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Application of Grobner Basis Technique in Economic Models and Markov Process

V.James[1,*], **B.Sivakumar**[2]

[1]Department of Mathematics, Rajalakshmi Engineering College, Chennai, India

[2]Department of Mathematics, SSN College of Engineering, Chennai, India

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Abstract The past decade has witnessed enormous advances in the applications of matrix algebra in the life sciences. These advances have significant implications for the future practice of economics, Network analysis, Data analysis, Image processing, Optimization problems, and Quantum mechanics. The main objective of matrix algebra is to solve a system of equations using classical algorithm methods like Gauss elimination or matrix inversion. In recent years, handling a large number of data or finding the solutions to a large number of systems of equations is one of the problems in matrix algebra techniques. One of the effective methods to solve system of polynomial is Grobner basis techniques. It is a powerful tool in matrix algebra and in this method the leading terms of the polynomials are eliminated until no further reductions are possible. The objective of this paper is to solve Input-Output economic problems using the Grobner basis method which is one of the matrix algebra techniques. In real world, many decisions are taken based on the output of Input-Output table or Leontief models problems which have a large number of data or industries. There are two types of Leontief models, namely closed and open models. In this article we consider the closed Leontief model. Also, we present one simple algorithm based on Grobner basis techniques to find the solution of a system of equations that occur on the Markov model.

Keywords Input-Output Economics, Open Model, Grobner Basis, Buchberger Algorithm, Markov Process, Transition Probabilities

1 Introduction

Wassily Leontief [1] was a economist who made significant contributions to the field of economics, particularly in the area of macro economics. It is a quantitative method in economics which interrelates the different sectors on their relationship. This model consists of a matrix that shows the input requirements of each sector and the outputs generated by each sector. The matrix is known as the Leontief matrix or the Input-Output table. By applying matrix algebra and using the Input-Output table, economists can calculate the direct and indirect effects of changes in demand or technology on the outputs and inputs of each sector. In 1992, Danyang Xie [2] proved the existence and solvability conditions of the Leontief system using the analysis of various sized system. Bidard [3], Ten Raa [4],[5], Rueda [6], and Zhang [7], have analysed Leontief model. Recently, Przybylinski [8] forecast inflation using input-output models. Also Seyit Kerimkhulle [9] studied a two- sector(wholesale trade and retail trade) Input-Output model of Kazakhstan. In all the above analyses, the solution was obtained by the matrix inversion method. Since the dimension of the Input-Output table is large, the computational time is intensive. In this paper we approach the solution of Input-Output model using Grobner basis technique. The basic concepts of Grobner basis and Input-output models are explained in section 2 and section 3.
In section 4, we focus on Markov process models as given in [10],[11],[12]. One of the problems of an analysis of Markov process is how the initial distribution of probabilities affects the state of the system after n – transition in the study of steady state. To answer these questions, we use the Grobner basis algorithm.

2 Grobner Basis

A Grobner basis [13], named by the mathematician Bruno Buchberger, is a fundamental concept in commutative algebra and algebraic geometry. Given a set of polynomials, a Grobner basis is a set of polynomials with a special property: if we divide any polynomial in the set by the Grobner basis, the remainder has a specific form that depends only on the variables in the original polynomial. This means that a Grobner basis allows us to systematically reduce a system of polynomial equations to a simpler form, and to study its properties in a more structured way. That is K being any field. We consider a polynomial $f(x_1, x_2, x_3, \ldots, x_n)$ in n variables with coefficients in field K. Let $K[x_1, x_2, x_3, \ldots, x_n]$ denote the polynomial ring. In the field K, we define n-dimensional affine space over K to be the set $K^n = \{(a_1, a_2, a_3, \ldots, a_n)\colon a_1, a_2, a_3, \ldots, a_n \in K\}$.

$$V(f) = \{(a_1, a_2, a_3, \ldots, a_n)\colon f(a_1, a_2, a_3, \ldots, a_n) = 0 \in K\}$$

then $V(f)$ is called the variety defined by f.
Let $f_1, f_2, f_3, \ldots, f_k$ be the polynomial in $K[x_1, x_2, x_3, \ldots, x_n]$.Then we define a set

$$< f_1, f_2, f_3, \ldots, f_k >= \{\sum_{i=1}^{k} h_i f_i\colon h_1, h_2, h_3, \ldots, h_k \in K[x_1, x_2, x_3, \ldots, x_n]\}$$

and it is said to be ideal generated by $f_1, f_2, f_3, \ldots, f_k$

Let $\{g_1, g_2, g_3, \ldots, g_t\}$ be a set of non-zero polynomials inside an ideal I. This set is a Grobner basis if the leading term of any non -zero polynomial is divisible by the leading term of any one of the g_i.

3 Input-Output Models

In this section, we have discussed the model of Leontief matrix through an example and focused on the method of solution of the system of equations and not studied the direct, indirect, and induced impacts.
We have considered an economy of 15 sectors or industries denoted by F_i. The names of the sectors are

1. Products of agriculture, hunting and related services
2. Mining support services
3. Fast moving consumer goods
4. Textile products and related services
5. Products of woods
6. Paper products and related to printing services
7. Refined petroleum products
8. Chemical products
9. Rubber and plastics products
10. Products made from Non-metallic mineral
11. Basic metals
12. Products made from Fabricated metal
13. Machinery and equipment
14. Electronic equipment
15. Electrical machinery and apparatus

Let $x_1, x_2, x_3, \ldots, x_{16}$ be the total output of industry $F_1, F_2, F_3, \ldots, F_{16}$ respectively. Assume that j^{th} sector needs to use a_{ij} unit from the i^{th} sector in order to create 1 unit. Additionally, suppose that each sector sells some of its output to other sectors (Intermediate output) and some of its output to consumers (final demand). Let d_i denote the final demand.
Then the total output = Intermediate output + final demand. Mathematically

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{i16}x_{16} + d_i \tag{1}$$

In general, consider the 'n' industries, denoted by $F_1, F_2, F_3, \ldots, F_n$. Then

$$x_i = \sum_{j=1}^{n} a_{ij}x_j + d_i \tag{2}$$

Let $X = (x_1, x_2, x_3, \ldots, x_n)^T$,

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$$

$d = (d_1, d_2, \ldots, d_n)^T$ Here X is called output matrix and A is called input–output matrix. If $d \neq 0$, then this model is called the open Leontief model. The expression can be rewritten as

$$X = AX + d \tag{3}$$

$$X - AX = d(I - A)X = d$$

Let $(I - A) = T$, then

$$TX = d \tag{4}$$

From the above equations, we have constructed the following polynomial ideal.

$$f_1 = (1 - a_{11})x_1 + a_{12}x_2 + \cdots + a_{1n}x_n - d_1$$
$$f_2 = a_{21}x_1 + (1 - a_{22})x_2 + \cdots + a_{2n}x_n - d_2$$

$$\vdots$$

$$f_n = a_{n1}x_1 + a_{n2}x_2 + \cdots + (1 - a_{nn})x_n - d_n$$

Now, if the number of equations is small, that is the order of the matrix A is small, then we can solve the system of equations using the inverse method. Suppose the order of the matrix is large or the matrix is sparse, then the classical method takes more computational time. Also if the diagonal element is near zero, then we will apply the partial pivoting concept. So we prefer the Grobner basis algorithm since the technique reduces complicated algebraic calculations. Let f_i be the polynomial constructed from the equation (4).

1. Construct polynomial set $f_1, f_2, \cdots, f_n$
2. Generate the ideal $I =< f_1, f_2, \cdots, f_n >$
3. Compute Grobner basis using Buchberger algorithm.

Consider the data set of Input – Output (presented in the table 1 and table 2) which can be viewed or downloaded from the website **https://stats.oecd.org.**

Based on the consumption matrix and demand vector, we construct the following polynomials.

1. $f_1 = 0.86221x_1 + 0.0000x_2 + 0.18522x_3 + 0.03006x_4 + 0.00303x_5 + 0.00343x_6 + 0.00002x_7 + 0.00339x_8 + 0.00452x_9 + 0.00016x_{10} + 0.00002x_{11} + 0.00002x_{12} + 0.00009x_{13} + 0.00002x_{14} + 0.00003x_{15} - 0.63221$
2. $f_2 = 0.00001x_1 + 0.99892x_2 + 0.00045x_3 + 0.00156x_4 + 0.00008x_5 + 0.00099x_6 + 0.44574x_7 + 0.01050x_8 + 0.00015x_9 + 0.01713x_x10 + 0.09108x_{11} + 0.00115x_{12} + 0.00091x_{13} + 0.00017x_{14} + 0.00072x_{15} - 0.42828$
3. $f_3 = 0.01915x_1 + 0.00007x_2 + 0.81216x_3 + 0.00288x_4 + 0.00012x_5 + 0.00101x_6 + 0.00054x_7 + 0.02062x_8 + 0.00210x_9 + 0.00249x_{10} + 0.00157x_{11} + 0.00053x_{12} + 0.00011x_{13} + 0.00006x_{14} + 0.00014x_{15} - 0.76077$
4. $f_4 = 0.01218x_1 + 0.00049x_2 + 0.00368x_3 + 0.78509x_4 + 0.00061x_5 + 0.00182x_6 + 0.00041x_7 + 0.00901x_8 + 0.00916x_9 + 0.00182x_{10} + 0.00050x_{11} + 0.00065x_{12} + 0.00089x_{13} + 0.00138x_{14} + 0.00279x_{15} - 0.73970$
5. $f_5 = 0.00283x_1 + 0.00773x_2 + 0.06951x_3 + 0.03186x_4 + 0.98283x_5 + 0.02816x_6 + 0.00296x_7 + 0.04924x_8 + 0.00558x_9 + 0.00914x_{10} + 0.00489x_{11} + 0.00455x_{12} + 0.00846x_{13} + 0.00786x_{14} + 0.00734x_{15} - 0.74271$
6. $f_6 = 0.00992x_1 + 0.00217x_2 + 0.15194x_3 + 0.04883x_4 + 0.00343x_5 + 0.60339x_6 + 0.00212x_7 + 0.04597x_8 + 0.01131x_9 + 0.01269x_{10} + 0.00359x_{11} + 0.00440x_{12} + 0.00571x_{13} + 0.00561x_{14} + 0.01481x_{15} - 0.28091$
7. $f_7 = 0.00093x_1 + 0.00672x_2 + 0.00020x_3 + 0.00608x_4 + 0.00056x_5 + 0.00471x_6 + 0.9513x_7 + 0.03294x_8 + 0.00911x_9 + 0.02087x_{10} + 0.04757x_{11} + 0.00537x_{12} + 0.01074x_{13} + 0.00727x_{14} + 0.00938x_{15} - 0.78886$
8. $f_8 = 0.09736x_1 + 0.00970x_2 + 0.02190x_3 + 0.06575x_4 + 0.00198x_5 + 0.01497x_6 + 0.02661x_7 + 0.58537x_8 + 0.11479x_9 + 0.01581x_{10} + 0.01866x_{11} + 0.00326x_{12} + 0.00369x_{13} + 0.00403x_{14} + 0.01564x_{15} - 0.17122$
9. $f_9 = 0.01277x_1 + 0.00424x_2 + 0.02445x_3 + 0.04167x_4 + 0.00249x_5 + 0.01099x_6 + 0.00310x_7 + 0.06628x_8 + 0.83813x_9 + 0.03311x_{10} + 0.00799x_{11} + 0.00962x_{12} + 0.01056x_{13} + 0.00814x_{14} + 0.01694x_{15} - 0.58578$
10. $f_{10} = 0.00133x_1 + 0.00667x_2 + 0.01787x_3 + 0.00289x_4 + 0.00056x_5 + 0.00284x_6 + 0.00383x_7 + 0.01147x_8 + 0.00294x_9 + 0.72529x_{10} + 0.04511x_{11} + 0.00972x_{12} + 0.01881x_{13} + 0.02203x_{14} + 0.01699x_{15} - 0.56221$
11. $f_{11} = 0.00036x_1 + 0.00060x_2 + 0.00020x_3 + 0.00074x_4 + 0.00180x_5 + 0.00072x_6 + 0.00134x_7 + 0.00426x_8 + 0.00203x_9 + 0.00469x_{10} + 0.45358x_{11} + 0.12194x_{12} + 0.12833x_{13} + 0.01262x_{14} + 0.11972x_{15} - 0.05423$
12. $f_{12} = 0.00147x_1 + 0.00924x_2 + 0.00270x_3 + 0.00513x_4 + 0.00155x_5 + 0.00248x_6 + 0.00057x_7 + 0.00344x_8 + 0.00428x_9 + 0.00461x_{10} + 0.19978x_{11} + 0.92920x_{12} + 0.09002x_{13} + 0.01457x_{14} + 0.07700x_{15} - 0.51236$
13. $f_{13} = 0.00851x_1 + 0.02131x_2 + 0.00662x_3 + 0.01976x_4 + 0.00055x_5 + 0.00151x_6 + 0.00162x_7 + 0.00964x_8 + 0.00242x_9 + 0.00257x_{10} + 0.01580x_{11} + 0.02306x_{12} + 0.85281x_{13} + 0.01450x_{14} + 0.02716x_{15} - 0.69780$
14. $f_{14} = 0.00178x_1 + 0.00177x_2 + 0.00156x_3 + 0.00433x_4 + 0.00011x_5 + 0.00441x_6 + 0.00103x_7 + 0.00395x_8 + 0.00207x_9 + 0.00360x_{10} + 0.01674x_{11} + 0.01026x_{12} + 0.02252x_{13} + 0.72969x_{14} + 0.04675x_{15} - 0.60883$

15. $f_{15} = 0.00164x_1 + 0.00326x_2 + 0.00131x_3 + 0.00384x_4 + 0.00032x_5 + 0.00049x_6 + 0.00028x_7 + 0.00479x_8 + 0.00112x_9 + 0.00165x_{10} + 0.03781x_{11} + 0.02346x_{12} + 0.05297x_{13} + 0.07122x_{14} + 0.88047x_{15} - 0.67631$

The main idea behind a Grobner basis is to choose a monomial order, so we fix dictionary order. To order the monomials in the variables of the polynomials, such that all polynomials in the basis have the same leading term. Then, we use a process called polynomial division to eliminate the leading term of each polynomial in the set, until we obtain a reduced form in which no polynomial has a leading term divisible by the leading term of any other polynomial in the set. This reduced form is the Grobner basis. This can be easily obtained by sage math (computer algebra [14]) and we get the following Grobner basis.

$x_1 + 4.105, x_2 - 18.34, x_3 - 8.4240, x_4 - 0.0000, x_5 - 17.960, x_6 + 21.01, x_7 + 0.000, x_8 + 7.713, x_9 - 0.05822, x_{10} - 0.2739, x_{11} + 0.9817, x_{12} - 0.12680, x_{13} + 0.0000, x_{14} - 0.8220, x_{15} - 0.57990.$

From this basis easily we can obtain the solution of the system of equations. From this solution, we observe that mining support services, products of woods, and its related industries sectors play a vital role.

4 Asymptotic behaviour of a Markov Process

A decision is made based on current trends and not on past experience. In particular business decisions turn out to be productive only if the optimal strategy is adopted in the appropriate situation. The effect of the strategy on production and consumption in the long run also is to be evaluated. Once the industries start production and consumption, does the transition from one state to another reach a steady state? Are the state probabilities independent of the initial probabilities to answer these questions we analyze the associated Markov processes.

4.1 Steady state Probabilities

Consider the transition matrix P of Markov process and its entries are non-negative after some steps. Let S be the unique matrix such that it satisfies the following condition.

$$SP = S \tag{5}$$

Also, the sum of each entry of a row in S should be one. From equation (4),

$$S(P - I) = 0$$

Now several methods are available to solve the system of equations. Suppose the dimension of P is large, then direct methods of matrix theory take the computation time to be large. We use Grobner basis techniques and try to solve the above system of equations in the following manner. Let P be the one-step transition probability matrix. **Algorithm**

1. Input: $S = [S_{1i}]_{1\times n}, P = [p_{ij}]_{n\times n}$
2. Initialization: $I = \emptyset$
3. For $i : 1 \to n$
4. $f_i \leftarrow \{\sum_{j=1}^{n} s_{1j} \times p_{ji}\} - s_{1i}$
5. $I = I \cup f_s$
6. End For
7. $f_s - \{\sum_{i=1}^{n} s_{1i}\} - 1$
8. $I = I \cup f_s$
9. Initialization: $G = I, T = \{\{f_i, f_j\} : f_i \neq f_j \in G\}$
10. While $T \neq \emptyset$ do
11. Choose any $\{f, g\} \in T$
12. Updating T : $T = T - f, g$
13. Computation of S polynomials: $S(f, g) \to_+ h$
14. If $h \neq 0$, then
15. $T = T \cup \{\{u, h\} : \forall u \in G\}$
16. $G = G \cup \{h\}$
17. Endif
18. End While
19. End Procedure

This algorithm can be explained through the following simple model. Suppose we consider two states S = (sweet1, sweet2) and they are denoted by $S = (s_{11}, s_{12})$ with a transition probability matrix

$$P = \begin{bmatrix} 0.4 & 0.6 \\ 0.35 & 0.65 \end{bmatrix}$$

The interpretation of this matrix is that a person's purchase was sweet1 there is a 40 percent chance that his next purchase will also be Sweet1. If a person's purchase was sweet2 there is a 65 percent chance that his next purchase will also be sweet2. Now our objective is to analyze the purchase habits of that person in the long run.
Now, we solve this problem using this Grobner basis algorithm.

Initially $I = \emptyset$ Computation of polynomials

$$For \quad i : 1- \rightarrow 2$$
$$f_1 \leftarrow s_{11} \times 0.4 + s_{12} \times 0.35 - s_{11}$$
$$f_2 \leftarrow s_{11} \times 0.6 + s_{12} \times 0.65 - s_{12}$$
$$I = I \cup \{f_1, f_2\}$$
$$End \quad Forloop$$
$$f_s = s_{11} + s_{12} - 1$$
$$I = \{f_1, f_2, f_s\}$$
$$G = \{f_1, f_2, f_s\} \quad T : \{\{f_1, f_2\}, \{f_1, f_s\}, \{f_2, f_s\}$$
$$While \quad T \neq \emptyset$$

First Pass through While Loop
Choose $\{f_1, f_2\}$

$$T = \{\{f_1, f_s\}, \{f_2, f_s\}\}$$
$$S(f_1, f_2) \rightarrow 0$$

Second Pass through While Loop
Choose $\{f_1, f_s\}$

$$T = \{\{f_2, f_s\}\}$$
$$S(f_1, f_s) \rightarrow -1.5833 s_{12} + 1 \quad (sayh)$$

If $h \neq 0$ then

$$T = \{\{f_2, f_s\}, \{f_1, h\}, \{f_2, h\}\}$$
$$G = \{f_1, f_2, h\}$$

End If
By continuing this process we get this set of Grobner basis $G = \{s_{11} - 0.3684, s_{12} - 0.6316\}$. From this, we conclude that a person purchases 37 percent sweet1.

5 Conclusions

In the present paper, we have shown that the Grobner bases method is an effective technique for solving system of equations existing in Input-Output models and Markov processes models. In general, when the matrix A is large, the Grobner basis technique can be effectively applied in the system of equations $AX = B$ and $AX = 0$.

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Table 1. Input-Output tables and demand vector(Unit: US Dollar Millions)

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	50271.3	0.8	67575.3	10966.8	1105.8	1252.6	7.1	1235.3	1648.4	56.9	7.8	5.9
2	2.5	283.7	116.6	408.3	21.2	258.8	116580.5	2746.2	38.9	4480.3	23822.4	301.2
3	2876.5	11.1	28217.9	432.3	17.3	152.3	81	3098.4	316.1	374.7	235.4	79.1
4	1206.1	48.8	363.9	21277	59.9	180.2	40.7	891.8	906.8	180.2	49.5	64.8
5	6.6	18	161.9	74.2	40	65.6	6.9	114.7	13	21.3	11.4	10.6
6	146.8	32.1	2249.6	722.9	50.8	5872.1	31.4	680.6	167.4	187.9	53.2	65.1
7	124.5	902.4	26.2	816.9	75.3	633	6540.9	4424.4	1223.2	2803.2	6388.9	720.9
8	11627.6	1158.3	2615.3	7852	236.7	1787.9	3177.5	49519.3	13709.5	1887.8	2228.7	389.7
9	455.7	151.3	872	1486.5	89	391.9	110.6	2364.3	5774.2	1181	285	343
10	13.8	69.4	186	30.1	5.8	29.6	39.9	119.4	30.6	2858.9	469.5	101.2
11	42.5	70.4	23.6	86.5	211.6	84.9	156.9	499.7	237.8	550.4	64088.9	14302
12	35.1	221.2	64.6	122.8	37	59.4	13.6	82.3	102.5	110.4	4781.1	1694.4
13	411.7	1031.5	320.2	956.7	26.5	72.9	78.4	466.6	117	124.3	764.9	1116.1
14	39.6	39.5	34.7	96.5	2.4	98.3	23	88.2	46.1	80.3	373.4	228.9
15	66.3	131.6	53	154.9	12.9	19.6	11.5	193.2	45.2	66.8	1526.3	947.2
Sector	13	14	15	Demand								
1	32.4	5.9	12.1	230652								
2	236.7	43.3	187.3	112014								
3	16	8.8	21.7	114288236.7								
4	88.4	136.5	276.2	7323216								
5	19.7	18.3	17.1	173088.4								
6	84.5	83	219.2	4159								
7	1442.7	976.1	1260.2	105955								
8	440.8	481	1867.9	20449								
9	376.7	290.2	604.4	20896								
10	195.8	229.3	176.8	5851								
11	15051.3	1480.7	14041.4	6360								
12	2154.4	348.7	1842.7	12262								
13	7124.6	701.8	1314.7	33776								
14	502.4	6029.8	1042.8	13581								
15	2138.2	2874.9	4825.2	27301								

Table 2. Consumption tables

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	0.13779	0.0000	0.18522	0.03006	0.00303	0.00343	0.00002	0.00339	0.00452	0.000165.8	0.00002	0.00002
2	0.00001	0.00108	0.00045	0.00156	0.00008	0.00099	0.44574	0.01050	0.00015	0.01713	0.09108	0.00115
3	0.01915	0.00007	0.18784	0.00288	0.00012	0.00101	0.00054	0.02062	0.00210	0.00249	0.00157	0.00053
4	0.01218	0.00049	0.00368	0.21491	0.00061	0.00182	0.00041	0.00901	0.00916	0.00182	0.00050	0.00065
5	0.00283	0.00773	0.06951	0.03186	0.01717	0.02816	0.00296	0.04924	0.00558	0.00914	0.00489	0.00455
6	0.00992	0.00217	0.15194	0.04883	0.00343	0.39661	0.00212	0.04597	0.01131	0.01269	0.00359	0.00440
7	0.00093	0.00672	0.00020	0.00608	0.00056	0.00471	0.04870	0.03294	0.00911	0.02087	0.04757	0.00537
8	0.09736	0.00970	0.02190	0.06575	0.00198	0.01497	0.02661	0.41463	0.11479	0.01581	0.01866	0.00326
9	0.01277	0.00424	0.02445	0.04167	0.00249	0.01099	0.00310	0.06628	0.16187	0.03311	0.00799	0.00962
10	0.00133	0.00667	0.01787	0.00289	0.00056	0.00284	0.00383	0.01147	0.00294	0.27471	0.04511	0.00972
11	0.00036	0.00060	0.00020	0.00074	0.00180	0.00072	0.00134	0.00426	0.00203	0.00469	0.54642	0.12194
12	0.00147	0.00924	0.00270	0.00513	0.00155	0.00248	0.00057	0.00344	0.00428	0.00461	0.19978	0.07080
13	0.00851	0.02131	0.00662	0.01976	0.00055	0.00151	0.00162	0.00964	0.00242	0.00257	0.01580	0.02306
14	0.00178	0.00177	0.00156	0.00433	0.00011	0.00441	0.00103	0.00395	0.00207	0.00360	0.01674	0.01026
15	0.00164	0.00326	0.00131	0.00384	0.00032	0.00049	0.00028	0.00479	0.00112	0.00165	0.03781	0.02346
Sector	13	14	15	Demand	5	6	7	8	9	10	11	12
1	0.00009	0.00002	0.00003	230652								
2	0.00091	0.00017	0.00072	112014236.7								
3	0.00011	0.00006	0.00014	114288								
4	0.00089	0.00138	0.00279	73232								
5	0.00846	0.00786	0.00734	173019.7								
6	0.00571	0.00561	0.01481	4159								
7	0.01074	0.00727	0.00938	105955								
8	0.00369	0.00403	0.01564	20449								
9	0.01056	0.00814	0.01694	20896								
10	0.01881	0.02203	0.01699	5851								
11	0.12833	0.01262	0.11972	6360								
12	0.09002	0.01457	0.07700	12262								
13	0.14719	0.01450	0.02716	33776								
14	0.02252	0.27031	0.04675	13581								
15	0.05297	0.07122	0.11953	27301								