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Other New Versions of Generalized Neutrosophic Connectedness and Compactness and Their Applications

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Abstract The concepts of neutrosophic connectedness and compactness between neutrosophic sets find extensive applications in various fields, including sensor networks, physics, mechanical engineering, robotics and data analysis involving numerous variables. Neutrosophic set theory also plays a pivotal role in addressing complex problems in engineering, environment science, economics, and advanced mathematical disciplines. Hence, this paper aims to extend the classical definitions of neutrosophic connectedness and compactness within neutrosophic topological spaces. We introduce new classes of neutrosophic connectedness and compactness, specifically, neutrosophic δ-ß-connectedness and neutrosophic δ-ß-compactness, defined using a generalized neutrosophic open set known as neutrosophic δ-ß-open sets . We explore several essential properties and characterizations of these spaces and introduce new notions of neutrosophic covers, which lead to the concept of neutrosophic compact spaces. Additionally, we present characterizations related to neutrosophic δ-ß-separated sets. A noteworthy feature of these concepts is their ability to model intricate connectedness networks and facilitate optimal solutions for problems involving a multitude of variables, each with degrees of acceptance, rejection, and indeterminacy. We provide relevant examples to illustrate our main findings.

Keywords Neutrosophic δ-ß-Open Sets, Neutrosophic δ-ß-Connectedness, Neutrosophic δ-ß-Compactness, Neutrosophic δ-ß-Separated-Sets

1. Introduction

The modern notions of neutrosophy and neutrosophic sets have paved the way for a diverse range of mathematical theories that extend both their classical interpretations and their counterparts in fuzzy set theory. While the study of fuzzy sets and fuzzy logic, initiated by L. Zadeh [1], explored their logic and foundational aspects, their topological structures were investigated by [2]. Building on this foundation, K. Atanassov [3] introduced the concept of intuitionistic fuzzy sets, and their intuitionistic fuzzy topological structures were further developed by Coker [4]. Neutrosophy and neutrosophic set theory were suggested via F. Smarandache in [5], and in 2012, A. A. Salama and S. A. Alblowi [6] presented the concept of neutrosophic topological spaces, building upon the framework of neutrosophic sets. Concurrently, in a general topology the δ-ß-open set concept and δ-ß-continuity were introduced by E. Hatir and T. Noiri [7]. Subsequent research by Al-Jumaili et al. [8] explored new types of maps with strongly closed graphs, while S. H. Abdulwahid and AL. Jumaili [9] proposed novel ideas of generalized cont-maps using a new generalized open set. Al. Sharqi et al. [10, 11] applied the concept of neutrosophic sets with complex values to explain real-life applications. Further contributions by A. Vadivel et al. [12] extended the idea of δ-open sets in neutrosophic topological spaces and introduced another class of generalized neutrosophic open sets, namely, neutrosophic δ-ß-open sets, along with a study of their essential properties. Al-Jumaili [13] applied the concept of neutrosophic δ-ß-open sets and proposed a novel idea of generalized neutrosophic continuous maps. Recent research by T. Ozturk et al. [14] defined various types of covers on neutrosophic soft sets and presented the concept of compactness in neutrosophic topological spaces. Subsequently, M. Arar [15] investigated countable compactness, and M. Parimala et al. [16] introduced the notion of neutrosophic $\alpha\psi$- connectedness, discussing its properties. Finally, A. Acikgoze and F. Esenbel [17] explored neutrosophic connectedness, including the notions of neutrosophic super-connected and neutrosophic strongly connected spaces, and studied their properties.

The structure of our present paper is as follows: In the first section, we provide a brief historical overview. In the second section, we introduce fundamental definitions and key results within the framework of neutrosophic topological spaces. The third section explores a new class of neutrosophic connectedness. The fourth section presents several characterizations related to neutrosophic δ-ß-separated sets. Lastly, the fifth section delves into essential characterizations concerning δ-ß-compactness.

2. Materials and Methods

In this part, recall the following required conclusions of generalized neutrosophic soft sets which play very important role throughout this manuscript.

Definition 2.1: [6] Let $\mathbb{Q} \neq \emptyset$. A neutrosophic set (concisely, $\mathcal{N}_S S$) $\mathcal{K}$ is an object having the model $\mathcal{K} = \{\langle w, \Gamma_{\mathcal{K}}(w), \lambda_{\mathcal{K}}(w), q_{\mathcal{K}}(w)\rangle : w \in \mathbb{Q}\}$ wherever $\Gamma_{\mathcal{K}} \longrightarrow [0,1]$ refer to grade of member-ship map, $\lambda_{\mathcal{K}} \longrightarrow [0,1]$ refer to grade of indeterminacy and $q_{\mathcal{K}} \longrightarrow [0,1]$ refer to grade of nonmembership map respectively, $\forall\ w \in \mathbb{Q}$ to the set $\mathcal{K}$ with $0 \leq \Gamma_{\mathcal{K}}(w) + \lambda_{\mathcal{K}}(w) + q_{\mathcal{K}}(w) \leq 3\ \forall\ w \in \mathbb{Q}$.

Remark 2.2: [6] A neutrosophic subset $\mathcal{K} = \{\langle w, \Gamma_{\mathcal{K}}(w), \lambda_{\mathcal{K}}(w), q_{\mathcal{K}}(w)\rangle : w \in \mathbb{Q}\}$ can be specified to ordered triple $\Gamma_{\mathcal{K}}(w), \lambda_{\mathcal{K}}(w), q_{\mathcal{K}}(w)$ in [0,1] on $\mathbb{Q}$.

Definition 2.3: [6] Let $\mathbb{Q} \neq \emptyset$ with the $\mathcal{N}_S S\ S, \mathcal{K}$ and $\mathcal{W}$ in the shape $\mathcal{K} = \{\langle w, \Gamma_{\mathcal{K}}(w), \lambda_{\mathcal{K}}(w), q_{\mathcal{K}}(w)\rangle : w \in \mathbb{Q}\}$, $\mathcal{W} = \{\langle w, \Gamma_{\mathcal{W}}(w), \lambda_{\mathcal{W}}(w), q_{\mathcal{W}}(w)\rangle : w \in \mathbb{Q}\}$, then

- **(a)** $0_{\mathcal{N}} = \langle w, 0,0,1\rangle$ and $1_{\mathcal{N}} = \langle w, 1,1,0\rangle$;
- **(b)** $\mathcal{K} \subseteq \mathcal{W} \Leftrightarrow \Gamma_{\mathcal{K}}(w) \leq \Gamma_{\mathcal{W}}(w), \lambda_{\mathcal{K}}(w) \leq \lambda_{\mathcal{W}}(w)$ and $q_{\mathcal{K}}(w) \geq q_{\mathcal{W}}(w) : w \in \mathbb{Q}$;
- **(c)** $\mathcal{K} = \mathcal{W} \Leftrightarrow \mathcal{K} \subseteq \mathcal{W}$ and $\mathcal{W} \subseteq \mathcal{K}$;
- **(d)** $1_{\mathcal{N}} - \mathcal{K} = \{\langle w, q_{\mathcal{K}}(w), 1 - \lambda_{\mathcal{K}}(w), \Gamma_{\mathcal{K}}(w)\rangle : w \in \mathbb{Q}\} = \mathcal{K}^c$;
- **(e)** $\mathcal{K} \cap \mathcal{W} = \{w, min(\Gamma_{\mathcal{K}}(w), \Gamma_{\mathcal{W}}(w)), min(\lambda_{\mathcal{K}}(w), \lambda_{\mathcal{W}}(w)), max(q_{\mathcal{K}}(w), q_{\mathcal{W}}(w)) : w \in \mathbb{Q}\}$;
- **(f)** $\mathcal{K} \cup \mathcal{W} = \{w, max(\Gamma_{\mathcal{K}}(w), \Gamma_{\mathcal{W}}(w)), max(\lambda_{\mathcal{K}}(w), \lambda_{\mathcal{W}}(w)), min\ (q_{\mathcal{K}}(w), q_{\mathcal{W}}(w)) : w \in \mathbb{Q}\}$.

Definition 2.4: [6] A neutrosophic Topology (shortly, $\mathcal{N}_S\mathcal{T}$) on $\mathbb{Q} \neq \emptyset$ is a family $\Phi_{\mathcal{N}}$ of neutrosophic sub-sets of $\mathbb{Q}$ satisfying:

- **(i)** $0_{\mathcal{N}}, 1_{\mathcal{N}} \in \Phi_{\mathcal{N}}$;
- **(ii)** $\mathcal{K}_1 \cap \mathcal{K}_2 \in \Phi_{\mathcal{N}}, \forall\ \mathcal{K}_1, \mathcal{K}_2 \in \Phi_{\mathcal{N}}$;
- **(iii)** $\cup \mathcal{K}_x \in \Phi_{\mathcal{N}}, \forall\ \{\mathcal{K}_x : x \in \Delta\} \subseteq \Phi_{\mathcal{N}}$.

Then, $(\mathbb{Q}, \Phi_{\mathcal{N}})$ called a neutrosiphic Topological space (shortly, $\mathcal{N}_S\mathcal{TS}$) in $\mathbb{Q}$.

The $\Phi_{\mathcal{N}}$ elements are described as neutrosophic open set (shortly, $\mathcal{N}_S\mathcal{OS}$) in $\mathbb{Q}$.A $\mathcal{N}_S S\ \mathcal{D}$ is called a neutrosophic closed (shortly, $\mathcal{N}_S\mathcal{CS}$)if and only if $\mathcal{D}^c$ is $\mathcal{N}_S\mathcal{OS}$.

Definition 2.5: Let $(\mathbb{Q}, \Phi_{\mathcal{N}})$ be $\mathcal{N}_S\mathcal{TS}$ on $\mathbb{Q}$ and $\mathcal{F}$ be an $\mathcal{N}_S S$ on $\mathbb{Q}$. So $\mathcal{F}$ is called:

- **(i)** A $\mathcal{N}_S$-regular open set (concisely, $\mathcal{N}_S\mathcal{ROS}$) [18] if $\mathcal{F} = \mathcal{N}_S Int(\mathcal{N}_S Cl(\mathcal{F}))$.
- **(ii)** Neutrosophic δ-closure of $\mathcal{F}$ [19] (concisely, $\mathcal{N}_S\delta Cl(\mathcal{F})$) is defined by $\mathcal{N}_S\delta Cl(\mathcal{F}) = \cap\{\mathcal{A} : \mathcal{F} \subseteq \mathcal{A}\ \&A\ is\ \mathcal{N}_S\mathcal{RCS}$ in $\mathbb{Q}\}$.
- **(iii)** Neutrosophic δ-interior of $\mathcal{F}$ [19] (concisely, $\mathcal{N}_S\delta Int(\mathcal{F})$) is defined by $\mathcal{N}_S\delta Int(\mathcal{F}) = \cup\{\mathcal{B} : \mathcal{B} \subseteq \mathcal{F}\ \&B\ is\ \mathcal{N}_S\mathcal{ROS}$ in $\mathbb{Q}\}$.

Definition 2.6: A subset $\mathcal{F}$ is said to be:

- **(a)** $\mathcal{N}_S$ Pre-open (shortly, $\mathcal{N}_S\mathcal{POS}$) [20] if $\mathcal{F} \subseteq \mathcal{N}_S Int(\mathcal{N}_S Cl(\mathcal{F}))$
- **(b)** $\mathcal{N}_S$ Semi-open (shortly, $\mathcal{N}_S\mathcal{SOS}$) [20] if $\mathcal{F} \subseteq \mathcal{N}_S Cl(\mathcal{N}_S Int(\mathcal{F}))$
- **(c)** $\mathcal{N}_S$ α-open (shortly, $\mathcal{N}_S\alpha\mathcal{OS}$) [20] if $\mathcal{F} \subseteq \mathcal{N}_S Int(\mathcal{N}_S Cl(\mathcal{N}_S Int(\mathcal{F})))$
- **(d)** $\mathcal{N}_S$ δ-open(shortly, $\mathcal{N}_S\delta\mathcal{OS}$) [19] if $\mathcal{F} = \mathcal{N}_S\delta Int(\mathcal{F})$;
- **(e)** $\mathcal{N}_S$ δ-Pre open (shortly, $\mathcal{N}_S\delta\mathcal{POS}$) [19] if $\mathcal{F} \subseteq \mathcal{N}_S Int(\mathcal{N}_S\delta Cl(\mathcal{F}))$;
- **(f)** $\mathcal{N}_S$ δ-Semi open (shortly, $\mathcal{N}_S\delta\mathcal{SOS}$) [19] if $\mathcal{F} \subseteq \mathcal{N}_S Cl(\mathcal{N}_S\delta Int(\mathcal{F}))$;
- **(g)** $\mathcal{N}_S E$-open (shortly, $\mathcal{N}_S\mathcal{EOS}$)[12] if $\mathcal{F} \subseteq \mathcal{N}_S Cl(\mathcal{N}_S\delta Int(\mathcal{F})) \cup \mathcal{N}_S Int(\mathcal{N}_S\delta Cl(\mathcal{F}))$;
- **(h)** $\mathcal{N}_S$ δ-ß-open (shortly, $\mathcal{N}_S\delta -$ ß$\mathcal{OS}$) [19] if $\mathcal{F} \subseteq \mathcal{N}_S Cl\left(\mathcal{N}_S Int(\mathcal{N}_S\delta Cl(\mathcal{F}))\right)$.
- **(i)** $\mathcal{N}_S$ δ-ß-interior of $\mathcal{F}$ [19] (shortly, $\mathcal{N}_S\delta -$ ß$Int(\mathcal{F})$) is defined via $\mathcal{N}_S\delta -$ ß$Int(\mathcal{F}) = \cup\{\mathcal{B} : \mathcal{B} \subseteq \mathcal{F}\ \&\ \mathcal{B}$ is a $\mathcal{N}_S\delta -$ ßOS in $\mathbb{Q}\}$.

$\mathcal{N}_S$ δ-ß-closure of $\mathcal{F}$ [19] (shortly, $\mathcal{N}_S\delta -$ ß$Cl(\mathcal{F})$) is defined via $\mathcal{N}_S\delta -$ ß$Cl(\mathcal{F}) = \cap\{\mathcal{A} : \mathcal{F} \subseteq \mathcal{A}\ \&\ \mathcal{A}$ is a $\mathcal{N}_S\delta -$ ß$\mathcal{CS}$ in $\mathbb{Q}\}$.

Remark2.7: The complements of $\mathcal{N}_S\mathcal{POS}$:respectively. $\mathcal{N}_S\mathcal{SOS}, \mathcal{N}_S\alpha\mathcal{OS}, \mathcal{N}_S\delta\mathcal{OS}, \mathcal{N}_S\delta\mathcal{POS}, \mathcal{N}_S\delta\mathcal{SOS}, \mathcal{N}_S\mathcal{EOS}$, $\mathcal{N}_S\delta -$ ß$\mathcal{OS}$) is neutrosophic closed sets, with indicated via $\mathcal{N}_S\mathcal{PCS}$ (respectively , $\mathcal{N}_S\mathcal{SCS}, \mathcal{N}_S\alpha\mathcal{CS}, \mathcal{N}_S\delta\mathcal{CS}, \mathcal{N}_S\delta\mathcal{PCS}, \mathcal{N}_S\delta\mathcal{SCS}, \mathcal{N}_S\mathcal{ECS}, \mathcal{N}_S\delta -$ ß$\mathcal{CS}$) in $\mathbb{Q}$.

Remark 2.8: The diagram 1 describes relations among various generalized neutrosophic open sets. None of these implications is reversible as illustrated by examples [21].

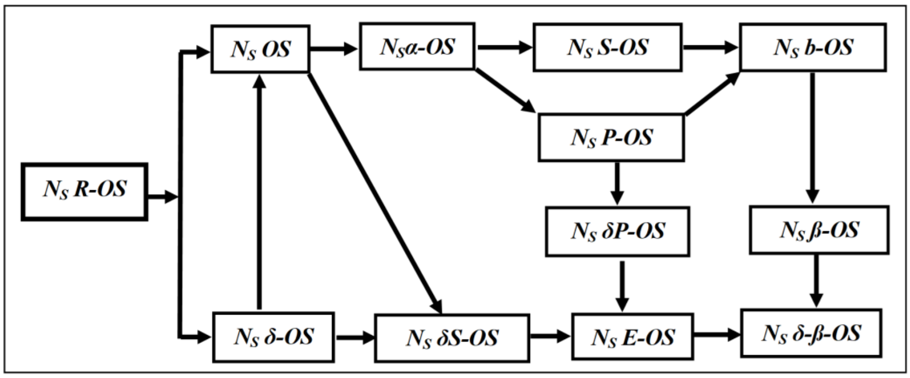

Diagram (1). The relationships among several generalized neutrosophic open sets

Definition 2.9: [19] Let $(\mathbb{Q}, \Phi_{\mathcal{N}})$ & $(\mathcal{Y}, \Psi_{\mathcal{N}})$ be any two $\mathcal{N}_{\mathcal{S}}$ $\mathcal{TS}$. A map $\mathfrak{F}: (\mathbb{Q}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ is defined as:

(a) $\mathcal{N}_{\mathcal{S}}$ δ-ß-open (concisely, $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{O}$) map if the image of each $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ of $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{OS}$ in$(\mathcal{Y}, \Psi_{\mathcal{N}})$.

(b) $\mathcal{N}_{\mathcal{S}}$ δ-ß-continuous(concisely, $\mathcal{N}_{\mathcal{S}}$ δ-ß-cont) map if $\mathfrak{F}^{-1}(\mathcal{K})$ is $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{OS}$ in $(\mathbb{Q}, \Phi_{\mathcal{N}})$ foe each $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ $\mathcal{K}$ of $(\mathcal{Y}, \Psi_{\mathcal{N}})$.

(c) $\mathcal{N}_{\mathcal{S}}$ δ-ß-irresolute (concisely, $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{I}rr$) map if $\mathfrak{F}^{-1}(\mathcal{K})$ is $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{OS}$ in $(\mathbb{Q}, \Phi_{\mathcal{N}})$, for all $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{OS}$ $\mathcal{K}$ of $(\mathcal{Y}, \Psi_{\mathcal{N}})$.

3. Various Properties of Neutrosophic δ-ß-Connectedness

This section is devoted to discussing various fundamental properties related to neutrosophic δ-ß-connectedness in neutrosophic topological spaces.

Definition 3.1: A $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$ $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is called a $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$ δ-ß-disconnected (concisely, $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}$ dconn) space if there exist $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{OS}$'s $\mathcal{F}, \mathcal{H}$ in $\mathbb{Q}$ & $\mathcal{F} \neq 0_{\mathcal{N}}, \mathcal{H} \neq 0_{\mathcal{N}}$ (s. t) $\mathcal{F} \cup \mathcal{H} = 1_{\mathcal{N}}$ and $\mathcal{F} \cap \mathcal{H} = 0_{\mathcal{N}}$. That is,

(a) $\Gamma_{\mathcal{F}}(w) \vee \Gamma_{\mathcal{H}}(w) = 1_{\mathcal{N}},\ \lambda_{\mathcal{F}}(w) \wedge \lambda_{\mathcal{H}}(w) = 1_{\mathcal{N}},\ q_{\mathcal{F}}(w) \wedge q_{\mathcal{H}}(w) = 1_{\mathcal{N}}$.

(b) $\Gamma_{\mathcal{F}}(w) \vee \Gamma_{\mathcal{H}}(w) = 1_{\mathcal{N}},\ \lambda_{\mathcal{F}}(w) \vee \lambda_{\mathcal{H}}(w) = 1_{\mathcal{N}}, q_{\mathcal{F}}(u) \wedge q_{\mathcal{H}}(w) = 0_{\mathcal{N}}$.

(c) $\Gamma_{\mathcal{F}}(w) \wedge \Gamma_{\mathcal{H}}(w) = 1_{\mathcal{N}},\ \lambda_{\mathcal{F}}(w) \wedge \lambda_{\mathcal{H}}(w) = 1_{\mathcal{N}}, q_{\mathcal{F}}(w) \vee q_{\mathcal{H}}(w) = 1_{\mathcal{N}}$.

(d) $\Gamma_{\mathcal{F}}(w) \wedge \Gamma_{\mathcal{H}}(w) = 1_{\mathcal{N}},\ \lambda_{\mathcal{F}}(w) \vee \lambda_{\mathcal{H}}(w) = 1_{\mathcal{N}}, q_{\mathcal{F}}(w) \vee q_{\mathcal{H}}(w) = 1_{\mathcal{N}}$.

If $\mathbb{Q}$ is not $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}$ dconn, then it is said to be neutrosophic δ-ß-connected (concisely, $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}$ conn) spaces.

Example 3.2: by using the same example which is mentioned in [21] assume $\mathbb{Q} = \{r, p, z\}$, and define $\mathcal{N}_{\mathcal{S}}\mathcal{S}$'s $\mathbb{Q}_1, \mathbb{Q}_2, \mathbb{Q}_3$, & $\mathbb{Q}_4$ in $\mathbb{Q}$ as

$$\mathbb{Q}_1 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.1}, \frac{\Gamma_p}{0.3}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.9}, \frac{q_p}{0.7}, \frac{q_z}{0.6}\right)\right)$$

$$\mathbb{Q}_2 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.2}, \frac{\Gamma_p}{0.3}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.8}, \frac{q_p}{0.7}, \frac{q_z}{0.6}\right)\right),$$

$$\mathbb{Q}_3 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.2}, \frac{\Gamma_p}{0.3}, \frac{\Gamma_z}{0.7}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.8}, \frac{q_p}{0.7}, \frac{q_z}{0.3}\right)\right),$$

$$\mathbb{Q}_4 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.7}, \frac{\Gamma_p}{0.6}, \frac{\Gamma_z}{0.8}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.3}, \frac{q_p}{0.4}, \frac{q_z}{0.2}\right)\right)$$

We have $\Phi = \{0, \mathbb{Q}_1, \mathbb{Q}_2, 1\}$. $\mathbb{Q}_3$ & $\mathbb{Q}_4$ are $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{OS}$'s . Then, $\mathbb{Q}$ is $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}$conn.

Example 3.3: Utilizing Example **3.2** and assume:

$$\mathbb{Q}_3 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.0}, \frac{\Gamma_p}{0.0}, \frac{\Gamma_z}{1.0}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{1.0}, \frac{q_p}{1.0}, \frac{q_z}{0.0}\right)\right),$$

$$\mathbb{Q}_4 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{1.0}, \frac{\Gamma_p}{1.0}, \frac{\Gamma_z}{0.0}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.0}, \frac{q_p}{0.0}, \frac{q_z}{1.0}\right)\right).$$

We get $\mathbb{Q}_3$ & $\mathbb{Q}_4$ are $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{OS}$'s . In that case, $\mathbb{Q}$ is $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}$ dconn.

Definition 3.4: Let $(\mathbb{Q}, \Phi_{\mathcal{N}})$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$ on $\mathbb{Q}$ and $\mathcal{F}$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{S}$ of $\mathbb{Q}$. If $\exists$ $\mathcal{N}_{\mathcal{S}}\delta - \text{ß}\mathcal{OS}$'s sets $\mathcal{K}_1$ and $\mathcal{K}_2$ in $\mathbb{Q}$ verification of the next statements, so $\mathcal{F}$ is said neutrosophic δ-ß-$\mathcal{C}_i$-disconnected $(i = 1,2,3,4)$ (s. t):

(a) $\mathcal{C}_1:\ \mathcal{F} \subseteq \mathcal{K}_1 \cup \mathcal{K}_2,\ \mathcal{K}_1 \cap \mathcal{K}_2 \subseteq \mathcal{F}^c,\ \mathcal{F} \cap \mathcal{K}_1 \neq 0_{\mathcal{N}},\ \mathcal{F} \cap \mathcal{K}_2 \neq 0_{\mathcal{N}}$.

(b) $\mathcal{C}_2:\ \mathcal{F} \subseteq \mathcal{K}_1 \cup \mathcal{K}_2,\ \mathcal{F} \cap \mathcal{K}_1 \cap \mathcal{K}_2 \neq 0_{\mathcal{N}},\ \mathcal{F} \cap \mathcal{K}_1 \neq 0_{\mathcal{N}},\ \mathcal{F} \cap \mathcal{K}_2 \neq 0_{\mathcal{N}}$.

(c) $\mathcal{C}_3:\ \mathcal{F} \subseteq \mathcal{K}_1 \cup \mathcal{K}_2,\ \mathcal{K}_1 \cap \mathcal{K}_2 \subseteq \mathcal{F}^c,\ \mathcal{K}_1 \nsubseteq \mathcal{F}^c,\ \mathcal{K}_2 \nsubseteq \mathcal{F}^c$.

(d) $\mathcal{C}_4:\ \mathcal{F} \subseteq \mathcal{K}_1 \cup \mathcal{K}_2,\ \mathcal{F} \cap \mathcal{K}_1 \cap \mathcal{K}_2 = 0_{\mathcal{N}}, \mathcal{K}_1 \nsubseteq \mathcal{F}^c, \mathcal{K}_2 \nsubseteq \mathcal{F}^c$.

On the other hand, $\mathcal{F}$ is said neutrosophic δ-ß-$\mathcal{C}_i$-connected $(i = 1,2,3,4)$ if $\mathcal{F}$ isn't neutrosophic δ-ß-$\mathcal{C}_i$-disconnected $(i = 1,2,3,4)$.

Remark 3.5: Evidently, the following propositions are held:

(a) $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{C}_1\text{conn} \Longrightarrow \mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{C}_2\text{conn}$.

(b) $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{C}_1\text{conn} \Longrightarrow \mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{C}_3\text{conn}$.

(c) $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{C}_3\text{conn} \Longrightarrow \mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{C}_4\text{conn}$.

(d) $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{C}_1\text{conn} \Longrightarrow \mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{C}_4\text{conn}$.

Not any of above implications are reversible as illustrated in the following examples.

Example 3.6: Utilizing Example **3.2** and assume:

$$\mathbb{Q}_3 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.8}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.6}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.2}, \frac{q_p}{0.5}, \frac{q_z}{0.4}\right)\right)$$

$$\mathbb{Q}_4 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.8}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.2}, \frac{q_p}{0.5}, \frac{q_z}{0.6}\right)\right),$$

$$\mathbb{Q}_5 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.2}, \frac{\Gamma_p}{0.4}, \frac{\Gamma_z}{0.9}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.8}, \frac{q_p}{0.6}, \frac{q_z}{0.1}\right)\right).$$

We get $\mathbb{Q}_4$ & $\mathbb{Q}_5$ are $\mathcal{N}_S\delta - ß\mathcal{OS}'s$. In that case, $\mathbb{Q}_3$ is $\mathcal{N}_S\delta - ß\mathcal{C}_2$conn. , $\mathcal{N}_S\delta - \beta\mathcal{C}_3$conn. and $\mathcal{N}_S\delta - \beta\mathcal{C}_4$conn. but not $\mathcal{N}_S\delta - \beta\mathcal{C}_1$conn.

Example 3.7: Utilizing Example **3.2** and assume:

$$\mathbb{Q}_3 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.8}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.6}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.2}, \frac{q_p}{0.5}, \frac{q_z}{0.4}\right)\right)$$

$$\mathbb{Q}_4 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.8}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.2}, \frac{q_p}{0.5}, \frac{q_z}{0.6}\right)\right),$$

$$\mathbb{Q}_5 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.2}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.9}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.8}, \frac{q_p}{0.5}, \frac{q_z}{0.1}\right)\right).$$

We obtain $\mathbb{Q}_4$ and $\mathbb{Q}_5$ are $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$. In that case, $\mathbb{Q}_3$ is $\mathcal{N}_S\delta - \beta\mathcal{C}_4$conn. but neither $\mathcal{N}_S\delta - \beta\mathcal{C}_3$conn. nor $\mathcal{N}_S\delta - ß\mathcal{C}_1$conn.

Definition 3.8: A $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q}, \Phi_\mathcal{N})$ is said to be a neutrosophic δ-ß- $\mathcal{C}_5$ -disconnected (concisely, $\mathcal{N}_S\delta - ß\mathcal{C}_5$dconn.) if $\exists$ neutrosophic sub-set $\mathcal{H}$ in $\mathbb{Q}$ which is $\mathcal{N}_S\delta - \beta\mathcal{OS}$ & $\mathcal{N}_S\delta - \beta\mathcal{CS}$ in $\mathbb{Q}$, (s. t) $\mathcal{H} \neq 0_\mathcal{N}, \mathcal{H} \neq 1_\mathcal{N}$. If $\mathbb{Q}$ is not $\mathcal{N}_S\delta - \beta\mathcal{C}_5$dconn., then it is called a neutrosophic δ-ß- $\mathcal{C}_5$ -connected (concisely, $\mathcal{N}_S\delta - ß\mathcal{C}_5$conn.).

Example 3.9: Utilizing Example **3.2** and assume:

$$\mathbb{Q}_3 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.1}, \frac{\Gamma_p}{0.2}, \frac{\Gamma_z}{0.3}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.9}, \frac{q_p}{0.8}, \frac{q_z}{0.7}\right)\right),$$ is $\mathcal{N}_S\delta - ß\mathcal{C}_5$dconn.

Theorem 3.10: If $(\mathbb{Q}, \Phi_\mathcal{N})$ is a $\mathcal{N}_S\mathcal{TS}$ on $\mathbb{Q}$. Then, $\mathcal{N}_S\delta - ß\mathcal{C}_5$dconn-ness implies $\mathcal{N}_S\delta - ß$conn-ness.

Proof: Presume that there exists non-empty $\mathcal{N}_S\delta - ß\mathcal{OS}'s$ $\mathcal{H}$ & $\mathcal{W} \ni \mathcal{H} \cup \mathcal{W} = 1_\mathcal{N}$ & $\mathcal{H} \cap \mathcal{W} = 0_\mathcal{N}$. So, $\Gamma_\mathcal{H} \vee \Gamma_\mathcal{W} = 1_\mathcal{N}$, $\lambda_\mathcal{H} \wedge \lambda_\mathcal{W} = 0_\mathcal{N}$, $q_\mathcal{H} \wedge q_\mathcal{W} = 0_\mathcal{N}$, and $\Gamma_\mathcal{H} \vee \Gamma_\mathcal{W} = 0_\mathcal{N}$, $\lambda_\mathcal{H} \wedge \lambda_\mathcal{W} = 1_\mathcal{N}$, and $q_\mathcal{H} \wedge q_\mathcal{W} = 1_\mathcal{N}$. In another meaning, $\mathcal{W}^c = \mathcal{H}$. Consequently, $\mathcal{H}$ is $\mathcal{N}_S\delta - ß$-clopen which implies $\mathbb{Q}$ is $\mathcal{N}_S\delta - ß\mathcal{C}_5$conn.

The converse isn't true as shown in the next example:

Example 3.11: Utilizing Example **3.2** and assume:

$$\mathbb{Q}_3 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.2}, \frac{\Gamma_p}{0.3}, \frac{\Gamma_z}{0.7}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.8}, \frac{q_p}{0.7}, \frac{q_z}{0.3}\right)\right),$$

$$\mathbb{Q}_4 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.7}, \frac{\Gamma_p}{0.6}, \frac{\Gamma_z}{0.8}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.3}, \frac{q_p}{0.4}, \frac{q_z}{0.2}\right)\right).$$

So, $\mathbb{Q}_3$ & $\mathbb{Q}_4$ are $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$. In that case, $\mathbb{Q}$ is $\mathcal{N}_S\delta - ß$conn. but not $\mathcal{N}_S\delta - ß\mathcal{C}_5$dconn.

Theorem 3.12: Let $\mathfrak{F}: (\mathbb{Q}, \Phi_\mathcal{N}) \longrightarrow (\mathcal{Y}, \Psi_\mathcal{N})$ be a $\mathcal{N}_S\delta - ß\mathcal{I}rr$ surjection map and $\mathbb{Q}$ be a $\mathcal{N}_S\delta - ß$conn. Then, $\mathcal{Y}$ is $\mathcal{N}_S\delta - ß$ conn.

Proof: Presume that $\mathcal{Y}$ is not $\mathcal{N}_S\delta - \beta$conn, then $\exists$ non-empty $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$ $\mathcal{K}_1$ and $\mathcal{K}_2$ in $\mathcal{Y}$ (s. t) $\mathcal{K}_1 \cup \mathcal{K}_2 = 1_\mathcal{N}$ & $\mathcal{K}_1 \cap \mathcal{K}_2 = 0_\mathcal{N}$. Since, $\mathfrak{F}$ is $\mathcal{N}_S\delta - ß\mathcal{I}rr$ map, so $\mathcal{K} = \mathfrak{F}^{-1}(\mathcal{K}_1) \neq 0_\mathcal{N}$, & $\mathcal{W} = \mathfrak{F}^{-1}(\mathcal{K}_2) \neq 0_\mathcal{N}$, which are $\mathcal{N}_S\delta - ß\mathcal{OS}'s$ in $\mathbb{Q}$, & $\mathfrak{F}^{-1}(\mathcal{K}_1) \cup \mathfrak{F}^{-1}(\mathcal{K}_2) = \mathfrak{F}^{-1}(1_N) = 1_\mathcal{N}, \Longrightarrow \mathcal{K} \cup \mathcal{W} = 1_\mathcal{N}$. As well, $\mathfrak{F}^{-1}(\mathcal{K}_1) \cap \mathfrak{F}^{-1}(\mathcal{K}_2) = \mathfrak{F}^{-1}(0_\mathcal{N}) = 0_\mathcal{N}$, which implies $\mathcal{K} \cap \mathcal{W} = 0_\mathcal{N}$. By supposition, that is a contradiction to $\mathbb{Q}$ which is $\mathcal{N}_S\delta - ß$dconn. Consequently, $\mathcal{Y}$ is $\mathcal{N}_S\delta - ß$conn.

Theorem 3.13: Let $(\mathbb{Q}, \Phi_\mathcal{N})$ be a $\mathcal{N}_S\mathcal{TS}$ which is $\mathcal{N}_S\delta - ß\mathcal{C}_5$conn. if and only if there exists no non-empty $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$ $\mathcal{H}$ & $\mathcal{W}$ in $\mathbb{Q} \ni \mathcal{W}^c = \mathcal{H}$.

Proof: Assume that $\mathcal{H}$ and $\mathcal{W}$ are two $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$ in $\mathbb{Q}$ (s. t) $\mathcal{H} \neq 0_\mathcal{N}, \mathcal{W} \neq 0_\mathcal{N}$, & $\mathcal{H} = \mathcal{W}^c$. Since $\mathcal{H} = \mathcal{W}^c$, so $\mathcal{W}^c$ is a $\mathcal{N}_S\delta - ß\mathcal{OS}$, and $\mathcal{W}$ is a $\mathcal{N}_S\delta - ß\mathcal{CS}$, and $\mathcal{H} \neq 0_\mathcal{N} \Longrightarrow \mathcal{W} \neq 1_\mathcal{N}$. But it's a contradiction to $\mathbb{Q}$ is $\mathcal{N}_S\delta - \beta\mathcal{C}_5$conn.

Conversely, presume $\mathcal{H}$ & $\mathcal{W}$ are both $\mathcal{N}_S\delta - \beta\mathcal{OS}$ & $\mathcal{N}_S\delta - ß\mathcal{CS}$ in $\mathbb{Q}$ (s. t) $\mathcal{H} \neq 0_\mathcal{N}, \mathcal{H} \neq 1_\mathcal{N}$. Currently take $\mathcal{H}^c = \mathcal{W}$ as a $\mathcal{N}_S\delta - \beta\mathcal{OS}$ and $\mathcal{H} \neq 1_\mathcal{N}$ which implies $\mathcal{H}^c = \mathcal{W} \neq 0_\mathcal{N}$, a contradiction. Thus, $\mathbb{Q}$ is $\mathcal{N}_S\delta - ß\mathcal{C}_5$conn.

Theorem 3.14: Let $(\mathbb{Q}, \Phi_\mathcal{N})$ be a $\mathcal{N}_S\mathcal{TS}$ which is $\mathcal{N}_S\delta - ß$ conn. if and only if there exist no non-zero $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$ $\mathcal{H}$ and $\mathcal{W}$ in $\mathbb{Q} \ni \mathcal{W}^c = \mathcal{H}$.

Proof: Necessity: Assume that $\mathcal{H}$ and $\mathcal{W}$ are two $\mathcal{N}_S\delta - ß\mathcal{OS}'s$ in $\mathbb{Q}$ (s. t) $\mathcal{H} \neq 0_\mathcal{N}, \mathcal{W} \neq 0_\mathcal{N}$, & $\mathcal{H} = \mathcal{W}^c$. Consequently, $\mathcal{W}^c$ is a $\mathcal{N}_S\delta - ß\mathcal{CS}$, Since $\mathcal{H} \neq 0_\mathcal{N}, \mathcal{W} \neq 1_\mathcal{N}$. This $\Longrightarrow \mathcal{W}$ is proper neutrosophic subset which is $\mathcal{N}_S\delta - ß\mathcal{OS}$ & $\mathcal{N}_S\delta - \beta\mathcal{CS}$ in $\mathbb{Q}$. Therefore, $\mathbb{Q}$ is not a $\mathcal{N}_S\delta - \beta$conn. by supposition, it's contradiction. So, there is no nonzero $\nexists$ $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$ $\mathcal{H}$ & $\mathcal{W}$ in $\mathbb{Q} \ni \mathcal{H} = \mathcal{W}^c$.

Sufficiency: Presume $\mathcal{H}$ is both $\mathcal{N}_S\delta - \beta\mathcal{OS}$ & $\mathcal{N}_S\delta - \beta\mathcal{CS} \subseteq \mathbb{Q} \ni \mathcal{H} \neq 0_\mathcal{N}$, and $\mathcal{H} \neq 1_\mathcal{N}$. At this time let $\mathcal{W} = \mathcal{H}^c$. So, $\mathcal{W}$ is $\mathcal{N}_S\delta - ß\mathcal{OS}$ & $\mathcal{W} \neq 1_\mathcal{N}$. This $\Longrightarrow \mathcal{H}^c = \mathcal{W} \neq 0_\mathcal{N}$; via supposition, it's contradiction. Consequently, $\mathbb{Q}$ is $\mathcal{N}_S\delta - ß$conn.

Theorem 3.15: A $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q}, \Phi_\mathcal{N})$ is $\mathcal{N}_S\delta - ß$conn. if and only if there exists no nonzero neutronsophic sub-sets $\mathcal{H}$ & $\mathcal{W}$ in $\mathbb{Q}, \ni \mathcal{H} = \mathcal{W}^c, \mathcal{W} = (\mathcal{N}_S\delta - \beta\mathcal{C}l(\mathcal{H}))^C$ and $\mathcal{H} = (\mathcal{N}_S\delta - \beta\mathcal{C}l(\mathcal{W}))^C$.

Proof: Necessity, Assume $\mathcal{H}$ & $\mathcal{W}$ are two neutrosophic subsets in $\mathbb{Q} \ni \mathcal{H} \neq 0_\mathcal{N}$, $\mathcal{W} \neq 0_\mathcal{N}$, and $\mathcal{H} = \mathcal{W}^c, \mathcal{W} = (\mathcal{N}_S\delta - ß\mathcal{C}l(\mathcal{H}))^C$, & $\mathcal{H} = (\mathcal{N}_S\delta - ß\mathcal{C}l(\mathcal{W}))^C$. Since $(\mathcal{N}_S\delta - \beta\mathcal{C}l(\mathcal{H}))^C$ and $(\mathcal{N}_S\delta - ß\mathcal{C}l(\mathcal{W}))^C$ are $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$ in $\mathbb{Q}$, so $\mathcal{H}$ and $\mathcal{W}$ are $\mathcal{N}_S\delta - \beta\mathcal{OS}'s$ in $\mathbb{Q}$. This $\Longrightarrow \mathbb{Q}$ isn't $\mathcal{N}_S\delta - \beta$conn., a contradiction. Consequently, there is no nonzero $\mathcal{N}_S\delta - ß\mathcal{OS}'s$ $\mathcal{H}$ & $\mathcal{W}$ in $\mathbb{Q}, \ni \mathcal{H} = \mathcal{W}^c, \mathcal{W} = (\mathcal{N}_S\delta - ß\mathcal{C}l(\mathcal{H}))^C$ & $\mathcal{H} = (\mathcal{N}_S\delta - \beta\mathcal{C}l(\mathcal{W}))^C$.

Sufficiency: Suppose $\mathcal{H}$ is both $\mathcal{N}_S\delta - ß\mathcal{OS}$ & $\mathcal{N}_S\delta - \beta\mathcal{CS}$, in $\mathbb{Q}$ (s. t) $\mathcal{H} \neq 0_\mathcal{N}$ and $\mathcal{H} \neq 1_\mathcal{N}$. Let $\mathcal{W} = \mathcal{H}^c$; by supposition, obtain a contradiction. Consequently, $\mathbb{Q}$ is $\mathcal{N}_S\delta - ß$conn. space.

Definition 3.16: A $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q}, \Phi_\mathcal{N})$ is called neutrosophic δ-ß-strongly connected (concisely, $\mathcal{N}_S\delta - ß\mathcal{S}t$conn) if $\exists$ no non-empty $\mathcal{N}_S\delta - \beta\mathcal{CS}'s$ $\mathcal{H}$ and $\mathcal{W} \subseteq \mathbb{Q} \ni \Gamma_\mathcal{H} + \Gamma_\mathcal{W} \geq 1_\mathcal{N}, \lambda_\mathcal{H} + \lambda_\mathcal{W} \geq 1_\mathcal{N}, q_\mathcal{H} + q_\mathcal{W} \leq 1_\mathcal{N}$ or $\Gamma_\mathcal{H} + \Gamma_\mathcal{W} \geq 1_\mathcal{N}, \lambda_\mathcal{H} + \lambda_\mathcal{W} \leq 1_\mathcal{N}, q_\mathcal{H} + q_\mathcal{W} \leq 1_\mathcal{N}$.

In another meaning, a $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q}, \Phi_\mathcal{N})$ is called $\mathcal{N}_S\delta -$

βStconn, if there exists no non-empty $\mathcal{N}_S\delta-\beta CS's$ $\mathcal{H}$ & $\mathcal{W}$ in $\mathbb{Q} \ni \mathcal{H}\cap\mathcal{W}=0_{\mathcal{N}}$.

Example 3.17: Utilizing Example (**3.2**) and presume:

$$\mathbb{Q}_3=\left(\mathbb{Q},\left(\frac{\Gamma_r}{0.0},\frac{\Gamma_p}{0.0},\frac{\Gamma_z}{1.0}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{1.0},\frac{q_p}{1.0},\frac{q_z}{0.0}\right)\right),$$

$$\mathbb{Q}_4=\left(\mathbb{Q},\left(\frac{\Gamma_r}{1.0},\frac{\Gamma_p}{1.0},\frac{\Gamma_z}{0.0}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.0},\frac{q_p}{0.0},\frac{q_z}{1.0}\right)\right)$$

In that case $\mathbb{Q}_3$ & $\mathbb{Q}_4$ are $\mathcal{N}_S\delta-\beta OS's$. So, $\mathbb{Q}$ is $\mathcal{N}_S\delta-\beta St$conn.

Theorem 3.18: Let $(\mathbb{Q},\Phi_{\mathcal{N}})$ be a a $\mathcal{N}_S\mathcal{TS}$ which is $\mathcal{N}_S\delta-ßSt$conn., if there exists no nonempty $\mathcal{N}_S\delta-ßOS's$ $\mathcal{H}$ and $\mathcal{W}$ in $\mathbb{Q},\ni \mathcal{H}\neq 1_{\mathcal{N}}\neq\mathcal{W}\ni \Gamma_{\mathcal{H}}+\Gamma_{\mathcal{W}}\geq 1_{\mathcal{N}},\lambda_{\mathcal{H}}+\lambda_{\mathcal{W}}\geq 1_{\mathcal{N}}$ and $q_{\mathcal{H}}+q_{\mathcal{W}}\leq 1_{\mathcal{N}}$.

Proof: Let $\mathcal{H}$ & $\mathcal{W}$ be a $\mathcal{N}_S\delta-ßOS's$ in $\mathbb{Q}$ (s. t) $\mathcal{H}\neq 1\neq\mathcal{W}$ and $\Gamma_{\mathcal{H}}+\Gamma_{\mathcal{W}}\geq 1_{\mathcal{N}},\lambda_{\mathcal{H}}+\lambda_{\mathcal{W}}\geq 1_{\mathcal{N}}$, and $q_{\mathcal{H}}+q_{\mathcal{W}}\leq 1_{\mathcal{N}}$. If we put $\mathcal{D}=\mathcal{H}^c$ and $\mathcal{C}=\mathcal{W}^c$, then $\mathcal{D}$ & $\mathcal{C}$ have $\mathcal{N}_S\delta-\beta CS's$ in $\mathbb{Q}$ and $\mathcal{D}\neq 0_{\mathcal{N}}\neq\mathcal{C}, q_{\mathcal{D}}+q_{\mathcal{C}}=\ \Gamma_{\mathcal{H}}+\Gamma_{\mathcal{W}}\geq 1_{\mathcal{N}},\Gamma_{\mathcal{D}}+\Gamma_{\mathcal{C}}=\lambda_{\mathcal{H}}+\lambda_{\mathcal{W}}\geq 1_{\mathcal{N}}$ and $\lambda_{\mathcal{D}}+\lambda_{\mathcal{C}}=q_{\mathcal{H}}+q_{\mathcal{W}}\leq 1_{\mathcal{N}}$,contradiction.

Conversely, by using the same method we obtain the opposite direction**.**

Theorem 3.19: Let$\mathfrak{F}:(\mathbb{Q},\Phi_{\mathcal{N}})\longrightarrow(\mathcal{Y},\Psi_{\mathcal{N}})$ be $\mathcal{N}_S\delta-ßIrr$ surjection map and $\mathbb{Q}$ be $\mathcal{N}_S\delta-ßSt$conn. Then, $\mathcal{Y}$ is $\mathcal{N}_S\delta-\beta St$conn.

Proof: Suppose $\mathcal{Y}$ isn't $\mathcal{N}_S\delta-ßSt$conn., so $\exists$ non-empty $\mathcal{N}_S\delta-\beta CS's$ $\mathcal{H}$ & $\mathcal{W}$ in $\mathcal{Y}\ni\mathcal{H}\neq 0_{\mathcal{N}},\mathcal{W}\neq 0_{\mathcal{N}}$ & $\mathcal{H}\cap\mathcal{W}=0_{\mathcal{N}}$.Because $\mathfrak{F}$ is $\mathcal{N}_S\delta-ßIrr$ map, consequently $\mathcal{A}=\mathfrak{F}^{-1}(\mathcal{H})\neq 0_{\mathcal{N}}$, & $\mathcal{B}=\mathfrak{F}^{-1}(\mathcal{W})\neq 0_{\mathcal{N}}$, which are $\mathcal{N}_S\delta-ßCS's$ in $\mathbb{Q}$, & $\mathfrak{F}^{-1}(\mathcal{H})\cap\ \mathfrak{F}^{-1}(\mathcal{W})=\mathfrak{F}^{-1}(0_{\mathcal{N}})=0_{\mathcal{N}}$, which $\Longrightarrow$ $\mathcal{A}\cap\mathcal{B}=0_{\mathcal{N}}$, via supposition, this contradiction to $\mathbb{Q}$ which isn't$\mathcal{N}_S\delta-ßSt$conn. Consequently, $\mathcal{Y}$ is $\mathcal{N}_S\delta-ßSt$conn.

Remark 3.20: The concepts of $\mathcal{N}_S\delta-ßSt$conn. and $\mathcal{N}_S\delta-\beta C_5$conn. are independent as shown in the following examples:

Example 3.21: By using Example (**3.2**) and suppose:

$$\mathbb{Q}_3=\left(\mathbb{Q},\left(\frac{\Gamma_r}{0.0},\frac{\Gamma_p}{0.0},\frac{\Gamma_z}{1.0}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{1.0},\frac{q_p}{1.0},\frac{q_z}{0.0}\right)\right),$$

$$\mathbb{Q}_4=\left(\mathbb{Q},\left(\frac{\Gamma_r}{1.0},\frac{\Gamma_p}{1.0},\frac{\Gamma_z}{0.0}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.0},\frac{q_p}{0.0},\frac{q_z}{1.0}\right)\right)$$

In that case, $\mathbb{Q}_3$ & $\mathbb{Q}_4$ are$\mathcal{N}_S\delta-\beta OS's$. So, $\mathbb{Q}$ is $\mathcal{N}_S\delta-\beta St$conn. but not $\mathcal{N}_S\delta-ßC_5$conn.

Example 3.22: By using Example (**3.2**) and suppose:

$$\mathbb{Q}_3=\left(\mathbb{Q},\left(\frac{\Gamma_r}{0.8},\frac{\Gamma_p}{0.6},\frac{\Gamma_z}{0.7}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.2},\frac{q_p}{0.4},\frac{q_z}{0.3}\right)\right)$$

$$\mathbb{Q}_4=\left(\mathbb{Q},\left(\frac{\Gamma_r}{0.3},\frac{\Gamma_p}{0.6},\frac{\Gamma_z}{0.8}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.7},\frac{q_p}{0.4},\frac{q_z}{0.2}\right)\right)$$

In that case, $\mathbb{Q}_3$ and $\mathbb{Q}_4$ are$\mathcal{N}_S\delta-ßOS's$. So, $\mathbb{Q}$ is $\mathcal{N}_S\delta-\beta C_5$conn. but not $\mathcal{N}_S\delta-\beta St$conn.

4. Several Properties of Neutrosophic *δ-ß*-Separated Sets

In this section, several characterizations related to neutrosophic *δ-ß*-separated sets have been introduced in neutrosophic Topological spaces.

Definition 4.1: If subsets $\mathcal{H}$ and $\mathcal{W}$ are non-zero neutrosophic in $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q},\Phi_{\mathcal{N}})$, then $\mathcal{H}$ & $\mathcal{W}$ are called:

(a) neutrosophic δ-*ß*-separated (concisely, $\mathcal{N}_S\delta-ßSep$) if $\mathcal{N}_S\delta-\beta Cl(\mathcal{H})\cap\mathcal{W}=\mathcal{H}\cap\mathcal{N}_S\delta-\beta Cl(\mathcal{W})=0_{\mathcal{N}}$

(b) neutrosophic δ-*ß*-weakly separated (concisely, $\mathcal{N}_S\delta-\beta WSep$) if $\mathcal{N}_S\delta-\beta Cl(\mathcal{H})\subseteq\mathcal{W}^c$ and $\mathcal{N}_S\delta-ßCl(\mathcal{W})\subseteq\mathcal{H}^c$.

Remark 4.2: Any two disjoint non-empty $\mathcal{N}_S\delta-ßCS's$ are $\mathcal{N}_S\delta-ßSep.$ in $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q},\Phi_{\mathcal{N}})$.

Proof: Assume $\mathcal{H}$ and $\mathcal{W}$ are disjoint non-empty $\mathcal{N}_S\delta-ßCS's$. So $\mathcal{N}_S\delta-ßCl(\mathcal{H})\cap\mathcal{W}=\mathcal{H}\cap\mathcal{N}_S\delta-ßCl(\mathcal{W})=\mathcal{H}\cap\mathcal{W}=0_{\mathcal{N}}$. This illustrates that $\mathcal{H}$ & $\mathcal{W}$ is $\mathcal{N}_S\delta-ßSep$.

Theorem 4.3: If $\mathcal{H}$ and $\mathcal{W}$ are nonzero neutrosophic subsets in $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q},\Phi_{\mathcal{N}})$. Then, the following statements are held:

(a) If $\mathcal{H}$ and $\mathcal{W}$ are $\mathcal{N}_S\delta-ßSep.$ with $\mathcal{D}\subseteq\mathcal{H},\mathcal{C}\subseteq\mathcal{W}$, then $\mathcal{D}$ and $\mathcal{C}$ are also $\mathcal{N}_S\delta-ßSep.$

(b) If $\mathcal{H}$ and $\mathcal{W}$ are both $\mathcal{N}_S\delta-ßOS's$ and if $\mathcal{F}=\mathcal{H}\cap\mathcal{W}^c$ & $\mathcal{G}=\mathcal{W}\cap\mathcal{H}^c$, then $\mathcal{F}$ & $\mathcal{G}$ are $\mathcal{N}_S\delta-ßSep.$

Proof: $(\boldsymbol{a})$ Assume $\mathcal{H}$ and $\mathcal{W}$ are $\mathcal{N}_S\delta-\beta Sep.$ sets in $(\mathbb{Q},\Phi_{\mathcal{N}})$. In that case $\mathcal{N}_S\delta-\beta Cl(\mathcal{H})\cap\mathcal{W}=0_{\mathcal{N}}=\mathcal{H}\cap\mathcal{N}_S\delta-\beta Cl(\mathcal{W})$. Since $\mathcal{D}\subseteq\mathcal{H}$ & $\mathcal{C}\subseteq\mathcal{W}$, so $\mathcal{N}_S\delta-ßCl(\mathcal{D})\subseteq\mathcal{N}_S\delta-ßCl(\mathcal{H})$ and $\mathcal{N}_S\delta-\beta Cl(\mathcal{C})\subseteq\mathcal{N}_S\delta-\beta Cl(\mathcal{W})$. This implies $\mathcal{N}_S\delta-\beta Cl(\mathcal{D})\cap\mathcal{C}\subseteq\mathcal{N}_S\delta-\beta Cl(\mathcal{H})\cap\mathcal{W}=0_{\mathcal{N}}$ and thus $\mathcal{N}_S\delta-\beta Cl(\mathcal{D})\cap\mathcal{C}=0_{\mathcal{N}}$. In the same way, $\mathcal{N}_S\delta-ßCl(\mathcal{C})\cap\mathcal{D}\subseteq\mathcal{N}_S\delta-ßCl(\mathcal{W})\cap\mathcal{H}=0_{\mathcal{N}}$ and so $\mathcal{N}_S\delta-\beta Cl(\mathcal{C})\cap\mathcal{D}=0_{\mathcal{N}}$. Consequently, $\mathcal{D}$ and $\mathcal{C}$ are $\mathcal{N}_S\delta-\beta Sep.$

Proof: $(\boldsymbol{b})$ Assume $\mathcal{H}$ and $\mathcal{W}$ are $\mathcal{N}_S\delta-\beta OS's$ in $(\mathbb{Q},\Phi_{\mathcal{N}})$. Then, $\mathcal{H}^c$ and $\mathcal{W}^c$ are $\mathcal{N}_S\delta-\beta CS's$. Since $\mathcal{F}\subseteq\mathcal{W}^c$, so $\mathcal{N}_S\delta-\beta Cl(\mathcal{F})\subseteq\mathcal{N}_S\delta-\beta Cl(\mathcal{W}^c)=\mathcal{W}^c$ and so $\mathcal{N}_S\delta-ßCl(\mathcal{F})\cap\mathcal{W}=0_{\mathcal{N}}$. Since $\mathcal{G}\subseteq\mathcal{W}$, so $\mathcal{N}_S\delta-\beta Cl(\mathcal{F})\cap\mathcal{G}\subseteq\mathcal{N}_S\delta-\beta Cl(\mathcal{F})\cap\mathcal{W}=0_{\mathcal{N}}$.
Therefore, $\mathcal{N}_S\delta-\beta Cl(\mathcal{F})\cap\mathcal{G}=0_{\mathcal{N}}$. In the same way, $\mathcal{N}_S\delta-\beta Cl(\mathcal{G})\cap\mathcal{F}=0_{\mathcal{N}}$. Consequently, $\mathcal{F}$ & $\mathcal{G}$ are $\mathcal{N}_S\delta-\beta Sep.$

Proposition 4.4: Each two $\mathcal{N}_S\delta-ßSep.$ sets are always disjoint in $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q},\Phi_{\mathcal{N}})$.

Proof: Let $\mathcal{H}$ & $\mathcal{W}$ be $\mathcal{N}_S\delta-ßSep.$ sets. In that case $\mathcal{H}\cap\mathcal{N}_S\delta-\beta Cl(\mathcal{W})=0_{\mathcal{N}}=\mathcal{N}_S\delta-ßCl(\mathcal{H})\cap\mathcal{W}$. Now, $\mathcal{H}\cap\mathcal{W}\subseteq\mathcal{H}\cap\mathcal{N}_S\delta-ßCl(\mathcal{W})=0_{\mathcal{N}}$. so, $\mathcal{H}\cap\mathcal{W}=0_{\mathcal{N}}$, and thus, $\mathcal{H}$ & $\mathcal{W}$ are disjoint.

Theorem 4.5: Let $(\mathbb{Q},\Phi_{\mathcal{N}})$ be a $\mathcal{N}_S\mathcal{TS}$. Then, $(\mathbb{Q},\Phi_{\mathcal{N}})$ is $\mathcal{N}_S\delta-\beta$conn. if and only if $\mathcal{H}\cup\mathcal{W}\neq 1_{\mathcal{N}}$, wherever $\mathcal{H}$ & $\mathcal{W}$ are $\mathcal{N}_S\delta-\beta Sep.$ sets.

Proof: Assume that $\mathbb{Q}$ is $\mathcal{N}_S\delta-\beta$conn. space. Presume, $1_{\mathcal{N}}=\mathcal{H}\cup\mathcal{W}$, where $\mathcal{H}$ and $\mathcal{W}$ are $\mathcal{N}_S\delta-ßSep.$ sets. In that case $\mathcal{N}_S\delta-ßCl(\mathcal{H})\cap\mathcal{W}=\mathcal{H}\cap\mathcal{N}_S\delta-ßCl(\mathcal{W})=0_{\mathcal{N}}$. Since $\mathcal{H}\subseteq\mathcal{N}_S\delta-\beta Cl(\mathcal{H})$, we have $\mathcal{H} \cap \mathcal{W} \subseteq \mathcal{N}_S\delta - \beta Cl(\mathcal{H}) \cap \mathcal{W} = 0_{\mathcal{N}}$. Consequently, $\mathcal{N}_S\delta - \beta Cl(\mathcal{H}) \subseteq \mathcal{W}^c = \mathcal{H}$ & $\mathcal{N}_S\delta -$ ß$Cl(\mathcal{W}) \subseteq \mathcal{H}^c = \mathcal{W}$. Thus, $\mathcal{H} = \mathcal{N}_S\delta - \beta Cl(\mathcal{H})$ and $\mathcal{W} = \mathcal{N}_S\delta - \beta Cl(\mathcal{W})$. Consequently, $\mathcal{H}$ & $\mathcal{W}$ are $\mathcal{N}_S\delta - \beta CS's$, and so, $\mathcal{H} = \mathcal{W}^c$ and $\mathcal{W} = \mathcal{H}^c$ are disjoint $\mathcal{N}_S\delta - \beta OS's$. So $\mathcal{H} \neq 0_{\mathcal{N}}, \mathcal{W} \neq 0_{\mathcal{N}} \ni \mathcal{H} \cup \mathcal{W} = 1_{\mathcal{N}}$ & $\mathcal{H} \cap \mathcal{W} = 0_{\mathcal{N}}, \mathcal{H}$ & $\mathcal{W}$ are $\mathcal{N}_S\delta - \beta OS's$. This means, $\mathbb{Q}$ is not $\mathcal{N}_S\delta - \beta$conn., a contradiction to $\mathbb{Q}$ which is a $\mathcal{N}_S\delta - \beta$conn. Thus, $1_{\mathcal{N}}$ isn't the union of every two $\mathcal{N}_S\delta - \beta Sep.$ sets.

Conversely, suppose $1_{\mathcal{N}}$ isn't the union of any two $\mathcal{N}_S\delta -$ ß$Sep.$ sets. Presume $\mathbb{Q}$ isn't $\mathcal{N}_S\delta -$ ßconn. In that case, $1_{\mathcal{N}} = \mathcal{H} \cup \mathcal{W}$, where $\mathcal{H} \neq 0_{\mathcal{N}}, \mathcal{W} \neq 0_{\mathcal{N}}$ (s. t) $\mathcal{H} \cap \mathcal{W} = 0_{\mathcal{N}}, \mathcal{H}$ and $\mathcal{W}$ are $\mathcal{N}_S\delta - \beta OS's$ in $\mathbb{Q}$. Since $\mathcal{H} \subseteq \mathcal{W}^c$ & $\mathcal{W} \subseteq \mathcal{H}^c, \mathcal{N}_S\delta -$ ß$Cl(\mathcal{H}) \cap \mathcal{W} \subseteq \mathcal{W}^c \cap \mathcal{W} = 0_{\mathcal{N}}$ and $\mathcal{H} \cap \mathcal{N}_S\delta - \beta Cl(\mathcal{W}) \subseteq \mathcal{H} \cap \mathcal{H}^c = 0_{\mathcal{N}}$. This means, $\mathcal{H}$& $\mathcal{W}$ are $\mathcal{N}_S\delta - \beta Sep.$ sets. That is a contradiction. Consequently, $\mathbb{Q}$ is $\mathcal{N}_S\delta - \beta$conn.

Definition 4.6: If a neutrosophic subset $\mathcal{H}$ in $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is nonzero, so

(a) neutrosophic δ-ß-regular open set(shortly, $\mathcal{N}_S\delta - \beta\mathcal{ROS}$) if $\mathcal{H} = \mathcal{N}_S\delta - \beta Int(\mathcal{N}_S\delta - \beta Cl(\mathcal{H}))$.
(b) neutrosophic δ-ß-regular closed set (concisely, $\mathcal{N}_S\delta - \beta\mathcal{RCS}$) if $\mathcal{H} = \mathcal{N}_S\delta - \beta Cl(\mathcal{N}_S\delta - \beta Int(\mathcal{H}))$.
(c) The complement of $\mathcal{N}_S\delta - \beta\mathcal{ROS}$ is $\mathcal{N}_S\delta - \beta\mathcal{RCS}$.

Definition 4.7: A $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is called:

(a) Neutrosophic δ-ß-super dis-connected (concisely, $\mathcal{N}_S\delta - \beta sup.dconn$) if $\exists$ $\mathcal{N}_S\delta -$ ß$\mathcal{ROS}$ $\mathcal{H}$ in $\mathbb{Q} \ni \mathcal{H} \neq 0_{\mathcal{N}}$ & $\mathcal{H} \neq 1_{\mathcal{N}}$.
(b) $\mathcal{N}_S$ δ-ß-super connected (concisely, $\mathcal{N}_S\delta - \beta sup.conn$) if there is no proper $\mathcal{N}_S\delta -$ ß$\mathcal{ROS}$ in $\mathbb{Q}$ that is (if $\mathbb{Q}$ is not $\mathcal{N}_S\delta -$ ß$sup.dconn.$)

Example 4.8: By using Example **3.2** and assuming:
$\mathbb{Q}_3 = \left(\mathbb{Q}, \left(\frac{\Gamma_r}{0.1}, \frac{\Gamma_p}{0.2}, \frac{\Gamma_z}{0.3}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.9}, \frac{q_p}{0.8}, \frac{q_z}{0.7}\right)\right)$, is $\mathcal{N}_S\delta -$ ß$sup.dconn.$

Theorem 4.9: The following properties are equivalent *in* $\mathcal{N}_S\mathcal{TS}$ $(\mathbb{Q}, \Phi_{\mathcal{N}})$:

(a) $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_S\delta - \beta sup.conn.$
(b) $\forall$ $\mathcal{N}_S\delta -$ ß$\mathcal{OS}$ $\mathcal{H} \neq 0_{\mathcal{N}}$ in $\mathbb{Q}$, we obtain $\mathcal{N}_S\delta -$ ß$Cl(\mathcal{H}) = 1_{\mathcal{N}}$.
(c) $\forall$ $\mathcal{N}_S\delta - \beta CS$ $\mathcal{H} \neq 1_{\mathcal{N}}$ in $\mathbb{Q}$, we obtain $\mathcal{N}_S\delta -$ ß$Int(\mathcal{H}) = 0_{\mathcal{N}}$.
(d) There exists no $\mathcal{N}_S\delta - \beta OS's$ $\mathcal{H}$ and $\mathcal{W}$ in $\mathbb{Q}, \ni$ $\mathcal{H} \neq 0_{\mathcal{N}}, \mathcal{W} \neq 0_{\mathcal{N}}$, & $\mathcal{H} \subseteq \mathcal{W}^c$.
(e) There exist no $\mathcal{N}_S\delta - \beta OS's$ $\mathcal{H}$ and $\mathcal{W}$ in $\mathbb{Q}, \ni$ $\mathcal{H} \neq 0_{\mathcal{N}}, \mathcal{W} \neq 0_{\mathcal{N}}, \mathcal{W} = (\mathcal{N}_S\delta - \beta Cl(\mathcal{H}))^c$, and $\mathcal{H} = (\mathcal{N}_S\delta - \beta Cl(\mathcal{W}))^c$.
(f) There exist no $\mathcal{N}_S\delta - \beta CS's$ $\mathcal{H}$ & $\mathcal{W}$ in $\mathbb{Q}, \ni$ $\mathcal{H} \neq 1_{\mathcal{N}}, \mathcal{W} \neq 1_{\mathcal{N}}, \mathcal{W} = (\mathcal{N}_S\delta - \beta Cl(\mathcal{H}))^c$, and $\mathcal{H} = (\mathcal{N}_S\delta -$ ß$Cl(\mathcal{W}))^c$.

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$: Suppose that, $\exists$ $\mathcal{N}_S\mathcal{OS}$ $\mathcal{H} \neq 0_{\mathcal{N}}$ (s. t) $\mathcal{N}_S\delta -$ ß$Cl(\mathcal{H}) \neq 1_{\mathcal{N}}$. Now choose $\mathcal{W} = \mathcal{N}_S\delta -$ ß$Int(\mathcal{N}_S\delta -$ ß$Cl(\mathcal{H}))$. In that case $\mathcal{W}$ is proper $\mathcal{N}_S\mathcal{ROS}$ in $\mathbb{Q}$ which contradicts that $\mathbb{Q}$ is $\mathcal{N}_S\delta -$ ß$sup.conn$-ness.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{c})$: Assume $\mathcal{H} \neq 1_{\mathcal{N}}$ is a $\mathcal{N}_S CS$ in $\mathbb{Q}$. If $\mathcal{W} = \mathcal{H}^c$. So, $\mathcal{W}$ is $\mathcal{N}_S\mathcal{OS}$ in $\mathbb{Q}$ and $\mathcal{W} \neq 0_{\mathcal{N}}$. Thus, $\mathcal{N}_S\delta - \beta Cl(\mathcal{H}) = 1_{\mathcal{N}}$, and $(\mathcal{N}_S\delta -$ ß$Cl(\mathcal{W}))^c = 0_{\mathcal{N}} \Longrightarrow$ $\mathcal{N}_S\delta -$ ß$Int(\mathcal{W}^c) = 0_{\mathcal{N}} \Longrightarrow \mathcal{N}_S\delta -$ ß$Int(\mathcal{H}) = 0_{\mathcal{N}}$.

$(\boldsymbol{c}) \Longrightarrow (\boldsymbol{d})$: Presume $\mathcal{H}$ and $\mathcal{W}$ are $\mathcal{N}_S\mathcal{OS}'s$ in $\mathbb{Q}$ (s. t) $\mathcal{H} \neq 0_{\mathcal{N}} \neq \mathcal{W}$ and $\mathcal{H} \subseteq \mathcal{W}^c$. Since $\mathcal{W}^c$ is an $\mathcal{N}_S CS$ in $\mathbb{Q}$ and $\mathcal{W} \neq 0_{\mathcal{N}}$ implies $\mathcal{W}^c \neq 1_{\mathcal{N}}$, we get $\mathcal{N}_S\delta -$ ß$Int(\mathcal{W}^c) = 0_{\mathcal{N}}$. But, from $\mathcal{H} \subseteq \mathcal{W}^c, 0_{\mathcal{N}} \neq \mathcal{H} = \mathcal{N}_S\delta - \beta Int(\mathcal{H}) \subseteq \mathcal{N}_S\delta - \beta Int(\mathcal{W}^c) = 0_{\mathcal{N}}$, which is a contradiction.

$(\boldsymbol{d}) \Longrightarrow (\boldsymbol{a})$: Assume $0_{\mathcal{N}} \neq \mathcal{H} \neq 1_{\mathcal{N}}$ is $\mathcal{N}_S\mathcal{ROS}$ in $\mathbb{Q}$. If we choose $\mathcal{W} = (\mathcal{N}_S\delta - \beta Cl(\mathcal{H}))^c$, we obtain $\mathcal{W} \neq 0_{\mathcal{N}}$. Otherwise, we get $\mathcal{W} \neq 0_N$ which $\Longrightarrow$ $(\mathcal{N}_S\delta - \beta Cl(\mathcal{W}))^c = 0_{\mathcal{N}}$. That implies $\mathcal{N}_S\delta -$ ß$Cl(\mathcal{H}) = 1_{\mathcal{N}}$. That explains $\mathcal{H} = \mathcal{N}_S\delta - \beta Int(\mathcal{N}_S\delta - \beta Cl(\mathcal{H})) = \mathcal{N}_S\delta - \beta Int(1_{\mathcal{N}}) = 1_{\mathcal{N}}$. But this contradiction to $\mathcal{H} \neq 1_{\mathcal{N}}$. Additionally, $\mathcal{H} \subseteq \mathcal{W}^c$, this is as well contradiction.

$(\boldsymbol{a}) \Longrightarrow (\boldsymbol{e})$: Assume $\mathcal{H}$ & $\mathcal{W}$ are $\mathcal{N}_S\mathcal{OS}'s$ in $\mathbb{Q}$ (s. t) $\mathcal{H} \neq 0_{\mathcal{N}} \neq \mathcal{W}$ and $\mathcal{W} = (\mathcal{N}_S\delta - \beta Cl(\mathcal{H}))^C$ and $\mathcal{H} = (\mathcal{N}_S\delta -$ ß$Int(\mathcal{W}))^C$. Now,

$\mathcal{N}_S\delta - \beta Int(\mathcal{N}_S\delta -$ ß$Cl(\mathcal{H})) = \mathcal{N}_S\delta - \beta Int(\mathcal{W}^c) = (\mathcal{N}_S\delta -$ ß$Cl(\mathcal{W}))^c = \mathcal{H}$ and $\mathcal{H} \neq 0_{\mathcal{N}}$ and $\mathcal{H} \neq 1_{\mathcal{N}}$. Presume not; if $\mathcal{H} = 1_{\mathcal{N}}$, so $1_{\mathcal{N}} = (\mathcal{N}_S\delta - \beta Cl(\mathcal{W}))^C$ implies $0_{\mathcal{N}} = \mathcal{N}_S\delta - \beta Cl(\mathcal{W}) \Longrightarrow \mathcal{W} = 0_{\mathcal{N}}$. This is a contradiction.

$(\boldsymbol{e}) \Longrightarrow (\boldsymbol{a})$: Suppose $\mathcal{H}$ is $\mathcal{N}_S\mathcal{OS}$ in $\mathbb{Q}$ (s.t) $\mathcal{H} = \mathcal{N}_S\delta -$ ß$Int(\mathcal{N}_S\delta - \beta Cl(\mathcal{H}))$, $0_{\mathcal{N}} \neq \mathcal{H} \neq 1_{\mathcal{N}}$.

Now $\mathcal{W} = (\mathcal{N}_S\delta -$ ß$Cl(\mathcal{H}))^c$ and $(\mathcal{N}_S\delta - \beta Cl(\mathcal{W}))^c = (\mathcal{N}_S\delta - \beta Cl(\mathcal{N}_S\delta - \beta Cl(\mathcal{H}))^c)^c = \mathcal{N}_S\delta - \beta Int(\mathcal{N}_S\delta -$ ß$Cl(\mathcal{H})) = \mathcal{H}$. This is a contradiction.

$(\boldsymbol{e}) \Longrightarrow (\boldsymbol{f})$: Presume that, $\mathcal{H}$ and $\mathcal{W}$ be $\mathcal{N}_S CS's$ in $\mathbb{Q}$, (s. t) $\mathcal{H} \neq 1_N \neq \mathcal{W}$ and $\mathcal{W} = (\mathcal{N}_S\delta - \beta Int(\mathcal{H}))^C$, and $\mathcal{H} = (\mathcal{N}_S\delta -$ ß$Int(\mathcal{W}))^C$. Putting $\mathcal{D} = \mathcal{H}^c$ & $\mathcal{C} = \mathcal{W}^c, \mathcal{D}$ & $\mathcal{C}$ become $\mathcal{N}_S\mathcal{OS}'s$ in $\mathbb{Q}$ and $\mathcal{D} \neq 0_{\mathcal{N}} \neq \mathcal{C}, (\mathcal{N}_S\delta -$ ß$Cl(\mathcal{D}))^C = (\mathcal{N}_S\delta -$ ß$Cl(\mathcal{H}^c))^C = ((\mathcal{N}_S\delta -$ ß$Int(\mathcal{H}))^c)^c = \mathcal{N}_S\delta -$ ß$Int(\mathcal{H}) = \mathcal{W}^c = \mathcal{C}$, and in the same way , $(\mathcal{N}_S\delta - \beta Cl(\mathcal{C}))^C = \mathcal{D}$. However this contradiction. $(\boldsymbol{f}) \Longrightarrow (\boldsymbol{e})$: The proof of this part is similar to the above.

5. Diverse Properties of Neutrosophic δ-ß-Compactness

In this part, several fundamental characterizations related to δ-ß-compactness have been investigated in neutrosophic Topological spaces.

Definition 5.1: Let $(\mathbb{Q}, \Phi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$. Then,

(a) A collection $\mathcal{W}$ of $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$'s in $\mathbb{Q}$ is said to be a neutrosophic open cover (concisely, $\mathcal{N}_{\mathcal{S}}\mathcal{O}Cov$) of a subset $\mathcal{W}$ of $\mathbb{Q}$ if $\mathcal{W} \subseteq \cup\{\mathbb{L}_\beta : \mathbb{L}_\beta \in \mathcal{W}\}$.

(b) A collection $\mathcal{W}$ of $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{OS}$'s in $\mathbb{Q}$ is said to be a neutrosophic δ-ß-open cover (concisely, $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{O}Cov$) of a subset $\mathcal{W}$ of $\mathbb{Q}$ if $\mathcal{W} \subseteq \cup\{\mathbb{L}_\beta : \mathbb{L}_\beta \in \mathcal{W}\}$.

Definition 5.2: Let $(\mathbb{Q}, \Phi_{\mathcal{N}})$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$. Then,

(a) $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is said to neutrosophic compact (concisely, $\mathcal{N}_{\mathcal{S}}$Com) if each $\mathcal{N}_{\mathcal{S}}\mathcal{O}Cov$ of $\mathbb{Q}$ has a finite sub-cover.

(b) $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is called neutrosophic δ-ß-compact (concisely, $\mathcal{N}_{\mathcal{S}}\delta -$ ß Com) *if* each $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}Cov$ of $\mathbb{Q}$ has finite sub-cover.

Definition 5.3: A subset $\mathcal{H}$ of $\mathcal{N}_{\mathcal{S}}\mathcal{TS}(\mathbb{Q}, \Phi_{\mathcal{N}})$ is said to:

(a) Neutrosophic compact (concisely, $\mathcal{N}_{\mathcal{S}}$Com) relative to $\mathbb{Q}$ if each $\mathcal{N}_{\mathcal{S}}\mathcal{O}Cov$ of $\mathbb{Q}$ has a finite sub-cover.

(b) Neutrosophic δ-ß-compact (concisely, $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom) with respect to $\mathbb{Q}$ if each $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}Cov$ of $\mathbb{Q}$ has finite sub-cover.

Theorem 5.4: Every $\mathcal{N}_{\mathcal{S}}\delta - \beta$Com -space is $\mathcal{N}_{\mathcal{S}}$Com-space.

Proof: Suppose, $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is a $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom-space and not $\mathcal{N}_{\mathcal{S}}$Com. Consequently, $\nexists$ nonzero $\mathcal{N}_{\mathcal{S}}\mathcal{O}\ Cov\ \mathcal{W}$ of $\mathbb{Q}$ hasn't finite sub-cover. As all $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{OS}$, so obtain $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}Cov\ \mathcal{W}$ of $\mathbb{Q}$, which hasn't finite sub-cover. This a contradiction to $\mathbb{Q}$ is $\mathcal{N}_{\mathcal{S}}$Com-space. Therefore, $\mathbb{Q}$ is $\mathcal{N}_{\mathcal{S}}$Com-space.

Theorem 5.5: A $\mathcal{N}_{\mathcal{S}}\delta - \beta C$ subset of $\mathcal{N}_{\mathcal{S}}\delta -$ ß Com-space $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta - \beta$Com relative to $\mathbb{Q}$.

Proof: Presume $\mathcal{H}$ is a $\mathcal{N}_{\mathcal{S}}C$ subset of $\mathcal{N}_{\mathcal{S}}$Com-space $(\mathbb{Q}, \Phi_{\mathcal{N}})$. In that case $\mathcal{H}^c$ is $\mathcal{N}_{\mathcal{S}}\mathcal{O}$ in $\mathbb{Q}$. Assume $\mathcal{W} = \{\mathbb{L}_i : i \in \Delta\}$ is $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}Cov$ of $\mathcal{H}$. So, $\mathcal{W}\cup\{\mathcal{H}^c\}$ is a $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}Cov$ of $\mathbb{Q}$. Since, $\mathbb{Q}$ is $\mathcal{N}_{\mathcal{S}}$ Com, it has finite sub-cover say $\{\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_n, \mathcal{H}^c\}$. In that case, $\{\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_n\}$ is finite $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}Cov$. Consequently, $\mathcal{H}$ is $\mathcal{N}_{\mathcal{S}}\delta - \beta$Com relative to $\mathbb{Q}$.

Theorem 5.6: If $\mathfrak{F}: (\mathbb{Q}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta - \beta$cont. surjection map and $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta - \beta$Com-space. Then, $(\mathcal{Y}, \Psi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom.

Proof: Presume a map $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßcont. surjection & $\mathbb{Q}$ is $\mathcal{N}_{\mathcal{S}}\delta - \beta$Com. Suppose $\{\mathcal{W}_\beta\}$ be a $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}Cov$ for $\mathcal{Y}$. As $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßcont., consequently $\{\mathfrak{F}^{-1}(\mathcal{W}_\beta)\}$ is a $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}Cov$ of $\mathbb{Q}$.

Since $\mathbb{Q}$ is $\mathcal{N}_{\mathcal{S}}\delta - \beta$Com. so $\{\mathfrak{F}^{-1}(\mathcal{W}_\beta)\}$ contains a finite sub-cover, say, $\{\mathfrak{F}^{-1}(\mathcal{W}_{\beta_1}), \mathfrak{F}^{-1}(\mathcal{W}_{\beta_2}), \dots, \mathfrak{F}^{-1}(\mathcal{W}_{\beta_n})\}$. Since $\mathfrak{F}$ is surjection, so $\{\mathcal{W}_{\beta_1}, \mathcal{W}_{\beta_2}, \dots, \mathcal{W}_{\beta_n}\}$ is finite sub-cover for $\mathcal{Y}$. Consequently, $\mathcal{Y}$ is $\mathcal{N}_{\mathcal{S}}\delta - \beta$Com.

Theorem 5.7: Let $\mathfrak{F}: (\mathbb{Q}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{O}$ map and $(\mathcal{Y}, \Psi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom. Then, $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom.

Proof: Suppose that, a map $\mathfrak{F}$ is a $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{O}$ & $\mathcal{Y}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom. Presume $\{\mathcal{W}_\beta\}$ is a $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{O}Cov$ for $\mathbb{Q}$. As $\mathfrak{F}$ is a $\mathcal{N}_{\mathcal{S}}\delta - \beta\mathcal{O}$ map, as a result $\{\mathfrak{F}(\mathcal{W}_\beta)\}$ is a $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{O}Cov$ of $\mathcal{Y}$. Since $\mathcal{Y}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom. so $\{\mathfrak{F}(\mathcal{W}_\beta)\}$ contains a finite sub-cover, called, $\{\mathfrak{F}(\mathcal{W}_{\beta_1}), \mathfrak{F}(\mathcal{W}_{\beta_2}), \dots, \mathfrak{F}(\mathcal{W}_{\beta_n})\}$. In that case $\{\mathcal{W}_{\beta_1}, \mathcal{W}_{\beta_2}, \dots, \mathcal{W}_{\beta_n}\}$ is a finite sub-cover for $\mathbb{Q}$. Consequently, $\mathbb{Q}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom.

Theorem 5.8: Let $\mathfrak{F}: (\mathbb{Q}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}\delta - \beta$cont -map from a $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom. space $(\mathbb{Q}, \Phi_{\mathcal{N}})$ onto $(\mathcal{Y}, \Psi_{\mathcal{N}})$ .Then the image of a $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom -space is $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom-space.

Proof: Presume that a map $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßcont, from a $\mathcal{N}_{\mathcal{S}}\delta - \beta$Com -space $(\mathbb{Q}, \Phi_{\mathcal{N}})$ onto $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Presume, $\{\mathbb{L}_i : i \in \Delta\}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{O}Cov$ of $(\mathcal{Y}, \Psi_{\mathcal{N}})$. As $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßcont., thus $\{\mathfrak{F}^{-1}(\mathbb{L}_i) : i \in \Delta\}$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{O}Cov$ of $(\mathbb{Q}, \Phi_{\mathcal{N}})$. As $(\mathbb{Q}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom, the $\mathcal{N}_{\mathcal{S}}\delta -$ ß$\mathcal{O}Cov\{\mathfrak{F}^{-1}(\mathbb{L}_i) : i \in \Delta\}$ of $(\mathbb{Q}, \Phi_{\mathcal{N}})$ has finite sub-cover $\{\mathfrak{F}^{-1}(\mathbb{L}_i) : i = 1,2,3, \dots, n\}$. So, $\mathbb{L} = \cup_{i \in \Delta} \mathfrak{F}^{-1}(\mathbb{L}_i)$.

In that case $\mathfrak{F}(\mathbb{L}) = \cup_{i \in \Delta} \mathbb{L}_i$ that is, $\mathcal{W} = \cup_{i \in \Delta} \mathbb{L}_i$. Consequently, $\{\mathbb{L}_1, \mathbb{L}_2, \dots, \mathbb{L}_n\}$ is finite sub-cover of $\{\mathbb{L}_i : i \in \Delta\}$ for $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Thus, $\mathcal{Y}, \Psi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ßCom-space.

6. Conclusions

Neutrosophic set theory plays a pivotal role in addressing numerous complex applications across engineering, environment science, economics and various advanced branches of mathematics. In this manuscript, we have explored novel classes of neutrosophic connectedness and compactness, specifically, neutrosophic δ-ß-connectedness and neutrosophic δ-ß-compactness, defined in the context of neutrosophic δ-ß-open sets. Furthermore, we have established the normality and regularity of neutrosophic δ-ß-open sets through this investigation. We anticipate that the findings presented in this manuscript will contribute to the development of information systems and have a positive impact on diverse fields within engineering, physics, and computer sciences. Additionally, we hope this research will stimulate further scientific exploration in the realm of neutrosophic topology, ultimately fostering the development of a comprehensive framework for practical implementations in these fields.

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