

Proof. Let w be an eventually nonnegative solution of (1) and (B_1). First assume that w > 0. Multiplying by $\psi(x)=sin\left(\frac{\pi}{L}\right)x$ and integrating over Ω yields

$$\int_\Omega \frac{\partial}{\partial t}\left(r\frac{\partial w}{\partial t}\right)\psi(x)dx+\int_\Omega q\frac{\partial w}{\partial t}\psi(x)dx + \int_\Omega p\frac{\partial^4 w}{\partial x^4}\psi(x)dx=\int_\Omega f(x,t)\psi(x)dx. \quad (3)$$

Using Jensen's inequality and (A_2), we get

$$\int_\Omega q\frac{\partial w}{\partial t}\psi(x)dx\geq q(t)W'(t). \quad (4)$$

Taking integrating by parts and using (B_1), we find

$$\int_\Omega \frac{\partial^4 w}{\partial x^4}\psi(x)dx=\int_\Omega w\psi^4(x)dx. \quad (5)$$

Equations (4), (5) are substituted in (3) and we obtain

$$\frac{d}{dt}\left(r\frac{dW(t)}{dt}\right)+q\frac{dW(t)}{dt}+ p\int_{\Omega} w\psi^4(x)dx \le 0,$$

where $W(t)=\int_{\Omega} w(x,t)\psi(x)dx$, i.e. $W(t)>0$ is a solution of (2).

Theorem 2

If $\boldsymbol{g}\in\mathbb{C}'([\mathbf{0},\infty),\mathbb{R})$ is such that

$$\limsup_{t\to\infty}\int_{t_1}^{t}\left(\phi(s)-\frac{q^2(s)\chi(s)}{4r(s)}\right)ds=\infty, \quad (6)$$

where $\Psi(s)=\exp(-2\int^{s} g(\eta)d\eta)$,
$\phi(s)=-\Psi(s)\Big(g(s)\big(g(s)-r(s)q(s)\big)-\big(r(s)g(s)\big)'+m\Big)$. Then every solution w of (1) and (B_1) is oscillatory.

Proof. Let $w(x,t)$ be an eventually non negative solution. We define the Riccati transformation,

$$U(t)=\Psi(t)\left(\frac{rW'(t)}{W(t)}+rg(t)\right), \qquad t\ge t_0,$$

$$U'(t) \quad \le -\frac{q(t)U(t)}{r}-\frac{U^2(t)}{r(t)\Psi(t)}-\phi(t). \quad (7)$$

From t_0 to t, integrate this inequality,

$$U(t_0)\ge U(t)+\int_{t_0}^{t}\left(\phi(s)-\frac{q^2(s)\Psi(s)}{4r(s)}\right)ds.$$

Taking $\limsup_{t\to\infty}$, we get

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left(\phi(s)-\frac{q^2(s)\Psi(s)}{4r(s)}\right)ds\le U(t_0)<\infty,$$

which leads to a contradiction.

Theorem 3

If $g\in\mathbb{C}^1([0,\infty),\mathbb{R})$,

$$\boldsymbol{\limsup_{t\to\infty}\frac{1}{\mathbb{J}(t,s)}A_\tau^t\left(\phi(s)-\frac{1}{4}\left(\frac{q(s)}{r(s)}+\mu_2-\frac{\rho'(s)}{\rho(s)}\right)^2\Psi(s)r(s)\right)}=\infty. \quad (8)$$

Then any solution $\boldsymbol{w}$ of (1) and (B_1) tends to zero.

Proof. Suppose that w is an eventually nonnegative solution. Applying A_τ^t to both sides of (7), we obtain

$$A_\tau^t\big(U'(s)\big)\le -\quad A_\tau^t\left(\frac{q(s)U(s)}{r(s)}-\frac{U^2(s)}{r(s)\Psi(s)}\right)-A_\tau^t\ \phi(s),$$

$$-\rho(\tau)\mathbb{J}(t,s)U(\tau)\le -A_\tau^t\big(\phi(s)\big)$$

$$-A_\tau^t\left(\left(\frac{q(s)}{r(s)}+\mu_2-\frac{\rho'(s)}{\rho(s)}\right)U(s)+\frac{U^2(s)}{r(s)\Psi(s)}\right)$$

$$A_\tau^t\left(\phi(s)-\frac{1}{4}\left(\left(\frac{q(s)}{r(s)}+\mu_2-\frac{\rho'(s)}{\rho(s)}\right)^2 r(s)\Psi(s)\right)\right)\le\rho(\tau)\mathbb{J}(t,s)U(\tau).$$

$$\limsup_{t\to\infty}\frac{1}{\mathbb{J}(t,s)}A_\tau^t\left(\phi(s)-\frac{1}{4}\left(\frac{q(s)}{r(s)}+\mu_2(t,s)-\frac{\rho'(s)}{\rho(s)}\right)^2\Psi(s)r(s)\right)\le\rho(\tau)\mathbb{J}(t,s)U(\tau),$$

which leads to a contradiction.

Theorem 4

If $a,b,c\in\mathbb{R}$ with $T_0\le a<c<b$, T_0 to t_0 such that

$$\frac{1}{\mathbb{J}(c,a)}A_a^c\left(\phi(s)-\frac{1}{4}\left(\frac{q(s)}{r(s)}+\mu_1-\frac{\rho'(s)}{\rho(s)}\right)^2\Psi(s)r(s)\right)+\frac{1}{\mathbb{J}(b,c)}A_c^b\left(\phi(s)-\frac{1}{4}\left(\frac{q(s)}{r(s)}+\mu_2+\frac{\rho'(s)}{\rho(s)}\right)^2\Psi(s)r(s)\right)>0. \quad (9)$$

Therefore, any solution w of (1) and (B_1) is oscillatory in G.

Corollary 1

Assume that Theorem 3's conditions have been satisfied and that (8) is substituted with

$$\limsup_{t\to\infty}\frac{1}{\mathbb{J}(t,s)}A_\tau^t\phi(s)=\infty,$$

$$\limsup_{t\to\infty}\frac{1}{\mathbb{J}(t,s)}A_\tau^t\left(\left(\frac{q(s)}{r(s)}+\mu_2-\frac{\rho'(s)}{\rho(s)}\right)^2\Psi(s)r(s)\right)<\infty.$$

Then any solution (1) and (B_1) is oscillatory.

Corollary 2

Let the functions $\mathbb{J},\mu_1,\mu_2,r,g$ and ϕ be the same as in Theorem 4. Moreover, suppose that (9) is replaced by

$$\frac{1}{\mathbb{J}(c,a)}A_a^c\phi(s)+\frac{1}{\mathbb{J}(b,c)}A_c^b\phi(s)> \frac{1}{\mathbb{J}(c,a)}A_a^c\left(\left(\frac{q(s)}{r(s)}+\mu_1-\frac{\rho'(s)}{\rho(s)}\right)^2\Psi(s)r(s)\right)+\frac{1}{\mathbb{J}(b,c)}A_c^b\left(\left(\frac{q(s)}{r(s)}+\mu_2-\frac{\rho'(s)}{\rho(s)}\right)^2\Psi(s)r(s)\right).$$

Then any solution w of (1) and (B_1) is oscillatory in G.

Example 1

Consider the Euler-Bernoulli beam equations,

$$\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial t}\right)+t\frac{\partial w}{\partial t}+\left(\frac{L}{\pi}\right)^4\frac{\partial^4 w}{\partial x^4}=t\cos t\sin\left(\frac{\pi}{L}\right)x, \quad (x,t)\in G. \quad (10)$$

Here $r=1, q=t, p=\left(\frac{L}{\pi}\right)^4$,

$f = t\cos t \sin\left(\frac{\pi}{L}\right)x, \in = \left(\frac{L}{\pi}\right)^4, g = \frac{-1}{2s}, \Psi(s) = s$ and $\phi(s) = \left(\frac{3}{2} + \frac{3}{4}s^2\right)$s with (B_1).

$$\limsup_{t\to\infty} \int_{t_0}^{t} \left(\left(\frac{3}{2} + \frac{3s^2}{4}\right)s - \frac{s^3}{4}\right) ds = \infty.$$

Therefore, Theorem 2's conditions are all satisfied. Hence, all solution of (10) is oscillatory. In fact, the one such solution of (10) is $w(x,t) = \sin t \sin\left(\frac{\pi}{L}\right)x$.

Example 2

Assume the Euler-Bernoulli beam equations,

$$\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial t}\right) + t\frac{\partial w}{\partial t} + \left(\frac{L}{\pi}\right)^4 \frac{\partial^4 w}{\partial x^4} = (2 + 3t + t^2)e^t \sin\left(\frac{\pi}{L}\right)x, (x,t) \in G. \quad (11)$$

Here $r = 1, q = t, p = \left(\frac{L}{\pi}\right)^4, \in = 1, g = \frac{1}{2s}$,

$$f = (2 + 3t + t^2)e^t \sin\left(\frac{\pi}{L}\right)x, \Psi = \frac{1}{s}$$

and $\phi = \left(\left(\frac{L}{\pi}\right)^4 - 1\right)\frac{1}{s^2}$ with (B_1).

$$\limsup_{t\to\infty} \int_{t_0}^{t} \left(\left(\left(\frac{L}{\pi}\right)^4 - 1\right)\frac{1}{s^2} - \frac{1}{4}\right) ds < \limsup_{t\to\infty} \int_{t_0}^{t} \left(\frac{1}{s^2}\right) ds < \infty.$$

Therefore, Theorem 2's conditions are not satisfied. In fact, $w(x,t) = te^t \sin\left(\frac{\pi}{L}\right)x$ is a nonoscillatory solution of (11).

3. Oscillation of Euler-Bernoulli Beam with Sliding

The case of sliding with boundary conditions is discussed,

$$\frac{\partial^3 w(0,t)}{\partial x^3} = \frac{\partial^3 w(L,t)}{\partial x^3} = \frac{\partial w}{\partial t}(0,t) = \frac{\partial w}{\partial t}(L,t) = 0. \quad (B_2)$$

The new oscillation is demonstrated in the following theorem utilizing the Integral operator type and the Riccati techniques.

Theorem 5

Every solution $\boldsymbol{w}$ of (1) satisfying the conditions (B_2) is oscillatory, if

$$\left(\boldsymbol{rW'(t)}\right)' + \boldsymbol{qW'(t)} \leq \mathbf{0}, \quad (12)$$

has no eventually nonnegative solution, then each solution of (1) and (B_2) tends to zero.

Proof. Suppose that w is a nonoscillatory solution of (1). First assume that w > 0 and integrate over Ω.

$$\int_{\Omega} \frac{\partial}{\partial t}\left(r\frac{\partial w}{\partial t}\right) dx + \int_{\Omega} q\frac{\partial w}{\partial t} dx + \int_{\Omega} p\frac{\partial^4 w}{\partial x^4} dx = \int_{\Omega} f dx. \quad (13)$$

Using Jensen's inequality and (A_2), we define

$$\int_{\Omega} q\frac{\partial w}{\partial t} dx \geq q(t)W'(t). \quad (14)$$

Taking integrating by parts and using (B_2), we get

$$\int_{\Omega} \frac{\partial^4 w(x,t)}{\partial x^4} dx = 0. \quad (15)$$

In (13), equations (14), (15) are substituted,

$$\frac{d}{dt}\left(r\frac{dW(t)}{dt}\right) + q\frac{dW(t)}{dt} \leq 0,$$

here $W(t) = \int_{\Omega} w(x,t)\psi(x)dx$, and $\psi(x) = 1$ ie, $W(t) > 0$ is a solution of (12).

Theorem 6

If $g \in \mathbb{C}^1([0,\infty),\mathbb{R})$ such that

$$\limsup_{t\to\infty} \frac{1}{\mathbb{J}(t,s)} A_\tau^t \left(\phi(s) - \frac{1}{4}\left(\frac{q(s)}{r(s)} + \mu_2 - \frac{\rho'(s)}{\rho(s)}\right)^2 \Psi(s)r(s)\right) = \infty, \quad (16)$$

where $\Psi(s) = \exp(-2\int^s g(\eta)d\eta)$,

$\phi(s) = \Psi(s)\Big(g(s)(r(s)q(s) - g(s)) - \left(r(s)g(s)\right)'\Big)$.

Then every solution w of (1) and (B_2) is oscillatory.

Proof. Assume that w is an eventually non negative solution and the Riccati transformation,

$$U(t) = \Psi(t)\left(\frac{r(t)W'(t)}{W(t)} + r(t)g(t)\right), \qquad t \geq t_0,$$

$$\leq -\frac{q(t)U(t)}{r(t)} - \frac{U^2(t)}{r(t)\Psi(t)} - \phi(t). \quad (17)$$

Applying A_τ^t both sides of (17), we obtain

$$A_\tau^t\left(U'(s)\right) \leq -A_\tau^t\left(\frac{q(s)U(s)}{r(s)} - \frac{U^2(s)}{r(s)\Psi(s)}\right) - A_\tau^t\ \phi(s),$$

Taking $\limsup_{t\to\infty}$, we get

$$\limsup_{t\to\infty} \frac{1}{\mathbb{J}(t,s)} A_\tau^t \left(\phi(s) - \frac{1}{4}\left(\frac{q(s)}{r(s)} + \mu_2 - \frac{\rho'(s)}{\rho(s)}\right)^2 \Psi(s)r(s)\right) \leq \rho(\tau)\mathbb{J}(t,\tau)U(\tau) < \infty,$$

which leads to a contradiction with (16).

Theorem 7

Assume that there exist $a, b, c \in \mathbb{R}$ with $T_0 \leq a < c < b$, T_0 to t_0 such that

$$\frac{1}{\mathbb{J}(c,a)} A_a^c\left(\phi(s) - \frac{1}{4}\left(\frac{q(s)}{r(s)} + \mu_1 - \frac{\rho'(s)}{\rho(s)}\right)^2 \Psi(s) r(s)\right) + \frac{1}{\mathbb{J}(b,c)} A_c^b\left(\phi(s) - \frac{1}{4}\left(\frac{q(s)}{r(s)} + \mu_2 + \frac{\rho'(s)}{\rho(s)}\right)^2 \Psi(s) r(s)\right) > 0.$$

Then any solution w of (1) and (B_2) tends to zero.

Corollary 3

Assume that Theorem 6's conditions have been satisfied and that (16) is substituted with

$$\limsup_{t\to\infty} \frac{1}{\mathbb{J}(t,s)} A_\tau^t \phi(s) = \infty, \text{ and}$$

$$\limsup_{t\to\infty} \frac{1}{\mathbb{J}(t,s)} A_\tau^t\left(\left(\frac{q(s)}{r(s)} + \mu_2 - \frac{\rho'(s)}{\rho(s)}\right)^2 \Psi(s) r(s)\right) < \infty.$$

Then all solution $w(x,t)$ of solution (1) and (B_2) is oscillatory.

Example 3

Consider the equations,

$$\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial t}\right) + t\frac{\partial w}{\partial t} + \left(\frac{L}{\pi}\right)^4 \frac{\partial^4 w}{\partial x^4} = (2 + t + t^2)e^t \cos\left(\frac{\pi}{L}\right)x, (x,t) \in G \quad (18)$$

with (B_2). Here $r = 1, q = t, p = \left(\frac{L}{\pi}\right)^4$,

$H = (t - s), \mu_2 = (t - s)^{-1}$, $\rho = \frac{1}{s}$,

$f = (2 + 3t + t^2)e^t \sin\left(\frac{\pi}{L}\right)x, g = \frac{1}{s}$, $\Psi = \frac{1}{s^2}$ and $\phi = \left(\frac{2}{s^2} - 1\right)\frac{1}{s^2}$.

$$\limsup_{t\to\infty} \frac{1}{\mathbb{J}(t,s)} A_\tau^t\left(\left(\frac{q(s)}{r(s)} + \mu_2 - \frac{\rho'(s)}{\rho(s)}\right)^2 \Psi(s) r(s)\right) < \infty.$$

Therefore, Theorem 6's assumptions are all unsatisfied. In fact, $w(x,t) = te^t \sin\left(\frac{\pi}{L}\right)x$ is a nonoscillatory solution of (18).

4. Oscillation of Euler-Bernoulli Beam with Hinged-sliding

We deal the case of hinged-sliding ends,

$$\frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^3 w(L,t)}{\partial x^3} = w(0,t) = \frac{\partial w}{\partial t}(L,t) = 0. \quad (B_3)$$

Theorem 8

Consider that $\psi^4(x) \geq \in \psi(x)$ in Ω for some $\in \geq 0$ and $\psi''(0) = \psi''(L) = \psi(0) = \psi(L) = 0$. If

$$\left(\boldsymbol{rW'(t)}\right)' + \boldsymbol{p} \in \boldsymbol{W(t)} \leq \mathbf{0}, \quad (19)$$

has no solution which is nonnegative, then each solution of (1) and (B_3) is oscillatory.

Proof. Assume that w is a nonoscillatory in G. We consider that w > 0. Multiplying by $\psi(x) = sin\left(\frac{\pi}{2L}\right)x$ and integrating over Ω, we find

$$\int_\Omega \frac{\partial}{\partial t}\left(r\frac{\partial w}{\partial t}\right)\psi dx + \int_\Omega q\frac{\partial w}{\partial t}\psi dx + \int_\Omega p\frac{\partial^4 w}{\partial x^4}\psi dx = \int_\Omega f\psi dx. \quad (20)$$

Using Jensen's inequality and (A_2),

$$\int_\Omega q\frac{\partial w}{\partial t}\psi dx \geq q(t)W'(t). \quad (21)$$

Taking integrating by parts and using (B_2), we get

$$\int_\Omega \frac{\partial^4 w}{\partial x^4}\psi dx = 0. \quad (22)$$

Equations (21), (22) are substituted in (20),

$$\frac{d}{dt}\left(r\frac{dW(t)}{dt}\right) + p \in W(t) \leq 0,$$

where $W(t) = \int_\Omega w(x,t)\psi(x)dx$,

Hence, Theorem 8 is proved.

Theorem 9

If $\boldsymbol{g} \in \mathbb{C}^1([\mathbf{0},\infty),\mathbb{R})$ such that

$$\limsup_{t\to\infty} \frac{1}{\mathbb{J}(t,s)} A_\tau^t\left(\phi(s) - \frac{1}{4}\left(\frac{q(s)}{r(s)} + \mu_2 - \frac{\rho'(s)}{\rho(s)}\right)^2 \Psi(s) r(s)\right) = \infty.$$

Then every solution w of (1) and (B_3) is oscillatory.

Example 4

Assume the Euler-Bernoulli beam equations,

$$\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial t}\right) + t^2\frac{\partial w}{\partial t} + \left(\frac{2L}{\pi}\right)^4 \frac{\partial^4 w}{\partial x^4} = t^2 \cos t \sin\left(\frac{\pi}{2L}\right)x, \quad (x,t) \in G \quad (23)$$

with (B_3). Here $r = 1, q = t^2, p = \left(\frac{2L}{\pi}\right)^4$,

$\in = 1, H = (t,s), \mu_2 = (t - s) - 1, \rho = s, g = \frac{-1}{2s}$, $\Psi = s$ and $\phi = \left(\frac{2L}{\pi} + \frac{1}{2} + \frac{1}{2}s^2\right)$s.

$$\limsup_{t\to\infty}\frac{1}{(t-s)}\int_{t_0}^{t} sds=\infty.$$

So, each condition of Theorem 9 is satisfied. Therefore, all the solution of (23) is oscillatory. In fact, $w = \sin t \sin\left(\frac{\pi}{2L}\right) x$ is one such solution of (23).

5. Conclusions

The major goal of this paper is to study a new condition for the oscillation criteria for Euler-Bernoulli beam equations with some boundary conditions. The newly derived result also includes required examples.

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