Mathematics and Statistics 12(1): 31-40, 2024 http://www.hrpub.org
DOI: 10.13189/ms.2024.120105

A New Robust Interval Estimation for the Median of An Exponential Population When Some of the Observations are Extreme Values

Faris Muslim Al-Athari

Department of Financial and Banking Sciences, AL-Turath University College, Iraq

Received July 31, 2023; Revised November 25, 2023; Accepted December 21, 2023

Cite This Paper in the following Citation Styles

(a): *[1] Faris Muslim Al-Athari , "A New Robust Interval Estimation for the Median of An Exponential Population When Some of the Observations Are Extreme Values," Mathematics and Statistics, Vol. 12, No. 1, pp. 31 - 40, 2024. DOI: 10.13189/ms.2024.120105.*

(b): *Faris Muslim Al-Athari (2024). A New Robust Interval Estimation for the Median of An Exponential Population When Some of the Observations Are Extreme Values. Mathematics and Statistics, 12(1), 31 - 40. DOI: 10.13189/ms.2024.120105.*

Abstract The issue of obtaining accurate interval estimates for the median of an exponential population when some of the observations are extreme values is an important issue for researchers in the fields of reliability applications and survival analysis. In this research paper, a new method is proposed for obtaining a robust confidence interval which is a substitute for the known ordinary (classical) confidence interval when there are extreme values in the sample. The proposed method is simply a result of changing the sample mean by a constant multiple of a sample median and adjusting the upper percentile point of the chi-square of the ordinary confidence interval formula. Further, the performance of the proposed method is evaluated and compared with the ordinary one by using Monte Carlo simulation based on 100,000 trials for each sample size with 5% and 10% extreme values showing that the proposed method, under the contaminated exponential distribution, is always performing better than the ordinary method in the sense of having simulated confidence probability quite close to the aimed confidence level with shorter width and smaller standard error. The use and the application of the proposed method to real-life data are presented and compared with the simulation results.

Keywords Exponential Distribution, Robustness, Median Estimates, Confidence Probability, Standard Error, Extreme Values, Coverage Probability

1. Introduction

The exponential distribution is known to be a popular model to analyze the lifetimes or failure times of individuals or items in different fields of study such as engineering, industry, electronics, medicine, etc. It is frequently used in measuring the reliability of machines, the reliability of the electronic systems, the waiting time in a queuing system, the duration of time between failures of a machine, the length of the queue, and the remission times of chronic diseases [1-10]. Moreover, it has useful properties and easy mathematical and statistical issues [11]. It represents the density of interarrival times between events and it is a continuous synonym of geometric probability distribution [6, 12]. The generalized exponential distribution is used in attribute sampling inspection with a new group chain given by [13]. Fathi H. Riad [14] obtained point and interval estimates for the so-called modified censored Kaies exponential population.

A random variable X is said to have an exponential probability density function of mean θ if its probability density function is of the form

$$f(x;\theta)=\frac{1}{\theta}e^{-\frac{x}{\theta}}, x\geq 0, \theta>0, \quad (1)$$

and zero otherwise.

It is easy to show that its median is:

$$X_{0.5}=\theta\ln 2=0.69315\theta \quad (2)$$

and the maximum likelihood estimator of its mean is the sample mean, $\bar{X} = \sum X / n$ which is consistent and efficient [15]. The maximum likelihood estimator of (2) is $\bar{X}\ \ln 2 = 0.69315\bar{X}$, which is known to be sensitive to outliers [6, 16].

Frequently, researchers prefer estimating the population median by using the known ordinary confidence interval rather than using a point estimate [17, 18]. But, unfortunately, real data aren't as clean as simulated exponential data and hence the ordinary (classical) confidence interval will give incorrect results due to the existence of the sample mean, which is sensitive to extreme values, and to the violations of the exponential and chi-squares distributions assumptions in the sense that the sample mean will not be the best estimator of the mean and the distribution of the pivotal quantity, $2n\bar{X}/\theta$, will be no more a chi-square distribution. These outliers may affect both the length and the confidence probability $(1-\alpha)$ in the sense that they pull the value of the confidence interval's length too much in their direction and alter the confidence probability. So, we seek a robust confidence interval that does almost as well as possible if the assumptions we make about the distribution are true but performs that much better within a range of alternatives to those assumptions due to the existence of the extreme values. The problem of finding robust interval estimates when the underlying data contains a wide range of extreme observations is very important in many statistical applications [19-24]. The statistical literature shows that the sample median might give a more meaningful measure of location than the sample mean when the data contain extreme values and is indeed more resistant to the influence of a few extreme observations [6, 16]. For instance, many descriptive measures of location reported by government agencies are medians, like household income, years of school completed, and resale home prices [25]. Abu-Shawiesh [22] proposed a modified (not robust) confidence interval for the median of the exponential population by using the sample median. His research paper was just a method of comparison with the usual (ordinary) confidence interval and was not an attempt to find a robust confidence interval. [22] used a completely different type of approach to find results close to the exact ones, but unfortunately, his results showed that the modified confidence interval, is poor. Ahmad et al. [20] used the method of weighted likelihood to obtain a robust point estimator for the parameter of the exponential distribution. The simulation results showed that their estimator is more efficient than the maximum likelihood estimator.

In this research paper, I propose a new method for obtaining robust interval estimation to the median of the exponential distribution to be a substitute for the ordinary estimation method when the sample contains extreme values. The effect of the extreme values on the behavior of the proposed confidence interval is studied and compared with the ordinary confidence interval by using three evaluation criteria, the simulated confidence probability (CP), the simulated confidence width (CW), and the standard error of the simulated width (SE). All calculations are done by writing a program with a simulation study using MATLAB [26].

2. The Ordinary (Classical) Method

It is well known that the ordinary interval estimation method is used to estimate the median of the exponential density when the data are free from outliers because it provides good information about the population median and the accuracy of its estimate. The basis of the confidence interval is: If X is an exponentially distributed random variable with mean θ then for samples of size n, the variable

$$\frac{2n\bar{X}}{\theta} = \frac{2n(\ln 2)\bar{X}}{X_{0.50}} \quad (3)$$

has a chi-square distribution. Using this fact and the fact that

$$P\left(\chi^2_{\alpha/2} \leq 2n(\ln 2)\bar{X}/X_{0.5} \leq \chi^2_{1-\alpha/2}\right) = 1-\alpha \quad (4)$$

We can write the ordinary $100(1-\alpha)\%$ Confidence interval for the median as

$$\left(2n\,ln(2)\bar{X}/\chi^2_{(1-\alpha/2)},\, 2n\,ln(2)\bar{X}/\chi^2_{(\alpha/2)}\right) \quad (5)$$

$$\text{and,}\quad E(L) = E\left[2n\ln(2)\,\bar{X}\left(\frac{1}{\chi^2_{(\alpha/2)}} - \frac{1}{\chi^2_{(1-\alpha/2)}}\right)\right]$$

$$= 2n\,ln(2)\left(\frac{1}{\chi^2_{\left(\frac{\alpha}{2}\right)}} - \frac{1}{\chi^2_{\left(1-\frac{\alpha}{2}\right)}}\right)\theta \quad (6)$$

is the exact width of this confidence interval and

$$S_e(L) = 2\sqrt{n}(\ln 2)\left(\frac{1}{\chi^2_{\frac{\alpha}{2}}} - \frac{1}{\chi^2_{1-\frac{\alpha}{2}}}\right)\theta$$

is the standard error of the exact width.

Such that χ^2_p is the p^th percentile of the chi-square distribution with 2n degrees of freedom. The ordinary confidence interval in (5) must have a simulation confidence probability equal to the confidence probability $(1-\alpha)$ under the assumption that the distribution of the data is exponential with no extreme values. Because the ordinary confidence interval in (5) is specific to the exponential data and the real data often aren't as clean as simulated exponential data and because the sample mean is sensitive to outliers, we don't anticipate that the ordinary confidence interval will give satisfactory results for the simulated confidence probability (CP) and for the simulated confidence width (CW) when the exponential data have some extreme values. Therefore, we seek a robust estimate for the median that does almost as well as possible if the assumptions about the exponential and the chi-square distribution are true but performs that much better within a range of alternatives to those assumptions due to the existence of the extreme values.

3. The Proposed Method

To get a robust confidence interval to extreme values, we suggested utilizing the following approximate relationship between the mean, median, and mode;

$$\bar{X} - \text{mod} = 3(\bar{X} - med) \quad (8)$$

such that the sample median

$$med = \begin{cases} (X_{(n/2)} + X_{(n/2+1)})/2, \ldots \text{ if n is even} \\ \ldots \\ X_{(n+1)/2}, \ldots \quad \text{if n is odd} \end{cases} \quad (9)$$

and $X_{(n)}$ is the value of the n^{th}-order statistic.

Assuming that the value of the mode, mod, in (8) equals zero we get the following important relationship between the sample mean and the sample median

$$\bar{X} = \frac{3med}{2} \quad (10)$$

Substituting (10) into (4) and (5) and making suitable adjustments by using $1.5\chi^2_{1-\alpha/2}$ instead of $\chi^2_{1-\alpha/2}$ to get the following robust confidence interval for the median

$$\left(\frac{2n(ln\,2)(1.5med)}{1.5\chi^2_{1-\frac{\alpha}{2}}}, \frac{2n(ln\,2)(1.5med)}{\chi^2_{\frac{\alpha}{2}}}\right)$$

$$= \left(\frac{(2n(ln\,2)med}{\chi^2_{1-\frac{\alpha}{2}}}, 3n(ln\,2)med/\chi^2_{\frac{\alpha}{2}}\right) \quad (11)$$

According to [27], the sample median has a maximal 50% breakdown point. It is easy and frequently used when the distribution is skewed and it is known for being insensitive to extreme values.

We expect that this proposed confidence interval in (11) would not be sensitive to extreme values due to the existence of the sample median (med) and would be robust with coverage probability approximately equal to the aimed confidence probability $(1-\alpha)$ and with appropriate simulated average width (CW) due to the adjustments to the percentile point of the chi-square distribution and the multiplier (3/2) in (10).

4. The Simulation Study

In this section, we are interested in finding results for comparing and studying the behaviors of the proposed robust and the classical confidence intervals for the median of the exponential distribution with and without extreme values, and investigating how the presence of the extreme values affects them using a large-scale simulation study with the help of MATLAB, the language of computing version 6.5 [26]. The most direct common technique to generate random numbers from the exponential probability density function (1) is to solve for x in the relationship u=F(x) such that $x = F^{-1}(u) = -\theta \ln(1-u)$ where u is a uniform random number between zero and one and F(x) is the distribution function, and then used them to get random samples of sizes n=10, 20, 30, 40, and 50 from the exponential distribution with and without extreme values. This is simply obtained by generating Pseudo-random uniform numbers from the function RAND of MATLAB to yield the corresponding values for the variable X using the inverse function using MATLAB, version 6.5.

The simulation is based on the following two situations:

(1) 100,000 replicates have been done to generate 100,000 samples for each sample size of n=10, 20, 30, 40, and 50 from the exponential distribution (1) without extreme values by using the probability transform $X = -\theta \ln(1-U)$

(2) For each combination of (n, δ), 100,000 trials have been done to obtain 100,000 samples for n=10, 20, 30, 40, and 50 and δ = 5% and 10% of the observations being extreme points and $1-\delta$ = 95% and 90% of the observations being none extreme points from the mixed exponential population using the transformation

$$Z = \begin{cases} X, if\, U \geq \delta \\ \ldots \\ a + bX, if\, U < \delta \text{ and } a,b > 0 \end{cases} \quad (12)$$

where Z is a contaminated exponential variable and X is an exponential variable.

Note that situation (2) is the same as generating random numbers from mixed exponential distribution given by (13)

$$g(x) = (1-\delta) f(x;\theta) + \delta f(x-a; b\theta) \quad (13)$$

where $0 \leq \delta \leq 1$.

For each combination (n, δ) the simulation results are based on 100,000 replicates, $a = b = 1000$, $\theta = 1$, and $\delta = 5\%\, and\, 10\%$ to find 100,000 95% and 99% confidence intervals of each method. These confidence limits are then used to estimate the following performance criteria:

(a) The simulated confidence probability

$$CP = \#(L_i(X) \leq \ln 2 \leq U_i(X))/100{,}000 \quad (14)$$

(b) The simulated confidence width (CW)

$$CW = \frac{\sum_{i=1}^{100{,}000}(U_i(X) - L_i(X))}{100{,}000} \quad (15)$$

And (c) The standard error of (CW), SE is

$$SE = \frac{\sum_{i=1}^{100,000}(U_i(X) - L_i(X) - CW)^2}{100,000} \quad (16)$$

where Li(X) and Ui(X) are respectively the lower and the upper confidence limits of the ith confidence interval for i = 1, 2… 100,000. The performance of the proposed robust method given in (11) is evaluated and compared with the ordinary confidence interval given in (5) by using the last performance criteria (14-16).

5. Results and Discussion

This research aims to construct an accurate and stable interval achieving coverage probability near the nominal one and informative having a short length even if the data contains some extreme values. I was surprised by the results of the simulation study. These results will benefit all applied researchers in the fields of studying reliability and variables of survival analysis without bothering to search for outliers and extreme points to remove them. The following results will show researchers the accuracy and importance of the proposed method in the presence or absence of extreme values.

The simulation results for all cases are presented in Tables 1-5 and Figures 1-4. Table 1 and Figures 1-4 show the simulated results for the performance criteria when sampling from an exponential distribution without extreme values. The simulated results for the coverage probability, average width, and the standard error with the two proportions (5% and 10%) of the extreme values, are given in Tables 2 and 3 and presented in Figures 1-4. For ease of comparison, the estimated coverage probabilities for the classical and the proposed methods under all types of data are given in Tables 4 and 5.

The results of Tables 1, 4, and 5 and Figures 1-4 show that the values of the simulated confidence probability (CP) of the proposed method are close to but a little bit shorter than the aimed confidence probability and the simulated confidence probability of the ordinary confidence interval. The values of the performance criterion (CW) of the proposed method are a little bit larger than the simulated confidence width (CW) of the ordinary confidence interval when the data are from exponential distribution without extreme values.

The results of Tables 2 to 5 and Figures 1 to 4 showed that, when the data are from the exponential distribution with some extreme values, the proposed confidence interval is more resistant to the extreme values than the ordinary confidence interval, and the extreme values are greatly changed the simulated confidence probabilities and increased the simulated confidence widths of the ordinary confidence interval. Lastly, when the data are from an exponential distribution with some extreme values, we suggest using the proposed robust confidence interval while the ordinary confidence interval should not be used at all. On the other hand, when the data are from the exponential distribution without extreme values, the ordinary confidence interval method followed by the proposed robust confidence interval should be used.

Table 1. The simulation results of the performance criteria under exponential distribution

		Approaches					
		Ordinary method (Classical)			Proposed method (Robust)		
Confidence probability		Performance Criteria			Performance Criteria		
$1-\alpha$	Sample size (n)	CP	CW	SE	CP	CW	SE
0.95	10	0.9518	1.0403	0.3281	0.9436	1.3155	0.5457
	20	0.9511	0.6677	0.1485	0.9415	0.8876	0.2721
	30	0.9515	0.5280	0.0957	0.9428	0.7376	0.1881
	40	0.9500	0.4502	0.0711	0.9442	0.6604	0.1466
	50	0.9504	0.3989	0.0563	0.9446	0.6087	0.1217
0.99	10	0.9902	1.5191	0.4790	0.9802	1.8291	0.7587
	20	0.9900	0.9239	0.2055	0.9792	1.2196	0.3739
	30	0.9907	0.7181	0.1302	0.9781	0.9253	0.2354
	40	0.9899	0.6070	0.0958	0.9788	0.8111	0.1800
	50	0.9902	0.5351	0.0755	0.9787	0.7381	0.1476

Table 2. The simulated results of the performance criteria under the 5% contaminated Exponential distribution

		Approaches					
		Ordinary method			Proposed method (Robust)		
Confidence probability		Performance Criteria			Performance Criteria		
$1-\alpha$	Sample size (n)	CP	CW	SE	CP	CW	SE
0.95	10	0.5767	259.3809	365.4920	0.9430	1.6886	37.5103
	20	0.3356	166.7980	165.2574	0.9415	0.8876	0.2721
	30	0.1933	132.1016	106.6085	0.9428	0.7396	0.1881
	40	0.1128	112.6150	79.1616	0.9442	0.6604	0.1466
	50	0.0647	99.8102	62.8191	0.9446	0.6087	0.1217
0.99	10	0.5933	379.9405	532.0420	0.9802	2.1331	45.6941
	20	0.3536	230.8008	228.6690	0.9786	1.1450	0.3511
	30	0.2089	179.6469	144.9785	0.9781	0.9253	0.2354
	40	0.1250	151.8221	106.7218	0.9788	0.8111	0.1800
	50	0.0738	133.8730	84.2577	0.9787	0.7381	0.1476

Table 3. The simulated results of the performance criteria under the 10% contaminated exponential distribution

		Approaches					
		Ordinary method			Proposed method (Robust)		
Confidence probability		Performance Criteria			Performance Criteria		
$1-\alpha$	Sample size (n)	CP	CW	SE	CP	CW	SE
0.95	10	0.3245	448.2952	437.0894	0.9434	7.1830	150.6811
	20	0.1033	287.6284	197.5751	0.9415	0.8876	0.2721
	30	0.0305	227.4410	127.2535	0.9428	0.7397	0.1881
	40	0.0091	193.9005	94.4074	0.9442	0.6604	0.1466
	50	0.0028	171.8147	74.8391	0.9446	0.6087	0.1217
0.99	10	0.3434	654.6109	638.2479	0.9798	09.9878	209.5195
	20	0.1171	397.9956	273.3875	0.9786	1.1450	0.3511
	30	0.0384	309.3046	173.0539	0.9781	0.9253	0.2354
	40	0.0128	261.4073	127.2755	0.9788	0.8111	0.1800
	50	0.0042	230.4508	100.3798	0.9787	0.7381	0.1476

Table 4. The simulated confidence probability for all approaches and all types of data with a confidence probability of $1-\alpha=0.95$

		Sample Size				
Types of data	Approach	10	20	30	40	50
Exponential	Ordinary	0.9518	0.9511	0.9515	0.9500	0.9504
	Proposed	0.9436	0.9415	0.9428	09442	0.9446
Contaminated exponential with $\delta=5\%$	Ordinary	0.5739	0.3356	0.1933	0.1128	0.0647
	Proposed	0.9434	0.9415	0.9428	0.9442	0.9446
Contaminated exponential with $\delta=10\%$	Ordinary	0.5767	0.3356	0.1933	0.1128	0.0647
	Proposed	0.9434	0.9415	0.9428	0.9442	0.9446

Table 5. The simulated confidence probability for all approaches and all types of data with a confidence probability of $1-\alpha=99\%$

		Sample Size				
Types of data	Approach	10	20	30	40	50
Exponential	Ordinary	0.9902	0.9900	0.9907	0.9899	0.9902
	Proposed	0.9802	0.9786	0.9781	09788	0.9787
Contaminated exponential with $\delta=5\%$	Ordinary	0.5933	0.3536	0.2089	0.1250	0.0738
	Proposed	0.9802	0.9786	0.9781	0.9788	0.9787
Contaminated exponential with $\delta=10\%$	Ordinary	0.3434	0.1171	0.0384	0.0128	0.0042
	Proposed	0.9798	0.9786	0.9781	0.9788	0.9787

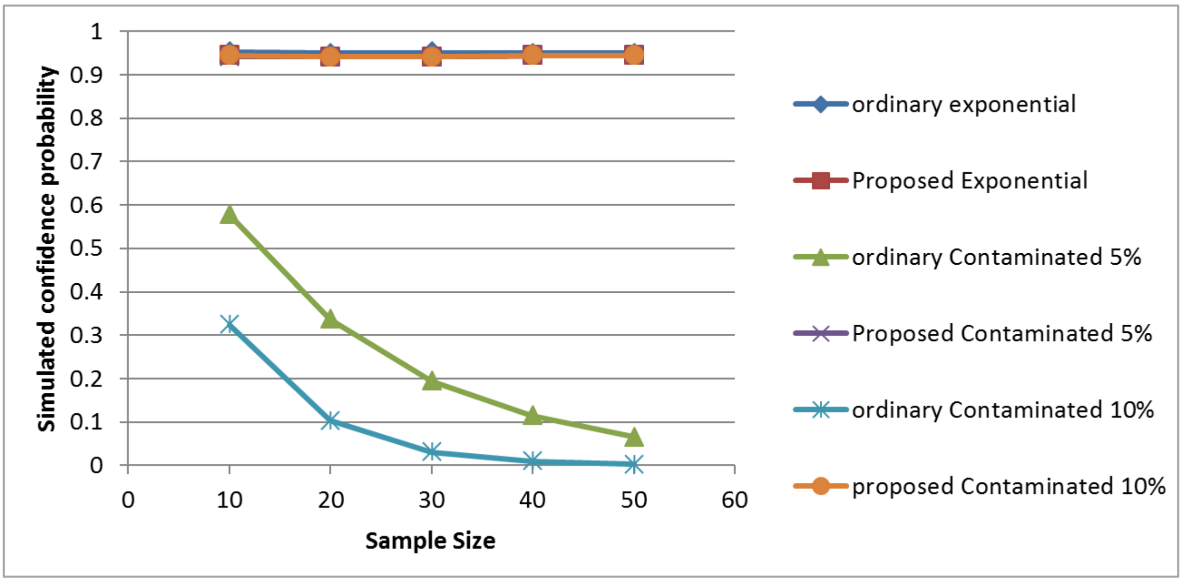

Figure 1. The simulated confidence probabilities of the proposed and ordinary confidence Intervals for all types of data sets with a confidence level of 95%

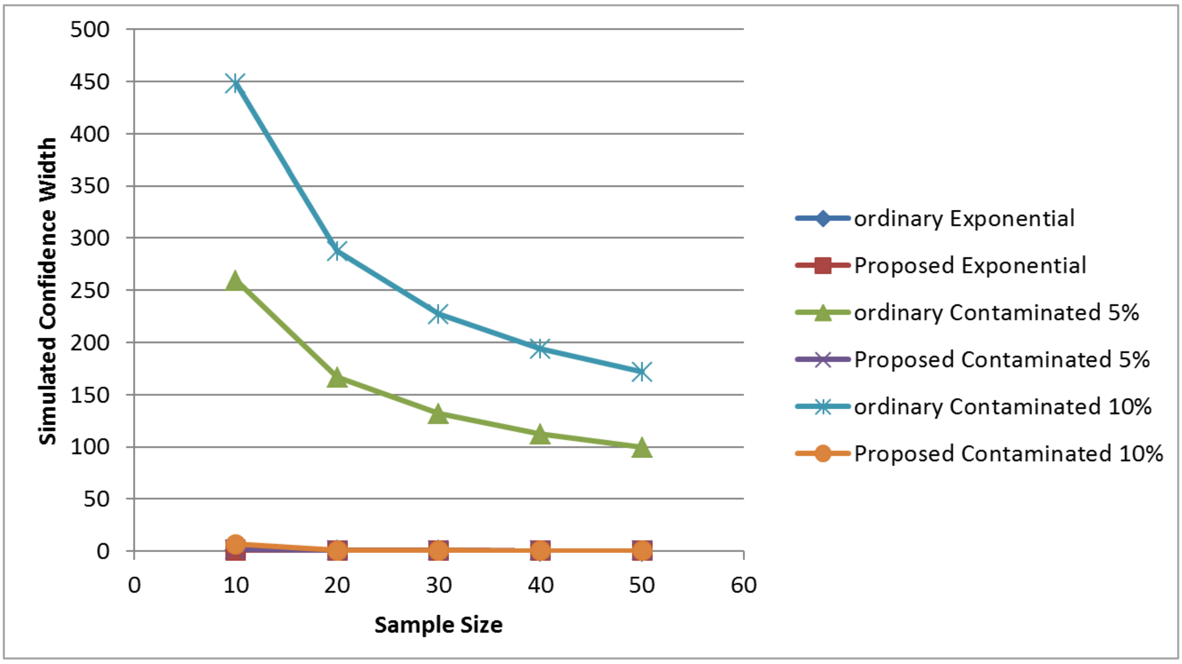

Figure 2. The simulated confidence widths of the proposed and ordinary confidence Intervals for all types of data sets with a confidence level of 95%

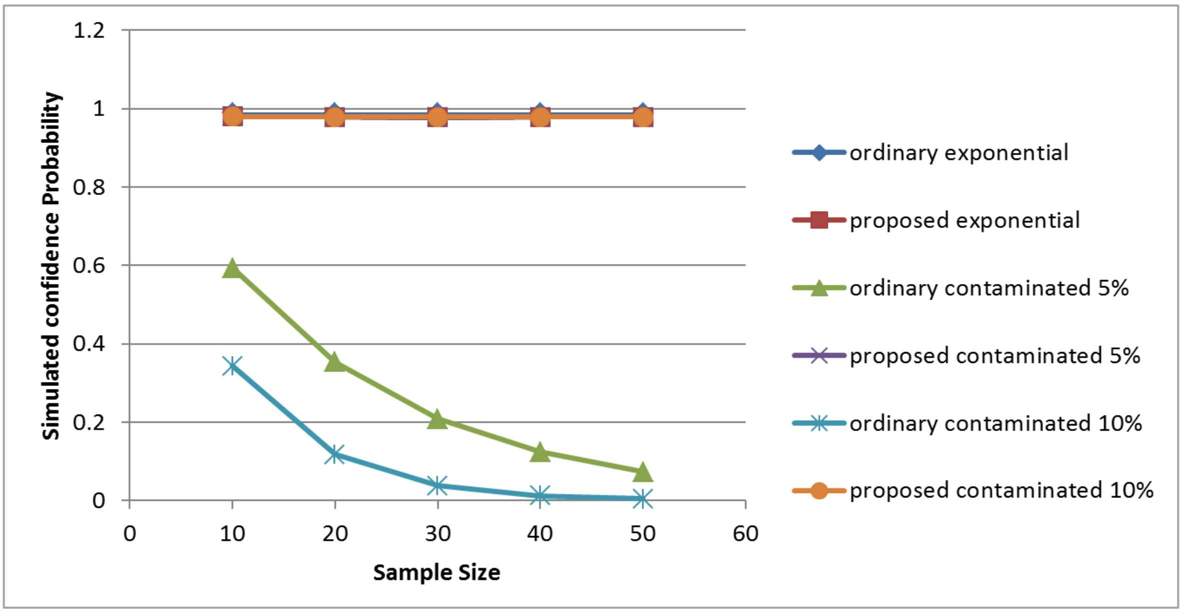

Figure 3. The simulated confidence probabilities of the proposed and the ordinary Confidence intervals for all types of data sets with a confidence level of 99%

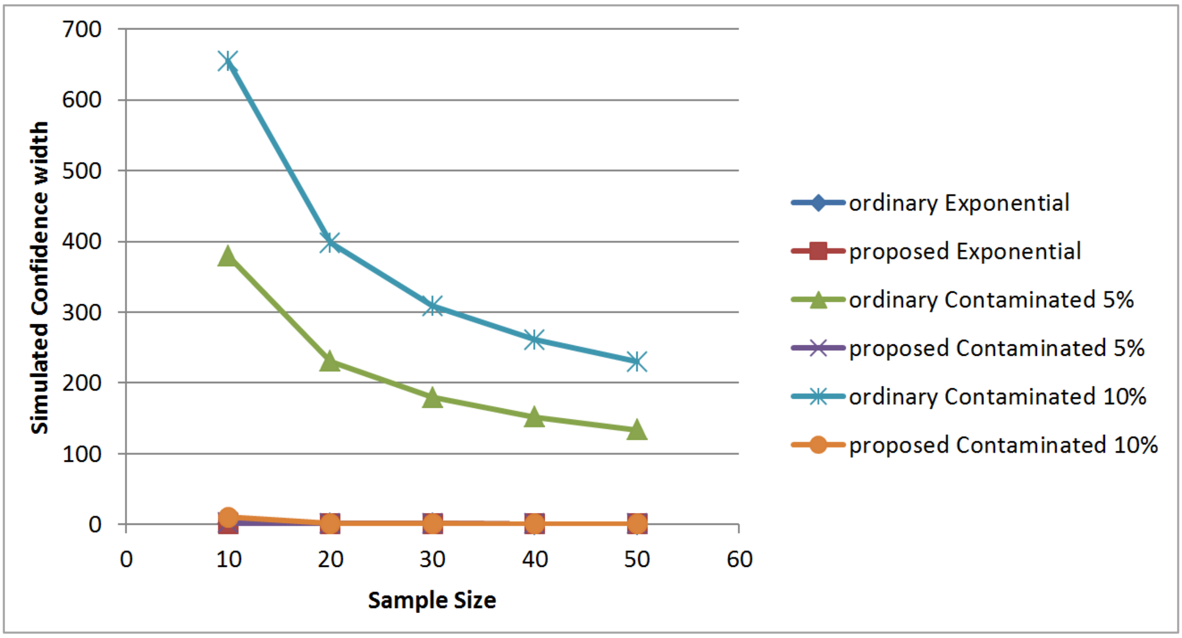

Figure 4. The simulated confidence widths of the proposed and ordinary confidence intervals for all types of data sets with a confidence level of 99%

6. Real-Life Examples

6.1. Example 1: from real data

The following data set was collected by Wilk et al. [28] in 1962 consisting of 34 survival times of transistors (in weeks)

3, 4, 5, 6, 6, 7, 8, 8, 9, 9, 9
10, 10, 11, 11, 11, 13, 13, 13, 13, 13, 17
17, 19, 19, 25, 29, 33, 42, 42, 52, 52, 52, 52

Kibria [29] in 2006 Showed that the data followed an exponential distribution of mean $\theta = 21$ weeks. By equation 2, it is easy to show that the value of the exponential median is $X_{0.5} = 14.556$ weeks. The sample mean and the sample median of this data set are respectively $\bar{X}$=18.912 weeks and med=13 weeks. Based on these data sets the 95% confidence intervals and their widths are calculated using the **ordinary** and the proposed methods (5) and (11) given in Table 6 below.

Table 6. The 95% interval estimates for all approaches and their widths for the lifetime data without extreme values

Approach	Interval estimate	Confidence width
Ordinary	(9.617, 18.929)	9.312
Proposed	(0.099, 19.517)	19.418

From the above table, we observe that both types of interval estimates include the population median 14.556 weeks, and the confidence width of the ordinary confidence interval as expected is shorter than the width of the proposed confidence interval, a result which supports the simulation study to some extent.

Let us now suppose that the two of these lifetimes 42 and 52 have been wrongly listed so that the data have two extreme values 420 and 520 as follows:

3, 4, 5, 6, 6, 7, 8, 8, 9, 9, 9,
10, 10, 11, 11, 11, 13, 13, 13, 13, 13, 17
17, 19, 19, 25, 29, 33, 420, 42, 520, 52, 52, 52

The sample mean and the sample median of this data set are respectively 43.794 and 13. From this, we notice that the values of the extreme points have changed the value of the mean. The robustness of the sample median is clear where its values are the same for the two data sets.

The results of 95% interval estimates and the corresponding confidence widths of the ordinary confidence interval and the proposed confidence interval for the population median under the data set with extreme values are calculated and given in the following Table 7.

Table 7. The 95% interval estimations for the lifetime data with extreme values

Approach	Interval estimate	Confidence width
Ordinary	(22.270, 43.833)	21.563
Proposed	(0.099, 19.517)	19.418

This table showed that the ordinary interval estimate of the median does not include the population median $X_{0.5} = 14.556$ weeks but otherwise the proposed interval estimate captured it. However, the proposed confidence interval for the median of the exponential population provided a shorter confidence interval width. Hence the proposed method performed well and therefore the ordinary method should be avoided in the presence of extreme values. These results are consistent with those of the simulation study.

6.2. Example 2: from simulated data

The following data of size 20 are generated by the author from an exponential distribution of mean 1 and median ln2=0.69315 without extreme values by using the Minitab program given as follows:

0.558450, 0.891250, 1.968860, 1.050725, 0.899730, 0.347540, 0.692271, 0.319676
0.253431, 0.602069, 1.290112, 3.211899, 4.728852, 0.450172, 1.056224, 1.691052
0.709048, 0.677309, 1.221596, 0.081867

The sample mean and the sample median are respectively $\bar{X}$= 1.135 and med= 0.800. The interval estimates and the corresponding widths of the median are calculated by using the ordinary and the proposed methods under the exponential data without extreme values are given in the following Table 8.

Table 8. The 95% interval estimates with their widths of the median for the simulated exponential data without extreme values

Approach	Interval estimate	Confidence width
Ordinary	(0.67, 1.288)	0.618
Proposed	(0.472, 1.362)	0.89

This table shows that both confidence intervals include the population median $X_{0.5} = 0.69315$, and the confidence width of the ordinary confidence interval as expected is shorter than the proposed confidence interval. These results are consistent with the simulation study.

Let us now suppose that two values of this data set 1.050725 and 1.056224 have been wrongly listed as 10.507250 and 10.562240 so that the data have two extreme values as follows:

0.55845, 0.891250, 1.968860, 10.507250, 0.899730, 0.347540, 0.69227, 0.319676
0.253431, 0.602069, 1.290112, 3.211899, 4.728852, 0.450172, 10.562240, 1.691052
0.709048, 0.677309, 1.221596, 0.081867.

Based on this data set, the sample mean and the sample median are respectively 2.083 and 0.800. From this, we notice that the values of the extreme points have changed the value of $\overline{X}$. The robustness of the sample median is clear where its values are the same for the two data sets.

Table 9. The 95% interval estimates of the median for the simulated exponential data with extreme values

Approach	Interval estimate	Confidence width
Ordinary	(1.229, 2.364)	1.135
Proposed	(0.472, 1.362)	0.89

The interval estimates and the corresponding confidence widths for the ordinary confidence interval and the proposed confidence interval for the median of the exponential population under the data set with extreme values are calculated and given in the Table 9.

This table showed that the ordinary interval estimate of the median does not include the population median $X_{0.5} = 0.69315$ but otherwise the proposed interval estimate captured it. However, the proposed confidence interval for the exponential median provided a shorter confidence interval width. Hence the proposed method performed well and therefore the ordinary method should be avoided in the presence of extreme values. These results are consistent with those of the simulation study.

7. Conclusions

The proposed method when the data contain a proportion of extreme values is quite important to reliability and lifetime studies. There are some research papers in a literature review that suggested the analogous idea, but these papers used different types of robust strategic approaches which do not use the same idea as this research paper. Hence the proposed confidence interval of this research paper gives more accurate results in the sense of having very close simulated confidence probability and simulated confidence width to the aimed values. This research paper suggested a new robust confidence interval to estimate the median when a sample from an exponential density contains a proportion of extreme values. The performance criteria (CP and CW) of the proposed robust confidence interval are evaluated and compared with those of the ordinary confidence interval and indicated, through the simulation results and presentation of some figures, that when the data are from an exponential distribution with some extreme values, we suggest using the proposed robust confidence interval while the ordinary confidence interval should not be used at all. On the other hand, when the data are from the exponential distribution without extreme values, the ordinary confidence interval method followed by the proposed robust method should be used.

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