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Forecasts with SPR Model Using Bootstrap-Reversible Jump MCMC

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Abstract Polynomial regression (PR) is a stochastic model that has been widely used in forecasting in various fields. Stationary stochastic models play a very important role in forecasting. Generally, PR model parameter estimation methods have been developed for non-stationary PR models. This article aims to develop an algorithm to estimate the parameters of a stationary polynomial regression (SPR) model. The SPR model parameters are estimated using the Bayesian method. The Bayes estimator cannot be determined analytically because the posterior distribution for the SPR model parameters has a complex structure. The complexity of the posterior distribution is caused by the SPR model parameters which have a variable dimensional space. Therefore, this article uses the reversible jump MCMC algorithm which is suitable for estimating the parameters of variable-dimensional models. Applying the reversible jump MCMC algorithm to big data requires many iterations. To reduce the number of iterations, the reversible jump MCMC algorithm is combined with the Bootstrap algorithm via the resampling method. The performance of the Bootstrap-reversible jump MCMC algorithm is validated using 2 simulated data sets. These findings show that the Bootstrap-reversible jump MCMC algorithm can estimate the SPR model parameters well. These findings contribute to the development of SPR models and SPR model parameter estimation methods. In addition, these findings contribute to big data modeling. Further research can be done by replacing Gaussian noise in SPR with non-Gaussian noise.

Keywords Big Data, Bootstrap, Reversible Jump MCMC, Stationary Polynomial Regression

1. Introduction

Polynomial regression (PR) is a form of regression in which the relationship between the independent variable and the dependent variable is modeled as a polynomial of degree n [1]. PR has been applied in various fields. Among them, PR has been used for color signal processing [1], PR has been used to predict the total infected cases with COVID-19 [2], PR has been used as a machine learning model [3], and PR has been applied to predict the grip strength [4]. Thus, PR has become an important and interesting topic for researchers in various countries to this day (for example Germany [5], Russia [6], USA [7], Canada [8] and Egypt [9]).

Stationary polynomial regression (SRP) models have played an important role in forecasting. However, the SPR model has not been widely studied. This is supported by the visualization of a network map (Figure 1) with the help of VOSviewer of 7736 scientific articles from 1950 to 2024. This data has been taken from the dimensions database (https://app.dimensions.ai/) by typing in the topic "polynomial regression”. Data was taken on December 17, 2023.

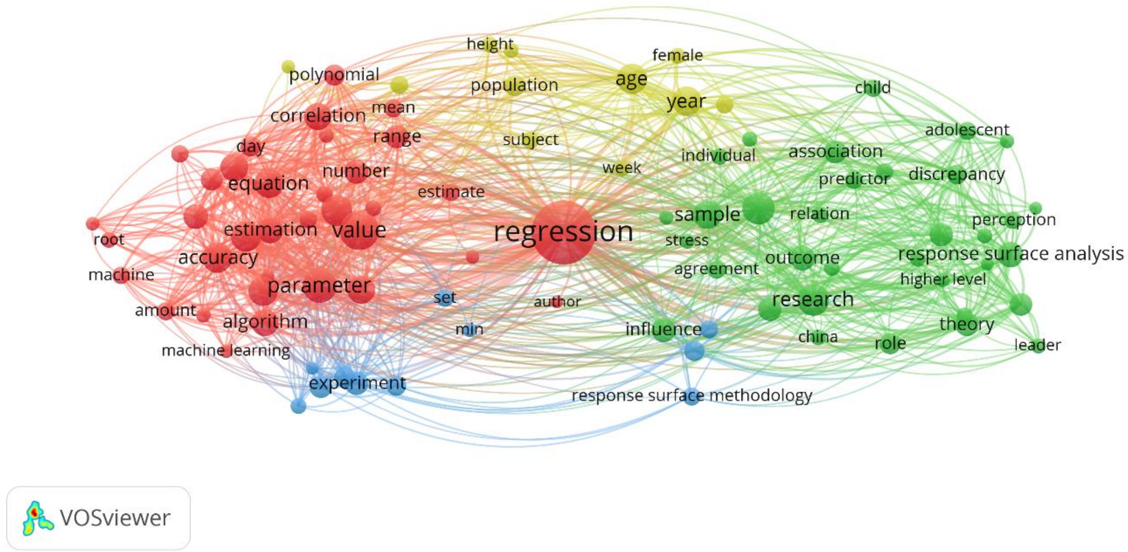

Figure 1. Network map visualization

In Figure 1, two topics are connected by a curve indicating that they have appeared together in the title and abstract of an article. In other words, these two topics have been linked in a study. Figure 1 shows that the PR model topics are not connected to the stationary topics. This shows that there has been no research related to SPR.

In SPR, the order of the SPR model is unknown so that the SPR model parameters have a complex structure because the SPR model parameters have a variable dimension space. Therefore, the reversible jump Markov Chain Monte Carlo (MCMC) algorithm is used. The reversible jump MCMC algorithm discovered by P.J. Green [10] is an algorithm designed to estimate model parameters with variable dimensions. This reversible jump MCMC algorithm uses a transformation that allows the Markov chain to move from one space to another space that has different dimensions. A more detailed description of reversible jump MCMC algorithm can be found in various literature, for example [10]. This algorithm does not guarantee to produce parameter estimators from a stationary PR model. For this reason, reparameterization [11] is applied to the PR coefficients so that the algorithm produces a stationary PR. Applying the Markov chain Monte Carlo method to big data requires a large number of iterations [12]. A combination of the Metropolis-Hastings algorithm and the Bootstrap method has been applied to big data [12]. To expand this idea, the reversible jump MCMC algorithm is combined with the Bootstrap method [13]. The Bootstrap method is a tool that can be used to analyze large data.

This article aims to propose an SPR model along with a method for estimating SPR model parameters. These findings contribute to the development of the SPR model. In addition, this article also contributes to the development of the reversible jump MCMC algorithm as an estimate of SPR model parameters.

2. Materials and Methods

This article uses n data pairs that have an SRP model with Gaussian noise. Suppose these n data pairs are $(x_1, y_1), \dots, (x_n, y_n)$. Initially, this data pair is used to create B Bootstrap samples by taking n points from $\{(x_1, y_1), \dots, (x_n, y_n)\}$ with replacement. For $j = 1, \dots, B$, let $\left(x_1^{j*}, y_1^{j*}\right), \dots, \left(x_n^{*j}, y_n^{*j}\right)$ is the jth Bootstrap sample. Each jth Bootstrap sample is used to estimate the SPR model parameters. Furthermore, parameters are estimated using Bayesian methods. In Bayesian methods, a likelihood function for the data is combined with a prior distribution for the parameters. Using Bayes' Theorem, this combination produces a posterior distribution for the parameters. The SPR model order includes parameters so that the posterior distribution for the parameters has a complex structure. Therefore, the Bayes estimator is calculated using the reversible jump MCMC algorithm. Then, the jth Bayes estimator $j = 1, \dots, B$ is used to determine the Bootstrap estimator. Next, estimators for the parameters are substituted into the SPR model. Finally, the SPR model is used to predict the value of y_{n+1} if the value of x_{n+1} is known.

3. Results and Discussion

3.1. The SPR Parameter Estimation Procedure is Based on the Bootstrap-Reversible Jump MCMC Algorithm

Let $(x_1, y_1), \dots, (x_n, y_n)$ be n pairs of observation data and $\left(x_1^{j*}, y_1^{j*}\right), \dots, \left(x_n^{*j}, y_n^{*j}\right)$ is the jth Bootstrap sample $(j = 1, \dots, B)$. The jth Bootstrap sample is modeled by polynomial regression of order p. Thus, the jth Bootstrap sample satisfies the stochastic equation:

$$y_t^{j*} = c_p(x_t^{j*})^p + \cdots + c_1 x_t^{j*} + c_0 + \varepsilon_t^{j*}, \qquad t = \quad 1, \dots., n. \tag{1}$$

In (1), $c = \{c_p, \dots., c_0\}$ is the coefficients of the SPR model and $\varepsilon^{*j} = \{\varepsilon_1^{*j}, \dots, \varepsilon_n^{*j}\}$ is the noise corresponding to the jth Bootstrap sample. In this SPR model, noise is assumed to be independent and normally distributed with mean 0 and variance σ^2. Therefore, the likelihood function for the jth Bootstrap sample can be written as

$$L(x^{*j}, y^{*j} | p, c) = (2\pi\sigma^2)^{-\frac{n}{2}}\ exp - \frac{1}{2\sigma^2}\sum_{t=1}^{n}\left(y_t^{*j} - c_p(x_t^{j*})^p - \cdots - c_1 x_t^{j*} - c_0\right)^2. \tag{2}$$

In (2), $x^{*j} = \{x_1^{*j}, \dots, x_n^{*j}\}$ and $y^{*j} = \{y_1^{*j}, \dots, y_n^{*j}\}$.

This article focuses on polynomial regression where the independent variable is stationary. Let S_{p+1} be the stationarity region for the independent variable y and F is a one-to-one transformation from $c \in S_{p+1}$ to $r = \{r_1, \dots, r_{p+1}\} \in (-1,1)^{p+1}$ [11]. In terms of coefficient r, the likelihood function for Bootstrap samples $\left(x_1^{j*}, y_1^{j*}\right), \dots, \left(x_n^{*j}, y_n^{*j}\right)$ can be written as

$$L(x^{*j}, y^{*j} | p, c) = (2\pi\sigma^2)^{-\frac{n}{2}}\ exp - \frac{1}{2\sigma^2}\sum_{t=1}^{n}\left(y_t^{*j} - F^{-1}(r_p)\left(x_t^{j*}\right)^p - \cdots - F^{-1}(r_1)x_t^{j*} - F^{-1}(c_0)\right)^2. \tag{3}$$

In (3), F^{-1} is the inverse transformation of F.

The prior distribution for order p is a Binomial distribution with parameters p_{max} and λ. The maximum order (p_{max}) value is chosen among the set of natural numbers. For value of p_{max} is set equal to 3. The prior distribution for the coefficient r is a uniform distribution on $(-1,1)^{p+1}$. The prior distribution for σ^2 is an inverse Gamma distribution with parameters α and β. For the value of α, the value of α is set equal to 1. The prior distribution for λ is a uniform distribution on the interval (0,1). The prior distribution for β is the Jeffreys prior distribution. Let $\vartheta = (p, r, \sigma^2, \lambda, \beta)$. Therefore, the prior distribution for parameter ϑ can be written as

$$\pi(\vartheta) = C_p^{p_{max}} \lambda^p (1-\lambda)^{p_{max}-p} \left(\frac{1}{2}\right)^{p+1} \frac{\beta^\alpha}{\Gamma(\alpha)} (\sigma^2)^{-\alpha-1} exp - \frac{\beta}{\sigma^2}\frac{1}{\beta}. \tag{4}$$

The likelihood function in equation (3) and the prior distribution in equation (4) are used to obtain the posterior distribution. According to Bayes' Theorem, the posterior distribution for the parameter ϑ can be written as

$$\pi(\vartheta | x^{*j}, y^{*j}) \propto (2\pi)^{-\frac{n}{2}}\ C_p^{p_{max}} \left(\frac{1}{2}\right)^{p+1} \frac{\beta^{\alpha-1}}{\Gamma(\alpha)} \lambda^{a^*-1}(1-\lambda)^{b^*-1}(\sigma^2)^{-u^*-1} exp - \frac{v^*}{\sigma^2}. \tag{5}$$

In (4), $a^* = p + 1$, $b^* = p_{max} - p + 1$, $u^* = \left(\alpha + \frac{n}{2}\right)$ and $v^* = \beta + \frac{1}{2}\sum_{t=1}^{n}\left(y_t^{*j} - F^{-1}(r_p)\left(x_t^{j*}\right)^p - \cdots - F^{-1}(r_1)x_t^{*j} - F^{-1}(r_0)\right)^2$. The posterior marginal distribution for (p, r, σ^2) is

$$\pi(p, r, \sigma^2 | x^{*j}, y^{*j}) \propto (2\pi)^{-\frac{n}{2}}\ C_p^{p_{max}} \left(\frac{1}{2}\right)^{p+1} \frac{\beta^{\alpha-1}}{\Gamma(\alpha)} \frac{\Gamma(a^*)\Gamma(b^*)}{\Gamma(a^*+b^*)} \frac{\Gamma(u^*)}{(v^*)^{u^*}}. \tag{6}$$

The parameter ϑ is estimated with a Markov chain generated via Gibbs Sampler [14]. Gibbs Sampler stages are

- $\lambda \sim Be(a^*, b^*)$
- $\sigma^2 \sim IG(u^*, v^*)$
- $(\beta | p, r, \sigma^2) \sim Ga\left(\alpha, \frac{1}{\sigma^2}\right)$
- $(p, r | \beta, \sigma^2) \sim \pi(p, r, \sigma^2 | x^{*j}, y^{*j})$

Since $\pi(p, r, \sigma^2 | x^{*j}, y^{*j})$ in equation (5) has a complex structure, the parameters (p, r, σ^2) are estimated with a Markov chain generated through the reversible jump Markov MCMC algorithm [10]. As in [15-16], reversible jump MCMC algorithm uses three types of transformations, namely: coefficient change, coefficient birth and coefficient death.

3.2. Validation of Bootstrap-Reversible Jump MCMC Algorithm

The Bootstrap-reversible jump MCMC algorithm is validated using simulated data. In this simulation, the dependent variable x and the parameters of the polynomial regression model are determined. Then, the dependent variable and parameters are substituted into equation (1) to produce the independent variable. Then, the Bootstrap-reversible jump MCMC algorithm is run using the dependent variable and independent variable as input. The output of the Bootstrap-reversible jump MCMC algorithm is an estimator of the SPR model parameters. The Bootstrap-reversible jump MCMC algorithm is said to be valid if the SPR model parameter estimator value is close to the SPR model parameter value.

The first simulation uses the SPR model parameters in Table 1 second row and the dependent variable in Figure 2.

Table 1. Parameter values from simulated data

p	c	σ^2
2	$(-1, 0.65, -0.25)$	4
3	$(-0.08, -0.07, -0.30, 0.83)$	4

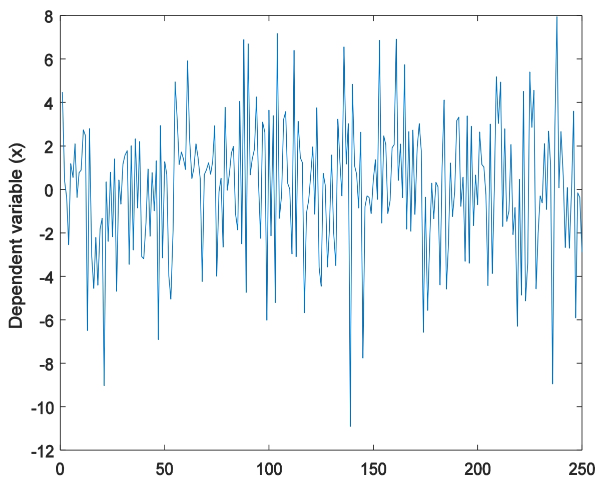

Figure 2. Simulated dependent data (p =2)

The SPR model parameter values in Table 1 second row and the dependent variable in Figure 2 are substituted into equation (1) to produce simulated independent data. Simulated independent data is presented in Figure 3.

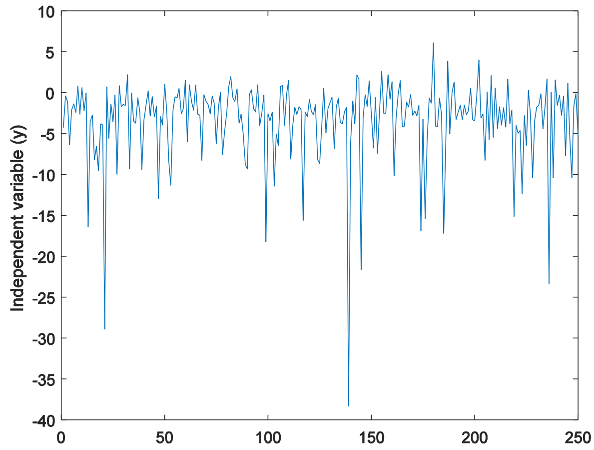

Figure 3. Simulated independent data (p=2)

The relationship between the independent variable in Figure 3 and the dependent variable in Figure 4 is presented in Figure 4.

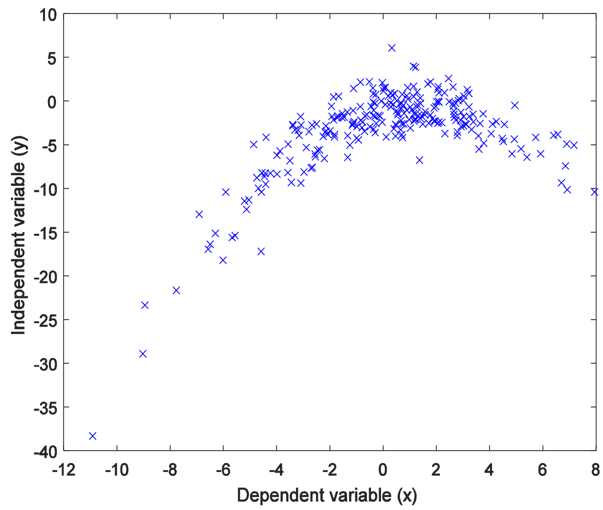

Figure 4. Simulated independent data vs Simulated dependent data (p=2)

Resampling of the data in Figure 4 was carried out at B=15 to obtain Bootstrap samples. Each jth Bootstrap sample (j = 1, …, 15) is used as input for the reversible jump MCMC algorithm and the algorithm is run with the number of interactions and the number of burn-in periods, respectively 100 thousand iterations and 20 thousand iterations. The output of the reversible jump MCMC algorithm is presented in Table 2.

Table 2. Reversible jump MCMC estimator using Bootstrap samples (first simulated data)

j	$\hat{p}$	$\hat{c}$	$\hat{\sigma}^2$
1	2	$(-1.02, 0.64, -0.23)$	4.67
2	2	$(-0.97, 0.66, -0.24)$	5.23
3	2	$(-1.15, 0.61, -0.23)$	4.80
4	2	$(-1.07, 0.66, -0.25)$	4.52
5	2	$(-1.07, 0.68, -0.25)$	4.44
6	2	$(-0.86, 0.64, -0.24)$	3.96
7	2	$(-0.73, 0.68, -0.24)$	4.87
8	2	$(-1.01, 0.72, -0.25)$	4.30
9	2	$(-0.97, 0.75, -0.24)$	4.84
10	2	$(-0.82, 0.61, -0.25)$	4.31
11	2	$(-1.15, 0.70, -0.24)$	3.88
12	2	$(-1.03, 0.68, -0.23)$	4.78
13	2	$(-0.77, 0.61, -0.25)$	4.84
14	2	$(-1.00, 0.68, -0.24)$	3.12
15	2	$(-0.91, 0.69, -0.25)$	4.27

The reversible jump MCMC estimators in Table 2 are used to calculate the Bootstrap estimator. Then, the Bootstrap-reversible jump MCMC estimator for the SPR model parameters is presented in Table 3 second row. Meanwhile, the confidence interval for the variance is (3.47, 5.44).

Table 3. Bootstrap-reversible jump MCMC estimator

p	c	σ^2
2	$(-0.97, 0.67, -0.24)$	4.46
3	$(-0.22, -0.1, -0.31, 0.82)$	3.65
2	$(0.17, 0.87, -0.06)$	1.09

In the same way, the second simulation is carried out. In the second simulation, the SRP model parameters are presented in Table 1 third row and the dependent variable in Figure 5. Simulated independent data is presented in Figure 6. The relationship between the independent variable in Figure 5 and the dependent variable in Figure 6 is presented in Figure 7.

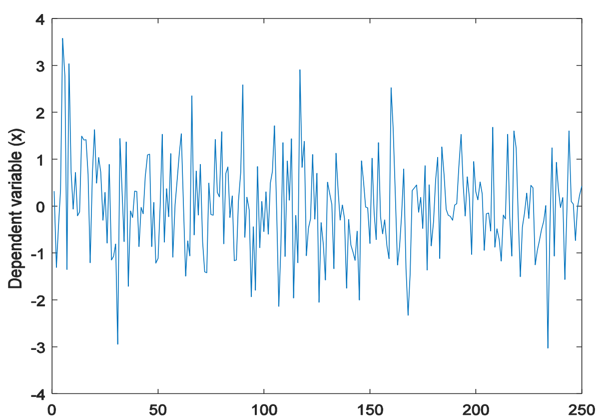

Figure 5. Simulated independent data (p=3)

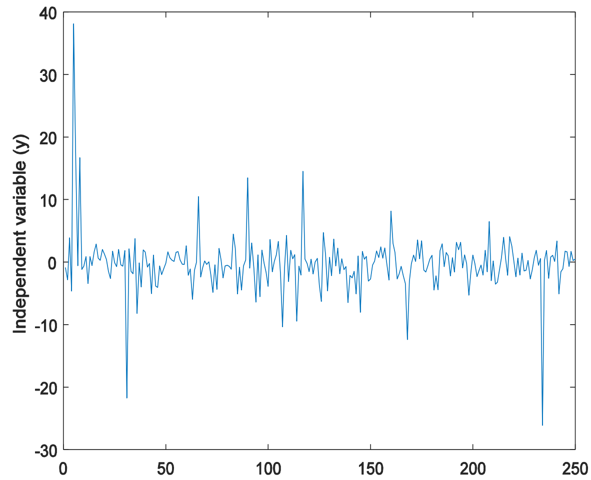

Figure 6. Simulated dependent data (p=3)

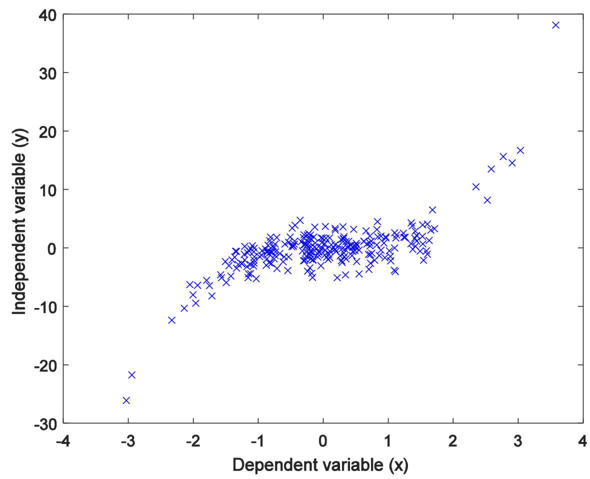

Figure 7. Simulated independent data vs simulated dependent data (p=3)

Table 4. Reversible jump MCMC estimator using Bootstrap samples (second simulated data)

j	$\hat{p}$	$\hat{c}2$	$\hat{\sigma}^2$
1	3	$(-0.22, 0.09, -0.34, 0.76)$	3.83
2	3	$(-0.15, 0.26, -0.32, 0.69)$	3.49
3	3	$(-0.20, -0.08, -0.31, 0.83)$	3.80
4	3	$(-0.30, -0.10, -0.28, 0.88)$	3.84
5	3	$(-0.18, 0.10, -0.30, 0.77)$	3.32
6	3	$(-0.24, -0.28, -0.29, 0.88)$	3.53
7	3	$(-0.22, -0.18, -0.30, 0.87)$	4.03
8	3	$(-0.24, -0.02, -0.37, 0.76)$	4.08
9	3	$(-0.25, -0.19, -0.29, 0.86)$	3.59
10	3	$(-0.13, -0.11, -0.27, 0.81)$	3.22
11	3	$(-0.14, -0.27, -0.30, 0.83)$	3.38
12	3	$(-0.22, -0.16, -0.23, 0.82)$	3.21
13	3	$(-0.25, -0.17, -0.30, 0.83)$	3.16
14	3	$(-0.35, -0.21, -0.37, 0.90)$	3.39
15	3	$(-0.25, -0.15, -0.34, 0.86)$	4.61

The output of the reversible jump MCMC algorithm is presented in Table 4. The Bootstrap-reversible jump MCMC estimator of the parameters is presented in the second row of Table 3. Meanwhile, the confidence interval for the variance is (2.87, 4.39).

3.3. Percentage of Covid-19 Cases vs Percentage of Deaths (Source: covid19.who.int)

The Corona virus appeared at the end of 2019 and spread in various countries. The Corona virus is one of the actual issues that has received the attention of writers, for example [17-21]. To find out more about this Corona virus, SPR is applied to model the relationship between the percentage of deaths and the percentage of COVID-19 cases. Data is taken from the percentage of deaths with the percentage of COVID-19 cases at https://covid19.who.int/ in August 2021. This data is presented in Figure 8.

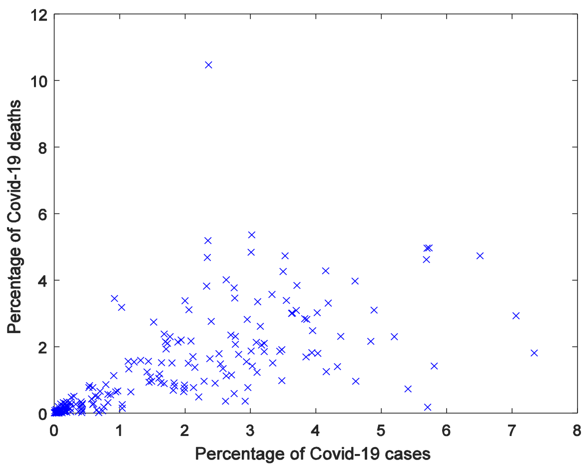

Figure 8. Percentage of Covid-19 cases vs percentage of Covid-19 deaths

Table 5. Reversible jump MCMC estimator using Bootstrap samples (Covid data)

j	$\hat{p}$	$\hat{c}$	$\hat{\sigma}^2$
1	2	$(0.20, 0.89, -0.04)$	1.17
2	2	$(0.09, 0.85, -0.05)$	0.82
3	2	$(0.05, 0.87, -0.05)$	0.59
4	2	$(0.18, 0.85, -0.06)$	1.57
5	2	$(0.15, 0.87, -0.07)$	0.99
6	2	$(0.23, 0.87, -0.08)$	1.13
7	2	$(0.37, 0.85, -0.07)$	2.32
8	2	$(0.15, 0.90, -0.07)$	0.66
9	2	$(0.21, 0.87, -0.05)$	1.52
10	2	$(0.09, 0.89, -0.05)$	0.69
11	2	$(0.25, 0.86, -0.07)$	1.60
12	2	$(0.17, 0.85, -0.06)$	1.00
13	2	$(0.13, 0.87, -0.06)$	0.96
14	2	$(0.09, 0.89, -0.07)$	0.72
15	2	$(0.17, 0.87, -0.06)$	0.93

Resampling of the data in Figure 8 was carried out at B = 15 to obtain Bootstrap samples. Each jth Bootstrap sample (j = 1, …, 15) is used as input for the reversible jump MCMC algorithm and the algorithm is run with the number of interactions and the number of burn-in periods, respectively 100 thousand iterations and 20 thousand iterations. The output of the reversible jump MCMC algorithm is presented in Table 5.

The reversible jump MCMC estimators in Table 5 are used to calculate the Bootstrap estimator. Then, the Bootstrap reversible jump MCMC estimator of the parameters is presented in Table 3 third row. Meanwhile, the confidence interval for the variance is (0.23, 2.00).

3.4. Discussion

This article develops the SPR model and develops a PR parameter estimation procedure using the reversible jump MCMC algorithm. Reversible jump MCMC has been used by other authors to estimate parameters that have variable dimensions, for example [10, 15, 22-23]. The parameter estimation procedure is validated using simulated data. Simulation studies show that the reversible jump MCMC algorithm is able to estimate the PR model parameters well. This can be proven by comparing the SPR model parameter values (Table 1) and the SPR model parameter estimator values (Table 3). Furthermore, reversible jump MCMC can estimate both the order and coefficients of the SPR model simultaneously. The algorithm developed has not been compared with other methods. Further research can be carried out by comparing the algorithm developed with other available methods to determine the quality of the algorithm developed.

The algorithm is applied to data on the percentage of deaths and the percentage of COVID-19 cases. The relationship between the percentage of deaths and the percentage of COVID-19 cases can be expressed by a 3rd order polynomial regression with the polynomial regression model coefficients presented in Table 3, third row. Previous studies that have used polynomial regression models on COVID-19 data can be found in various literature, for example [3, 24-27].

The algorithm developed is a combination of the Bootstrap algorithm and reversible jump MCMC so that this reparameterization [11] is applied to the SPR model coefficients to guarantee the stationarity of the independent variables. So, this algorithm finds estimates of the order and coefficients of the SPR model.

As in [28-30], the SPR model uses Gaussian distributed noise. In this SPR model, the order of the SPR model is limited to below 4. Further research can be extended to SPR models with non-Gaussian noise of order more than 3.

4. Conclusions

This article has proposed the development of the SPR model and the Bootstrap reversible jump MCMC algorithm to estimate the SPR model parameters. The order of the

SPR model is assumed to be unknown so that the parameter space has variable dimensions. Simulation studies show that the Bootstrap reversible jump MCMC algorithm has found good parameter estimators of the SPR model. In addition, this algorithm estimates the order and coefficients of the SPR model simultaneously. These findings contribute to the development of SPR models and the estimation of SPR model parameters.

The SPR model has been assumed to have Gaussian distributed noise. Further research can be extended to SPR models that have non-Gaussian distributed noise.

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