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On Graph Representation of Bee's Dances

Araceli Giménez Lorente

Higher Education School of Art and Design, EASD (ISEACV), Castelló de la Plana, Spain

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Abstract The application of graph theory in ethology is a relevant field, where the background of this study is focused only from the points of view of biology and chemistry. Our study is to find patterns in a specific behavior of bees, their dance. The purpose is to present the dance of bees as graphs, finding a mathematical character to animal ethology and a pattern that allows us to communicate with them. The methodology used is the observation of the dances of the hymenoptera through the viewing of videos, and bibliographic material. It is intended to analyze the communication of bees, translating their trajectory to the digraphs associated with the three different dances that we know, and through the associated matrix algebra, determine their homeomorphisms and main characteristics. The main result will be to extrapolate the schemes of the dances of the bees to directed graphs and their respective associated matrices to carry out a mathematical study. The dance of the bees is a very complex language that is analyzed with the digraphs in the sections that bear the name of the different dances, previously an introduction to these dances is made. Bees are capable of creating what we understand as a symbolic mathematical language to communicate. This is a point of connection with another intelligent species, and with the help of discrete mathematics, we can communicate; these results are partial since this study is preliminary, but necessary as a first phase. In the " discussion" section, some observations are made about this research and its possible implementation. Although this study of ethology from the mathematical point of view by approximation of graph theory is unprecedented, it is necessary for its possible future applications. For all these reasons, we see the need for this study.

Keywords Digraph Representation, Ethology, The Dance of Bees, Matrix Algebra

1 Introduction

The scout bees called *Apis mellifera*, commonly known as the honey bee, perform two types of dance, *the circle dance* and *the figure eight dance*, and there is an Italian subspecies the *Apis mellifera lingustica* that has developed a third type of dance, *the sickle dance*.

When a scout bee returns to its hive, it may perform movement patterns known as "the bee dance"; the bee moves left and right. These movements are accompanied by acoustic signals that are produced as part of the execution of the dances. In addition to transmitting this sound through the air, they also excite the Johnston organs located in the antenna of the follower bees with which the dancing bee communicates.[1]

This pattern can be considered a circuit and is made up of several phases. The first phase is called "wagging", in which the bee walks in a straight line and shakes its abdomen, from side to side, and the second phase is the return phase, in which he turns and walks back to the starting point.[2]

We are going to define and analyze the three different dances and their geometry. In the first place we have "the circle dance", the scout bee communicates to the others that the food source is within a radius of less than 10 meters. The intensity of the dance in relation to speed and duration tells the others how abundant the food source is, and the bee leaves behind the scent of the flower with which it has been in contact.

The second dance called "in sickle" indicates distances between $10 - 100$ meters from the power source.

The third is "the dance in eight", which tells us that the source is located in a radius greater than 100 meters. Here the central axis, the sinuous line according to the angle with respect to the sun, tells us the approximate position of the origin of the food.

But they not only perform this dance to indicate their new food source, but also to indicate a new location for the colony, and for this they must know how to measure in three dimensions, length, width and height, and know the arrival time at that location, that is, they have a very complex symbolic language to communicate. In this research work, the trajectory of these dances have been drawn as graphs to be able to analyze them mathematically.

As a graph definition we have the third reference, "A directed graph (or a digraph for short) G consists of a set of vertices $V = \{v_1, v_2, ...\}$, a set of edges $E = \{e_1, e_2, ...\}$, and a mapping ψ that maps every edge onto some ordered pair of vertices (v_i, v_j). As in the case of undirected graphs, a vertex is represented by a point and an edge by a line segment between v_i and v_j with an arrow directed from v_i to v_j.[...]. A digraph is also referred to as an oriented graph." [3]

2 The circle dance

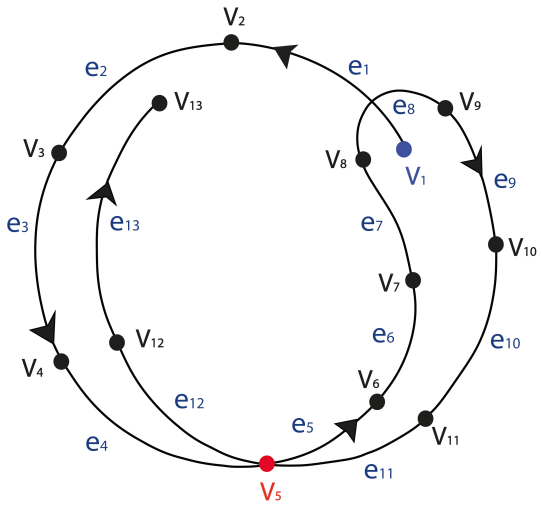

Figure 1. Graph of the circle dance of the bees.

Definition 2.1**: Walk**
Let G be a digraph. A path in G is a (finite) sequence in which vertices and edges in G appear alternately:

$$v_0, v_0v_1, v_1v_2, v_2, ..., v_{n-1}, v_{n-1}v_n, v_n,$$

where each edge v_iv_{i+1} has v_i as its origin and v_{i+1} as its destination. The vertices v_0 and v_n are referred to as the origin and the end of the path, respectively. The number of edges in the path is called its length. [4]

$v_1, v_1v_2, v_2, v_2v_3, v_3, v_3v_4, v_4, v_4v_5, v_5, v_5v_6, v_6, v_6v_7, v_7, v_7v_8, v_8, v_8v_9, v_9, v_9v_{10}, v_{10}, v_{10}v_{11}, v_{11}, v_{11}v_5, v_5, v_5v_{12}, v_{12}, v_{12}v_{13}, v_{13}$.

As we can see in Fig.1, this path is not closed because it starts at the first vertex and ends at the thirteenth.

Since the vertices are odd, it cannot be a closed path. We call the adjacency matrix of G the matrix $n \times n$,

$A = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1, & \text{if}(v_i, v_j) \in E; \\ 0, & \text{if}(v_i, v_j) \notin E. \end{cases}$$

Then, $A =$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The diagonal of the associated matrix are all zeros, so it has no loops.

It can be seen that vertex 5, which is marked in red, has a double entry, on the one hand it is the direction of vertex 4, and on the other the edge corresponding to vertex 11 of the digraph also passes through vertex 5. It is the only vertex that crosses twice in two opposite directions.

3 The sickle dance

Let $G = (V, E)$, with $|V| = n$.
The set of vertices and arrows associated with this digraph is:

$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

$$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_1), (v_1, v_4), (v_4, v_5), (v_5, v_1)\}$$

For the path, which is unique, of the sickle dance we will have:

$$v_1, v_1v_2, v_2, v_2v_3, v_3, v_3v_1, v_1, v_1v_4, v_4, v_4v_5, v_5, v_5v_1, v_1$$

This path is closed because as we can see it starts at the first vertex and ends at the same vertex.

For the adjacent matrix of G, we will define $n \times n$, and we name as the matrix B the adjacency matrix of *the sickle dance*, where

$$B = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Which can be represented in the next figure.

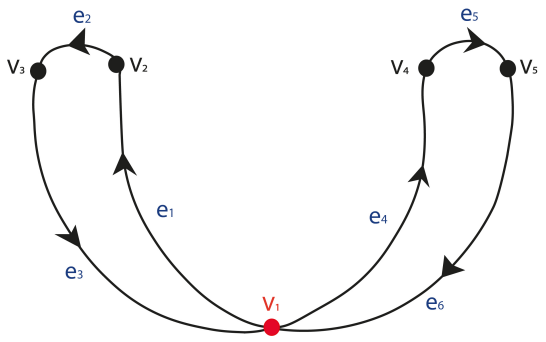

Figure 2. The sickle dance of the bees

In this digraph, (see Fig.2), we can also see a *self-loop vertex*, which corresponds to vertex 1 and has no additional loop, which in this case is the starting point of the sickle dance of the bees. It has two entrances in two opposite directions, since the route of the bees is first to the left, returning to the point of origin, to later change its trajectory to the right.

4 The figure eight dance

Let $G = (V, E)$, with $|V| = n$.

The set of vertices and arrows associated with this digraph, in relation of the figure eight dance, is:

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}\}$$

$$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_6, v_7), (v_7, v_8),$$

$$(v_8, v_9), (v_9, v_{10}), (v_{10}, v_{11}), (v_{11}, v_5), (v_5, v_{12}), (v_{12}, v_{13})\}$$

For the digraph corresponding to *the dance in eight*, we have:

$v_1, v_1v_2, v_2, v_2v_3, v_3, v_3v_4, v_4, v_4v_5, v_5, v_5v_6, v_6, v_6v_7, v_7,$
$v_7v_8, v_8, v_8v_9, v_9, v_9v_{10}, v_9v_{11}, v_{10}, v_{10}v_1, v_{11}, v_{11}v_1, v_1$

This graph is also closed (see Fig.3).

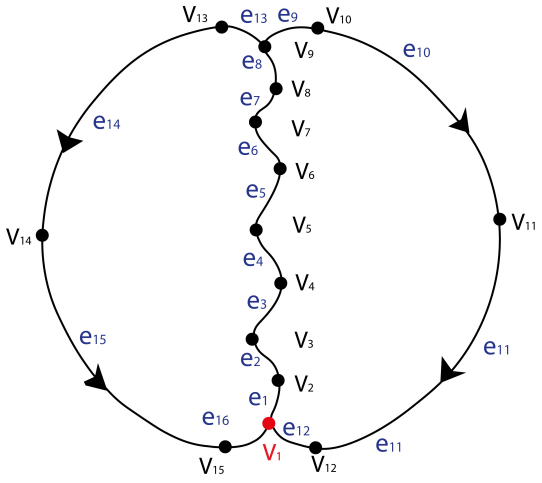

Figure 3. The figure eight dance of the bees

4.1 The figure eight dance. Another version

The second version of the digraph called dance in eight is too complex to associate an adjaciency matrix with it (see Fig. 4).

That is why the digraph is simplified to be able to study it (see Fig.5). For a simplified version, we have:

Adjacency matrix

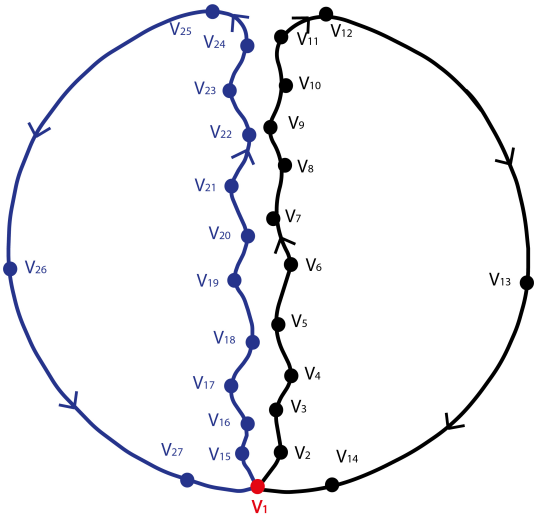

Figure 4. The eight dance of the bees, second version

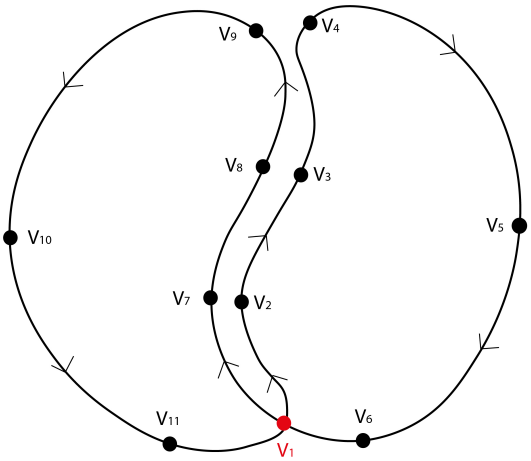

Figure 5. The eight dance of the bees, simplified version

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Following the pattern of the previous dances of the bees, we have a vertex v_1, also marked in red, which appears even in the variables and in the simplified version, through which it is passed twice. Vertex 1 is the origin of the dances; in addition, the segment from vertex 1 to vertex 9 is traversed twice, the first time the bee turns to the right, and the second time the hymenoptera turns to the left, thus completing a circle and closing the digraph.

4.2 The time represented by a subgraph

A connected component of a subgraph representing the dance in eight of the bees is related to time. The vibrations that leave a sinuous trajectory of the vertical of the graph, extrapolate the distance to seconds as one kilometer is equivalent to one second (see Fig.6).

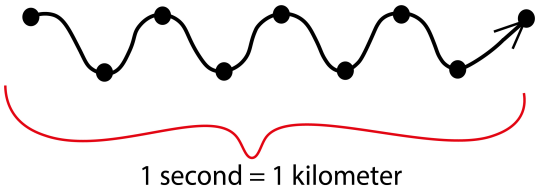

Figure 6. The relationship between the distance and the time in seconds

Karl von Frisch asserted that the vibration movement of the bee in the eight dance has a vibration of between 13 and 15 hertz [6]. Being such an intense vibration it is audible. The further away the target is, the tempo of the dance slows down. In addition to the relationship between the subgraph and the distance to the power supply is the relationship that the dance occupies in terms of the cells of the honeycomb.

Approximately if the food is at 500 m the movement occupies $2-3$ cells, at 1000 m around 3 cells, 2000 m over $3-4$ cells, to 3500 m over 4 cells, to 4500 m over $4-5$ cells, and so on in arithmetic progression [5].

A simplification of the formulas that relate time to distance, and that their original study is by Frisch Jander, has a reduced version in the Kohl and Rutschmann study [2] and [7], which is the following:

$$t = a + \frac{b}{c}(1 - exp^{-cd}), \tag{1}$$

where d is the distance from the power source. The variable t is actually three time variables tw (the swing phase), tr the retrace phase, and tc (the duration of the circuit) and their sum is t. And the last variable is c which is the circuit.

4.3 Coordinates

In addition to the temporal factors, we must study distance as a particular case. There is a research on three species of Asian honey bees in Sri Lanka and on geographic subspecies of the western honey bee, *Apis mellifera*; the researchers Lindauer and Bosch discovered that they communicate with subspecies-specific distance codes, which we will call distance dialects. A distance dialect is the slope of the linear function that relates the duration of the movement phase to the distance of a resource in the field. In other words, they are ecological adaptations that evolved to maximize the precision of communication [5].

The angle of the subgraph in Figure 6, which can be negative or positive on the x-axis, determines the position of the Sun in reference to its power source. It has been observed that they can be from 40 to 65, even 120, but if the Sun is aligned with the power source or the hive, this subgraph will appear vertical, that is, at an angle of 90 with respect to the x axis in a bidimensional representation (x, y) (see Figure 7).

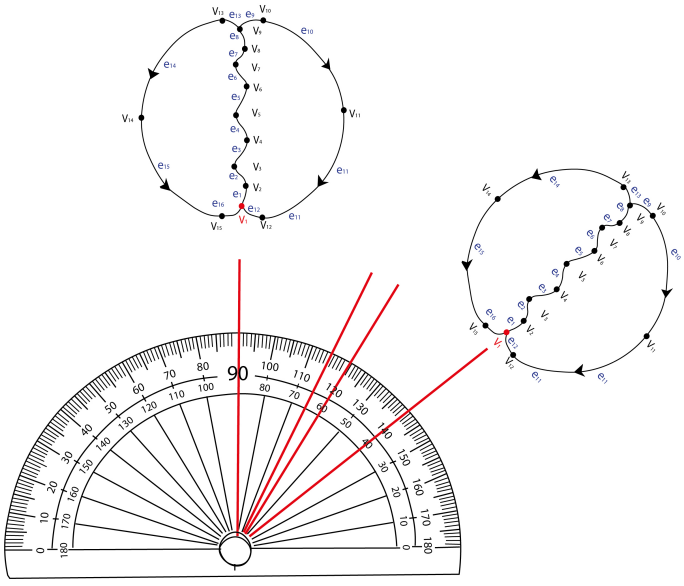

Figure 7. Different coordinates

According to M. Beekmann, bees can travel up to 9.5 km to notify their swarm of the presence of a food source [9].

It must be taken into account that bees not only collect nectar, but also pollen [10] and in the language of the dances, in some way that we do not know, they can

indicate the presence of these two components, nectar and pollen. They may also be able to travel a long distance and transport both components, despite their size.

The researchers R. De Marco and R. Menzel studied how the bees that perform the dances combine the compass (direction) and the odometric information (distance) [11], that they acquire during the search flight, this makes the spatial dimension more complex, since we are talking about a three-dimensional space and that situates, as has been commented before, the Sun as a point of reference.

5 Objectives

The main objective of this study is to apply mathematical representations as graph to ethology, because we have learned from nature by imitating it, for example when we observe a dragonfly fly it reminds us of the flight of a helicopter.

Experiments on the learning curve of *Apis Mellifica* bees carried out by R, Menzel, H. Freudel and U.Rühl, show us that bees can learn [8]. And the bees dances make the spectators not only find their power source, but also learn to communicate.

Human movement is related either to sports or in its artistic variable to ballet or dance, there are gestural communications such as sign language, but we have nothing comparable to the communication of bees, nothing with that precision, in our case we depend on technology for this, to find out some coordinates or the time we need to get somewhere.

Also if they are able to teach their language, we can learn to create a new method through observation based on this sophisticated language of movement.

6 Materials and Methods

Bibliographic material and videos have been used to study and observe the three dances of the bees. The methodology applied for this research is graph associated with matrix algebra. Specifically, d igraphs h ave been used in the sections 3, 4, 4.1 and 5. The Adobe Illustration vector drawing program has been used to draw the graphics, where the vertices represent the inflections, and the turns and edges of the trajectory of the dance.

The vertices marked with red color, regardless of how they are numbered, represent a reference point that is passed more than once or a crossroad.

An adjaciency matrix has been associated to each digraph, but in the case of the dance in eight, the graph had to be simplified because the associated matrix was very extensive.

Images not only clarify the text, but are necessary because they are graphic, and that is what this study is about.

These digraphs represent the symbolic language of the bee dance, since it is a mathematically coded language, which must be in mathematical language.

7 Results

As a main result we have that having passed the bee dance schemes to graphs, we can now study this phenomenon mathematically, and applying matrix notation we have been able to deduce factors that help us find new results. For example, if there are three independent types of dance, the circle dance, the figure eight dance, and the sickle dance. We can know it not only because their graphs are not equivalent, but also because their associated matrices are different from each other. Within the dances there are homeomorphisms that are permutations of a more optimized form of the trajectory that they describe during the movement.

Learning from other disciplines such as theoretical physics, that in the "cloud chamber" you can see the trajectory of the elementary subatomic particles, and that through said trajectory you can know if what you are seeing is a proton or an electron, among others particles. Extrapolating from graph theory, we can see the complexity of this study, since we are talking about very precise results, not only because of the dance we know the distance and time associated with its food source, but it seems that it also gives us another associated values such as the quantity and quality of said flowers.

8 Discussion

Although it seems that we understand the language of bees, at least their three dances, it is very likely that the bees, in addition to obtaining information from these three dances, also do so from the sounds that the scout bee makes, the smell they give off a nd w hat t hey do. they see, since bees can see the ultraviolet spectrum, and even through touch, since in the visual recordings, in the videos, we can appreciate how the observer bees touch the bee that is communicating their message with their antennae and legs. In addition, the concept of the hive mind that these Hymenoptera have, makes details escape us since these beings have not been shown to be singular individuals and are extremely sociable, in fact they act as one, they coordinate as one mind. From a mathematical point of view, it seems that all we can do is computer simulations based on these values, and introducing other factors such as weather, pheromones and other odor-related molecules, as well as touch pressure. But this is another study that should be related to computational mathematics.

9 Conclusions

We conclude that despite the complexity of this study, the first phase of this investigation has been carried out satisfactorily. Only three dances of the bees are determined, among them they are not homeomorphic, but all the others that find resemblance to these three unique patterns are permutations of these synchronized movements, as can be seen in the digraphs presented, since it is not possible to obtain them from the same graph (see Figures $1, 2$ and 3).

For all that we say about the results and the discussion, another study applied to the extrapolation of this language to an algorithm as a mathematical modeling is estimated.

Bees are capable of creating a symbolic mathematical language, based on the laws of science to communicate accurately; using the sun as his compass, and the vibratory motion as a wavelength, with its valleys and its peaks; they mathematically encode the coordinates and time, and communicate it to the other bees.

As far as we know, bees are the only ones, apart from our species, capable of communicating symbolically with geometric precision, therefore it deserves all the necessary studies to be able to decipher it.

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