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A Pivotal Operation on Triangular Fuzzy Number for Solving Fuzzy Nonlinear Programming Problems

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Abstract Fuzzy nonlinear programming plays a vital role in decision-making where uncertainties and nonlinearity significantly impact outcomes. Real-world situations often involve imprecise or vague information. Fuzzy nonlinear programming allows for the representation of uncertainty through fuzzy sets, enabling more accurate modeling of real-world complexities. Many optimization problems exhibit nonlinear relationships among variables. Fuzzy nonlinear programming addresses these complex relationships, providing solutions that linear programming methods cannot accommodate. The objective of this research article proposes Fuzzy Non-Linear Programming Problems (FNLPP) under environment of triangular Fuzzy numbers. This paper proposed a method based on the pivotal operation with aid of Wolfe's technique. Fuzzy non-linear programming is an area of study that deals with optimization problems in which the objective function and constraints involve fuzzy numbers, which represent uncertainty or vagueness in real-world data. We claim that the proposed method is easier to understand and apply compared to existing methods for solving similar problems that arise in real-life situations. To demonstrate the effectiveness of the method, the authors have solved a numerical example and provided illustrations in the paper. This proposed method in the paper aims to address such complexities and find solutions to these problems more efficiently.

Keywords Fuzzy Non Linear Programming Problem, Fuzzy Optimal Solution, Pivotal Operation, Triangular Fuzzy Number

1. Introduction

Nonlinear programming is indeed a fundamental and widely applied technique in Operations Optimization. Despite its long history and extensive research, there is still a need to develop new approaches to addressing real-world problems more effectively within the framework of nonlinear programming. In practical applications, nonlinear programming models often involve parameters whose values are provided by experts but may not be exact due to uncertainty in the data. Dealing with this uncertainty becomes crucial for decision-makers. One suitable approach to handling such uncertainty is to represent these parameters as fuzzy data and use concepts from fuzzy theory to tackle the problem. This study presents a literature review that focused on non-linear programming problems.

Allahviranloo et al. [1] proposed a new method for solving fully fuzzy linear programming problems using a ranking function. This method likely aims to find solutions that consider the fuzzy nature of the parameters and the associated uncertainties. On the other hand, Amit Kumar & Jagdeep Kaur [2] proposed a new method to find fuzzy optimal solutions for fully fuzzy linear programming problems with equality constraints. Their method likely focuses on optimizing solutions in the presence of fuzzy data and equality constraints. Both of these papers appear to contribute to the field of fuzzy linear programming, providing techniques to handle uncertainty and fuzziness in real-world situations where exact values for parameters are not available. Without access to the full papers, it's challenging to provide more details or insights into the specific methods and algorithms proposed in these studies. However, it's evident that fuzzy nonlinear programming plays an important role in tackling real-world optimization problems in the presence of uncertainty and imprecision in the data. B. Dharmaraj and S. Appasamy [3] explore the use of a modified Gauss elimination technique in addressing problems characterized by both fuzziness and nonlinearity. The authors aim to present and discuss the adaptation of the modified Gauss elimination method specifically for separable fuzzy nonlinear programming problems. Additionally, they seek to evaluate the effectiveness of this technique in solving such problems. The overarching goal is to contribute to the field of mathematical modeling in engineering problems by introducing a novel approach tailored to handle the intricacies arising from the combination of fuzziness and nonlinearity in optimization scenarios

Nayak [4] might present innovative algorithms, theoretical models, or computational methodologies designed specifically to address the resolution of fuzzy nonlinear programming problems under the inclusion of linear constraints. Additionally, they might provide practical demonstrations or case studies to showcase how their proposed techniques effectively derive optimal solutions within this specific problem context. Dubois, D., and Prade, H [5] proposed in the realm of fuzzy logic and mathematics, fuzzy numbers serve as a method for expressing uncertainty or imprecision. Diverging from classical (crisp) numbers, fuzzy numbers encompass a spectrum of values accompanied by corresponding degrees of membership. The procedures applied to fuzzy numbers entail the systematic manipulation of these fuzzy values.

Dhurai and Karpagam [6] could potentially introduce computational techniques, mathematical frameworks, or algorithms incorporating a new pivotal operation specifically designed to address fully fuzzy linear programming problems. Moreover, their paper might encompass discussions, analyses, or practical examples illustrating how their proposed approach effectively resolves optimization challenges within the domain of complete fuzziness.

K. Ganesan, and P. Veeramani [7] introduced a formulation for fuzzy linear programming problems incorporating trapezoidal fuzzy numbers. This formulation is likely to encompass the representation of objective functions, constraints, and decision variables utilizing the characteristics of trapezoidal fuzzy numbers. Jianjun Lu, and Shu-Cherng Fang [8] presented specialized methods or techniques designed to address optimization scenarios involving constraints represented by fuzzy relations. These methods aim to provide a solution strategy for this specific problem type. The focus might involve an examination of how fuzzy relation equations influence constraints in nonlinear optimization problems and the formulation of effective strategies to resolve such problems despite the inclusion of fuzzy constraints. Kirtiwant P. Ghadle [9] presents a survey of Wolfe's Modified Simplex Method, a specialized variant of the simplex method crafted for the resolution of linear programming problems. The discussion within the paper might encompass an exploration of the method's principles, advantages, and limitations. The primary emphasis of the paper appears to be on introducing an innovative approach or modification to Wolfe's Modified Simplex Method, specifically designed for addressing quadratic programming problems. This adaptation could include alterations to the algorithm or the integration of specific strategies aimed at effectively managing quadratic terms.

Lalitha and Loganathan [10, 11, 12] could potentially introduce specialized computational strategies or theoretical frameworks aimed at addressing nonlinear programming problems specifically in fuzzy environments. Moreover, their paper might incorporate practical examples or case studies showcasing how their proposed methods effectively solve optimization challenges within this context of fuzziness. Maleki et al. [13] solved linear programming problems where all decision parameters are fuzzy numbers using the comparison of fuzzy numbers. This method likely addresses optimization problems where all variables are represented as fuzzy numbers, incorporating the concept of fuzzy arithmetic. Nagoor Gani & Mohamed Assarudeen [14] proposed a new operation on Triangular fuzzy numbers specifically for solving fuzzy linear programming problems. This method probably focuses on addressing fuzzy linear programming problems that involve triangular fuzzy numbers as parameters. This research article proposed a mathematical approach related pivot division operations on triangular fuzzy numbers to solve a problem called Fuzzy Non-Linear Programming Problem (FNLPP) using a two-phase simplex method. This approach aims to find the fuzzy optimal solution of the FNLPP. In the context of fuzzy numbers, triangular fuzzy numbers are a type of representation where each fuzzy number is defined by three parameters: a lower bound, a modal value (peak), and an upper bound. This representation allows for the consideration of uncertainty or imprecision in mathematical modeling. The proposed pivot operation mentioned is likely a step in the two-phase simplex method for solving Fuzzy Non-Linear Programming Problem (FNLPP). The simplex method is an iterative optimization technique used to solve linear programming problems, and the two-phase approach is used when dealing with more complex problems or problems that involve inequalities.

Nasseri & Alizadeh [15] proposed a method for solving fuzzy linear programming problems by transforming them into classical linear programming problems. This method likely simplifies the solution process by leveraging classical linear programming techniques. Each of these papers contributes to the field of fuzzy linear programming and related areas, providing different techniques and approaches to handling uncertainty and fuzziness in optimization problems. The use of fuzzy logic and fuzzy sets allows decision-makers to incorporate vagueness and uncertainty into their models, making the solutions more realistic and adaptable to real-world situations. Please note that without access to the full papers, it's challenging to provide more detailed insights into the methodologies and algorithms proposed in each of these studies. Nonetheless, these papers have likely contributed significantly to the advancement of fuzzy optimization techniques.

The importance of Fuzzy Nonlinear programming problem (FNLPP) lies not only in the theoretical advancements related to fuzzy numbers and their properties but also in its practical applications. Fuzzy Nonlinear programming has found applications in a wide range of fields, including engineering, economics, decision-making, and resource allocation, where decision-makers often deal with imprecise and uncertain information. The continuous efforts of researchers in this area have led to the development of robust techniques and algorithms to handle fuzzy data efficiently. As a result, Fuzzy Nonlinear programming (FLPP) has become an essential part of the broader field of fuzzy optimization and has greatly enriched the theories of fuzzy sets and their practical applications. The constant research and development in fuzzy linear programming reflect the ongoing recognition of its importance in solving real-world problems that involve uncertain or imprecise data, and it continues to be an active area of study in the broader domain of operations research and applied mathematics. It appears that the paper you are referring to focuses on ranking fuzzy numbers and its applications in various fields, particularly in decision-making, data analysis, artificial intelligence, and operation research. Ranking fuzzy numbers is indeed an essential procedure in the fuzzy environment, as it allows decision-makers to handle uncertainty and make informed choices based on fuzzy data.

Raj and Ranjana [16] might introduce innovative solutions or computational methodologies designed specifically to handle the challenges posed by fuzzy non-linear programming problems incorporating linear inequality constraints. Additionally, their paper might feature practical demonstrations or case studies illustrating how their proposed methods effectively derive solutions within this problem domain.

Saeid Jafarzadeh Ghoushchi, E. Osgooei, Gholamreza Haseli, Hana Tomášková [17] proposed a strategy that relies on alpha-cut theory and adjusted triangular fuzzy fuzzy numbers to achieve optimal solutions for practical scenarios. In this approach, the problem is treated as fully fuzzy and tackled using the newly introduced definition of triangular fuzzy numbers to optimize both decision variables and the objective function. Various numerical examples are employed to demonstrate the application of this approach. The usage of fuzzy logic is widespread in many real-world applications, as mentioned, such as automobile engine and automatic gear control systems, air conditioners, video enhancement in TV sets, washing machines, mobile robots, information systems, traffic control systems, and more. Fuzzy logic's ability to handle imprecise or vague information makes it a valuable tool for modeling and controlling complex systems.

N. Safaeiour's [18] method does not rely on fuzzy ranking functions, the addition of nonnegative variables, or impose restrictions on the coefficient matrix elements. To illustrate the effectiveness and superiority of our proposed method, numerical examples are presented. H. Tanaka and K. Asai [19] might explore strategies or methodologies designed to address linear programming problems featuring parameters or coefficients represented by fuzzy numbers or encompassing uncertainty. They could focus on defining, analyzing, and deriving solutions while considering the inherent fuzziness present in both the objective function and constraints within the context of linear programming.

Behera and Walaa Ibrahim Gabr [20] potentially presented specific methodologies or approaches designed for quadratic and nonlinear programming problems where all elements be it parameters, variables, or constraints are treated as fuzzy quantities. The focus might revolve around developing strategies to effectively formulate and solve these optimization problems amidst pervasive uncertainty or imprecision, utilizing methodologies rooted in fuzzy logic or related techniques. The formulation of Fuzzy Linear Programming Problem (FLPP) by Zimmermann [21] in 1978 marked a significant milestone in the application of fuzzy sets and fuzzy logic to linear programming. The introduction of fuzzy variables and fuzzy numbers in linear programming opened up new possibilities for modeling real-world problems with uncertain data and vague constraints. Since the introduction of Fuzzy Nonlinear programming (FLPP), numerous researchers have contributed to the field by proposing various types of fuzzy linear programming problems and developing different methods to solve them. These methods leverage concepts from fuzzy set theory, fuzzy arithmetic, and other related theories to handle fuzzy parameters and constraints in optimization models.

Contribution and Motivation, Novelty

Here is the summary for the contribution, motivation and novelty.

The contribution of investigating a pivotal operation on triangular fuzzy numbers for addressing fuzzy nonlinear programming problems lies in its potential to revolutionize decision-making in complex, uncertain environments.

This study's motivation stems from the inadequacies of traditional approaches in handling the intricate interplay between nonlinear relationships and uncertainties within optimization frameworks. By introducing a specialized operation tailored to triangular fuzzy numbers, this research aims to fill this gap. The unique characteristics of triangular fuzzy numbers offer a promising avenue to more accurately represent uncertainties inherent in real-world scenarios while addressing the nonlinear nature of relationships between variables.

The core contribution lies in the development of a specific mathematical operation that streamlines computations involving triangular fuzzy numbers, potentially simplifying the complexity associated with fuzzy nonlinear programming. This innovation could significantly enhance the precision of modeling uncertainties and optimize decision-making processes in diverse fields such as engineering, economics, and logistics.

The motivation is to equip decision-makers with a more robust tool set that better captures the nuanced complexities of real-world problems. By refining the handling of uncertainties and non-linearities, this research endeavors to pave the way for more reliable, adaptable, and effective solutions within fuzzy systems, ultimately aiming to advance the capabilities of decision support in uncertain and complex decision landscapes.

The novelty of employing a pivotal operation on triangular fuzzy numbers for solving fuzzy nonlinear programming problems lies in its innovative approach to handling both uncertainty and nonlinearity simultaneously within optimization frameworks.

- Introducing a unique operation tailored for triangular fuzzy numbers, specifically designed to handle uncertainties within fuzzy systems.
- Addressing both uncertainty representation and nonlinear relationships concurrently within optimization frameworks, offering a comprehensive approach.

Providing a novel method to streamline computations and improve accuracy in modeling uncertainties, promising more adaptable and precise solutions in fuzzy nonlinear programming.

This research article is structured into different sections: Section 2 provides preliminary definitions related to fuzzy numbers and possibly other concepts essential for understanding the subsequent content of the paper. Section 3 describes existing methods for ranking fuzzy numbers, likely summarizing prior research and approaches used by other authors. Section 4 introduces a new pivot operation, which is likely a novel method or algorithm for ranking fuzzy numbers, possibly with some numerical examples to illustrate its effectiveness. Section 5 concludes the paper, summarizing the main findings, contributions, and possibly discussing potential avenues for further research.

2. Preliminaries

Definition 2.1: Let ***R*** be the real line, then a fuzzy set $\tilde{A}$ in ***R*** is defined to be a set of ordered pairs $\tilde{A} = \{(\tilde{x}, \mu_{\tilde{A}}(x)) | x \in \mathbf{R}\}$, which $\mu_{\tilde{A}}(x)$ is the membership function for the fuzzy set. The membership function maps each element of ***R*** to a membership value between 0 and 1.

Definition 2.2: A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be triangular fuzzy number. If its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ \ x \leq \ a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for\ \ a_1 < x \leq a_2 \\ 1 & for\ \ x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\ \ a_2 < x \leq a_3 \\ 0 & for\ \ x \geq \ a_3 \end{cases}$$

Definition 2.3: An effective approach for ordering the elements of F(R) is also to define a ranking fuction R:F(R) →R which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number then function $R(\tilde{A}) = \frac{a+4b+c}{6}$, we define orders on F(R) by $\tilde{a} \geq \tilde{b}$ if and only if $R(\tilde{a}) \geq R(\tilde{b})$, $\tilde{a} \leq \tilde{b}$ if and only if $R(\tilde{a}) \leq R(\tilde{b})$, $\tilde{a} = \tilde{b}$ if and only if $R(\tilde{a}) = R(\tilde{b})$.

Definition 2.4: Arithmetic operations on Triangular fuzzy number

Let $\tilde{A} = (a_1, a_2, a_3)$ & $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers then

(i) $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii) $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

(iii) $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$

(iv) $\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}\right)$

Definition 2.5: Fuzzy non-linear programming problem

In this segment, we address the optimization challenge involving a non-linear fuzzy objective function and fuzzy flexible nonlinear constraints. Let's delve into the specific nonlinear programming problem presented.

$$\text{Min/Max} \quad f(\tilde{x})$$

Subject to

$$\tilde{g_i}(x) \geq (\leq) \tilde{b_i} \qquad i=1,2,\ldots m.$$

$$\text{For all } \tilde{x} \in R^n \text{ and } \tilde{x} \geq 0.$$

3. Proposed Algorithm

Step1

Convert inequality constraints into equations by introducing slack variables $\tilde{x}_i^2 (i = 1,2 \ldots\ldots m)$ in the i^{th} constraints and the slack variables in the.j^{th}.constraints.

Step 2

Convert the Lagrangian function differentiating the Lagrangian function

$$L(\tilde{x},\tilde{s},\tilde{\lambda}\tilde{\mu}) = f(\tilde{x}) - \sum_{i=1}^{n} \tilde{\lambda}_i \left[\sum_{j=1}^{n} \tilde{p}_{ij}\,\tilde{x}_j - \tilde{q}_i + \tilde{s}_i^2\right] - \sum_{j=1}^{n} \tilde{\mu}_j - \tilde{x}_j + \tilde{s}_j^2$$

Differentiating the Lagrangian function $L(\tilde{x},\tilde{s},\tilde{\lambda},\tilde{\mu})$ with respect to the components of $\tilde{x},\tilde{s},\tilde{\lambda},\tilde{\mu}$ and equating the first order partial derivative to zero. Derive Kuhn tucker condition from the resulting equations.

Step 3

Introduce non- negative artificial variables $\tilde{y}_j(j = 1,2 \ldots\ldots n)$ in the Kuhn tucker condition

$$\tilde{c}_j + \sum \tilde{c}_{jk}\ \tilde{x}_k - \sum_{i=1}^{n} \tilde{\lambda}_i \tilde{a}_{ij} + \tilde{y}_j = 0$$

$$(j = 1,2 \ldots\ldots n)$$

for j=1,2…n and construct an objective function

$$\tilde{z} = \tilde{y}_1 + \tilde{y}_2 \ldots\ldots \tilde{y}_n$$

Step 4

Obtain an initial basic feasible solution to the Linear Programming Problem:

$$Min\ \ \tilde{z} = \tilde{y}_1 + \tilde{y}_2 \ldots\ldots \tilde{y}_n$$

subject to the constraints:

$$\sum_{k=1}^{n} \tilde{c}_{jk}\,\tilde{x}_k - \sum_{i=1}^{m} \tilde{\lambda}_i \tilde{a}_{ij} + \tilde{y}_j = -\tilde{c}_j; (j = 1,2 \ldots\ldots n)$$

$$\sum_{j=1}^{n} \tilde{a}_{ij}\,\tilde{x}_j + \tilde{s}_j^2 = \tilde{q}_i; (i = 1,2 \ldots\ldots m)$$

$$\tilde{\lambda}_j, \tilde{\mu}_j, \tilde{x}_{j,} \tilde{y}_j, \tilde{a}_i \geq 0, (i = 1,2 \ldots\ldots m)(j = 1,2 \ldots\ldots n)$$

and satisfy the slackness condition:

$$\tilde{\lambda}_i \tilde{s}_i = 0 \quad \text{and} \quad \tilde{\mu}_j, \tilde{x}_j = 0$$

Step 5

Solve this LPP by above steps. Choose greatest coefficients. If greatest coefficient is unique, then variable corresponding to this column becomes incoming variable. If greatest coefficient is not unique, then uses tie breaking technique.

Step 6

Compute the ranking function with $\tilde{x}_B$ (R.H.S).Choose minimum ranking, the variable corresponding to this row is Outgoing variable. If artificial variable is outgoing in the basis, it means corresponding artificial column also will be removed.

Step 7

Then proceed the table given by step 6 and go to next Step.

Step 8

Ignore corresponding row and column. Proceed to step5 for remaining elements and repeat the same procedure. Either an optimal solution is obtained or there is an indication of an unbounded solution.

Step 9

If all rows and columns are ignored, the existing solutions represent an optimal solution. Thus optimum solution is obtained.

4. Numerical Example

Here's an outline of the process described:

Conversion of Pivotal Triangular Fuzzy Number:

Begin with a pivotal triangular fuzzy number that represents a constraint or an objective function in the FNLPP.

Pivot Division Operation: The pivot division operation involves manipulating the fuzzy numbers in a way that helps you improve the current solution. This might involve adjusting the triangular fuzzy number parameters to move towards the optimal solution.

Repeat Procedure: After the pivot division operation, you repeat the procedure to refine the solution further. This iterative process continues until you reach an optimal or near-optimal solution for the FNLPP.

Two-Phase Simplex Method: The two-phase simplex method is a variation of the classical simplex method. The first phase involves finding a feasible starting solution, and the second phase optimizes the solution iteratively until the optimal solution is reached. It appears that you're applying this method to the FNLPP, incorporating the pivot division operation with triangular fuzzy numbers.

Optimum Solution: The process aims to converge towards the fuzzy optimal solution of the FNLPP. This solution takes into account the uncertainty inherent in the triangular fuzzy number representations.

It's important to note that solving optimization problems involving fuzzy numbers can be complex due to the inherent uncertainty and imprecision. The iterative approach you're describing, using the two-phase simplex method and pivot division operations, shows an effort to address these challenges and find solutions that consider the fuzziness of the input data.

The specific implementation details and mathematical formulas for the pivot division operation and the two-phase simplex method in the context of triangular fuzzy numbers would require more in-depth understanding of the mathematical model and problem domain.

We discuss an objective fuzzy non-linear programming problem.

$$\text{Max}\ \tilde{Z} = \widetilde{2x_1} + \widetilde{3x_2} - \widetilde{2x_1^2}$$

Subject to

$$\widetilde{x_1}+4\widetilde{x_2} \le \tilde{4}$$
$$\widetilde{x_1}+\widetilde{x_2} \le \tilde{2}$$
$$\widetilde{x_1}, \widetilde{x_2} \ge 0$$

First, we convert all the fuzzy coefficients into triangular fuzzy integers.

$$\text{Max } \tilde{Z} = (1,2,3)\widetilde{x_1} + (2,3,4)\widetilde{x_2} - (1,2,3)\widetilde{x_1^2}$$

Subject to

$$(0,1,2)\widetilde{x_1} + (3,4,5)\widetilde{x_2} \le (3,4,5)$$
$$(0,1,2)\widetilde{x_1} + (0,1,2)\widetilde{x_2} \le (1,2,3)$$

We convert the inequality constraints into equations by introducing slack variables, $\widetilde{x_3^2}$ *and* $\widetilde{x_4^2}$ respectively.

Considering $\widetilde{x_1} \ge 0$ and $\widetilde{x_2} \ge 0$ also as the inequality constraints, we convert then also into equations by introducing slack variables $\widetilde{x_5}^2 \ge 0$ and $\widetilde{x_6}^2 \ge 0$ in them. The problem thus becomes

$$\text{Max } \tilde{Z} = (1,2,3)\widetilde{x_1} + (2,3,4)\widetilde{x_2} - (1,2,3)\widetilde{x_1^2}$$

Subject to

$$(0,1,2)\widetilde{x_1} + (3,4,5)\widetilde{x_2} + (0,1,2)\widetilde{x_3^2} = (3,4,5)$$
$$(0,1,2)\widetilde{x_1} + (0,1,2)\widetilde{x_2} + (0,1,2)\widetilde{x_4^2} = (1,2,3)$$
$$(0,1,2)\widetilde{x_1} - (0,1,2)\widetilde{x_5}^2 = (0,0,0)$$
$$(0,1,2)\widetilde{x_1} - (0,1,2)\widetilde{x_6}^2 = (0,0,0)$$

Construct the Lagrangian function and equate the first order partial derivative of L with respect to the variables $x_1, x_2, x_3, x_4, x_5, x_6, \lambda_1, \lambda_2, \lambda_3, \lambda_4$.

$$\text{Max } \tilde{Z} = (1,2,3)\widetilde{x_1} + (2,3,4)\widetilde{x_2} - (1,2,3)\widetilde{x_1^2}$$

$$L(x_1, x_2, x_3, x_4, x_5, x_6, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = ((1,2,3)\widetilde{x_1} + (2,3,4)\widetilde{x_2} - (1,2,3)\widetilde{x_1^2}) - \widetilde{\lambda_1}\left((0,1,2)\widetilde{x_1} + (3,4,5)\widetilde{x_2} + (0,1,2)\widetilde{x_3^2} - (3,4,5)\right) - \widetilde{\lambda_2}\left((0,1,2)\widetilde{x_1} + (0,1,2)\widetilde{x_2} + (0,1,2)\widetilde{x_4^2} - (1,2,3)\right) - \widetilde{\lambda_3}\left((0,1,2)\widetilde{x_1} - (0,1,2)\widetilde{x_5^2} - (0,0,0)\right) - \widetilde{\lambda_4}((0,1,2)\widetilde{x_1} - (0,1,2)\widetilde{x_6^2} - (0,0,0))$$

$$\frac{\partial L}{\partial x_1} = (1,2,3) - 2(1,2,3)\widetilde{x_1} - (0,1,2)\widetilde{\lambda_1} - (0,1,2)\widetilde{\lambda_2} - (0,1,2)\widetilde{\lambda_3}$$

$$\frac{\partial L}{\partial x_2} = (2,3,4) - (3,4,5)\widetilde{\lambda_1} - (0,1,2)\widetilde{\lambda_2} - (0,1,2)\widetilde{\lambda_4}$$

$$\frac{\partial L}{\partial \lambda_1} = (0,1,2)\widetilde{x_1} + (3,4,5)\widetilde{x_2} + (0,1,2)\widetilde{x_3^2} - (3,4,5)$$

$$\frac{\partial L}{\partial \lambda_2} = \left((0,1,2)\widetilde{x_1} + (0,1,2)\widetilde{x_2} + (0,1,2)\widetilde{x_4^2} - (1,2,3)\right)$$

$$\frac{\partial L}{\partial \lambda_3} = (0,1,2)\widetilde{x_1} - (0,1,2)\widetilde{x_5^2} - (0,0,0)$$

$$\frac{\partial L}{\partial \lambda_4} = (0,1,2)\widetilde{x_2} - (0,1,2)\widetilde{x_6^2} - (0,0,0)$$

After simplification the problem becomes,

$$(2,4,6)\widetilde{x_1} + (0,1,2)\widetilde{\lambda_1} + (0,1,2)\widetilde{\lambda_2} + (0,1,2)\widetilde{\lambda_3} = (1,2,3)$$
$$(3,4,5)\widetilde{\lambda_1} - (0,1,2)\widetilde{\lambda_2} - (0,1,2)\widetilde{\lambda_4} = (2,3,4)$$
$$(0,1,2)\widetilde{x_1} + (3,4,5)\widetilde{x_2} + (0,1,2)\widetilde{x_3^2} = (3,4,5)$$
$$\left((0,1,2)\widetilde{x_1} + (0,1,2)\widetilde{x_2} + (0,1,2)\widetilde{x_4^2} = (1,2,3)\right)$$
$$(0,1,2)\widetilde{x_1} - (0,1,2)\widetilde{x_5^2} = (0,0,0)$$
$$(0,1,2)\widetilde{x_2} - (0,1,2)\widetilde{x_6^2} = (0,0,0)$$

Now Introducing Artificial variable, we get

$$\text{Min } \tilde{Z} = (0,1,2)\widetilde{A_1} + (0,1,2)\widetilde{A_2}$$

Subject to the constraints

$$(2,4,6)\widetilde{x_1} + (0,1,2)\widetilde{\lambda_1} + (0,1,2)\widetilde{\lambda_2} + (0,1,2)\widetilde{\lambda_3} + (0,1,2)\widetilde{A_1} = (1,2,3)$$
$$(3,4,5)\widetilde{\lambda_1} - (0,1,2)\widetilde{\lambda_2} - (0,1,2)\widetilde{\lambda_4} + (0,1,2)\widetilde{A_2} = (2,3,4)$$
$$(0,1,2)\widetilde{x_1} + (3,4,5)\widetilde{x_2} + (0,1,2)\widetilde{x_3^2} = (3,4,5)$$
$$(0,1,2)\widetilde{x_1} + (0,1,2)\widetilde{x_2} + (0,1,2)\widetilde{x_4^2} = (1,2,3)$$

Table 1. First iteration

		$\widetilde{C_j}$	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,1,2)	(0,1,2)	
$\widetilde{C_B}$	$\widetilde{B}$	$\widetilde{X_B}$	$\widetilde{x_1}$	$\widetilde{x_2}$	$\widetilde{x}_3^2$	$\widetilde{x}_4^2$	$\widetilde{\lambda_1}$	$\widetilde{\lambda_2}$	$\widetilde{\lambda_3}$	$\widetilde{\lambda_4}$	$\widetilde{A_1}$	$\widetilde{A_2}$	$F(\tilde{Z})$
(0,1,2)	$\widetilde{A_1}$	(1,2,3)	(2,4,6)	(0,0,0)	(0,0,0)	(0,0,0)	(0,1,2)	(0,1,2)	(0,1,2)	(0,0,0)	(0,1,2)	(0,0,0)	2.2
(0,1,2)	$\widetilde{A_2}$	(2,3,4)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(3,4,5)	(0,1,2)	(0,0,0)	(0,1,2)	(0,0,0)	(0,1,2)	∞
(0,0,0)	$\widetilde{x}_3^2$	(3,4,5)	(0,1,2)	(3,4,5)	(0,1,2)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	6.5
(0,0,0)	$\widetilde{x}_4^2$	(1,2,3)	(0,1,2)	(0,1,2)	(0,0,0)	(0,1,2)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	3.5
Z_j			(0,4,12)	(0,0,0)	(0,0,0)	(0,0,0)	(0,5,14)	(0,2,8)	(0,1,4)	(0,1,4)	(0,1,4)	(0,1,4)	
$Z_j - C_j$			-(0,4,12)	(0,0,0)	(0,0,0)	(0,0,0)	-(0,5,14)	-(0,2,8)	-(0,1,4)	-(0,1,4)	(-4,0,2)	(-4,0,2)	

Table 2. Last iteration

		C_j	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,1,2)	
$\widetilde{C_B}$	$\widetilde{B}$	$\widetilde{X_B}$	$\widetilde{x_1}$	$\widetilde{x_2}$	$\widetilde{x}_3^2$	$\widetilde{x}_4^2$	$\widetilde{\lambda_1}$	$\widetilde{\lambda_2}$	$\widetilde{\lambda_3}$	$\widetilde{\lambda_4}$	$\widetilde{A_2}$	$F(\tilde{Z})$
(0,0,0)	$\widetilde{x_1}$	(0.5,0.5,0.5)	(1,1,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0.25,0.33)	(0,0.25,0.33)	(0,0.25,0.33)	(0,0,0)	(0,0,0)	
(0,1,2)	$\widetilde{A_2}$	(2,3,4)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(3,4,5)	(0,1,2)	(0,0,0)	(0,1,2)	(0,1,2)	
(0,0,0)	$\widetilde{x}_3^2$	(3,3.5,4)	(0,0,0)	(3,4,5)	(0,1,2)	(0,0,0)	-(0.0.25,0.66	-(0.0.25,0.66	-(0.0.25,0.66	(0,0,0)	(0,0,0)	
(0,0,0)	$\widetilde{x}_4^2$	(-0.5,1.5,5.5)	(0,0,0)	(0,0,0)	(0,0,0)	(0,1,2)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	
Z_j			(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,4,10)	(0,1,4)	(0,0,0)	(0,1,4)	(0,1,4)	
$Z_j - C_j$			(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,4,10)	(0,1,4)	(0,0,0)	(0,1,4)	(0,0,2)	

Table 1 typically denotes the starting values of decision variables and constraints. Throughout the optimization iterations, these variables are modified and their values refined, progressively moving closer to the optimal solution. Consequently, Table 2 displays the updated values of variables and constraints at the final iteration, signifying the achievement of the optimal solution.

$$\widetilde{x_1} = (0.5,0.5,0.5) \text{ and } \widetilde{x_2} = (0,0,0)$$

Hence Max $\tilde{Z}$ = (0.25, 0.5, 0.75)

Advantages

An advantageous aspect of employing a pivotal operation on triangular fuzzy numbers for solving fuzzy nonlinear programming problems lies in its ability to offer enhanced modeling precision and computational efficiency.

Triangular fuzzy numbers provide a structured way to represent uncertain or imprecise information within the fuzzy nonlinear programming framework. This representation allows for a more nuanced and accurate modeling of uncertainties that are inherent in many real-world scenarios.

The application of a pivotal operation streamlines computational complexity. It potentially simplifies calculations involving triangular fuzzy numbers, enabling more efficient algorithms and reducing the computational burden in solving fuzzy nonlinear programming problems.

By leveraging this pivotal operation on triangular fuzzy numbers, there's a potential for better decision-making. It enables a more comprehensive analysis of nonlinear relationships while considering uncertainties, leading to more informed and robust decisions in complex optimization problems.

The methodology involving pivotal operations on triangular fuzzy numbers may offer versatility in its applicability across various domains. Its potential effectiveness in handling uncertainties and nonlinear relationships could make it a valuable tool in diverse fields such as engineering, economics, logistics, and more.

This approach could contribute to advancing solutions within fuzzy systems by providing a refined technique that specifically targets the challenges posed by nonlinear relationships and uncertainties, potentially paving the way for new methodologies or improvements in existing fuzzy system applications.

5. Conclusions

This research paper introduces a mathematical method designed to address fuzzy non-linear programming problems featuring triangular fuzzy numbers and potentially explores how triangular fuzzy numbers characterized by specific properties can be employed to model uncertainty and handle nonlinearity within optimization problems offering a potentially effective approach to tackling complex, uncertain scenarios in decision-making and problem-solving realms.

These numbers are utilized to manage uncertainty and imprecision within both the objective function and constraints. Each triangular fuzzy number is defined by a triangular membership function, providing a means to represent this uncertainty. The approach revolves around a new method paired with a ranking function to solve these problems effectively. The ranking function plays a crucial role in determining the importance or weights assigned to different criteria in the problem. The proposed Wolfe technique aims to efficiently handle fuzzy nonlinear programming problems that involve linear constraints. Furthermore, it hints at the potential expansion of this method to encompass problems where both the objective function and constraints exhibit non-linearity

Future Scope

The future scope of fuzzy nonlinear programming holds significant potential for advancements in several key areas:

Future research could focus on refining modeling techniques to better represent uncertainties and complex relationships. This might involve exploring new types of fuzzy sets or innovative mathematical frameworks to handle increasingly intricate real-world scenarios.

Advancements in computational methods and algorithms tailored for fuzzy nonlinear programming could enhance efficiency and scalability, allowing for more extensive and rapid problem-solving in larger-scale applications.

Integration with emerging technologies like machine learning and artificial intelligence could bolster the capabilities of fuzzy nonlinear programming. This fusion might lead to more adaptive, self-learning systems capable of addressing dynamic and evolving uncertainties.

Expanding applications in diverse fields such as healthcare, finance, climate modeling, and supply chain management could unlock new avenues for utilizing fuzzy nonlinear programming to solve complex problems specific to these domains.

Collaborations across disciplines, combining insights from mathematics, computer science, and domain-specific expertise, could result in novel approaches and practical implementations, further broadening the scope and applicability of fuzzy nonlinear programming. Future developments might focus on extending fuzzy nonlinear programming to tackle multi-objective optimization problems, considering conflicting goals and uncertainties simultaneously.

Exploring these avenues could lead to breakthroughs, making fuzzy nonlinear programming even more effective in addressing real-world complexities and fostering its broader adoption across various industries and problem-solving domains.

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