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Some New Kind of Contra Continuous Functions in Nano Ideal Topological Spaces

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Abstract The main objective of this paper is to introduce a new type of contra continuous function namely $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuous$ based on the concept of $NI\mathcal{M}_{\mathcal{V}}-open$ set and $NI\mathcal{M}_{\mathcal{V}}-continuous$ function in Nano Ideal Topological Spaces. The conceptualisation of contra continuous functions, which is an alteration of continuity that requires inverse images of open sets to be closed rather than open. We compare $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuous$ function with $CN-continuous$ function and establish the independent relation between $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuous$ and $CNI-continuous$ functions by providing suitable counter examples. Fundamental properties of $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuity$ with $NI\mathcal{M}_{\mathcal{V}}-closure$ and $N-Kernel$ are investigated. We study the behaviour of $NI\mathcal{M}_{\mathcal{V}}-interiority\ condition$ with $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuity$. We define $NI\mathcal{M}_{\mathcal{V}}-T_2$ space and describe its relation upon $nano-Urysohn$ space and $nano-Ultra\ Hausdorff$ space. Characterizations of $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuity$ based on $nano-Urysohn$ space, $nano-Ultra\ Hausdorff$ space and graph function namely $Contra\ NI\mathcal{M}_{\mathcal{V}}-closed$ are explored. As like the continuity, the $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuity$ preserves the property that it maps $NI\mathcal{M}_{\mathcal{V}}-connected$ and $NI\mathcal{M}_{\mathcal{V}}-compact$ sets to the same type of sets in co-domain. We defined $NI\mathcal{M}_{\mathcal{V}}-normal$ space and described its nature over $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuity$. Also we have introduced $Contra\ NI\mathcal{M}_{\mathcal{V}}-irresolute$ functions with an example and discussed its relation with $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuity$ and analysed its basic properties. Composition of functions under $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuous$, $Contra\ NI\mathcal{M}_{\mathcal{V}}-irresolute$ and $CN-continuous$ are examined.

Keywords $Contra\ NI\mathcal{M}_{\mathcal{V}}-continuous$, $NI\mathcal{M}_{\mathcal{V}}-normal$, $Contra\ NI\mathcal{M}_{\mathcal{V}}-irresolute$, $Contra\ NI\mathcal{M}_{\mathcal{V}}-closed$

1. Introduction

The conception of $ideal\ topology$ was intiated by Kuratowski [1]. Jankovic and Hamlett developed the idea of Kuratowksi closure operator in [2]. The concept of $contra\ continuous$ function was introduced by Dontchev J in [3]. The $Lower(upper)$ approximations were established by Z. Pawlak [4]. Lellis Thivagar and Carmel Richard introduced the $Nano\ Topological\ Spaces$ (briefly, NTS) in [5] and $nano\ continuity$ in [6]. Later Karthiksankar P [7] defined the $nano-clopen$ sets in NTS. M. Parimala introduced Nano Ideal Topological Spaces (briefly, $NITS$) in [8], $\mathcal{N}-local\ function$, $NI-open$ and $NI-continuous\ function$ in [9] and $closure\ operator$ $cl_N{}^*(.)$ in [10] which is defined as follows: $A_N{}^*(\mathcal{I},\mathcal{N}) = \{u \in \mathcal{U} : V \cap A \notin \mathcal{I}\ for\ every\ V \in V_N(u)\}$,where $V_N(u) = \{\ V : u \in V\ and\ V \in \mathcal{N}\}, cl_N{}^*(A) = A \cup A_N{}^*$, for $A \subseteq \mathcal{U}$ and analysed its properties. New variants of nano ideal continuous functions were introduced by Abd El-Fattah, A. El-Atik and Hanan Z. Hassan [11]. The connectivity in Nano Topological Spaces like Nano I-connectedness and Strongly Nano I-connectedness were defined by S. Gunavathy, R. Alagar, Aiyared Iampan, Vediyappan Govindan [12]. In this work, we introduce the classes of $Contra\ NI\mathcal{M}_\mathcal{V}-continuity$ and compare with existing functions and examine its characterizations. Also we have defined $Contra\ NI\mathcal{M}_\mathcal{V}-irresoluteness$ and studied the relationship among these functions.

2. Preliminaries

Definition 2.1: [13] A subset H of a $NTS\ (\mathcal{U},\mathcal{N})$ is said to be $N-connected$ if it cannot be written as the union of two $N-separated$ sets. Otherwise, the set S is called $N-disconnected$.

Definition 2.2: [14] A function $f: (\mathcal{V},\tau_\mathcal{R}(Y)) \to (\mathcal{W},\tau_{\mathcal{R}'}(Z))$ is $CN-Continuous$ (briefly $CN-Cts$) if $f^{-1}(M)$ is $nano-closed$ in $(V,\tau_\mathcal{R}(Y))$ for every $nano-open$ set M in $(\mathcal{W},\tau_{\mathcal{R}'}(Z))$.

Definition 2.3: [14] Let $(\mathcal{U},\tau_\mathcal{R}(Z))$ be a NTS and $H \subseteq \mathcal{U}$. Then $nano-kernel$ of H is defined as $Ker_N(H) = \cap\{V: H \subseteq V, V \in \tau_\mathcal{R}(Z)\}$.

Theorem 2.1: [14] Let $(\mathcal{U},\tau_\mathcal{R}(Z))$ be a NTS and $H_1, H_2 \subseteq \mathcal{U}$. Then

(i) $u \in Ker_N(H_1)$ if and only if for any $nano-closed$ set P containing u, $H_1 \cap P = \emptyset$.
(ii) If $H_1 \subseteq Ker_N(H_1)$ and then $H_1 = Ker(H_1)$ if H_1 is $nano-open$ in $\mathcal{U}$.
(iii) If $H_1 \subseteq H_2$, then $Ker_N(H_1) \subseteq Ker_N(H_2)$.

Definition 2.4: [15] A $NTS\ (\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is said to be $nano-Urysohn\ space$ if for any two distinct points $x,y \in \mathcal{U}$, there exists disjoint $nano-open$ subsets $x \in A, y \in B$ such that the $nano-closures\ \bar{A}\ and\ \bar{B}$ are disjoint $nano-closed$ subsets of $\mathcal{U}$.

Definition 2.5: [15] A $NTS\ (\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is said to be $nano-Ultra\ Hausdorff\ space$ if for every pair of distinct points x and y in $\mathcal{U}$, there exist disjoint $nano-clopen$ sets A and B in $\mathcal{U}$ containing x and y respectively.

Definition 2.6: [15] A $NTS\ (\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is said to be $nano-Ultra\ normal$ if for any two disjoint non-empty $nano-closed$ sets S and T, there exists $nano-clopen$ sets A of S and B of T such that $A \cap B = \emptyset$.

Definition 2.7: [16] Let $(\mathcal{U},\mathcal{N},\mathcal{I})$ be a $NITS$ and $H \subseteq \mathcal{U}$. Then $NIcl_{\mathcal{M}_\mathcal{V}}(H) = \{u \in \mathcal{U} : int^*_{\tau_N}(cl_N{}^*(V)) \cap H \neq \emptyset$, for each $V \in V_N(u)\}$, where $V_N(u) = \{V : u \in V\ and\ V \in \mathcal{N}\}$ and $\tau_N{}^*(\mathcal{I}) = \{G \subseteq \mathcal{U} : cl_N{}^*(\mathcal{U}-G) = \mathcal{U}-G\}$. Here $int^*_{\tau_N}(V)$ denotes the $interior$ of V in $\tau_N{}^*(\mathcal{I})$. If $NIcl_{\mathcal{M}_\mathcal{V}}(H) = H$ then H is a $NI\mathcal{M}_\mathcal{V}-closed\ set$ and its complement is $NI\mathcal{M}_\mathcal{V}-open$. The family of all $NI\mathcal{M}_\mathcal{V}-open$ sets of a $NITS$ is denoted by $NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$.

Definition 2.8: [17] A function $f: (\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y),\mathcal{J})$ is $NI\mathcal{M}_\mathcal{V}-continuous$ (briefly $NI\mathcal{M}_\mathcal{V}-continuous$) if $f^{-1}(M)$ is $NI\mathcal{M}_\mathcal{V}-open$ in $(\mathcal{U},\tau_{\mathcal{R}'}(Z),\mathcal{I})$ for every $nano\ open$ set M in $(\mathcal{V},\tau_{\mathcal{R}'}(Y),\mathcal{J})$ (i.e) if $f^{-1}(M) \in NI\mathcal{M}_\mathcal{V}O(\mathcal{U})$ for all $M \in \tau_{\mathcal{R}'}(Y)$.

Definition 2.9: [18] A function $g : (\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y),\mathcal{J})$ is said to be $NI\mathcal{M}_\mathcal{V}-irresolute$ if $g^{-1}(E)$ is $NI\mathcal{M}_\mathcal{V}-open$ set in $(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ for every $NI\mathcal{M}_\mathcal{V}-open$ set E in $(\mathcal{V},\tau_{\mathcal{R}'}(Y),\mathcal{J})$.

3. Contra Continuity via Nano Ideal

Definition 3.1: A function $g: (\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y))$ is $Contra\ NI\mathcal{M}_\mathcal{V}-continuous$ (briefly, $CNI_{\mathcal{M}_\mathcal{V}}-continuous$) if $g^{-1}(S)$ is $NI\mathcal{M}_\mathcal{V}-closed$ in $(\mathcal{U},\tau_{\mathcal{R}'}(Z),\mathcal{I})$ for every $nano-open$ set S in $(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ (i.e) if $g^{-1}(S) \in NI\mathcal{M}_\mathcal{V}C(\mathcal{U},Z)$ for all $S \in \tau_{\mathcal{R}'}(Y)$.

Example 3.1: Let $\mathcal{U} = \{e,k,p,t\}\ with\ \mathcal{U}/\mathcal{R} = \{\{e,k\},\{p\},\{t\}\}\ and\ Z = \{k,t\} \subseteq \mathcal{U}\ \tau_\mathcal{R}(Z) = \{\emptyset,\mathcal{U},\{t\},\{e,k\},\{e,k,t\}\}$ and the $ideal\ \mathcal{I} = \{\emptyset,\{e\},\{k\},\{p\},\{e,k\},\{k,p\},\{e,p\},\{e,k,p\}\}\ NI\mathcal{M}_\mathcal{V}O(Z) = \{\emptyset,\mathcal{U},\{e,k\}\}$ $NI\mathcal{M}_\mathcal{V}C(Z) = \{\emptyset,\mathcal{U},\{p,t\}\}$. Let $\mathcal{V} = \{e,k,p,t\}\ with\ \mathcal{V}/\mathcal{R}' = \{\{e,k\},\{p\},\{t\}\}\ and\ Y = \{e,k\} \subseteq \mathcal{V}\ \tau_{\mathcal{R}'}(Y) = \{\emptyset,\mathcal{V},\{e,k\}\}$. Define $g: \mathcal{U} \to \mathcal{V}$ as $g(e) = p, g(k) = t, g(p) = k, g(t) = e$. Then $g^{-1}(T)$ is $NI\mathcal{M}_\mathcal{V}-closed$ in $\mathcal{U}$ whenever T is $nano-open$ in $\mathcal{V}$. Therefore g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$.

Proposition 3.1: Any $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function is $CN-continuous$.

Proof: Let $g: (\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y))$ be any $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function and let T be $nano-open$ in $\mathcal{V}$. Since, g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$, $g^{-1}(T)$ is $NI\mathcal{M}_\mathcal{V}-closed$ in $\mathcal{U}$. Since each $NI\mathcal{M}_\mathcal{V}-closed$ set is $nano-closed$. Hence $g^{-1}(T)$ is $nano-closed$ in $\mathcal{U}$. Hence g is $CN-continuous$.

Remark 3.1: The converse of $Proposition$ 3.1 need not be true as is evidenced below.

Example 3.2: Let $\mathcal{U} = \{e,k,p,t\}\ with\ \mathcal{U}/\mathcal{R} = \{\{e,k\},\{p\},\{t\}\}\ and\ Z = \{k,p\} \subseteq \mathcal{U}$ $\tau_\mathcal{R}(Z) = \{\emptyset,\mathcal{U},\{p\},\{e,k\},\{e,k,p\}\}$, $\tau^c_\mathcal{R}(Z) = \{\emptyset,\mathcal{U},\{t\},\{p,t\},\{e,k,t\}\}$ and the $ideal\ \mathcal{I} = \{\emptyset,\{e\},\{k\},\{e,k\}\}\ NI\mathcal{M}_\mathcal{V}O(Z) = \{\emptyset,\mathcal{U},\{e,k\}\}\ NI\mathcal{M}_\mathcal{V}C(Z) = \{\emptyset,\mathcal{U},\{p,t\}\}$. Let $\mathcal{V} =$

$\{e,k,p,t\}$ with $\mathcal{V}/\mathcal{R}' = \{\{e,p,t\},\{k\}\}$ and $Y = \{e,p,t\} \subseteq \mathcal{V}$ $\tau_{\mathcal{R}'}(Y) = \{\emptyset,\mathcal{V},\{e,p,t\}\}$. Define $g:\mathcal{U} \to \mathcal{V}$ as $g(e) = t, g(k) = p, g(p) = k, g(t) = e$. Clearly g is $CN-continuous$ but not $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ as the inverse image of $\{e,p,t\}$ (i.e) $g^{-1}(\{e,p,t\}) = \{e,k,t\}$ is $nano-closed$ but not $NI\mathcal{M}_\mathcal{V}-closed$.

Theorem 3.1: $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ and $CNI-continuous$ functions are independent of each other.

Example 3.3: (i) Let $\mathcal{U} = \{e,k,p,t\}$ $with$ $\mathcal{U}/\mathcal{R} = \{\{e,k\},\{p\},\{t\}\}$ and $Z = \{k,p\} \subseteq \mathcal{U}$ $\tau_{\mathcal{R}}(Z) = \{\emptyset,\mathcal{U},\{p\},\{e,k\},\{e,k,p\}\}$ and the $ideal$ $\mathcal{I} = \{\emptyset,\{e\},\{k\},\{e,k\}\}$ $NIC(Z) = \{\emptyset,\mathcal{U},\{e,k,t\}\}$, $NI\mathcal{M}_\mathcal{V}C(Z) = \{\emptyset,\mathcal{U},\{p,t\}\}$. Let $\mathcal{V} = \{e,k,p,t\}$ $with$ $\mathcal{V}/\mathcal{R}' = \{\{e\},\{k,p,t\}\}$ and $Y = \{k\} \subseteq \mathcal{V}$, $\tau_{\mathcal{R}'}(Y) = \{\emptyset,\mathcal{V},\{k,p,t\}\}$. Define $g:\mathcal{U} \to \mathcal{V}$ as $g(e) = k, g(k) = p, g(p) = e, g(t) = t$. Clearly g is $CNI-continuous$ but not $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ as the inverse image of $\{k,p,t\}$ (i.e) $g^{-1}(\{k,p,t\}) = \{e,k,t\}$ is $NI-closed$ but not $NI\mathcal{M}_\mathcal{V}-closed$.

(ii) $\mathcal{U} = \{e,k,p,t\}$ $with$ $\mathcal{U}/\mathcal{R} = \{\{e\},\{k,p\},\{t\}\}$ and $Z = \{e,k\} \subseteq \mathcal{V}$ $\tau_{\mathcal{R}}(Z) = \{\emptyset,\mathcal{U},\{e\},\{k,p\},\{e,k,p\}\}$ and the $ideal$ $\mathcal{I} = \emptyset,\{e\},\{k\},\{\{p\},\{e,k\},\{k,p\},\{e,p\},\{e,k,p\}\}$ $NI\mathcal{M}_\mathcal{V}C(\mathcal{U},Z) = \{\emptyset,\mathcal{U},\{t\},\{e,t\},\{k,p,t\}\}$, $NIC(Z) = \{\emptyset,\mathcal{U}\}$ Let $\mathcal{V} = \{e,k,p,t\}$ $with$ $\mathcal{V}/\mathcal{R}' = \{\{e\},\{k\},\{p\},\{t\}\}$ and $Y = \{k,p\} \subseteq \mathcal{V}$, $\tau_{\mathcal{R}'}(Y) = \{\emptyset,\mathcal{V},\{k,p\}\}$. Define $g:\mathcal{U} \to \mathcal{V}$ as $g(e) = k, g(k) = t, g(p) = e, g(t) = p$. Clearly g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ but not $CNI-continuous$ as the inverse image of $\{k,p\}$ (i.e) $g^{-1}(\{k,p\}) = \{e,t\}$ is $NI\mathcal{M}_\mathcal{V}-closed$ but not $NI-closed$.

Theorem 3.2: Let $g:(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y))$, then the following statements are equivalent:

(i) g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$.

(ii) for each $nano-closed$ subset S of $\mathcal{V}$, $g^{-1}(S) \in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$.

(iii) for each $z \in \mathcal{U}$ and each $nano-closed$ set S of $\mathcal{V}$ containing $g(z)$, there exists $H \in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$ such that $g(H) \subset S$.

(iv) $g(NIcl_{\mathcal{M}_\mathcal{V}}(E)) \subset Ker_N(g(E))$ for each $E \subset \mathcal{U}$.

(v) $NIcl_{\mathcal{M}_\mathcal{V}}(g^{-1}(G)) \subset g^{-1}(Ker_N(G))$ for each $\mathcal{V} \subset \mathcal{U}$.

Proof: $(i) \Longrightarrow (ii)$ and $(ii) \Longrightarrow (iii)$ are obvious.

$(iii) \Longrightarrow (ii)$ Let S be any $nano-closed$ set of $\mathcal{V}$ and $z \in g^{-1}(S)$. Then $g(z) \in S$ and there exists $H_z \in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$ such that $g(H_z) \subset S$. Therefore, $g^{-1}(S) = \bigcup\{H_z : z \in g^{-1}(S)\}$. Hence, $g^{-1}(S) \in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$.

$(ii) \Longrightarrow (iv)$ Let $E \subset \mathcal{U}$. If $w \notin Ker_N(g(E))$, then by Theorem 2.1, there exists a $nano-closed$ set S of $\mathcal{V}$ containing w such that $g(E) \cap S = \emptyset$. Therefore $E \cap g^{-1}(S) = \emptyset$ and $NIcl_{\mathcal{M}_\mathcal{V}}(E) \cap g^{-1}(S) = \emptyset$. Hence $g(NIcl_{\mathcal{M}_\mathcal{V}}(E)) \cap S = \emptyset$ and $w \notin g(NIcl_{\mathcal{M}_\mathcal{V}}(E))$. Thus $g(NIcl_{\mathcal{M}_\mathcal{V}}(E)) \subset Ker_N(g(E))$.

$(iv) \Longrightarrow (v)$ Let $G \subset \mathcal{V}$. By hypothesis and Theorem 2.1, $g(NIcl_{\mathcal{M}_\mathcal{V}}(g^{-1}(G))) \subset Ker_N(g(g^{-1}(G))) \subset Ker_N(G)$ and $NIcl_{\mathcal{M}_\mathcal{V}}(g^{-1}(G)) \subset g^{-1}(Ker_N(G))$.

$(v) \Longrightarrow (i)$ Let G be a $nano-open$ set of $\mathcal{V}$. By Theorem 2.1, $NIcl_{\mathcal{M}_\mathcal{V}}(g^{-1}(G)) \subset g^{-1}(Ker_N(G)) = g^{-1}(G)$ and $NIcl_{\mathcal{M}_\mathcal{V}}(g^{-1}(G)) = g^{-1}(G)$. Therefore $g^{-1}(G)$ is $NI\mathcal{M}_\mathcal{V}-closed$ in $(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I})$.

Definition 3.2: A function $g:(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y))$ satisfy the $NI\mathcal{M}_\mathcal{V}-interiority\ condition$ if $NIint_{\mathcal{M}_\mathcal{V}}\left(g^{-1}(cl_N(O))\right) \subset g^{-1}(O)$ whenever O is $nano-open$ set of $(\mathcal{V},\tau_{\mathcal{R}'}(Y))$.

Theorem 3.3: If a function $g:(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y))$ is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ and satisfies $NI\mathcal{M}_\mathcal{V}-interiority\ condition$, then g is $NI\mathcal{M}_\mathcal{V}-continuous$.

Proof: Let W be any $nano-open$ set of $\mathcal{V}$. Since g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ and $cl_N(W)$ is $nano-closed$, by Theorem 3.2, $g^{-1}(cl_N(W))$ is $NI\mathcal{M}_\mathcal{V}-open$ in $(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I})$. By hypothesis of g, $g^{-1}(W) \subset g^{-1}(cl_N(W)) \subset NIint_{\mathcal{M}_\mathcal{V}}\left(g^{-1}(cl_N(W))\right) \subset NIint_{\mathcal{M}_\mathcal{V}}(g^{-1}(W)) \subset g^{-1}(W)$. Thus $g^{-1}(W) = NIint_{\mathcal{M}_\mathcal{V}}(g^{-1}(W))$ and so $g^{-1}(W) \in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$. Therefore g is $NI\mathcal{M}_\mathcal{V}-continuous$.

Theorem 3.4: Let $(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I})$ be any $NITS$ and $g:(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y))$ be a function and the $graph\ function$ $f:\mathcal{U} \to \mathcal{U} \times \mathcal{V}$, defined by $f(u) = (u,f(u))$ for each $u \in \mathcal{U}$. Then g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ if and only if f is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$.

Proof: Let $u \in \mathcal{U}$ and F be any $nano-closed$ set in $\mathcal{U} \times \mathcal{V}$ containing $f(u)$. Then $F \cap (\{u\} \times \mathcal{V})$ is $nano-closed$ in $\{u\} \times \mathcal{V}$ containing $f(u)$. Also $\{u\} \times \mathcal{V}$ is homeomorphic to $\mathcal{V}$. Hence $\{v \in \mathcal{V} : (u,v) \in F\}$ is a $nano-closed$ subset of $\mathcal{V}$. Since g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$, $\bigcup\{g^{-1}(V) \in \mathcal{U} : (u,v) \in F\}$ is a $NI\mathcal{M}_\mathcal{V}-open$ subset of $(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I})$. Further, $u \in \bigcup\{g^{-1}(V) \in \mathcal{U} : (u,v) \in F\} \subset f^{-1}(F)$. Hence $f^{-1}(F)$ is $NI\mathcal{M}_\mathcal{V}-open$. Then f is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$.

Conversely, let B be a $nano-closed$ subset of $\mathcal{V}$. Then $\mathcal{U} \times B$ is a $nano-closed$ subset of $\mathcal{U} \times \mathcal{V}$. Since f is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$, $f^{-1}(\mathcal{U} \times B)$ is a $NI\mathcal{M}_\mathcal{V}-open$ subset of $\mathcal{U}$. Also, $f^{-1}(\mathcal{U} \times B) = g^{-1}(B)$. Hence g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$.

Definition 3.3: A $NITS$ $(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I})$ is said to be $NI\mathcal{M}_\mathcal{V}-T_2$ if for any two distinct points $v,w \in \mathcal{U}$, there exists $NI\mathcal{M}_\mathcal{V}-open$ sets V and W containing v and w such that $V \cap W = \emptyset$.

Theorem 3.5: If $(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I})$ is a $NITS$ and for any two points $u_1,u_2 \in \mathcal{U}$ with $u_1 \neq u_2$, there exists a function g into a $nano-Urysohn$ space $(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ such that $g(u_1) \neq g(u_2)$ and g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ at u_1,u_2. Then $(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I})$ is $NI\mathcal{M}_\mathcal{V}-T_2$.

Proof: Let $u_1,u_2 \in \mathcal{U}$ with $u_1 \neq u_2$. Then by hypothesis, there exists a $nano-Urysohn$ space $(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ and a function $g:(\mathcal{U},\tau_{\mathcal{R}}(Z),\mathcal{I}) \to (\mathcal{V},\tau_{\mathcal{R}'}(Y))$ which satisfies the required condition. Let $v_j = g(u_j)$ for $j = 1,2$. Then $v_1 \neq v_2$. Since

$(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ is $nano-Urysohn$, there exists a $nano-open\,nbds\,V_{v_1}$ and V_{v_2} of v_1 and v_2 in $\mathcal{V}$ such that $cl_N(V_{v_1})\cap cl_N(V_{v_2})=\emptyset$. Since g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ at u_j, there exists $NI\mathcal{M}_\mathcal{V}-open\,nbds\,U_{u_1}$ of u_j in $\mathcal{U}$ such that $g(U_{u_j})\subset cl_N\left(V_{v_j}\right)$ for $j=1,2$. Hence we get $U_{u_1}\cap U_{u_2}=\emptyset$ because $cl_N(V_{v_1})\cap cl_N(V_{v_2})=\emptyset$. Therefore $(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is $NI\mathcal{M}_\mathcal{V}-T_2$.

Corollary 3.1: If g is a $injective\,CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function of a $NITS\,(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ into a $nano-Urysohn$ space $(\mathcal{V},\tau_{\mathcal{R}'}(Y))$, then $(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is $NI\mathcal{M}_\mathcal{V}-T_2$.

Proof: For any two points u_1,u_2 in $\mathcal{U}$ with $u_1\neq u_2$, g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function of $\mathcal{U}$ into a $nano-Urysohn$ space $(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ such that $g(u_1)\neq g(u_2)$ because g is $injective$. By Theorem 3.5, $(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is $NI\mathcal{M}_\mathcal{V}-T_2$.

Theorem 3.6: If g is a $injective\,CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function of a $NITS\,(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ into a $nano-Ultra\,Hausdorff$ space $(\mathcal{V},\tau_{\mathcal{R}'}(Y))$, then $(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is $NI\mathcal{M}_\mathcal{V}-T_2$.

Proof: Let u_1,u_2 be the pair of distinct points in $\mathcal{U}$. Since g is $injective$, $\mathcal{V}$ is $nano-Ultra\,Hausdorff, g(u_1)\neq g(u_2)$ there exists $nano-clopen$ sets X_1,X_2 such that $g(u_1)\in X_1, g(u_2)\in X_2$ and $X_1\cap X_2=\emptyset$. Then $u_j\in g^{-1}(X_j)\in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$ for $j=1,2$ and $g^{-1}(X_1)\cap g^{-1}(X_2)=\emptyset$. Therefore $(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is $NI\mathcal{M}_\mathcal{V}-T_2$.

Definition 3.4: Let $g:(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})\to(\mathcal{V},\tau_{\mathcal{R}'}(Y))$. The graph $G(g)$ of the function g is $CNI_{\mathcal{M}_\mathcal{V}}-closed$ in $\mathcal{U}\times\mathcal{V}$ if for any $(u_1,u_2)\in(\mathcal{U}\times\mathcal{V})\backslash G(g)$, there exists $H\in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$ and a $nano-closed$ set S of $\mathcal{V}$ containing u_2 such that $(\mathcal{U}\times\mathcal{V})\cap G(g)=\emptyset$.

Lemma 3.1: Let $g:(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})\to(\mathcal{V},\tau_{\mathcal{R}'}(Y))$. The graph $G(g)$ of the function g is $CNI_{\mathcal{M}_\mathcal{V}}-closed$ in $\mathcal{U}\times\mathcal{V}$ if and only if for each $(u_1,u_2)\in(\mathcal{U}\times\mathcal{V})\backslash G(g)$, there exists $H\in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},Z)$ such that $g(H)\cap cl_N(S)=\emptyset$ where S is a $nano-closed$ subset of $\mathcal{U}\times\mathcal{V}$ containing $g(u_1)$.

Proof: We shall prove that $g(H)\cap cl_N(S)=\emptyset$, $(H\times S)\cap G(g)=\emptyset$. Let $(H\times S)\cap G(g)\neq\emptyset$. Then there exists $(u_1,u_2)\in(H\times S)$ and $(u_1,u_2)\in G(g)$. Hence $u_1\in H, u_2\in S$ and $u_2=g(u_1)\in S$. Therefore, $g(H)\cap cl_N(S)=\emptyset$. Converse part can be done in a similar way.

Theorem 3.7: If $g:(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})\to(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ is a $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function and $\mathcal{V}$ is a $nano-Urysohn$ space, then $G(g)$ is $CNI_{\mathcal{M}_\mathcal{V}}-closed$ in $\mathcal{U}\times\mathcal{V}$.

Proof: Let $(u_1,u_2)\in(\mathcal{U}\times\mathcal{V})\backslash G(g)$. Then $u_2\neq g(u_1)$ and there exists $nano-open$ sets E,F of $\mathcal{V}$ such that $g(u_1)\in E$, $u_2\in F$ and $cl_N(E)\cap cl_N(F)=\emptyset$. Since g is $CNI_{\mathcal{M}_\mathcal{V}}-Cts$, there exists $W\in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},u_1)$ such that $g(W)\subset cl_N(E)$. Therefore $g(W)\cap cl_N(F)=\emptyset$. Hence $G(g)$ is $CNI_{\mathcal{M}_\mathcal{V}}-closed$.

Theorem 3.8: If $g:(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})\to(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ is a $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function and $(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ is T_2, then $G(g)$ is $CNI_{\mathcal{M}_\mathcal{V}}-closed$.

Proof: Let $(u_1,u_2)\in(\mathcal{U}\times\mathcal{V})\backslash G(g)$. Then $u_2\neq g(u_1)$ and there exists a $nano-open$ set E of $\mathcal{V}$ such that $g(u_1)\in E$, $u_2\notin E$. Since g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$, there exists $W\in NI\mathcal{M}_\mathcal{V}O(\mathcal{U},u_1)$ such that $g(W)\subset cl_N(E)$. Therefore $g(W)\cap(\mathcal{V}-E)=\emptyset$ and $\mathcal{V}-E$ is a $nano-closed$ set of $\mathcal{V}$ containing u_2. Hence $G(g)$ is $CNI_{\mathcal{M}_\mathcal{V}}-closed$.

Theorem 3.9: If $g:(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})\to(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ is a $surjective\,CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function and $\mathcal{U}$ is $NI\mathcal{M}_\mathcal{V}-connected$, then $\mathcal{V}$ is $N-connected$.

Proof: Let $g:(\mathcal{U},\tau_\mathcal{R}(Z_1),\mathcal{I})\to\left(\mathcal{V},\tau_{\mathcal{R}'}(Z_2)\right)$ be a $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function from a $NI\mathcal{M}_\mathcal{V}-connected$ space $\mathcal{U}$ onto a $NTS\,\mathcal{V}$. Assume that $\mathcal{V}$ is $N-disconnected$. Then $\mathcal{V}=H\cup T$, where H and T are non-empty $nano-clopen$ sets in $\mathcal{V}$ with $H\cap T=\emptyset$. Since g is $CNI_{\mathcal{M}_\mathcal{V}}-Cts$, we have that $g^{-1}(H)$ and $g^{-1}(T)$ are non-empty $NI\mathcal{M}_\mathcal{V}-open$ sets in $\mathcal{U}$ with $g^{-1}(H)\cap g^{-1}(T)=g^{-1}(H\cup T)=g^{-1}(\mathcal{V})=\mathcal{U}$ and $g^{-1}(H)\cap g^{-1}(T)=g^{-1}(H\cap T)=g^{-1}(\emptyset)=\emptyset$. Hence $\mathcal{U}$ is not $NI\mathcal{M}_\mathcal{V}-connected$, which is a contradiction. Therefore, $\mathcal{V}$ is $N-connected$.

Definition 3.5: A $NITS\,(\mathcal{U},\tau_\mathcal{R}(Z_1),\mathcal{I})$ is said to be $NI\mathcal{M}_\mathcal{V}-space$ if every $nano-open$ set is $NI\mathcal{M}_\mathcal{V}-open$ in $(\mathcal{U},\tau_\mathcal{R}(Z_1),\mathcal{I})$.

Theorem 3.10: A function $g:(\mathcal{U},\tau_\mathcal{R}(Z_1),\mathcal{I})\to(\mathcal{V},\tau_{\mathcal{R}'}(Z_2),\mathcal{J})$ is $CN-continuous$ and $\mathcal{U}$ is $NI\mathcal{M}_\mathcal{V}-space$, then g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$.

Proof: Let T be $nano-closed$ set in $\mathcal{V}$. Since g is $CN-continuous$, $g^{-1}(T)$ is $nano-open$ in $\mathcal{U}$. Since $\mathcal{U}$ is a $NI\mathcal{M}_\mathcal{V}-space$, $g^{-1}(T)$ is $NI\mathcal{M}_\mathcal{V}-open$ in $\mathcal{U}$. Therefore, g is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$.

Definition 3.6: A collection of $closed$ sets $H=\{H_\lambda:\lambda\in\wedge\}$ in a $NITS\,(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is termed to be $NI\mathcal{M}_\mathcal{V}-closed\,cover$ of subset E of $\mathcal{U}$ if $E\subset\bigcup\{H_\lambda:H_\lambda\in NI\mathcal{M}_\mathcal{V}C(\mathcal{U},Z),\lambda\in\wedge\}$.

Definition 3.7: A $NITS\,(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is said to be $NI\mathcal{M}_\mathcal{V}-closed\,compact$ if for every $NI\mathcal{M}_\mathcal{V}-closed\,cover\,\{S_j:j\in\Delta\}$ of $\mathcal{U}$, there exists a finite subset $\Delta_1\subset\Delta$ such that $\mathcal{U}-\bigcup\{V_j:j\in\Delta_1\}\in\mathcal{I}$.

Theorem 3.11: If $g:(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})\longrightarrow(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ is $CNI_{\mathcal{M}_\mathcal{V}}-continuous$ and the set E is $NI\mathcal{M}_\mathcal{V}-closed\,compact\,relative$ to $\mathcal{U}$, then $g(E)$ is $g(\mathcal{I})-compact$ in $\mathcal{V}$.

Proof: Let $g:(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})\longrightarrow(\mathcal{V},\tau_{\mathcal{R}'}(Y))$ be a $surjective\,CNI_{\mathcal{M}_\mathcal{V}}-continuous$ function and $\{W_j:j\in\Delta\}$ be any $nano-open\,cover$ of $\mathcal{V}$. Then $\{g^{-1}(W_j):j\in\Delta\}$ is a $NI\mathcal{M}_\mathcal{V}-closed\,cover$ of $\mathcal{U}$. By our assumption, there exists a finite subset Δ_1 of Δ such that $\mathcal{U}-\bigcup\{W_j:j\in\Delta_1\}\in\mathcal{I}$. Hence, $\mathcal{V}-\bigcup\{W_j:j\in\Delta_1\}\in g(\mathcal{I})$. Therefore $\mathcal{V}$ is $g(\mathcal{I})-compact$.

Definition 3.8: A $NITS\,(\mathcal{U},\tau_\mathcal{R}(Z),\mathcal{I})$ is said to be

$NI\mathcal{M}_{\mathcal{V}} - normal\, space$ if for any pair of disjoint $nano - closed$ sets P and T such that there exists $NI\mathcal{M}_{\mathcal{V}} - open$ sets C of P and F of T such that $P \cap T = \emptyset$.

Theorem 3.12: If $g: (\mathcal{U}, \tau_{\mathcal{R}}(Z), \mathcal{I}) \longrightarrow (\mathcal{V}, \tau_{\mathcal{R}'}(Y))$ is a $injective\, CNI_{\mathcal{M}_{\mathcal{V}}} - continuous\, closed$ function and $(\mathcal{V}, \tau_{\mathcal{R}'}(Y))$ is a $nano - Ultra\, normal$ space, then $(\mathcal{U}, \tau_{\mathcal{R}}(Z), \mathcal{I})$ is a $NI\mathcal{M}_{\mathcal{V}} - normal\, space$.

Proof: Let P and T be the pair of disjoint $nano - closed$ sets of $\mathcal{U}$. Since g is $injective\, and\, nano - closed$, then $g(P) \cap g(T) = \emptyset$ where $g(P)$ and $g(T)$ are $nano - closed$ subsets of $\mathcal{V}$. Since $(\mathcal{V}, \tau_{\mathcal{R}'}(Y))$ is $nano - Ultra\, normal$, there exists $nano - clopen$ sets F of $g(P)$ and S of $g(T)$ in $\mathcal{V}$ such that $F \cap S = \emptyset$. Hence $P \subset g^{-1}(F)$, $g^{-1}(F) \in NI\mathcal{M}_{\mathcal{V}}O(\mathcal{U}, Z)$ and $T \subset g^{-1}(S)$, $g^{-1}(S) \in NI\mathcal{M}_{\mathcal{V}}O(\mathcal{U}, Z)$. So $g^{-1}(F) \cap g^{-1}(S) = \emptyset$. Therefore $(\mathcal{U}, \tau_{\mathcal{R}}(Z), \mathcal{I})$ is a $NI\mathcal{M}_{\mathcal{V}} - normal\, space$.

Definition 3.9: A function $g: (\mathcal{U}, \tau_{\mathcal{R}}(Z), \mathcal{I}) \to (\mathcal{V}, \tau_{\mathcal{R}'}(Y), \mathcal{J})$ is $Contra\, NI\mathcal{M}_{\mathcal{V}} - irresolute$ (briefly, $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$) if $g^{-1}(T)$ is $NI\mathcal{M}_{\mathcal{V}} - closed$ in $(\mathcal{U}, \tau_{\mathcal{R}'}(Z), \mathcal{I})$ for every $NI\mathcal{M}_{\mathcal{V}} - open$ set T in $(\mathcal{V}, \tau_{\mathcal{R}'}(Y), \mathcal{J})$ (i.e) if $g^{-1}(T) \in NI\mathcal{M}_{\mathcal{V}}C(\mathcal{U}, Z)$ for all $T \in NI\mathcal{M}_{\mathcal{V}}O(\mathcal{U}, Y)$.

Example 3.4: Let $\mathcal{U} = \{e, k, p, t\}\, with\, \mathcal{U}/\mathcal{R} = \{\{e\}, \{k, t\}, \{p\}\}\, and\ Z = \{e, k\} \subseteq \mathcal{U}$ $\tau_{\mathcal{R}}(Z) = \{\emptyset, \mathcal{U}, \{e\}, \{k, t\}, \{e, k, t\}\}$ and the $ideal\ \mathcal{I} = \{\emptyset, \{e\}, \{k\}, \{p\}, \{e, k\}, \{k, p\}, \{e, p\}, \{e, k, p\}\}$ $NI\mathcal{M}_{\mathcal{V}}O(\mathcal{U}, Z) = \{\emptyset, \mathcal{U}, \{e\}\}$ $NI\mathcal{M}_{\mathcal{V}}C(\mathcal{U}, Z) = \{\emptyset, \mathcal{U}, \{k, p, t\}\}$. Let $\mathcal{V} = \{e, k, p, t\}\, with\, \mathcal{V}/\mathcal{R}' = \{\{e, k\}, \{p\}, \{t\}\}$ $and\ Z = \{k, p\} \subseteq \mathcal{U}\ \tau_{\mathcal{R}'}(Z) = \{\emptyset, \mathcal{U}, \{p\}, \{e, k\}, \{e, k, p\}\}$ and the $ideal\ \mathcal{I} = \{\emptyset, \{e\}, \{k\}, \{e, k\}\}$ $NI\mathcal{M}_{\mathcal{V}}O(\mathcal{V}, Y) = \{\emptyset, \mathcal{U}, \{e, k\}\}$ Define $g: \mathcal{U} \to \mathcal{V}$ as $g(e) = p, g(k) = e = g(p), g(t) = k$. Then $g^{-1}(T)$ is $NI\mathcal{M}_{\mathcal{V}} - closed$ in $\mathcal{U}$ whenever T is $NI\mathcal{M}_{\mathcal{V}} - open$ in $\mathcal{V}$. Therefore g is $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$.

Theorem 3.13: Let $g: (\mathcal{U}, \tau_{\mathcal{R}}(Z), \mathcal{I}) \to (\mathcal{V}, \tau_{\mathcal{R}'}(Y), \mathcal{J})$, then the following statements are equivalent:

(i) g is $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$.
(ii) inverse image of every $NI\mathcal{M}_{\mathcal{V}} - open$ set in $(\mathcal{V}, \tau_{\mathcal{R}'}(Y), \mathcal{J})$ is $NI\mathcal{M}_{\mathcal{V}} - closed$ in $(\mathcal{U}, \tau_{\mathcal{R}}(Z), \mathcal{I})$
(iii) for each $NI\mathcal{M}_{\mathcal{V}} - closed$ subset S of $\mathcal{V}$, $g^{-1}(S) \in NI\mathcal{M}_{\mathcal{V}}O(\mathcal{U}, Z)$.
(iv) for each $z \in \mathcal{U}$ and each $NI\mathcal{M}_{\mathcal{V}} - closed$ set S of $\mathcal{V}$ containing $g(z)$, there exists $H \in NI\mathcal{M}_{\mathcal{V}}O(\mathcal{U}, Z)$ such that $g(H) \subset S$.

Proof: The proof is similar to Theorem 3.2.

Theorem 3.14: Let $g: (\mathcal{U}, \mathcal{N}, \mathcal{I}) \longrightarrow (\mathcal{V}, \mathcal{M}, \mathcal{J})$ be any function in $NITS$. If g is $CNI_{\mathcal{M}_{\mathcal{V}}} - continuous$ then it is $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$.

Proof: Let E be any $NI\mathcal{M}_{\mathcal{V}} - open$ subset of $\mathcal{V}$, then E is $nano - open$ set in $\mathcal{V}$. $CNI_{\mathcal{M}_{\mathcal{V}}} - continuity$ of g implies that $g^{-1}(E)$ is $NI\mathcal{M}_{\mathcal{V}} - closed$ in $\mathcal{U}$. Hence g is $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$.

Theorem 3.15: Let $f: (\mathcal{U}, \mathcal{N}, \mathcal{I}) \longrightarrow (\mathcal{V}, \mathcal{M}, \mathcal{J})$ and $g: (\mathcal{V}, \mathcal{M}, \mathcal{J}) \longrightarrow (\mathcal{W}, \mathcal{P}, \mathcal{K})$ be any two functions in $NITS$. Then the following statements hold.

(i) $g \circ f$ is $CNI_{\mathcal{M}_{\mathcal{V}}} - continuous$ if f is $NI\mathcal{M}_{\mathcal{V}} - continuous$ and g is $CN - continuous$.
(ii) $g \circ f$ is $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$ if f is $NI\mathcal{M}_{\mathcal{V}} - continuous$ and g is $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$.
(iii) $g \circ f$ is $CNI_{\mathcal{M}_{\mathcal{V}}} - continuous$ if f is $NI\mathcal{M}_{\mathcal{V}} - irresolute$ and g is $CNI_{\mathcal{M}_{\mathcal{V}}} - continuous$.

Proof: (i) Let F be a $nano - open$ set in $\mathcal{W}$. Since g is $CN - continuous$, $g^{-1}(F)$ is $nano - closed$ in $\mathcal{V}$. $NI\mathcal{M}_{\mathcal{V}} - continuous$ of f implies, $f^{-1}(g^{-1}(F))$ is $NI\mathcal{M}_{\mathcal{V}} - closed$ in $\mathcal{U}$ and hence $g \circ f$ is $CNI_{\mathcal{M}_{\mathcal{V}}} - continuous$.

(ii) Let F be a $NI\mathcal{M}_{\mathcal{V}} - open$ set in $\mathcal{W}$. Since g is $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$, $g^{-1}(F)$ is $NI\mathcal{M}_{\mathcal{V}} - closed$ in $\mathcal{V}$. Since every $NI\mathcal{M}_{\mathcal{V}} - closed$ set is $nano - closed$ and f is $NI\mathcal{M}_{\mathcal{V}} - continuous$, $f^{-1}(g^{-1}(F))$ is $NI\mathcal{M}_{\mathcal{V}} - closed$ in $\mathcal{U}$. Hence $g \circ f$ is $CNI_{\mathcal{M}_{\mathcal{V}}} - irresolute$.

(iii) Let F be a $nano - closed$ set in $\mathcal{W}$. Since g is $CNI_{\mathcal{M}_{\mathcal{V}}} - continuous$, $g^{-1}(F)$ is $NI\mathcal{M}_{\mathcal{V}} - open$ in $\mathcal{V}$. Since f is $NI\mathcal{M}_{\mathcal{V}} - irresolute$, $f^{-1}(g^{-1}(F))$ is $NI\mathcal{M}_{\mathcal{V}} - open$ in $\mathcal{U}$. Hence $g \circ f$ is $CNI_{\mathcal{M}_{\mathcal{V}}} - continuous$.

4. Conclusions

This paper concentrates on introducing a new variant of $Contra\, continuous$ function namely $Contra\, NI\mathcal{M}_{\mathcal{V}} - continuous$ functions in $nano\, ideal$ topological spaces. The relation between this function and existing $contra\, continuous$ functions in NTS and $NITS$ are explored. Also we have defined $NI\mathcal{M}_{\mathcal{V}} - interiority\, condition$ and examined the behaviour of $Contra\, NI\mathcal{M}_{\mathcal{V}} - continuity$ upon $nano - Urysohn$, $nano - Ultra\, Hausdorff$ and $N - connected$ space. For future work, this may be moved in defining $Contra\, NI\mathcal{M}_{\mathcal{V}} - open$ functions and $Contra\, NI\mathcal{M}_{\mathcal{V}} - homeomorphism$ in Nano ideal topological spaces.

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