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# Resolution of Linear Systems Using Interval Arithmetic and Cholesky Decomposition

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**Abstract** This article presents an innovative approach to solving linear systems with interval coefficients efficiently. The use of intervals allows the uncertainty and measurement errors inherent in many practical applications to be considered. We focus on the solution algorithm based on the Cholesky decomposition applied to positive symmetric matrices and illustrate its efficiency by applying it to the Leontief economic model. First, we use Sylvester's criterion to check whether a symmetric matrix is positive, which is an essential condition for the Cholesky decomposition to be applicable. It guarantees the validity of our solution algorithm and avoids undesirable errors. Using theoretical analyses and numerical simulations, we show that our algorithm based on the Cholesky decomposition performs remarkably well in terms of accuracy. To evaluate our method in concrete terms, we apply it to the Leontief economic model. This model is widely used to analyze the economic interdependencies between different sectors of an economy. By considering the uncertainty in the coefficients, our approach offers a more realistic and reliable solution to the Leontief model. The results obtained demonstrate the relevance and effectiveness of our algorithm for solving linear systems with interval coefficients, as well as its successful application to the Leontief model. These advances are crucial for fields such as economics, engineering, and the social sciences, where data uncertainty can greatly affect the results of analyses. In summary, this article highlights the importance of interval arithmetic and Cholesky's method in solving linear systems with interval coefficients. Applying these tools to the Leontief model can help you better understand the impact of uncertainty and make informed decisions in a variety of fields, including economics and engineering.

**Keywords** Arithmetic Interval, Interval Matrix, System of Interval Linear Equations, Decomposition of Cholesky

## 1 Introduction

Interval arithmetic is the branch of mathematics concerned with the properties and operations of numerical intervals. Although it may seem abstract at first glance, interval arithmetic has practical applications in many fields, from computing and engineering to the physics and economics. It allows the manipulation of intervals rather than exact numbers. This mathematical discipline allows uncertainty, imprecision, or measurement error to be expressed formally and rigorously. In addition, interval arithmetic offers a new approach to complex mathematical problems where the quantities involved are uncertain or difficult to evaluate accurately, which can lead to a deeper understanding of the concepts and the exploration of innovative solutions.

This article focuses on solving the $AX = B$ system applied to matrices with interval coefficients using the Cholesky decomposition, a method used to factor a symmetric positive definite matrix into a product of two lower triangular matrices and its transpose with interval coefficients. More precisely, this factorization makes it possible to solve the system more efficiently.

Using this approach in the Leontief model, the relationships between the different economic sectors are generally represented by a system of linear equations [1] which describe the total demand of each sector as a function of the total production of the other sectors. However, there are often uncertainties and errors in the data used to construct these equations. Where interval arithmetic is used, uncertain or poorly measured values can be represented by intervals instead of precise numbers. This allows uncertainties in the results to be considered and the impact of errors on economic predictions to be quantified.

In this article, we will discuss the Cholesky decomposition to solve a linear system with interval coefficients and apply this decomposition to an economic model called the Leontief model.

# 2 Interval arithmetic

## 2.1 Elementary operations on intervals

Let $I\mathbb{R} = \{\widehat{a} = [a_1; a_2] : a_1 \leq a_2 and a_1, a_2 \in \mathbb{R}\}$ be the set of all proper intervals and $\overline{I\mathbb{R}} = \{\widehat{a} = [a_1; a_2] : a_1 > a_2; a_1, a_2 \in \mathbb{R}\}$ be the set of all improper intervals on the real line R. If $a_1 = a_2 = a$, then $\widehat{a} = [a, a] = a$ is a real number (or a degenerate interval). We shall use the terms "interval" and "interval number" interchangeably. The mid-point and width(or half-width) of an interval number $\widehat{a} = [a_1, a_2]$ are defined as $m(\widehat{a}) = \frac{a_1 + a_2}{2}$ and $w(\widehat{a}) = \frac{a_2 - a_1}{2}$. We denote the set of generalized intervals(proper and improper) by :

$$K\mathbb{R} = I\mathbb{R} \cup \overline{I\mathbb{R}} = \{[a_1; a_2] : a_1, a_2 \in \mathbb{R}\}$$

The set of generalized intervals $K\mathbb{R}$ is a group with respect to addition and multiplication operations of zero free intervals, while maintaining the inclusion monotonicity.
The "dual" is an important monadic operator proposed by **kaucher** that reverses the end-points of the intervals in $K\mathbb{R}$. For $\widehat{a} = [a_1, a_2] \in K\mathbb{R}$, its dual is defined by $dual(\widehat{a}) = dual([a_1, a_2]) = [a_2, a_1]$. The opposite of an interval $\widehat{a} = [a_1, a_2]$ is $opp([a_1, a_2]) = [-a_1, -a_2]$ which is the additive inverse of $[a_1, a_2]$ and $\left[\frac{1}{a_1}, \frac{1}{a_2}\right]$ is the multiplicative inverse of $[a_1, a_2]$, provided $0 \notin [a_1, a_2]$.
That is, $\widehat{a} + (-dual(\widehat{a})) = [0, 0]$ and $\widehat{a} \times \frac{1}{dual(\widehat{a})} = [1, 1]$.

**Ganesan and Veeramani** [2] proposed new interval arithmetic on $\mathbb{IR}$. We extend these arithmetic operations to the set of generalized interval numbers $K\mathbb{R}$ and incorporate the concept of dual.
For $\widehat{a} = [a_1; a_2]$ , $\widehat{b} = [b_1; b_2] \in K\mathbb{R}$ and for $* \in \{+; -; \times; \div\}$ we define :

$$\widehat{a} * \widehat{b} = [m(\widehat{a}) * m(\widehat{b}) - k; m(\widehat{a}) * m(\widehat{b}) + k] \text{ and}$$
$$k = min\{(m(\widehat{a}) * m(\widehat{b}) - \alpha; \beta - (m(\widehat{a}) * m(\widehat{b}))\}$$

$\alpha$ and $\beta$ are the end points of the interval $\widehat{a}$ and $\widehat{b}$

If $\widehat{a} = [a_1; a_2] \in K\mathbb{R}$ is positive, we define $\sqrt{\widehat{a}}$ as

$$\sqrt{\widehat{a}} = [\sqrt{a_1}; \sqrt{a_2}]$$

It is clear that by this notation, we have $\sqrt{\widehat{a}} \times \sqrt{\widehat{a}} = \widehat{a}$

## 2.2 Interval matrices

### 2.2.1 Definitions

A square interval matrix $\widehat{A_{n,n}}$ of order n is defined as a matrix and can be written in the form [3] :

$$\widehat{A_{n,n}} = (\widehat{a_{i,j}})_{1 \leq i \leq n, 1 \leq j \leq n} = \begin{pmatrix} \widehat{a_{1,1}} & \widehat{a_{1,2}} & \cdots & \widehat{a_{1,n}} \\ \widehat{a_{2,1}} & \widehat{a_{2,2}} & \cdots & \widehat{a_{2,n}} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{a_{n,1}} & \widehat{a_{n,2}} & \cdots & \widehat{a_{n,n}} \end{pmatrix}$$

If $\widehat{A_{n,n}}$ and $\widehat{B_{n,n}}$ are interval matrices and $\alpha \in \mathbb{R}$, then :
- $\alpha \widehat{A_{n,n}} = \alpha(\widehat{a_{i,j}})_{1 \leq i \leq n, 1 \leq j \leq n}$
- $\widehat{A_{n,n}} + \widehat{B_{n,n}} = (\widehat{a_{i,j}} + \widehat{b_{i,j}})_{1 \leq i \leq n, 1 \leq j \leq n}$
- $\widehat{A_{n,n}} - \widehat{B_{n,n}} = (\widehat{a_{i,j}} - \widehat{b_{i,j}})_{1 \leq i \leq n, 1 \leq j \leq n}$ if $\widehat{A} \neq \widehat{B}$, and $\widehat{A_{n,n}} - \widehat{B_{n,n}}$=[0; 0] if $\widehat{A} = \widehat{B}$
- $\widehat{A_{n,n}}.\widehat{B_{n,n}} = \left(\sum_{k=1}^{n} \widehat{a_{i,k}}.\widehat{b_{k,j}}\right)_{1 \leq i \leq n, 1 \leq j \leq n}$

**The transpose of a square matrix** $\widehat{A_{n,n}}$ of order n is an interval coefficient matrix denoted by: $\widehat{A_{n,n}}^T$, obtained by exchanging the rows and columns of $\widehat{A_{n,n}}$ .
**A symmetric matrix with interval coefficient**: A symmetric matrix $\widehat{A_{n,n}}$ of order n is a square matrix which is equal to its own transpose, i.e. such that $\widehat{a_{i,j}} = \widehat{a_{j,i}}$ for all $i$ and $j$ between 1 and n, where the $\widehat{a_{i,j}}$ are the interval matrix coefficients and n is its order.

### 2.2.2 Determinant of an interval coefficient matrix

For any square matrix $\widehat{A_{n,n}}$ of order n with interval coefficient corresponds a value called the determinant of $\widehat{A_{n,n}}$ noted $det(\widehat{A_{n,n}})$, the method of calculating the determinant remains the same from the case of matrices with interval coefficients except the determinant of un interval matrix in an interval [3].
It is easy to see that most of the properties of the determinant of a classical matrix are valid for the determinant of the interval matrix.

# 3 Solving the $\widehat{A}$ $\widehat{X}$=$\widehat{B}$ system using the Choleski decomposition

## 3.1 Positive definite matrix and the Sylvester criterion

### 3.1.1 Definitions

Let $\widehat{A_{n,n}}$ be a symmetric square matrix with interval coefficient of size n . We call principal minors the determinants of the n matrices $A_p = (\widehat{a_{ij}})$, for p ranging from 1 to n.
Sylvester's criterion provides a simple method for testing the positive definiteness of a matrix $\widehat{A_{n,n}}$.

#### 3.1.2 Sylvester's criterion :

For a symmetric matrix with interval coefficient $\widehat{A_{n,n}}$ of size n to be positive definite, it is necessary and sufficient that the n principal minors $(A_p)_{1\leq p\leq n}$ are strictly positive intervals.

#### 3.1.3 Example 1:

Consider the symmetric interval matrix

$$\widehat{A}=\begin{pmatrix}[3.7;4.3] & [-1.5;-0.5] & [0;0]\\ [-1.5;-0.5] & [3.7;4.3] & [-1.5;-0.5]\\ [0;0] & [-1.5;-0.5] & [3.7;4.3]\end{pmatrix}$$

Let's check if $\widehat{A}$ is a positive definite square symmetric matrix using **Sylvester**'s criterion:
We have :

$|[3.7;4.3]| = [3.7;4.3] > 0$
and $\begin{vmatrix}[3.7;4.3] & [-1.5;-0.5]\\ [-1.5;-0.5] & [3.7;4.3]\end{vmatrix} = [11.94;18.06] > 0$
and
$$\begin{vmatrix}[3.7;4.3] & [-1.5;-0.5] & [0;0]\\ [-1.5;-0.5] & [3.7;4.3] & [-1.5;-0.5]\\ [0;0] & [-1.5;-0.5] & [3.7;4.3]\end{vmatrix} = [37.10;74.89] > 0$$

So $\widehat{A}$ is symmetric positive definite.

## 3.2 Cholesky decomposition

If A is a square matrix with interval coefficient symmetric and positive definite, then there exists a lower triangular matrix with interval coefficient F which satisfies:

$$A = F.F^T$$

This decomposition, called the factorization of **Cholesky**, is the product of a lower triangular matrix $F$ by its transpose.
Let $\widehat{A_{n,n}}$ be a square matrix of order n and interval coefficient such that:

$$\widehat{A_{n,n}}=\begin{pmatrix}\widehat{a_{1,1}} & \widehat{a_{1,2}} & \cdots & \widehat{a_{1,n}}\\ \widehat{a_{2,1}} & \widehat{a_{2,2}} & \cdots & \widehat{a_{2,n}}\\ \vdots & \vdots & \ddots & \vdots\\ \widehat{a_{n,1}} & \widehat{a_{n,2}} & \cdots & \widehat{a_{n,n}}\end{pmatrix}$$

If $F$ a lower triangular Matrix with interval coefficient that satisfies $A = F.F^T$:
So :

$$\widehat{F_{n,n}}=\begin{pmatrix}\widehat{b_{1,1}} & \widehat{0} & \cdots & \widehat{0}\\ \widehat{b_{2,1}} & \widehat{b_{2,2}} & \cdots & \widehat{0}\\ \vdots & \vdots & \ddots & \vdots\\ \widehat{b_{n,1}} & \widehat{b_{n,2}} & \cdots & \widehat{b_{n,n}}\end{pmatrix}$$

Such as :

$$\widehat{b_{i,j}} = \frac{1}{\widehat{b_{j,j}}}\left(\widehat{a_{j,j}} - \sum_{k=1}^{j-1}\widehat{b_{i,k}}.\widehat{b_{j,k}}\right) \quad (1)$$

With $i = j+1, j+2, ...., n$
And

$$\widehat{b_{j,j}} = \sqrt{\widehat{a_{j,j}} - \sum_{k=1}^{j-1}(\widehat{b_{j,k}})^2} \quad (2)$$

With $j = 1, 2, ...., n$

#### Consequences

If A is a square matrix of order 3 and satisfies all the conditions, then from (1) and (2) the interval coefficients of the matrix F are defined by:

$$\widehat{b_{1,1}} = \sqrt{\widehat{a_{1,1}}}$$
$$\widehat{b_{2,1}} = \frac{\widehat{a_{2,1}}}{\widehat{b_{1,1}}}$$
$$\widehat{b_{2,2}} = \sqrt{\widehat{a_{2,2}} - (\widehat{b_{2,1}})^2}$$
$$\widehat{b_{3,1}} = \frac{\widehat{a_{3,1}}}{\widehat{b_{1,1}}}$$
$$\widehat{b_{3,2}} = \frac{\widehat{a_{3,2}} - \widehat{b_{3,1}} \times \widehat{b_{2,1}}}{\widehat{b_{2,2}}}$$
$$\widehat{b_{3,3}} = \sqrt{\widehat{a_{3,3}} - (\widehat{b_{3,1}})^2 - (\widehat{b_{3,2}})^2}$$

#### Solving the $\widehat{A}\ \widehat{X}=\widehat{B}$ system

To solve a linear system involving interval matrices, we seek to find the smallest interval vector containing the set of vectors $\widehat{X}$ such that there exists a point matrix $A \in \widehat{A}$ and $B \in \widehat{B}$ and we have the equality $Ax = B$.

#### Calculation algorithm

Let $\widehat{A}$ and $\widehat{B}$ be two square matrices of order $n$ with interval coefficients. Solving the system $\widehat{A}\widehat{X} = \widehat{B}$ consists of going through the following steps:
**Step 1** : Check if $\widehat{A}$ is a positive definite symmetric matrix using Sylvester's criterion.
**Step 2** : Decompose $\widehat{A}$ as $\widehat{F} \times \widehat{F}^T$ using Cholesky decomposition.
**Step 3** : We set $\widehat{F}^T\widehat{X} = \widehat{Y}$.
**Step 4** : Solve the system $\widehat{F}\widehat{Y} = \widehat{B}$.
**Step 5** : Solve the system $\widehat{F}^T\widehat{X} = \widehat{Y}$.

#### Complexity of Cholesky decomposition

The Cholesky decomposition for matrices with interval coefficients has specific characteristics compared with the case of real matrices. In terms of complexity, the Cholesky decomposition for matrices with interval coefficients is generally higher than for real matrices. This is due to the need to manipulate

intervals during arithmetic operations (sum, subtraction, multiplication, etc.), which increases the number of operations required. If n is the size of the matrix, the complexity is necessarily greater than the order $O(n^3)$.
Operations with intervals can be more costly in terms of calculation time. Interval propagation during Cholesky decomposition may require additional computations to maintain the validity of the intervals. This can lead to an increase in execution time compared to the case of real matrices.

# 4 Application and comparison

## 4.1 Application

We take the symmetric matrix :

$\widehat{A} = \begin{pmatrix} [3.7;4.3] & [-1.5;-0.5] & [0;0] \\ [-1.5;-0.5] & [3.7;4.3] & [-1.5;-0.5] \\ [0;0] & [-1.5;-0.5] & [3.7;4.3] \end{pmatrix}$ and $\widehat{B} = \begin{pmatrix} [-14;0] \\ [-9;0] \\ [-3;0] \end{pmatrix}$ in example 1, we have shown that $\widehat{A}$ is symmetric positive definite, so we can decompose it as a product of a triangular matrix by its transpose using the decomposition of **Cholesky**

$$\widehat{F} = \begin{pmatrix} \widehat{b_{1,1}} & \widehat{0} & \widehat{0} \\ \widehat{b_{2,1}} & \widehat{b_{2,2}} & \widehat{0} \\ \widehat{b_{3,1}} & \widehat{b_{3,2}} & \widehat{b_{3,3}} \end{pmatrix} = \begin{pmatrix} [1.9;2.07] & [0;0] & [0;0] \\ [-0.76;-0.24] & [1.76;2.06] & [0;0] \\ [0;0] & [-0.8;-0.24] & [1.74;2.06] \end{pmatrix}$$
:

We can notice that:

$$\widehat{F} \times \widehat{F}^T \approx \begin{pmatrix} [3.60;4.27] & [-1.52;-0.45] & [0;0] \\ [-1.52;-0.45] & [3.15;4.64] & [-1.56;-0.42] \\ [0;0] & [-1.56;-0.42] & [3.08;4.67] \end{pmatrix} \approx \widehat{A}$$

let's put $\widehat{F}^T \widehat{X} = \widehat{Y}$ and we solve the system $\widehat{F}\widehat{Y} = \widehat{B}$, we find :

$$\widehat{Y} = \begin{pmatrix} [-7.05;0] \\ [-6.54;0] \\ [-3.35;0] \end{pmatrix}$$

System resolution $\widehat{A}\widehat{X} = \widehat{B}$ is to solve $\widehat{F}^T \widehat{X} = \widehat{Y}$.

$$\widehat{F}^T \widehat{X} = \widehat{Y} \text{ so } \widehat{X} = \begin{pmatrix} [-4.53;0] \\ [-3.9;0] \\ [-1.76;0] \end{pmatrix}$$

## 4.2 Comparison of results:

$\widehat{X} = \begin{pmatrix} [-6.38;0.] \\ [-6.40;1.32] \\ [-3.40;0] \end{pmatrix}$ the result found by **Ning et al** using Gauss elimination [4]

$\widehat{X} = \begin{pmatrix} [-6.38;1.12] \\ [-6.40;1.54] \\ [-3.40;1.40] \end{pmatrix}$ the result found by **Ning et al** using **Hansen**'s technique [5]

$\widehat{X} = \begin{pmatrix} [-4.482;0] \\ [-3.816;0] \\ [-1.776;0.006] \end{pmatrix}$ the result found by **karkar nora - Benmohamed Khier** using Gauss elimination [6]

We notice that the method of **Choleski** with the arithmetic of the intervals gives more precise results and is very close compared to the others.

# 5 An application in input-output (I-O) model

## 5.1 The Leontief model

Leontief's model [1] helps analyze inter-industry production and economic relationships in an economy. The model assumes that each industry uses a combination of goods and services produced by other industries to produce its own goods and services.
Leontief's model uses an input-output matrix, also known as an "input-output matrix," which shows the quantity of each product needed to produce one unit of each final product. This matrix is used to calculate inter-industry linkages and multiplier effects in the economy. The Leontief model can be used to assess the impact of disruptions on specific industries or on the economy as a whole. It can also be used to assess the impact of economic policies, such as industrial development policies or trade policies.

## 5.2 A symmetric input-output matrix

A symmetric input-output matrix is one in which the quantity of each final product needed to produce a unit is the same regardless of the final product under consideration.
In a symmetric input-output matrix, the diagonal elements represent the share of total output that each industry uses to produce its own product. The off-diagonal elements represent the quantities of each product needed to produce one unit of each final product.
A symmetric input-output matrix represents an economy in which all industries are equally interrelated and interdependent. In other words, each industry depends on other industries to produce its own products, and each industry also contributes to the production of other industries, which can have a larger multiplier effect in the economy. Indeed, a disruption in one industry can have an impact on the entire economy, as each industry is closely linked to other industries.

## 5.3 Input-output matrix with interval coefficient

If the coefficients of the input-output matrix are substituted with intervals instead of precise numerical values, it means that the exact quantities of each product needed to produce one unit of each final product are unknown or uncertain. The intervals may represent a range of possible values or uncertainty about the exact quantities needed. In this case, the interpretation of the input-output matrix must be modified accordingly. Instead of representing exact quantities, the input-output matrix represents qualitative relationships between industries and products. The interval coefficients can be used to perform sensitivity analyses or simulations to assess the impact of different uncertainties on the economy. For example, by modifying the intervals, it is possible to determine how changes in the output of one industry affect overall output and other industries. However, results obtained from an input-output matrix with interval coefficients may be less accurate than those obtained from a matrix with accurate numerical coefficients. It is therefore important to take into account the margin of error associated with the intervals when interpreting the results obtained from this matrix.

### 5.4 Application

Consider the input-output table (**table 1**) for this economic system with 3 industries A, B and C, where the coefficients are represented by intervals:
Let

**Table 1.** Input-output table

| ***** | A | B | C | Z |
|---|---|---|---|---|
| A | [0.1 ;0.2] | [0.2 ;0.3] | [0.1 ;0.2] | [7;9] |
| B | [0.2 ;0.3] | [0.3 ;0.4] | [0.2 ;0.3] | [2;3] |
| C | [0.1 ;0.2] | [0.2 ;0.3] | [0.3 ;0.4] | [0;0] |

- $\widehat{X}$ : be the production matrix
- $\widehat{A}$ : the domestic consumption matrix
- $\widehat{Z}$ : the export matrix
- $\widehat{I}$ : the identity matrix

To determine the level of production we solve the system

$$\widehat{X} = \widehat{AX} + \widehat{Z} \tag{3}$$

$$(3) \Leftrightarrow \widehat{Z} = (\widehat{I} - \widehat{A})\widehat{X}$$

We put :

$$\widehat{B} = (\widehat{I} - \widehat{A}) = \begin{pmatrix} [0.8;0.9] & [-0.3;-0.2] & [-0.2;-0.1] \\ [-0.3;-0.2] & [0.6;0.7] & [-0.3;-0.2] \\ [-0.2;-0.1] & [-0.3;-0.2] & [0.6;0.7] \end{pmatrix} :$$

Using the Sylvester criterion
$|[0.8;0.9]| = [0.8;0.9] > 0$ ,

$$\begin{vmatrix} [0.8;0.9] & [-0.3;-0.2] \\ [-0.3;-0.2] & [0.6;0.7] \end{vmatrix} = [0.4;0.58] > 0$$

And we have :

$$det(\widehat{B}) = \begin{vmatrix} [0.8;0.9] & [-0.3;-0.2] & [-0.2;-0.1] \\ [-0.3;-0.2] & [0.6;0.7] & [-0.3;-0.2] \\ [-0.2;-0.1] & [-0.3;-0.2] & [0.6;0.7] \end{vmatrix}$$

$= [0.12;0.33] > 0$

The matrix $\widehat{B}$ verifies the Sylvester criterion so it admits a Cholesky decomposition.
Applying the algorithm, we get:
$\widehat{B} = \widehat{F} \times \widehat{F}^T$

With $\widehat{F} = \begin{pmatrix} [0.89;0.94] & [0;0] & [0;0] \\ [-0.33;-0.21] & [0.7;0.8] & [0;0] \\ [-0.22;-0.11] & [-0.5;-0.27] & [0.57;0.78] \end{pmatrix}$

To determine the level of production of each sector, we must solve the system $\widehat{B}\widehat{X} = \widehat{Z}$
We find :

$$\widehat{X} = \begin{pmatrix} [9.71;19.98] \\ [6.61;19.33] \\ [3.15;13.85] \end{pmatrix}$$

# 6 Conclusions

The **Cholesky** decomposition is a numerically stable method to efficiently solve linear systems with interval coefficients. It also reduces the number of operations required to solve the system compared to other methods such as the Gauss-Jordan and Hansen methods.

In summary, the **Cholesky** decomposition is an important method for solving linear systems with interval coefficients because it guarantees the positive definition of the matrix, guarantees that the solution is also an interval, and can solve the system efficiently and numerically stable.

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