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To Enhance New Interval Arithmetic Operations in Solving Linear Programming Problem Using Interval-valued Trapezoidal Neutrosophic Numbers

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Abstract Now in real-life scenarios, indeterminacy arises everywhere in various fields, i ncluding p hysics, mathematics, economics, philosophy, social sciences, etc. It occurs whenever prediction is difficult, w hen w e d idn't g et a predetermined outcome or obtain fixed o r m ultiple possible outcomes etc. Overcoming indeterminacy is one of the most prominent duties for everyone to lead a confusionless society. Hence a neutrosophic concept came into force to analyze indeterminacy explicitly. In contrast, a fuzzy set assigns only membership grade, and an intuitionistic set allocates membership and non-membership to elements. Decision-makers can use neutrosophic settings to model uncertainty and ambiguity in complex systems for flexible analysis. The neutrosophic environment with interval num-bers makes one handle the situations efficiently. Hence we utilize interval-valued trapezoidal neutrosophic numbers for more flexibility. Trapezoidal number together with interval truth, interval indeterminacy, and interval falsity are the parameters of these neutrosophic numbers. Considering a de-neutrosophication technique in crisp numbers again leads to vagueness in real-life circumstances. Hence our primary goal is to develop a new de-neutrosophication strategy in the form of an interval number instead of the crisp number. This paper provides an overview of the de-neutrosophication and a new ranking technique based on an interval number, and some extended neutrosophic linear programming theorems. Further, an interval version of simplex and Robust Two-Step method (RTSM) are used to answer an interval-valued trapezoidal neutrosophic linear programming problem. Finally, this paper highlights the limitations and advantages of the proposed technique to improve problem-solving in a wide range of fields.

Keywords Simplex Method, Neutrosophic Linear Programming Problem, Interval-valued Trapezoidal Neutrosophic Number, Interval Numbers

1 Introduction

In recent years, the application domain of optimization techniques has expanded significantly in all engineering and science disciplines. It offers a powerful toolkit for resolving challenges that occurs in the problem that arises in the real-world. A technique of mathematical modeling called linear programming is utilized to accomplish an objective while taking into account constraints. A Soviet mathematician, L. V. Kantorovich, suggested a technique to solving linear programming problems in 1939. He created it as a method of budgeting expenses and gains during the period of Second World War to lower army costs and also to enhance causalities inflicted on the enemy. Eventually, it was the Dutch-American economist who was named as T. C. Koopmans created linear programs for traditional economic issues. Hitchcock developed transportation problems as linear programs and the explanation given by him is very similar to the simplex method in 1941. Later in 1947, it was Dantzig who shaped the simplex method, which efficiently tackles linear programming problems in the majority of cases. The parameters used in past linear programming techniques are precise and definite. The basic first step in linear programming domains is to create a suitable mathematical model that significantly depends on the data. Since vagueness and imprecision occur everywhere in real-world circumstances, the data could be fuzzy. Because of these circumstances, linear programming problems evolved into fuzzy linear programming problems and it is one of the best tools for decision-making.

When Zadeh [1] announced the idea of fuzzy set theory in 1965, it was indeed a miracle in an uncertain environment and it now plays an important role wherever imprecise data exists. Fuzzy linear programming approaches are currently receiving a lot of attention from various researchers for their efficiency, accuracy, high-quality realistic outcomes, etc. It has a massive exploration of food, agriculture, transportation, construction, engineering, manufacturing, resource allocation, banking and finance, inventory management, game theory, the energy industry, etc. In 1974, Tanaka et al. [2] was the one who brought the notion of decision-making in an ambigious environment for answering fuzzy mathematical programming problems. In 1978, Zimmermann [3] combines individual objective functions in different ways for determining optimal solutions to answer fuzzy linear programming problems. In 1984, Tanaka and Asai [4] formulate a fuzzy linear programming problem to find a solution in ambiguous parameters and they found that fuzzy numbers are important in modeling decision problems. In 2001, Jamison and Lodwick [5] proposed a penalty method to solve fuzzy linear programming problems. In the early 2000s, Maleki et al. [6, 7] proposed an effective method to probe linear programming problems with fuzzy variables and a fuzzy ranking technique. In 2001, Itoh et al. [8] proposed a linear programming problem with fuzzy values in agricultural management. During 2011-2014, Kumar et al. [9, 10, 11] contributed enormously to solve fuzzy linear programming in both equality and inequality constraints using various methods namely ranking techniques, the Mehar method, different parameters, etc. In various years, Ganesan and Veeramani [12], Ebrahimnejad et al. [13, 14], Das et al. [15] and many others contributed to the development of a novel method with some significant and intriguing outcomes for the solution of linear programming problems involving symmetric trapezoidal fuzzy numbers. In 2020, Ghanbari et al. [16] investigate and summarise different types of models and solutions that were done by various researchers in fuzzy linear programming.

Due to high complexity, the current research work has been concentrating on interval numbers rather than crisp ones. Expressing the values is more adaptable than the conventional approach to work within the boundary. For solving interval linear programming problems, in 1981 Steuer [17] presented three algorithms with three cases based on the boundedness of the feasible region, number of zero, and nonzero objective functions. In 1989, Ishibuchi and Tanaka [18] stated and scrutinized the linear programming problem, where the parameters are formulated with interval coefficients. In 1992, Nakahara et al. [19] considered a linear programming problem in an interval environment and a new concept of constraints based on probability was investigated. In 1994, Shaocheng [20] concentrated on two kinds of linear programming in both fuzzy and interval parameters. In 2001, Sengupta et al. [21] described an extension of a linear programming problem in interval form using inequality constraints. To avoid complexity based on certainty, Moore [22] pioneered the concept of interval numbers in 2009. In most situations, the coefficients of the constraints or objective functions are not always in crisp numbers due to vagueness. Ramesh and Ganesan [23] handled this in 2011 by solving linear programming problems using generalized interval arithmetic operations. Later in 2013, Alolyan [24] proposed a novel approach for addressing linear programming problems with the help of fuzzy parameters within the objective function and also restrictions based on the inclining between intervals. In 2018, Ashayerinasab et al. [25] developed a new algorithm, obtained solutions to several interval linear programming problems and finally compared it with a Monte Carlo simulation. In 2019, Wang and Peng [26] transformed fuzzy linear programming problems into interval linear programming problems to determine optimal solutions.

Moreover, the fuzzy set is not appropriate for handling vagueness and hesitation, therefore in 1986, Atanassov [27] comprehended the fuzzy sets called as intuitionistic fuzzy sets. It characterizes not only a single membership grade but also a non-membership grade and is useful to examine decision-making problems that exist in real life. The concept of intuitionistic optimization was extended by Angelov [28] in 1997 and it was based on the degrees that maximize the acceptance and minimize the rejection. In 2015, Nagoor Gani [29] developed a ranking technique for solving generalized intuitionistic linear programming problems using trapezoidal parameters. Kabiraj et al. [30] recommended a technique in 2019 for solving linear programming problems in an intuitionistic environment by comparing it with Zimmerman, Weight assignment, and Angelov's technique. In 2019, Sidhu and Kumar [31] proposed a new method for solving trapezoidal intuitionistic fuzzy linear programming problems using the Mehar method. In 2019, Ritika and Ratnesh [32] extended the concept based on value and ambiguity indices and proposed a ranking technique for comparing intuitionistic trapezoidal fuzzy numbers. In 2020, Bharati and Singh [33] suggested a fresh technique to evaluate interval-valued intuitionistic linear programming problems with trapezoidal numbers. Nishad and Singh [34], Sidhu [35], Wan and Dong [36] and various other researchers carried out the applications in the field of intuitionistic optimization in the trapezoidal environment.

Smarandache (1998) [37] introduced neutrosophic sets to shade the problem of fuzzy sets that handles less and inconsistent information. It was invented as an protracted system of fuzzy, intuitionistic and traditional sets. Here indeterminacy is included together with membership and non-membership. Hence each element is associated with three parameters i.e., truth, indeterminacy, and falsity. The neutrosophic sets and logic are the utmost way of handling real-life problems, which obliges a generalization of fuzzy sets. Using imprecise parameters to manage independent parameters, Hussain et al. [38] developed neutrosophic linear programming problems in 2017. With parameters expressed as trapezoidal neutrosophic numbers for solving fully neutrosophic linear programming problems, Abdel-Basset et al. [39] developed a novel ranking function in 2018. The contribution of Bera and Mahapatra

[40, 41] to the neutrosophic linear programming problem plays a vast role in real-life applications: introduced the concept of generalized single-valued neutrosophic number in 2020, constructed a ranking function based on the geometrical structure of the trapezium and extended Big-M Simplex method. In 2020, the research on neutrosophic linear programming problems attracted worldwide researchers to its need to handling indeterminacy explicitly. To solve mixed-constraint neutrosophic linear programming problems with parameters represented by triangular neutrosophic numbers with mixed constraints, Das and Dash [42] presented a new ranking algorithm. A new parametric ranking function was proposed by Darehmiraki [43] to address the neutrosophic linear programming problem. A different ranking technique was used to interpret triangular interval neutrosophic linear programming problems into crisp linear programming problems by Nafei et al. [44]. Basumatary and Broumi [45] proposed a method to solve interval-valued triangular neutrosophic linear programming problems. In 2021, Abdelfattah [46] enlarged the parametric approach in the triangular neutrosophic linear programming environment for all types of maximization, minimization, and mixed constraints.

Based on the literature study, we examine that the common de-neutrosophication approaches done in the interval-valued trapezoidal neutrosophic environments are in crisp numbers. There is no work related to the interval-based de-neutrosophication technique. The most effective method for illustrating the fuzziness and ambiguity between the boundaries (upper and lower) of any economic or technological significance shrewdness is to show it as an interval number. Therefore, analyzing the interval-valued trapezoidal neutrosophic numbers as intervals is more cost-effective than crisp ones when decision-makers need additional information. This encourages us to widen the concept of linear programming problems in the interval-valued trapezoidal neutrosophic environment by switching the problem into an interval rather than a crisp one.

Linear programming problems with the parameters as interval-valued trapezoidal neutrosophic fuzzy numbers are resolved in this paper. Then, the parameters are converted into interval numbers using the proposed de-neutrosophication technique. Hence the given neutrosophic linear programming problem is converted into an interval linear programming problem. To show the efficiency of the proposed technique, two examples are adapted from [47] and solved by using an interval version of linear programming problems namely, the simplex method and the Robust Two-Step method (RTSM) [48]. Later, the efficiency solution obtained by proposed technique is compared with [47] and [49].

The paper includes the subsequent format.

- Rudimentary descriptions related to neutrosophic, interval number and its arithmetic operations are deliberated in section 2.
- Definition and α, β and γ - cut of an interval-valued trapezoidal neutrosophic numbers are conferred in section 3.
- A new interval based de-neutrosophication technique based on α, β and γ - cut of the interval-valued trapezoidal neutrosophic numbers is presented in section 4.
- Mathematical formulation to an interval-valued trapezoidal neutrosophic linear programming problem with some definitions and basic theorems are delibrated in section 5.
- Using simplex algorithm and Robust Two-Step method (RTSM) [48], the computational method using interval numbers are elucidated and demonstrated with examples in sections 6 & 7.
- The contrast analysis and the significance with limitations are explained in section 8 and atlast, the conclusion is derived in section 9.

2 Preliminaries

It is time to recall some basic definitions of neutrosophic set, interval numbers, interval arithmetic operations etc.

2.1 Fuzzy set

Let A be a function that maps the values from the space or universe of discourse X to the unit interval $[0, 1]$. Then the fuzzy set A is the collection of ordered pairs in which each element $x \in X$ is associated with the membership function $\mu_A(x)$ and is defined as $A = \{(x, \mu_A(x)) : x \in X\}$. Here, $\mu_A : X \to [0, 1]$ is the membership function or grade of membership of $x \in X$ in the fuzzy set A.

2.2 Neutrosophic set [50]

Let us denote $\mu_{\widetilde{N}}$ as the truth degrees, $\pi_{\widetilde{N}}$ as the indeterminacy degrees and $\upsilon_{\widetilde{N}}$ as the falsity degrees. Then these three degrees are together assigned with an element a in an universal set X is called an Neutrosophic set $\left(\widetilde{N}\right)$ and is defined as,

$$\widetilde{N} = \left\{\left\langle a, \mu_{\widetilde{N}}(a), \pi_{\widetilde{N}}(a), \upsilon_{\widetilde{N}}(a) : a \in X\right\rangle\right\}. \quad (1)$$

Here $\mu_{\widetilde{N}}(a), \pi_{\widetilde{N}}(a)$ and $\upsilon_{\widetilde{N}}(a)$ belongs to the non-standard interval $]^{-}0, 1^{+}[$. And $1^{+} = 1 + \epsilon$, $^{-}0 = 1 - \epsilon$, where 1 is the standard part and ϵ is the non-standard part. Also $^{-}0 \leq \mu_{\widetilde{N}}(a) + \pi_{\widetilde{N}}(a) + \upsilon_{\widetilde{N}}(a) \leq 3^{+}$.

2.3 Single Valued Neutrosophic set [50]

Since the components in the neutrosophic set belongs to the non-standard interval, it is very challenging while applying these theories in the real life applications. Hence the concept of Single-valued neutrosophic set $\left(\widetilde{SN}\right)$ was enriched by Wang and he defined the components truth, indeterminacy and falsity in the real standard elements of [0,1] and is given as

$$\widetilde{SN} = \left\{\left\langle a, \mu_{\widetilde{N}}(a), \pi_{\widetilde{N}}(a), \upsilon_{\widetilde{N}}(a) : a \in X\right\rangle\right\}, \quad (2)$$

where, $\mu_{\widetilde{N}}(a), \pi_{\widetilde{N}}(a), \upsilon_{\widetilde{N}}(a) \in [0,1]$ and $0 \leq \mu_{\widetilde{N}}(a) + \pi_{\widetilde{N}}(a) + \upsilon_{\widetilde{N}}(a) \leq 3$.

2.4 Interval numbers [23]

Let $\widetilde{p} = [p^L, p^R] = \{p : p^L \leq p \leq p^R, p \in \mathbf{R}\}$, be any interval on $\mathbf{R}$, where p^L is the left limit and p^R is the right limit of $\widetilde{p}$ respectively.
Then $\text{mid}(\widetilde{p}) = \frac{p^L+p^R}{2}$ is the mid-point and $\text{wid}(\widetilde{s}) = \frac{p^R-p^L}{2}$ is the width (half-width) of an interval respectively.
It can also be represented based on mid-point and width (half-width) as

$$\begin{aligned}\widetilde{p} &= \langle \text{mid}(\widetilde{p}), \text{wid}(\widetilde{p}) \rangle \\ &= \{p : \text{mid}(\widetilde{p}) - \text{wid}(\widetilde{p}) \leq p \leq \text{mid}(\widetilde{p}) + \text{wid}(\widetilde{p}),\ p \in \mathbf{R}\}.\end{aligned} \quad (3)$$

2.5 Interval Arithmetic Operations[51]

The interval arithmetic operations proposed by Ming Ma et al. are given below.
If $\widetilde{p} = [p^L, p^R]$ and $\widetilde{s} = [s^L, s^R]$ and for $* \in \{+, -, \times, \div\}$, then

$$\begin{aligned}\widetilde{p} * \widetilde{s} &= \langle \text{mid}(\widetilde{p}), \text{wid}(\widetilde{p}) \rangle * \langle \text{mid}(\widetilde{s}), \text{wid}(\widetilde{s}) \rangle \\ &= \langle \text{mid}(\widetilde{p}) * \text{mid}(\widetilde{s}),\ \max\{\text{wid}(\widetilde{p}), \text{wid}(\widetilde{s})\} \rangle.\end{aligned} \quad (4)$$

2.6 Ranking of interval numbers [52]

Let $\widetilde{p} = [p^L, p^R]$ and $\widetilde{s} = [s^L, s^R]$ be any two intervals in real numbers. And let the extended order relation between those intervals be denoted as $\lessdot$. Then for $\text{mid}(\widetilde{p}) \leq \text{mid}(\widetilde{s})$, we create an assumptions,
$\widetilde{p} \lessdot \widetilde{s} \implies \widetilde{p}$ is inferior to $\widetilde{s}$ ($\widetilde{s}$ is superior to $\widetilde{p}$).

Let I be the set of all closed intervals on the real line $\mathbf{R}$. Define an acceptability function $A : I \times I \to [0, \infty)$ such that

$$A(\widetilde{p} \lessdot \widetilde{s}) = \frac{\text{mid}(\widetilde{s}) - \text{mid}(\widetilde{p})}{\text{wid}(\widetilde{s}) + \text{wid}(\widetilde{p})}, \text{ where } \text{wid}(\widetilde{s}) + \text{wid}(\widetilde{p}) \neq 0 \quad (5)$$

and we can depict $A(\widetilde{p} \lessdot \widetilde{s})$ as the grade of acceptibility for the first interval is inferior to the second interval.

The comparative position of mean and width of the interval $\widetilde{s}$ with respect to the interval $\widetilde{p}$ is categorized as follows.

$$A(\widetilde{p} \lessdot \widetilde{s}) = \begin{cases} 0, & \text{mid}(\widetilde{p}) = \text{mid}(\widetilde{s}) \\ \in (0,1), & \text{mid}(\widetilde{p}) < \text{mid}(\widetilde{s})\ \&\ p^R > s^L \\ \geq 1, & \text{mid}(\widetilde{p}) < \text{mid}(\widetilde{s})\ \&\ p^R < s^L \end{cases}$$

It is performed as given below:

(i) If $A(\widetilde{p} \lessdot \widetilde{s}) = 0$, then $\widetilde{p}$ inferior to $\widetilde{s}$ is not accepted.

(ii) If $0 < A(\widetilde{p} \lessdot \widetilde{s}) < 1\ \&\ A(\widetilde{p} \lessdot \widetilde{s}) \geq 1$, then $\widetilde{p}$ inferior to $\widetilde{s}$ is accepted.

3 Interval - valued trapezoidal Neutrosophic fuzzy number [33]

Bharati [33] presented the defintion of an interval-valued intuitionistic fuzzy number. In intuitionistic environment, each element is associated with membership and nonmembership. The disadvantage part is the hesitancy that depends on membership and non-membership. But discussing hesitancy explicitly is a major advantange in neutrosophic environment and hence we have three parameters truth, indeterminacy and falsity. Therefore, the extended version of the definition interval-valued intuitionistic fuzzy number into interval-valued trapezoidal neutrosophic fuzzy number is mention below.

Let $a_1, a_2, a_3, a_4 \in R$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. An interval-valued trapezoidal neutrosophic fuzzy number can be expressed as $\widetilde{T_n} = \langle (a_1, a_2, a_3, a_4); [\rho^L, \rho^R], [\kappa^L, \kappa^R], [\nu^L, \nu^R] \rangle$, where $\rho^L : X \to [0,1]$, $\rho^R : X \to [0,1]$ are the lower truth and upper truth degrees whose functions are defined as

$$\rho^L_{\widetilde{T_n}}(x) = \begin{cases} \rho^L \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \rho^L, & a_2 \leq x \leq a_3 \\ \rho^L \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

$$\rho^R_{\widetilde{T_n}}(x) = \begin{cases} \rho^R \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \rho^R, & a_2 \leq x \leq a_3 \\ \rho^R \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

Also, $\kappa^L : X \to [0,1]$, $\kappa^R : X \to [0,1]$ are the lower indeterminacy and upper indeterminacy degrees whose functions are defined as

$$\kappa^L_{\widetilde{T_n}}(x) = \begin{cases} \frac{(a_2-x)+\kappa^L(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \kappa^L, & a_2 \leq x \leq a_3 \\ \frac{(x-a_3)+\kappa^L(a_4-x)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases}$$

$$\kappa^R_{\widetilde{T_n}}(x) = \begin{cases} \frac{(a_2-x)+\kappa^R(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \kappa^R, & a_2 \leq x \leq a_3 \\ \frac{(x-a_3)+\kappa^R(a_4-x)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases}$$

Similarly, $\nu^L : X \to [0,1]$, and $\nu^R : X \to [0,1]$ are the lower falsity and upper falsity degrees whose functions are defined as

$$\nu^L_{\widetilde{T_n}}(x) = \begin{cases} \frac{(a_2-x)+\nu^L(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \nu^L, & a_2 \leq x \leq a_3 \\ \frac{(x-a_3)+\nu^L(a_4-x)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases}$$

$$\nu^R_{\widetilde{T_n}}(x) = \begin{cases} \frac{(a_2-x)+\nu^R(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \nu^R, & a_2 \leq x \leq a_3 \\ \frac{(x-a_3)+\nu^R(a_4-x)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases}$$

3.1 (α, β, γ) - cut sets of an Interval-valued trapezoidal neutrosophic fuzzy number

Let $\widetilde{T_n} = \left\langle (a_1, a_2, a_3, a_4); [\rho^L, \rho^R], [\kappa^L, \kappa^R], [\nu^L, \nu^R] \right\rangle$ be any interval-valued trapezoidal neutrosophic fuzzy number. For $\alpha \in [0, 1]$, α - cut of lower truth,

$$\rho^L \frac{x - a_1}{a_2 - a_1} \geq \alpha \implies x - a_1 \geq \frac{\alpha}{\rho^L}(a_2 - a_1)$$
$$\implies x \geq a_1 + \frac{\alpha}{\rho^L}(a_2 - a_1)$$
$$\rho^L \frac{a_4 - x}{a_4 - a_3} \geq \alpha \implies a_4 - x \geq \frac{\alpha}{\rho^L}(a_4 - a_3)$$
$$\implies x \leq a_4 - \frac{\alpha}{\rho^L}(a_4 - a_3)$$

Hence α - cut of lower truth is defined as

$$\alpha^L_{\widetilde{T_n}} = \left[a_1 + \frac{\alpha}{\rho^L}(a_2 - a_1), a_4 - \frac{\alpha}{\rho^L}(a_4 - a_3)\right]. \quad (6)$$

Similarly, α - cut of upper truth,

$$\rho^R \frac{x - a_1}{a_2 - a_1} \geq \alpha \implies x \geq a_1 + \frac{\alpha}{\rho^R}(a_2 - a_1)$$
$$\rho^R \frac{a_4 - x}{a_4 - a_3} \geq \alpha \implies x \leq a_4 - \frac{\alpha}{\rho^R}(a_4 - a_3)$$

Hence α - cut of upper truth is defined as

$$\alpha^R_{\widetilde{T_n}} = \left[a_1 + \frac{\alpha}{\rho^R}(a_2 - a_1), a_4 - \frac{\alpha}{\rho^R}(a_4 - a_3)\right] \quad (7)$$

Let $\beta \in [0, 1]$ and $\beta = 1 - \alpha$, β - cut of lower indeterminacy,

$$\frac{(a_2 - x) + \kappa^L(x - a_1)}{a_2 - a_1} \leq 1 - \alpha$$
$$\implies 1 - \frac{(1 - \kappa^L)(x - a_1)}{a_2 - a_1} \leq 1 - \alpha$$
$$\implies x - a_1 \leq \frac{\alpha(a_2 - a_1)}{1 - \kappa^L}$$
$$\implies x \leq a_1 + \frac{\alpha(a_2 - a_1)}{1 - \kappa^L}$$

Also, $\frac{(x - a_3) + \kappa^L(a_4 - x)}{a_4 - a_3} \leq 1 - \alpha$

$$\implies \frac{(x - a_3)(1 - \kappa^L)}{a_4 - a_3} + \kappa^L \leq 1 - \alpha$$
$$\implies x - a_3 \leq \left(1 - \frac{\alpha}{1 - \kappa^L}\right)(a_4 - a_3)$$
$$\implies x \leq a_4 - \frac{\alpha(a_4 - a_3)}{1 - \kappa^L}$$

Hence β- cut of lower indeterminacy is defined as

$$\beta^L_{\widetilde{T_n}} = \left[a_1 + \frac{\alpha(a_2 - a_1)}{1 - \kappa^L}, a_4 - \frac{\alpha(a_4 - a_3)}{1 - \kappa^L}\right] \quad (8)$$

Similarly, β - cut of upper indeterminacy,

$$\frac{(a_2 - x) + \kappa^R(x - a_1)}{a_2 - a_1} \leq 1 - \alpha$$
$$\implies x \leq a_1 + \frac{\alpha(a_2 - a_1)}{1 - \kappa^R}$$
$$\frac{(x - a_3) + \kappa^R(a_4 - x)}{a_4 - a_3} \leq 1 - \alpha$$
$$\implies x \leq a_4 - \frac{\alpha(a_4 - a_3)}{1 - \kappa^R}$$

Hence β - cut of upper indeterminacy is defined as

$$\beta^R_{\widetilde{T_n}} = \left[a_1 + \frac{\alpha(a_2 - a_1)}{1 - \kappa^R}, a_4 - \frac{\alpha(a_4 - a_3)}{1 - \kappa^R}\right] \quad (9)$$

Let $\gamma \in [0, 1]$ and $\gamma = 1 - \alpha$, γ - cut of lower falsity,

$$\frac{(a_2 - x) + \nu^L(x - a_1)}{a_2 - a_1} \leq 1 - \alpha$$
$$\implies 1 - \frac{(1 - \kappa^L)(x - a_1)}{a_2 - a_1} \leq 1 - \alpha$$
$$\implies x - a_1 \leq \frac{\alpha(a_2 - a_1)}{1 - \nu^L}$$
$$\implies x \leq a_1 + \frac{\alpha(a_2 - a_1)}{1 - \nu^L}$$
$$\frac{(x - a_3) + \nu^L(a_4 - x)}{a_4 - a_3} \leq 1 - \alpha$$
$$\implies \frac{(x - a_3)(1 - \nu^L)}{a_4 - a_3} + \nu^L \leq 1 - \alpha$$
$$\implies x - a_3 \leq \left(1 - \frac{\alpha}{1 - \nu^L}\right)(a_4 - a_3)$$
$$\implies x \leq a_4 - \frac{\alpha(a_4 - a_3)}{1 - \nu^L}$$

Hence γ - cut of lower falsity is defined as

$$\gamma^L_{\widetilde{T_n}} = \left[a_1 + \frac{\alpha(a_2 - a_1)}{1 - \nu^L}, a_4 - \frac{\alpha(a_4 - a_3)}{1 - \nu^L}\right] \quad (10)$$

Similarly, γ - cut of upper falsity,

$$\frac{(a_2 - x) + \nu^R(x - a_1)}{a_2 - a_1} \leq 1 - \alpha$$
$$\implies x \leq a_1 + \frac{\alpha(a_2 - a_1)}{1 - \nu^R}$$
$$\frac{(x - a_3) + \nu^R(a_4 - x)}{a_4 - a_3} \leq 1 - \alpha$$
$$\implies x \leq a_4 - \frac{\alpha(a_4 - a_3)}{1 - \nu^R}$$

Hence γ - cut of upper falsity is defined as

$$\gamma^R_{\widetilde{T_n}} = \left[a_1 + \frac{\alpha(a_2 - a_1)}{1 - \nu^R}, a_4 - \frac{\alpha(a_4 - a_3)}{1 - \nu^R}\right] \quad (11)$$

4 A new ranking - interval form using $(\alpha, \beta, \gamma)-$ cut sets of an interval-valued trapezoidal neutrosophic fuzzy number

In this segment, we proposed ranking of interval-valued trapezoidal neutrosophic fuzzy numbers. Most of the de-neutrosophication technique in neutrosophic environment is in crisp form but we represents in interval form. And it is formulated based on mid point and width (half-width) of (α, β, γ) - cut sets associated with truth, indeterminacy and falsity that includes for both lower and upper limits. Let $\text{mid}(\alpha^L_{\widetilde{T_n}}), \text{mid}(\beta^L_{\widetilde{T_n}}), \text{mid}(\gamma^L_{\widetilde{T_n}})$ $\text{mid}(\alpha^R_{\widetilde{T_n}}), \text{mid}(\beta^R_{\widetilde{T_n}})$ and $\text{mid}(\gamma^R_{\widetilde{T_n}})$ are the lower limits and upper limits of the midpoint of (α, β, γ) - cut corresponding to truth, indeterminacy and falsity respectively. Let $\text{wid}(\alpha^L_{\widetilde{T_n}}), \text{wid}(\beta^L_{\widetilde{T_n}}), \text{wid}(\gamma^L_{\widetilde{T_n}}), \text{wid}(\alpha^R_{\widetilde{T_n}}), \text{wid}(\beta^R_{\widetilde{T_n}})$ and $\text{wid}(\gamma^R_{\widetilde{T_n}})$ are the lower limits and upper limits of width (half-width) of (α, β, γ) - cut that corresponds to truth, indeterminacy and falsity respectively.

Consider $r \in [0, 1]$.
Based on equation 6, the mid point and width (half-width) based on the α - cut of lower truth is given as

$$\left\langle \text{mid}(\alpha^L_{\widetilde{T_n}}), \text{wid}(\alpha^L_{\widetilde{T_n}}) \right\rangle = \Big\langle \frac{1}{2}\{(a_1 + a_4) + \frac{\alpha}{\rho^L}(a_2 - a_1 - a_4 + a_3)\}, \frac{1}{2}\{(a_4 - a_1) - \frac{\alpha}{\rho^L}(a_4 - a_3 + a_2 - a_1)\} \Big\rangle \quad (12)$$

Similarly, using equation 7, the mid point and width (half-width) based on the α - cut of upper truth is expressed as

$$\left\langle \text{mid}(\alpha^R_{\widetilde{T_n}}), \text{wid}(\alpha^R_{\widetilde{T_n}}) \right\rangle = \Big\langle \frac{1}{2}\{(a_1 + a_4) + \frac{\alpha}{\rho^R}(a_2 - a_1 - a_4 + a_3)\}, \frac{1}{2}\{(a_4 - a_1) - \frac{\alpha}{\rho^R}(a_4 - a_3 + a_2 - a_1)\} \Big\rangle \quad (13)$$

From equations 12 & 13, the mid point and width(half-width) based on the truth membership using 2.5 is obtained as

$$\begin{aligned} f(\alpha_{\widetilde{T_n}}) &= r\left\langle \text{mid}(\alpha^L_{\widetilde{T_n}}), \text{wid}(\alpha^L_{\widetilde{T_n}}) \right\rangle + (1-r)\left\langle \text{mid}(\alpha^R_{\widetilde{T_n}}), \text{wid}(\alpha^R_{\widetilde{T_n}}) \right\rangle \\ &= \left\langle r\ \text{mid}(\alpha^L_{\widetilde{T_n}}) + (1-r)\ \text{mid}(\alpha^R_{\widetilde{T_n}}), \max\{\text{wid}(\alpha^L_{\widetilde{T_n}}), w(\alpha^R_{\widetilde{T_n}})\} \right\rangle \end{aligned} \quad (14)$$

Using equation 8, the mid point and width (half-width) based on the β - cut of lower indeterminacy is acquired as

$$\left\langle \text{mid}(\beta^L_{\widetilde{T_n}}), \text{wid}(\beta^L_{\widetilde{T_n}}) \right\rangle = \Big\langle \frac{1}{2}\{(a_1 + a_4) + \frac{\alpha}{1-\kappa^L}(a_2 - a_1 - a_4 + a_3)\}, \frac{1}{2}\{(a_4 - a_1) - \frac{\alpha}{1-\kappa^L}(a_4 - a_3 + a_2 - a_1)\} \Big\rangle \quad (15)$$

Similarly, using equation 9, the mid point and width (half-width) based on the β - cut of upper indeterminacy is expressed as

$$\left\langle \text{mid}(\beta^R_{\widetilde{T_n}}), \text{wid}(\beta^R_{\widetilde{T_n}}) \right\rangle = \Big\langle \frac{1}{2}\{(a_1 + a_4) + \frac{\alpha}{1-\kappa^R}(a_2 - a_1 - a_4 + a_3)\}, \frac{1}{2}\{(a_4 - a_1) - \frac{\alpha}{1-\kappa^R}(a_4 - a_3 + a_2 - a_1)\} \Big\rangle \quad (16)$$

From equations 15 & 16, the mid point and width(half-width) of indeterminacy using 2.5 is given as

$$\begin{aligned} f(\beta_{\widetilde{T_n}}) &= r\left\langle \text{mid}(\beta^L_{\widetilde{T_n}}), \text{wid}(\beta^L_{\widetilde{T_n}}) \right\rangle + (1-r)\left\langle \text{mid}(\beta^R_{\widetilde{T_n}}), w(\beta^R_{\widetilde{T_n}}) \right\rangle \\ &= \left\langle r\ \text{mid}(\beta^L_{\widetilde{T_n}}) + (1-r)\ \text{mid}(\beta^R_{\widetilde{T_n}}), \max\{\text{wid}(\beta^L_{\widetilde{T_n}}), \text{wid}(\beta^R_{\widetilde{T_n}})\} \right\rangle \end{aligned} \quad (17)$$

Using equation 10, the mid point and width (half-width) based on the γ - cut of lower falsity is expressed as

$$\left\langle \text{mid}(\gamma^L_{\widetilde{T_n}}), \text{wid}(\gamma^L_{\widetilde{T_n}}) \right\rangle = \Big\langle \frac{1}{2}\{(a_1 + a_4) + \frac{\alpha}{1-\nu^L}(a_2 - a_1 - a_4 + a_3)\}, \frac{1}{2}\{(a_4 - a_1) - \frac{\alpha}{1-\nu^L}(a_4 - a_3 + a_2 - a_1)\} \Big\rangle \quad (18)$$

Similarly, using equation 11, the mid point and width (half-width) based on the γ - cut of upper falsity is expressed as

$$\left\langle \text{mid}(\gamma^R_{\widetilde{T_n}}), \text{wid}(\gamma^R_{\widetilde{T_n}}) \right\rangle = \Big\langle \frac{1}{2}\{(a_1 + a_4) + \frac{\alpha}{1-\nu^R}(a_2 - a_1 - a_4 + a_3)\}, \frac{1}{2}\{(a_4 - a_1) - \frac{\alpha}{1-\nu^R}(a_4 - a_3 + a_2 - a_1)\} \Big\rangle \quad (19)$$

From equations 18 & 19, the mid point and width(half-width) of falsity using 2.5 is given as

$$\begin{aligned} f(\gamma_{\widetilde{T_n}}) &= r\left\langle \text{mid}(\gamma^L_{\widetilde{T_n}}), \text{wid}(\gamma^L_{\widetilde{T_n}}) \right\rangle + (1-r)\left\langle \text{mid}(\gamma^R_{\widetilde{T_n}}), \text{wid}(\gamma^R_{\widetilde{T_n}}) \right\rangle \\ &= \left\langle r\ \text{mid}(\gamma^L_{\widetilde{T_n}}) + (1-r)\ \text{mid}(\gamma^R_{\widetilde{T_n}}), \max\{\text{wid}(\gamma^L_{\widetilde{T_n}}), \text{wid}(\gamma^R_{\widetilde{T_n}})\} \right\rangle \end{aligned} \quad (20)$$

4.1 Ranking function based on interval number (midpoint and width (half-width))

Let us define a ranking function that allocates every interval-valued trapezoidal neutrosophic number to an interval number.

The ranking function based on interval number (midpoint and width (half-width)) is obtained by using 14, 17 & 20 and is defined as

$$R(\widetilde{T_n}) = sf(\alpha_{\widetilde{T_n}}) + (1-s)\{f(\beta_{\widetilde{T_n}}) + f(\gamma_{\widetilde{T_n}})\}, \quad (21)$$

where $s \in [0,1]$ denotes the various choices for the $(\alpha,\beta,\gamma)-$cut sets of the truth, indeterminacy and falsity membership degrees using interval numbers and it is purely depends on the decision makers preference that may be uncertainty, neutral or certainty.

4.2 Relation between interval-valued trapezoidal neutrosophic fuzzy numbers based on interval numbers

Sengupta and Pal [52] introduced ranking of interval numbers and it is discussed in section 2.6. Our aim is to rank two interval-valued trapezoidal neutrosophic number. It introduces the following ranking methodology for interval-valued trapezoidal neutrosophic numbers. And we apply it to relate two interval-valued trapezoidal neutrosophic fuzzy numbers based on interval numbers.

Let $\widetilde{T_n1} = \langle (a_1,a_2,a_3,a_4); [\rho1^L,\rho1^R],[\kappa1^L,\kappa1^R],[\nu1^L,\nu1^R]\rangle$ & $\widetilde{T_n2} = \langle (b_1,b_2,b_3,b_4); [\rho2^L,\rho2^R],[\kappa2^L,\kappa2^R],[\nu2^L,\nu2^R]\rangle$ be any two interval-valued trapezoidal neutrosophic fuzzy numbers.

Using the sections 4 & 4.1, we convert the interval-valued trapezoidal neutrosophic fuzzy numbers into interval numbers. Then we can define the relation between the ranking function as follows:

Let the interval form of the given two interval-valued trapezoidal neutrosophic fuzzy numbers be denoted as $R(\widetilde{T_n1}) = \langle \text{mid}(\widetilde{T_n1}), \text{wid}(\widetilde{T_n1})\rangle$ and $R(\widetilde{T_n2}) = \langle \text{mid}(\widetilde{T_n2}), \text{wid}(\widetilde{T_n2})\rangle$. Based on section 2.6, we can perform $A(\widetilde{T_n1} \otimes \widetilde{T_n2})$ as the grade of acceptibility of the first interval to be inferior to the second interval and is given as

$$A(\widetilde{T_n1} \otimes \widetilde{T_n2}) = \frac{\text{mid}(\widetilde{T_n2}) - \text{mid}(\widetilde{T_n1})}{\text{wid}(\widetilde{T_n2}) + \text{wid}(\widetilde{T_n1})},$$

where, $\text{wid}(\widetilde{T_n2}) + \text{wid}(\widetilde{T_n1}) \neq 0$

It is performed as given below:

(i) If $A(\widetilde{T_n1} \otimes \widetilde{T_n2}) = 0$, then $\widetilde{T_n1}$ inferior to $\widetilde{T_n2}$ is not accepted.

(ii) If $0 < A(\widetilde{T_n1} \otimes \widetilde{T_n2}) < 1$ & $A(\widetilde{T_n1} \otimes \widetilde{T_n2}) \geq 1$, then $\widetilde{T_n1}$ inferior to $\widetilde{T_n2}$ is accepted.

4.3 Representation of interval-valued trapezoidal neutrosophic fuzzy number into interval number

Let $\widetilde{T_n1} = \langle (30,35,45,50); [0.6,0.8],[0.3,0.5],[0.2,0.4]\rangle$ Figure 1 is the pictorial representation of the above interval-valued trapezoidal neutrosophic fuzzy number.

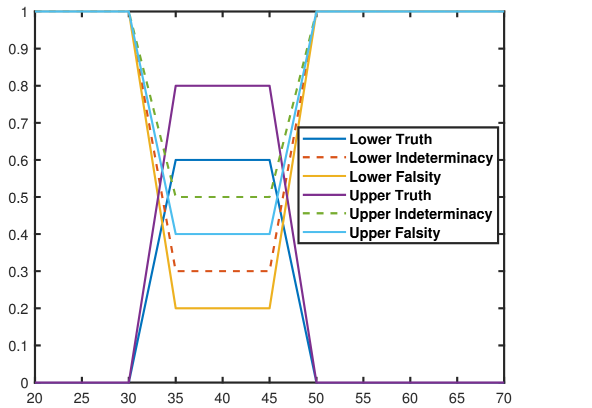

Figure 1. Pictorial representation of interval-valued trapezoidal neutrosophic fuzzy number

Choose α,β,γ,r & $s = 0.6$, using section4,

$$\langle \text{mid}(\alpha^L_{\widetilde{T_n1}}), \text{wid}(\alpha^L_{\widetilde{T_n1}})\rangle = \langle 40,5\rangle, \text{ and}$$
$$\langle \text{mid}(\alpha^R_{\widetilde{T_n1}}), \text{wid}(\alpha^R_{\widetilde{T_n1}})\rangle = \langle 40,6.25\rangle$$

$$\text{Hence } f(\alpha_{\widetilde{T_n1}}) = r\langle 40,5\rangle + (1-r)\langle 40,6.25\rangle = \langle 40r+40-40r,\ \max\{5,6.25\}\rangle = \langle 40,6.25\rangle$$

Also, $\langle \text{mid}(\beta^L_{\widetilde{T_n1}}), \text{wid}(\beta^L_{\widetilde{T_n1}})\rangle = \langle 40,5.75\rangle$ and

$$\langle \text{mid}(\beta^R_{\widetilde{T_n1}}), \text{wid}(\beta^R_{\widetilde{T_n1}})\rangle = \langle 40,4\rangle$$

$$\text{and } f(\beta_{\widetilde{T_n1}}) = r\langle 40,5.71\rangle + (1-r)\langle 40,4\rangle = \langle 40r+40-40r,\ \max\{5.71,4\}\rangle = \langle 40,5.71\rangle$$

Similarly, $\langle \text{mid}(\gamma^L_{\widetilde{T_n1}}), \text{wid}(\gamma^L_{\widetilde{T_n1}})\rangle = \langle 40,6.25\rangle$ and

$$\langle \text{mid}(\gamma^R_{\widetilde{T_n1}}), \text{wid}(\gamma^R_{\widetilde{T_n1}})\rangle = \langle 40,5\rangle$$

$$\text{and } f(\gamma_{\widetilde{T_n1}}) = r\langle 40,6.25\rangle + (1-r)\langle 40,5\rangle = \langle 40r+40-40r,\ \max\{6.25,5\}\rangle = \langle 40,6.25\rangle$$

Using section 4.1,

$$R(\widetilde{T_n1}) = s\langle 40,6.25\rangle + (1-s)\{\langle 40,5.71\rangle + \langle 40,6.25\rangle\} = \langle 40s+40(1-s)+40(1-s),\ \max\{6.25,5.71,6.25\}\rangle = \langle 56,6.25\rangle$$

4.4 Comparison of two interval-valued trapezoidal neutrosophic fuzzy number interms of interval number

Let $\widetilde{T_n 1} = \langle (30, 35, 45, 50); [0.6, 0.8], [0.3, 0.5], [0.2, 0.4] \rangle$,
then $R(\widetilde{T_n 1}) = \langle 56, 6.25 \rangle$
and $\widetilde{T_n 2} = \langle (40, 45, 55, 60); [0.5, 0.7], [0.4, 0.6], [0.1, 0.3] \rangle$,
then $R(\widetilde{T_n 2}) = \langle 70, 6.67 \rangle$

Using 4.2, $A(\widetilde{T_n 1} \oslash \widetilde{T_n 2}) = \dfrac{\text{mid}(\widetilde{T_n 2}) - \text{mid}(\widetilde{T_n 1})}{\text{wid}(\widetilde{T_n 2}) + \text{wid}(\widetilde{T_n 1})}$

$= \dfrac{70 - 56}{6.25 + 6.67} = \dfrac{14}{12.62} = 1.109 > 1$

Hence by 4.2, $\widetilde{T_n 1}$ is inferior to $\widetilde{T_n 2}$ is accepted.

Therefore, $\widetilde{T_n 1} < \widetilde{T_n 2}$

5 Interval-valued Trapezoidal Neutrosophic fuzzy linear programming problem $(NLPP_{ivt})$

This section provides some definitions and theorems based on $(NLPP_{ivt})$ before applying the proposed de-neutrosophication technique.

A linear programming problem in which all or some of the parameters are encountered as interval-valued trapezoidal neutrosophic fuzzy numbers is called an interval-valued trapezoidal neutrosophic fuzzy linear programming problem $(NLPP_{ivt})$. Considering the problem in a neutrosophic way makes the decision makers to analyze their set of values in truth, indeterminacy and falsity membership degrees. And adaption of interval-valued is flexible to analyze the values more efficient. The mathematical formulation is designed as follows.

$$\text{Maximize} \quad \widetilde{Z}^{N_{ivt}} = \sum_{j=1}^{n} \widetilde{c_j}^{N_{ivt}} \; \widetilde{x_j}^{N_{ivt}}$$

$$\text{subject to} \quad \sum_{j=1}^{n} \widetilde{a_{ij}}^{N_{ivt}} \; \widetilde{x}^{N_{ivt}} \; \leq \widetilde{b_i}^{N_{ivt}},$$

$$\widetilde{x_{ij}}^{N_{ivt}} \geq \widetilde{0} \;\; i = 1, 2...m \qquad (22)$$

where $\widetilde{c}^{N_{ivt}}, \widetilde{x}^{N_{ivt}}, \widetilde{b}^{N_{ivt}}$ and $\widetilde{a}^{N_{ivt}}$ are interval-valued trapezoidal neutrosophic fuzzy numbers.

5.1 Interval-valued trapezoidal neutrosophic fuzzy basic solution

Given a system of m simultaneous interval-valued trapezoidal neutrosophic fuzzy linear equations in n unknowns, $m \leq n$, $\widetilde{A}^{N_{ivt}} \widetilde{x}^{N_{ivt}} = \widetilde{b}^{N_{ivt}}$, $\widetilde{x}^{N^T_{ivt}} \in \mathbf{R^n}$, $\widetilde{A}^{N_{ivt}}$ is a $m \times n$ interval-valued trapezoidal neutrosophic fuzzy matrix of rank m.

Now, $\widetilde{A}^{N_{ivt}} = [\widetilde{B}^{N^T_{ivt}}, \widetilde{N}^{N^T_{ivt}}]$, where $\widetilde{B}^{N^T_{ivt}}$ be any $m \times m$ non-singular interval-valued trapezoidal neutrosophic fuzzy submatrix formed by m linearly independent columns of $\widetilde{A}^{N_{ivt}}$ and is named as basis matrix. And $\widetilde{N}^{N^T_{ivt}}$ is named as non-basis matrix. Then an interval-valued trapezoidal neutrosophic fuzzy basic solution $(\widetilde{X}_B^{N_{ivt}}, \widetilde{N}^{N^T_{ivt}})$ is obtained by setting $\widetilde{X}_B^{N_{ivt}} = (\widetilde{B}^{N^T_{ivt}})^{-1} \widetilde{b}^{N_{ivt}}$, $\widetilde{N}^{N^T_{ivt}} = 0^{N^T_{ivt}}$. The component $\widetilde{X}_B^{N_{ivt}}$ is called a basic variable and $\widetilde{N}^{N^T_{ivt}}$ is called a non-basic variable.

5.2 Interval-valued trapezoidal neutrosophic fuzzy feasible solution

If $\widetilde{x}^{N_{ivt}}$ satisfies all the given constraints, then $\widetilde{x}^{N_{ivt}}$ is called an interval-valued trapezoidal neutrosophic fuzzy feasible solution (NFS_{ivt}) of (22).

5.3 Interval-valued trapezoidal neutrosophic fuzzy basic feasible solution

An interval-valued trapezoidal neutrosophic fuzzy feasible solution which is also an interval-valued trapezoidal neutrosophic fuzzy basic solution is called an interval-valued trapezoidal neutrosophic fuzzy basic feasible solution.

5.4 Interval-valued trapezoidal neutrosophic fuzzy optimal solution

Any interval-valued trapezoidal neutrosophic fuzzy feasible solution (NFS_{ivt}) which optimizes the interval-valued trapezoidal neutrosophic objective function of the interval-valued trapezoidal neutrosophic linear programming problem $(NLPP_{ivt})$ is called an interval-valued trapezoidal neutrosophic fuzzy optimal solution.

i.e., An interval-valued trapezoidal neutrosophic fuzzy feasible solution $\widetilde{x}^{*N_{ivt}}$ is an interval-valued trapezoidal neutrosophic fuzzy optimal solution assuming that all solutions $\widetilde{x}^{N_{ivt}}$ to (22), $\widetilde{c}^{N_{ivt}} \widetilde{x}^{*N_{ivt}} \geq \widetilde{c}^{N_{ivt}} \widetilde{x}^{N_{ivt}}$.

Theorem 1 The set of all feasible solution to an interval-valued trapezoidal neutrosophic linear programming problem is a convex set.
Proof.
Consider the system defined in (22).
To show that the convex combination of all feasible solutions to an interval-valued trapezoidal neutrosophic linear programming problem is also a feasible solution.
Let the set of all feasible solution to the interval-valued trapezoidal neutrosophic linear programming problem be denoted as S.
To prove S is a convex set.
i.e., To prove $\widetilde{X_1}^{N_{ivt}}, \widetilde{X_2}^{N_{ivt}} \in S$
$\implies \lambda \widetilde{X_1}^{N_{ivt}} + (1 - \lambda) \widetilde{X_2}^{N_{ivt}} \in S$, where $0 \leq \lambda \leq 1$
Since $\widetilde{X_1}^{N_{ivt}}$, $\widetilde{X_2}^{N_{ivt}}$ are feasible solution to (22), that implies $\widetilde{A}^{N_{ivt}} \widetilde{X_1}^{N_{ivt}} = \widetilde{b}^{N_{ivt}}$ and $\widetilde{A}^{N_{ivt}} \widetilde{X_2}^{N_{ivt}} = \widetilde{b}^{N_{ivt}}$.
Also, $\widetilde{X_1}^{N_{ivt}} \geq 0$ and $\widetilde{X_2}^{N_{ivt}} \geq 0$.
Let $\widetilde{X^1}^{N_{ivt}} = \lambda \widetilde{X_1}^{N_{ivt}} + (1 - \lambda) \widetilde{X_2}^{N_{ivt}}$,

now $\widetilde{A}^{N_{ivt}}\widetilde{X^1}^{N_{ivt}} = \widetilde{A}^{N_{ivt}}\left[\lambda\widetilde{X_1}^{N_{ivt}} + (1-\lambda)\widetilde{X_2}^{N_{ivt}}\right]$
$= \lambda\widetilde{A}^{N_{ivt}}\widetilde{X_1}^{N_{ivt}} + (1-\lambda)\widetilde{A}^{N_{ivt}}\widetilde{X_2}^{N_{ivt}}$
$= \lambda\widetilde{b}^{N_{ivt}} + (1-\lambda)\widetilde{b}^{N_{ivt}} = \widetilde{b}^{N_{ivt}}$.
Hence $\widetilde{X^1}^{N_{ivt}}$ is a solution to the interval-valued trapezoidal neutrosophic linear programming problem.
Again, $\widetilde{X_1}^{N_{ivt}} \geq 0$ and $\widetilde{X_2}^{N_{ivt}} \geq 0$
$\implies \lambda\widetilde{X_1}^{N_{ivt}} + (1-\lambda)\widetilde{X_2}^{N_{ivt}} \geq 0.$
Hence $\widetilde{X^1}^{N_{ivt}}$ is a feasible solution to the interval-valued trapezoidal neutrosophic linear programming problem.
Hence $\widetilde{X^1}^{N_{ivt}} \in S \implies S$ is a convex set.

Theorem 2 Let the interval-valued trapezoidal neutrosophic fuzzy basic feasible solution to (22) be $\widetilde{X}_B^{N_{ivt}} \approx (\widetilde{B}^{N_{ivt}})^{-1}\widetilde{b}^{N_{ivt}}$. If for any column $\widetilde{a_j}^{N_{ivt}}$ in $\widetilde{A}^{N_{ivt}}$ which is not in $\widetilde{B}^{N_{ivt}}$, the condition $\widetilde{Z_j}^{N_{ivt}} - \widetilde{C_j}^{N_{ivt}} < \widetilde{0}^{N_{ivt}}$ hold and $\widetilde{y_{ij}}^{N_{ivt}} > \widetilde{0}^{N_{ivt}}$ for some $i,\ i \in \{1,2,...,m\}$ then it is viable to obtain a new interval-valued trapezoidal neutrosophic beasic feasible solution by displacing one of the columns in $\widetilde{B}^{N_{ivt}}$ by $\widetilde{a_j}^{N_{ivt}}$.

Proof.
Let $\widetilde{X}_B^{N_{ivt}} = (\widetilde{x}_{B_1}^{N_{ivt}}, \widetilde{x}_{B_2}^{N_{ivt}}, ..., \widetilde{x}_{B_m}^{N_{ivt}})$.
Suppose that $\widetilde{X}_B^{N_{ivt}}$ be an interval-valued trapezoidal neutrosophic fuzzy basic feasible solution and it has k positive components, such that $\widetilde{X}_B^{N_{ivt}} \approx (\widetilde{B}^{N_{ivt}})^{-1}\ \widetilde{b}^{N_{ivt}}$ or $\widetilde{B}^{N_{ivt}}\widetilde{X}_B^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}}$ where $\widetilde{x}_{B_i}^{N_{ivt}}$ belongs to the set of all interval-valued trapezoidal neutrosophic numbers for $i = 1,2,...m$ and $\widetilde{x}_{B_i}^{N_{ivt}} > \widetilde{0}^{N_{ivt}}$ for $i = 1,2,...k,\ k < m$.

Now, $\widetilde{B}^{N_{ivt}}\widetilde{X}_B^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}}$

$$\implies \sum_{i=1}^{k} \widetilde{x}_{B_i}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} + \widetilde{0}^{N_{ivt}}\widetilde{b}_{k+1}^{N_{ivt}} + \widetilde{0}^{N_{ivt}}\widetilde{b}_{k+2}^{N_{ivt}} + ... + \widetilde{0}^{N_{ivt}}\widetilde{b}_m^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}}$$

$$\implies \sum_{i=1}^{k} \widetilde{x}_{B_i}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} + \sum_{i=k+1}^{m} \widetilde{0}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}} \tag{23}$$

Then any column $\widetilde{a_j}^{N_{ivt}}$ in $\widetilde{A}^{N_{ivt}}$ which is not in $\widetilde{B}^{N_{ivt}}$ can be written as

$$\widetilde{a_j}^{N_{ivt}} \approx \sum_{i=1}^{m} \widetilde{y}_{ij}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} \approx \widetilde{y}_j^{N_{ivt}}\widetilde{B}^{N_{ivt}}$$

And we know that if the basis vector $\widetilde{b}_r^{N_{ivt}}$ for which $\widetilde{y}_{rj}^{N_{ivt}} \neq \widetilde{0}^{N_{ivt}}$ is displaced by $\widetilde{a_j}^{N_{ivt}}$ in $\widetilde{A}^{N_{ivt}}$, then the new set of vectors $\widetilde{b}_i^{N_{ivt}}$ $(i = 1,2,...,m)$ form a basis.
For $\widetilde{y}_{rj}^{N_{ivt}} \neq \widetilde{0}^{N_{ivt}}$ and $r \leq k$,

$$\widetilde{b}_r^{N_{ivt}} \approx \frac{\widetilde{a_j}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} - \sum_{i=1_{i\neq r}}^{m} \frac{\widetilde{y}_{ij}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\,\widetilde{b}_i^{N_{ivt}} \approx \frac{\widetilde{a_j}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} - \sum_{i=1_{i\neq r}}^{m} \frac{\widetilde{y}_{ij}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\,\widetilde{b}_i^{N_{ivt}} - \sum_{i=k+1}^{m} \frac{\widetilde{y}_{ij}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\,\widetilde{b}_i^{N_{ivt}}$$

From Equation 23,

$$\sum_{i=1_{i\neq r}}^{m} \widetilde{x}_{B_i}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} + \widetilde{x}_{B_r}^{N_{ivt}}\widetilde{b}_r^{N_{ivt}} + \sum_{i=k+1}^{m} \widetilde{0}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}}$$

$$\implies \sum_{i=1_{i\neq r}}^{m} \widetilde{x}_{B_i}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} + \widetilde{x}_{B_r}^{N_{ivt}} \left(\frac{\widetilde{a_j}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} - \sum_{i=1_{i\neq r}}^{k} \frac{\widetilde{y}_{ij}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\,\widetilde{b}_i^{N_{ivt}} - \sum_{i=k+1}^{m} \frac{\widetilde{y}_{ij}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\,\widetilde{b}_i^{N_{ivt}}\right) + \sum_{i=k+1}^{m} \widetilde{0}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}}$$

$$\implies \sum_{i=1_{i\neq r}}^{m} \widetilde{x}_{B_i}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} + \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\widetilde{a_j}^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\sum_{i=1_{i\neq r}}^{k} \widetilde{y}_{ij}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\sum_{i=k+1}^{m} \widetilde{y}_{ij}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} + \sum_{i=k+1}^{m} \widetilde{0}^{N_{ivt}}\widetilde{b}_i^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}}$$

$$\implies \sum_{i=1_{i\neq r}}^{m} \left(\widetilde{x}_{B_i}^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\widetilde{y}_{ij}^{N_{ivt}}\right)\widetilde{b}_i^{N_{ivt}} + \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\widetilde{a_j}^{N_{ivt}} + \sum_{i=k+1}^{m} \left(\widetilde{0}^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\widetilde{y}_{ij}^{N_{ivt}}\right)\widetilde{b}_i^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}}$$

Since $\widetilde{x}_{B_i}^{N_{ivt}} \approx \widetilde{0}^{N_{ivt}}$, for $i = k+1,...,m,$ then

$$\sum_{i=1_{i\neq r}}^{m} \left(\widetilde{x}_{B_i}^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\widetilde{y}_{ij}^{N_{ivt}}\right)\widetilde{b}_i^{N_{ivt}} + \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\widetilde{a_j}^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}}$$

$$\implies \sum_{i=1_{i\neq r}}^{m} \widehat{\widetilde{X}_{B_i}^{N_{ivt}}}\widetilde{b}_i^{N_{ivt}} - \widehat{\widetilde{x}_{B_r}}\widetilde{a_j}^{N_{ivt}} \approx \widetilde{b}^{N_{ivt}},$$

where $\widehat{\widetilde{X}_{B_i}^{N_{ivt}}} \approx \widetilde{x}_{B_i}^{N_{ivt}} - \dfrac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\widetilde{y}_{ij}^{N_{ivt}}, i \neq r$ and

$$\widehat{\widetilde{x}_{B_r}} \approx \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}},$$

which gives a new interval-valued trapezoidal neutrosophic fuzzy basic solution to $\widetilde{A}^{N_{ivt}}\widetilde{x}^{N_{ivt}} = \widetilde{b}^{N_{ivt}}$.
Now prove this new interval-valued trapezoidal neutrosophic fuzzy basic solution is also feasible.
This requires that $\widetilde{x}_{B_i}^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\widetilde{y}_{ij}^{N_{ivt}} \geq \widetilde{0}^{N_{ivt}},\ i \neq r$ and $\frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} \geq \widetilde{0}^{N_{ivt}}$.
Then choose $\widetilde{y}_{rj}^{N_{ivt}} \geq \widetilde{0}^{N_{ivt}}$ such that $\frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} \approx \min\limits_i \left\{\frac{\widetilde{x}_{B_i}^{N_{ivt}}}{\widetilde{y}_{ij}^{N_{ivt}}} : \widetilde{y}_{ij}^{N_{ivt}} \geq \widetilde{0}^{N_{ivt}}\right\}$.
Then $\frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} \leq \frac{\widetilde{x}_{B_i}^{N_{ivt}}}{\widetilde{y}_{ij}^{N_{ivt}}} \implies \frac{\widetilde{x}_{B_i}^{N_{ivt}}}{\widetilde{y}_{ij}^{N_{ivt}}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} \geq \widetilde{0}^{N_{ivt}}$.
Hence the new interval-valued trapezoidal neutrosophic fuzzy basic solution is also feasible.

Theorem 3 Let $\widetilde{X}_B^{N_{ivt}} \approx (\widetilde{B}^{N_{ivt}^T})^{-1}\widetilde{b}^{N_{ivt}}$ be an interval-valued trapezoidal neutrosophic fuzzy basic feasible solution to the (22). Let $\widehat{\widetilde{X}_B^{N_{ivt}}}$ be another interval-valued trapezoidal neutrosophic fuzzy basic feasible solution obtained by admitting a non-basis column vector $\widetilde{x_j}^{N_{ivt}}$ in the basis for which the net evaluation $\widetilde{Z_j}^{N_{ivt}} - \widetilde{C_j}^{N_{ivt}}$ is negative, then $\widehat{\widetilde{X}_B^{N_{ivt}}}$ is an improved interval-valued trapezoidal neutrosophic fuzzy basic feasible solution to the problem. i.e., $\widehat{\widetilde{C}_B^{N_{ivt}}}\widehat{\widetilde{X}_B^{N_{ivt}}} \geq \widetilde{C}_B^{N_{ivt}}\widetilde{X}_B^{N_{ivt}}$ i.e., $\widehat{\widetilde{Z}^{N_{ivt}}} \geq \widetilde{Z}_0^{N_{ivt}}$.

Proof.

Let $\widetilde{X}_B^{N_{ivt}}$ be the interval-valued trapezoidal neutrosophic fuzzy basic feasible solution and $\widetilde{Z}_0^{N_{ivt}} \approx \widetilde{C}_B^{N_{ivt}}\widetilde{X}_B^{N_{ivt}}$.

Let $\widetilde{x_j}^{N_{ivt}}$ be the column vector introduced in the basis such that $\widetilde{Z_j}^{N_{ivt}} - \widetilde{C_j}^{N_{ivt}} < \widetilde{0}^{N_{ivt}}$.

Let $\widetilde{b}_r^{N_{ivt}}$ be the vector removed from the basis.

Let $\widehat{\widetilde{X}_B^{N_{ivt}}}$ be the new interval-valued trapezoidal neutrosophic fuzzy basic feasible solution,

then $\widehat{\widetilde{x}_{B_i}^{N_{ivt}}} \approx \widetilde{x}_{B_i}^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{r_j}^{N_{ivt}}}\widetilde{y}_{ij}^{N_{ivt}}$, $i \neq r$ and $\widehat{\widetilde{x}_{B_r}^{N_{ivt}}} \approx \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{r_j}^{N_{ivt}}}$.

Since $\widehat{\widetilde{C}_{B_i}^{N_{ivt}}} \approx \widetilde{C}_{B_i}^{N_{ivt}}$, $i \neq r$ and $\widehat{\widetilde{C}_{B_r}^{N_{ivt}}} \approx \widetilde{C}_j^{N_{ivt}}$ and now the new value of the interval-valued trapezoidal neutrosophic fuzzy objective function is

$$\begin{aligned}
\widehat{\widetilde{Z}^{N_{ivt}}} &\approx \sum_{i=1}^{m} \widehat{\widetilde{C}_{B_i}^{N_{ivt}}}\ \widehat{\widetilde{X}_{B_i}^{N_{ivt}}} \\
&\approx \sum_{i=1}^{m} \widehat{\widetilde{C}_{B_i}^{N_{ivt}}}\ \widehat{\widetilde{x}_{B_i}^{N_{ivt}}} + \widehat{\widetilde{C}_{B_r}}^{N_{ivt}}\ \widehat{\widetilde{x}_{B_r}^{N_{ivt}}} \\
&\approx \sum_{i=1_{i\neq r}}^{m} \widetilde{C}_{B_i}^{N_{ivt}}\ \left[\widetilde{x}_{B_i}^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{r_j}^{N_{ivt}}}\ \widetilde{y}_{ij}^{N_{ivt}}\right] \\
&\quad + \widetilde{C}_{B_r}^{N_{ivt}}\ \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} \\
&\approx \sum_{i=1_{i\neq r}}^{m} \widetilde{C}_{B_i}^{N_{ivt}}\widetilde{x}_{B_i}^{N_{ivt}} - \sum_{i=1_{i\neq r}}^{m} \widetilde{y}_{ij}^{N_{ivt}}\ \widetilde{C}_{B_i}^{N_{ivt}}\ \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} \\
&\quad + \widetilde{C}_{B_r}^{N_{ivt}}\ \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{r_j}^{N_{ivt}}} \\
&\approx \sum_{i=1_{i\neq r}}^{m} \widetilde{C}_{B_i}^{N_{ivt}}\widetilde{x}_{B_i}^{N_{ivt}} - \sum_{i=1}^{m} \widetilde{y}_{ij}^{N_{ivt}}\ \widetilde{C}_{B_i}^{N_{ivt}}\ \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} \\
&\quad + \frac{\widetilde{y}_{ij}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}}\ \widetilde{C}_{B_r}^{N_{ivt}}\ \widetilde{x}_{B_r}^{N_{ivt}} + \widetilde{C}_{B_r}^{N_{ivt}}\ \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{r_j}^{N_{ivt}}} \\
&\approx \sum_{i=1}^{m} \widetilde{C}_{B_i}^{N_{ivt}}\widetilde{x}_{B_i}^{N_{ivt}} - \widetilde{Z_j}^{N_{ivt}}\ \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{rj}^{N_{ivt}}} + \widetilde{C}_{B_r}^{N_{ivt}}\ \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{r_j}^{N_{ivt}}} \\
&\approx \widetilde{Z}_0^{N_{ivt}} - \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{r_j}^{N_{ivt}}}\left[\widetilde{Z_j}^{N_{ivt}} - \widetilde{C_j}^{N_{ivt}}\right]
\end{aligned}$$

$$\text{Since } \widetilde{y}_{r_j}^{N_{ivt}} > 0,\ \widetilde{Z_j}^{N_{ivt}} - \widetilde{C_j}^{N_{ivt}} < \widetilde{0}^{N_{ivt}},$$

$$\text{and } \frac{\widetilde{x}_{B_r}^{N_{ivt}}}{\widetilde{y}_{r_j}^{N_{ivt}}} > \widetilde{0}^{N_{ivt}}$$

$$\approx \widetilde{Z}_0^{N_{ivt}} + \text{ positive quantity} > \widetilde{Z}_0^{N_{ivt}}.$$

Hence the new interval-valued trapezoidal neutrosophic fuzzy basic feasible solution $\widehat{\widetilde{X}_B^{N_{ivt}}}$ is a new improved value for the interval-valued trapezoidal neutrosophic fuzzy objective function.

6 Computational method

This section discusses about the computational part to apply the referred ranking function in modes of midpoint and width (half-width) of $(\alpha, \beta, \gamma)-$ cut sets. The interval-valued trapezoidal neutrosophic fuzzy linear programming problem shall be resolved efficiently. We utilize two methods for solving interval linear programming problem (LPP_{in}), one is simplex algorithm and the other one is RTSM [48].

The below steps describes the interval version of simplex algorithm.

Step 1: Using the proposed de-neutrosophication technique based on interval numbers, transform the interval-valued trapezoidal neutrosophic fuzzy linear programming problem into interval linear programming problem and it is defined as

$$\text{Maximize} \quad \widetilde{Z}^{in} = \sum_{j=1}^{n} \widetilde{c_j}^{in} \ \widetilde{x_j}^{in}$$

$$\text{subject to} \quad \sum_{j=1}^{n} \widetilde{a_{ij}}^{in} \ \widetilde{x}^{in} \ \leq \widetilde{b_i}^{in},$$

$$\widetilde{x_{ij}}^{in} \geq \widetilde{0} \ \ i = 1, 2...m \qquad (24)$$

where $\widetilde{c_j}^{in}, \widetilde{x_j}^{in}, \widetilde{b_i}^{in}$ and $\widetilde{a_{ij}}^{in}$ are interval numbers.

Step 2: If the interval linear programming problem is a minimization type, then convert it into maximization by using $Minimize\ (\widetilde{Z}^{in}) = -Maximize\ (-\widetilde{Z}^{in})$

Step 3: Introduce slack or surplus variables for converting the inequalities into equalities.

Step 4: Proceed the problem using the interval version of simplex method.

Step 5: Create an initial simplex table to identify the initial basic feasible solution to the interval linear programming problem.

Step 6: The rules for testing the optimality is that, if $\widetilde{Z_j}^{in} - \widetilde{C_j}^{in} \geq 0$ the current basic feasible solution is optimal. Otherwise, ie., if one $\widetilde{Z_j}^{in} - \widetilde{C_j}^{in} < 0$, proceed with the next step.

Step 7: The most negative $\widetilde{Z_j}^{in} - \widetilde{C_j}^{in}$ identifies the pivot column $\widetilde{a_{ik}}^{in} > 0, \ i = 1, 2, ...m$. Then based on that pivot column, find the ratio using $\frac{\widetilde{b_i}^{in}}{\widetilde{a_{ik}}^{in}}$ for $\widetilde{a_{ik}}^{in} > 0, \ i = 1, 2, ...m$. By choosing the minimum ratio, we can identify the pivot row.

Step 8: For calculating the new entries, let us proceed with the following steps.

(i) And the element ($\widetilde{a_{kr}}^{in}$) that intersects the pivot row and column is known as the leading element.

(ii) Find $\widetilde{a_{kj}}^{in} = \frac{\widetilde{a_{kj}}^{in}}{\widetilde{a_{kr}}^{in}}$ and $\widetilde{a_{ij}}^{in} = \widetilde{a_{ij}}^{in} - \frac{\widetilde{a_{kj}}^{in}}{\widetilde{a_{kr}}^{in}} \widetilde{a_{ir}}^{in}$ where $i = 1, 2, ..., m, \ \ i \neq k, \ \ j = 1, 2, ...n.$

Step 9: Proceed it until we get an optimal solution.

The second method we use for computation is RTSM which was proposed by Fan and Huang [48] for solving the interval linear programming problem (LPP_{in}). It has two submodels i.e., conservative and optimistic. The sub-problem 1 is designed as follows.

$$\text{Maximize } f^L = \sum_{j=1}^{k} c_j^L x_j^L + \sum_{j=k+1}^{n} c_j^L x_j^R$$

subject to

$$\sum_{j=1}^{k} |a_{ij}|^R sign(a_{ij}) x_j^L + \sum_{j=k+1}^{n} |a_{ij}|^L sign(a_{ij}) x_j^R \leq b_i^L, \quad i = 1, 2, ..., m$$

$$x_j^L \geq o, \ j = 1, 2, ..., k$$

$$x_j^R \geq o, \ j = k+1, k+2, ..., n$$

Solving the above submodel 1, we can obtain the solution as x_{jopt}^L, for $j = 1, 2, ..., k$ & x_{jopt}^R, for $j = k+1, k+2, ..., n$.

The submodel 2 is formulated as

$$\text{Maximize } f^R = \sum_{j=1}^{k} c_j^R x_j^R + \sum_{j=k+1}^{n} c_j^R x_j^L$$

subject to

$$\sum_{j=1}^{k} |a_{ij}|^L sign(a_{ij}) x_j^R + \sum_{j=k+1}^{n} |a_{ij}|^R sign(a_{ij}) x_j^L \leq b_i^R, \quad i = 1, 2, ..., m$$

$$\sum_{j=1}^{l_{i1}} a_{ij}^L x_j^R + \sum_{j=l_{i1}+1}^{k} a_{ij}^L x_{jopt}^L + \sum_{j=k+1}^{l_{i2}} a_{ij}^L x_j^L + \sum_{j=l_{i2}+1}^{n} a_{ij}^L x_{jopt}^R \leq b_i^R,$$

$$x_j^R \geq x_{jopt}^L \ j = 1, 2, ..., k$$

$$x_j^L \leq x_{jopt}^R \ j = k+1, k+2, ..., n$$

$$x_j^R \geq o, \ j = 1, 2, ..., k$$

$$x_j^L \geq o, \ j = k+1, k+2, ..., n$$

Solving the above submodel 2, we can obtain the solution as x_{jopt}^R, for $j = 1, 2, ..., k$ & x_{jopt}^L, for $j = k+1, k+2, ..., n$. Therefore, from the combination of both the solutions we obtain the solution for the given interval linear programming problem (LPP_{in}).

7 Illustrations

To show the effectiveness of the proposed de-neutrosophication technique based on interval numbers, we adapted two examples from Khatter [47]. Here the fuzzy parameters are assumed based on decision makers choice and we consider it as interval-valued trapezoidal neutrosophic linear programming problem. The first example is fully linear programming problem in interval-valued trapezoidal neutrosophic environment and all the parameters of this problem are assumed in interval-valued trapezoidal neutrosophic numbers. The second example is partially linear programming problem in interval-valued trapezoidal neutrosophic environment, in which the objective function is in interval-valued trapezoidal neutrosophic numbers.

7.1 Example: 1

A company manufactures clay bowls and clay mugs. For manufacturing the products, the resources that need are clay and labor. Our aim is to maximize the profit based on the items manufactured per day using the given resources. The resource requirements for production and profit per product are presented below.

For production, there are around 40 hours of labor and around 120 pounds of clay available each day. Here all the fuzzy parameters we consider are in the form of interval-valued

Table 1. Transformed interval numbers interms of mid-point and width (half-width) for different parameters of α, β, γ, r & s (example 1)

$\alpha, \beta, \gamma,$ r & s	$\widetilde{c_1}^{in}$	$\widetilde{c_2}^{in}$	$\widetilde{a_1}^{in}$	$\widetilde{a_2}^{in}$	$\widetilde{b_1}^{in}$	$\widetilde{a_3}^{in}$	$\widetilde{a_4}^{in}$	$\widetilde{b_2}^{in}$
0	$\langle 80, 10\rangle$	$\langle 100, 10\rangle$	$\langle 3.50, 1.25\rangle$	$\langle 6, 3\rangle$	$\langle 80, 20\rangle$	$\langle 13, 5.5\rangle$	$\langle 11, 4.5\rangle$	$\langle 240, 20\rangle$
0.1	$\langle 76, 9.38\rangle$	$\langle 95, 9.44\rangle$	$\langle 3.33, 1.23\rangle$	$\langle 5.70, 2.90\rangle$	$\langle 76, 18.33\rangle$	$\langle 12.35, 5.22\rangle$	$\langle 10.45, 4.31\rangle$	$\langle 228, 18.75\rangle$
0.2	$\langle 72, 8.75\rangle$	$\langle 90, 8.89\rangle$	$\langle 3.15, 1.20\rangle$	$\langle 5.40, 2.8\rangle$	$\langle 72, 16.67\rangle$	$\langle 11.7, 4.94\rangle$	$\langle 9.9, 4.12\rangle$	$\langle 216, 17.50\rangle$
0.3	$\langle 68, 8.12\rangle$	$\langle 85, 8.33\rangle$	$\langle 2.97, 1.18\rangle$	$\langle 5.10, 2.70\rangle$	$\langle 68, 15\rangle$	$\langle 11.05, 4.67\rangle$	$\langle 9.35, 3.94\rangle$	$\langle 204, 16.25\rangle$
0.4	$\langle 64, 7.5\rangle$	$\langle 80, 7.78\rangle$	$\langle 2.80, 1.15\rangle$	$\langle 4.80, 2.60\rangle$	$\langle 64, 13.33\rangle$	$\langle 10.4, 4.39\rangle$	$\langle 8.8, 3.75\rangle$	$\langle 192, 15\rangle$
0.5	$\langle 60, 6.88\rangle$	$\langle 75, 7.22\rangle$	$\langle 2.62, 1.12\rangle$	$\langle 4.50, 2.5\rangle$	$\langle 60, 11.67\rangle$	$\langle 9.75, 4.11\rangle$	$\langle 8.25, 3.56\rangle$	$\langle 180, 13.75\rangle$
0.6	$\langle 56, 6.25\rangle$	$\langle 70, 6.67\rangle$	$\langle 2.45, 1.10\rangle$	$\langle 4.20, 2.4\rangle$	$\langle 56, 10\rangle$	$\langle 9.10, 3.83\rangle$	$\langle 7.70, 3.38\rangle$	$\langle 168, 12.50\rangle$
0.7	$\langle 52, 5.62\rangle$	$\langle 65, 6.11\rangle$	$\langle 2.28, 1.07\rangle$	$\langle 3.90, 2.30\rangle$	$\langle 52, 8.33\rangle$	$\langle 8.45, 3.56\rangle$	$\langle 7.15, 3.19\rangle$	$\langle 156, 11.25\rangle$
0.8	$\langle 48, 5\rangle$	$\langle 60, 5.56\rangle$	$\langle 2.10, 1.05\rangle$	$\langle 3.60, 2.2\rangle$	$\langle 48, 6.67\rangle$	$\langle 7.80, 3.28\rangle$	$\langle 6.60, 3\rangle$	$\langle 144, 10\rangle$
0.9	$\langle 44, 4.38\rangle$	$\langle 55, 5\rangle$	$\langle 1.93, 1.02\rangle$	$\langle 3.30, 2.1\rangle$	$\langle 44, 5\rangle$	$\langle 7.15, 3\rangle$	$\langle 6.05, 2.81\rangle$	$\langle 132, 8.75\rangle$
1.0	$\langle 40, 3.75\rangle$	$\langle 50, 4.44\rangle$	$\langle 1.75, 1.00\rangle$	$\langle 3, 2\rangle$	$\langle 40, 3.33\rangle$	$\langle 6.50, 2.72\rangle$	$\langle 5.5, 2.62\rangle$	$\langle 120, 7.5\rangle$

Product	Labor (Hr/Unit)	Clay (Lb/Unit)	Profit (Dollar/Unit)
Bowl	$\widetilde{1}$	$\widetilde{4}$	$\widetilde{40}$
Mug	$\widetilde{2}$	$\widetilde{3}$	$\widetilde{50}$

trapezoidal neutrosophic numbers. The general form of the given interval-valued trapezoidal neutrosophic linear programming problem are given as follows:

$$\begin{aligned} \text{Maximize} \quad & \widetilde{Z}^{N_{ivt}} = \widetilde{c_1}^{N_{ivt}} \widetilde{x_1}^{N_{ivt}} + \widetilde{c_2}^{N_{ivt}} \widetilde{x_2}^{N_{ivt}} \\ \text{subject to} \quad & \widetilde{a_1}^{N_{ivt}} \widetilde{x_1}^{N_{ivt}} + \widetilde{a_2}^{N_{ivt}} \widetilde{x_2}^{N_{ivt}} \leq \widetilde{b_1}^{N_{ivt}}, \\ & \widetilde{a_3}^{N_{ivt}} \widetilde{x_1}^{N_{ivt}} + \widetilde{a_4}^{N_{ivt}} \widetilde{x_2}^{N_{ivt}} \leq \widetilde{b_2}^{N_{ivt}}, \\ & \widetilde{x_1}^{N_{ivt}}, \widetilde{x_2}^{N_{ivt}} \geq \widetilde{0}^{N_{ivt}} \end{aligned}$$

where $\widetilde{c_1}^{N_{ivt}}, \widetilde{c_2}^{N_{ivt}}, \widetilde{a_1}^{N_{ivt}}, \widetilde{a_2}^{N_{ivt}}, \widetilde{a_3}^{N_{ivt}}, \widetilde{a_4}^{N_{ivt}}, \widetilde{b_1}^{N_{ivt}}$ and $\widetilde{b_2}^{N_{ivt}}$ are all interval-valued trapezoidal neutrosophic numbers and are assumed as follows:

$$\begin{aligned} \widetilde{c_1}^{N_{ivt}} &= \big\langle (30, 35, 45, 50); [0.6, 0.8], [0.3, 0.5], [0.2, 0.4] \big\rangle, \\ \widetilde{c_2}^{N_{ivt}} &= \big\langle (40, 45, 55, 60); [0.5, 0.7], [0.4, 0.6], [0.1, 0.3] \big\rangle, \\ \widetilde{a_1}^{N_{ivt}} &= \big\langle (0.5, 0.75, 2.75, 3); [0.5, 0.7], [0.3, 0.5], [0.0, 0.2] \big\rangle, \\ \widetilde{a_2}^{N_{ivt}} &= \big\langle (0, 1, 5, 6); [0.5, 0.7], [0.3, 0.5], [0.0, 0.2] \big\rangle, \\ \widetilde{a_3}^{N_{ivt}} &= \big\langle (1, 3.5, 9.5, 12); [0.3, 0.5], [0.2, 0.4], [0.1, 0.3] \big\rangle, \\ \widetilde{a_4}^{N_{ivt}} &= \big\langle (1, 2.5, 8.5, 10); [0.6, 0.8], [0.3, 0.5], [0.2, 0.4] \big\rangle, \\ \widetilde{b_1}^{N_{ivt}} &= \big\langle (20, 35, 45, 60); [0.3, 0.5], [0.2, 0.4], [0.4, 0.6] \big\rangle, \\ \widetilde{b_2}^{N_{ivt}} &= \big\langle (100, 110, 130, 140); [0.6, 0.8], [0.3, 0.5], [0.2, 0.4] \big\rangle. \end{aligned}$$

The above interval-valued trapezoidal neutrosophic linear programming problem is then transformed into interval linear programming problem (LPP_{in}) using the proposed de-neutrosophication technique.

The coefficients of the objective function, constraints and the right-hand side of the interval-valued trapezoidal neutrosophic linear programming problem are converted into interval numbers for various degrees of α, β, γ, r & s and are displayed in table 1.

The reformed interval linear programming problem is solved further for various parameters α, β, γ, r & s (the values mentioned in table 1) using interval version of Simplex method and RTSM [48] and we obtain the optimal solution in the form of interval numbers and is tabulated in table 2 and then compare our results obtained using the proposed de-neutrosophication technique with existing results of Khatter [47] and Tamilarasi [49] and it is given in table 3.

For the degree 0, the objective values obtained by Khatter [47] is 1571.45 and Tamilarasi [49] is 1520.36, whereas by the proposed de-neutrosophication technique we got the objective values as [1620.94,1660.94] for simplex method and [2200,3120] for RTSM.

Similarly, for the degree 1, Khatter [47] obtained the objective values as 278.76 and Tamilarasi [49] as 945.18, whereas by the proposed approach we got the objective values as [812.94,827.94] for simplex method and [1475.69,1996.315] for RTSM.

By the comparison, we observed that for the simplex method, the objective values for all degrees between [0,1] are efficient with Khatter [47], whereas it is efficient between [0,0.5] with Tamilarasi [49]. And for RTSM, the objective values are efficient for all the values between [0,1].

Figure 2 displays graphically the comparison between the solutions obtained by new de-neutrosophication technique for different parameters with Khatter [47] and Tamilarasi [49].

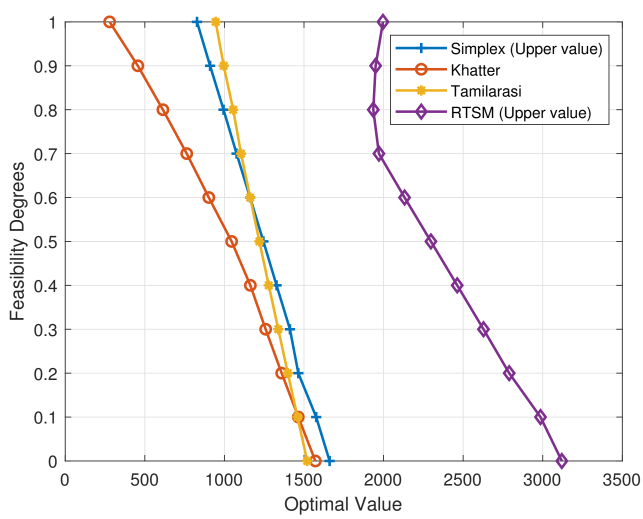

Figure 2. Comparison of example 1 with the existing methods

Table 2. Optimal solution interms of mid-point and width (half-width) for different parameters of α, β, γ, r & s (example 1)

$\alpha, \beta, \gamma,$ r & s	Simplex			RTSM [48]		
r & s	$\widetilde{x_1}^{in}$	$\widetilde{x_2}^{in}$	$\widetilde{Z}^{in}$	$\widetilde{x_1}^{in}$	$\widetilde{x_2}^{in}$	$\widetilde{Z}^{in}$
0	$\langle 14.178, 20\rangle$	$\langle 5.067, 20\rangle$	$\langle 1640.94, 20\rangle$	$\langle 17.334, 17.334\rangle$	$\langle 10, 10\rangle$	$\langle 2660, 460\rangle$
0.1	$\langle 14.198, 18.75\rangle$	$\langle 5.041, 18.75\rangle$	$\langle 1557.94, 18.75\rangle$	$\langle 17.304, 17.304\rangle$	$\langle 10.30, 10.30\rangle$	$\langle 2568.50, 417.41\rangle$
0.2	$\langle 14.177, 17.5\rangle$	$\langle 5.068, 17.5\rangle$	$\langle 1476.86, 17.5\rangle$	$\langle 17.27, 17.27\rangle$	$\langle 10.64, 10.64\rangle$	$\langle 2446.84, 342.38\rangle$
0.3	$\langle 14.154, 16.25\rangle$	$\langle 5.095, 16.25\rangle$	$\langle 1395.55, 16.25\rangle$	$\langle 17.26, 17.26\rangle$	$\langle 11.042, 11.042\rangle$	$\langle 2344.43, 283.39\rangle$
0.4	$\langle 14.176, 15\rangle$	$\langle 5.068, 15\rangle$	$\langle 1312.7, 15\rangle$	$\langle 17.22, 17.22\rangle$	$\langle 11.516, 11.516\rangle$	$\langle 2242.19, 220.46\rangle$
0.5	$\langle 14.15, 13.75\rangle$	$\langle 5.098, 13.75\rangle$	$\langle 1231.35, 13.75\rangle$	$\langle 17.175, 17.175\rangle$	$\langle 12.083, 12.083\rangle$	$\langle 2142.18, 155.33\rangle$
0.6	$\langle 14.178, 12.5\rangle$	$\langle 5.067, 12.5\rangle$	$\langle 1148.66, 12.5\rangle$	$\langle 17.125, 17.125\rangle$	$\langle 12.78, 12.78\rangle$	$\langle 2045.70, 86.40\rangle$
0.7	$\langle 14.208, 11.25\rangle$	$\langle 5.021, 11.25\rangle$	$\langle 1065.181, 11.25\rangle$	$\langle 17.1, 17.1\rangle$	$\langle 13.65, 13.65\rangle$	$\langle 1955.81, 14.95\rangle$
0.8	$\langle 14.177, 10\rangle$	$\langle 5.068, 10\rangle$	$\langle 984.576, 10\rangle$	$\langle 17.04, 17.04\rangle$	$\langle 14.76, 14.76\rangle$	$\langle 1870.58, 64.83\rangle$
0.9	$\langle 14.212, 8.75\rangle$	$\langle 5.019, 8.75\rangle$	$\langle 901.37, 8.75\rangle$	$\langle 16.958, 16.958\rangle$	$\langle 16.25, 16.25\rangle$	$\langle 1795.42, 154.58\rangle$
1.0	$\langle 14.176, 7.5\rangle$	$\langle 5.068, 7.5\rangle$	$\langle 820.44, 7.5\rangle$	$\langle 16.865, 16.865\rangle$	$\langle 18.335, 18.335\rangle$	$\langle 1736, 260.31\rangle$

Table 3. Comparison: Optimal solution based on new de-neutrosophication technique vs Existing method (example 1)

	Using proposed de-neutrosophication technique		Existing Method	
$\alpha, \beta, \gamma,$ r & s	Simplex	RTSM [48]	Khatter [47]	Tamilarasi [49]
0	[1620.94,1660.94]	[2200,3120]	1571.45	1520.36
0.1	[1539.19,1576.69]	[2151.09,2985.92]	1464.27	1459.16
0.2	[1459.36,1494.36]	[2104.46,2789.22]	1359.78	1398.54
0.3	[1379.3,1411.8]	[2061.04,2627.81]	1259.16	1338.43
0.4	[1297.7,1327.7]	[2021.73,2462.65]	1163.02	1278.74
0.5	[1217.6,1245.1]	[1986.85,2297.5]	1045.33	1219.77
0.6	[1136.16,1161.16]	[1959.3,2132.09]	902.02	1161.74
0.7	[1053.93,1076.43]	[1940.86,1970.75]	763.21	1104.64
0.8	[974.58,994.58]	[1805.75,1935.4]	613.61	1055.18
0.9	[892.62,910.12]	[1640.84,1950]	456.02	995.67
1.0	[812.94,827.94]	[1475.69,1996.31]	278.76	945.18

7.2 Example: 2

Let us consider another interval-valued trapezoidal neutrosophic linear programming problem and is taken from Khatter [47]. In this problem, the coefficients of the objective functions are alone considered as interval-valued trapezoidal neutrosophic numbers. And the general form is given below:

$$\begin{aligned} \text{Maximize} \quad & Z = \widetilde{c_1}^{N_{ivt}} x_1 + \widetilde{c_2}^{N_{ivt}} x_2 \\ \text{subject to} \quad & a_1 x_1 + a_2 x_2 \leq b_1, \\ & a_3 x_1 + a_4 x_2 \leq b_2, \\ & a_5 x_1 + a_6 x_2 \leq b_3, \\ & x_1, x_2 \leq 0 \end{aligned}$$

where,
$\widetilde{c_1}^{N_{ivt}} = \widetilde{25} = \big\langle (19, 22, 30, 33); [0.7, 0.9], [0.0, 0.2], [0.3, 0.5] \big\rangle$
$\widetilde{c_2}^{N_{ivt}} = \widetilde{48} = \big\langle (44, 47, 51, 54); [0.5, 1], [0.5, 0.75], [0, 0.5] \big\rangle$
$a_1 = 15;\ a_2 = 30;\ a_3 = 24;\ a_4 = 6;\ a_5 = 21;$
$a_6 = 14;\ b_1 = 45000;\ b_2 = 24000;\ b_3 = 28000.$

Since the objective function of the above problem is in interval-valued trapezoidal neutrosophic number, use the proposed de-neutrosophication technique to transform it into an interval number and the values are mentioned in table 4 for various parameters.

And the coefficients of the constraints are given as $a_1{}^{in} = \big\langle 15, 0 \big\rangle;\ a_2{}^{in} = \big\langle 30, 0 \big\rangle;\ b_1{}^{in} = \big\langle 45000, 0 \big\rangle;\ a_3{}^{in} = \big\langle 24, 0 \big\rangle;\ a_4{}^{in} = \big\langle 6, 0 \big\rangle;\ b_2{}^{in} = \big\langle 24000, 0 \big\rangle;\ a_5{}^{in} = \big\langle 21, 0 \big\rangle;\ a_6{}^{in} = \big\langle 14, 0 \big\rangle;\ b_3{}^{in} = \big\langle 28000, 0 \big\rangle$. And the values are constant for all parameters α, β, γ, r & s.

Table 4. Transformed Interval numbers interms of mid-point and width (half-width) for different parameters of α, β, γ, r & s (example 2)

$\alpha, \beta, \gamma,$ r & s	$\widetilde{c_1}^{in}$	$\widetilde{c_2}^{in}$
0	$\big\langle 52, 7 \big\rangle$	$\big\langle 98, 5 \big\rangle$
0.1	$\big\langle 49.40, 6.70 \big\rangle$	$\big\langle 93.10, 4.70 \big\rangle$
0.2	$\big\langle 46.80, 6.40 \big\rangle$	$\big\langle 88.20, 4.40 \big\rangle$
0.3	$\big\langle 44.20, 6.10 \big\rangle$	$\big\langle 83.30, 4.10 \big\rangle$
0.4	$\big\langle 41.60, 5.80 \big\rangle$	$\big\langle 78.40, 3.80 \big\rangle$
0.5	$\big\langle 39, 5.50 \big\rangle$	$\big\langle 73.70, 3.50 \big\rangle$
0.6	$\big\langle 36.40, 5.20 \big\rangle$	$\big\langle 68.60, 3.20 \big\rangle$
0.7	$\big\langle 33.80, 4.90 \big\rangle$	$\big\langle 63.70, 2.90 \big\rangle$
0.8	$\big\langle 31.20, 4.60 \big\rangle$	$\big\langle 58.80, 2.60 \big\rangle$
0.9	$\big\langle 28.60, 4.30 \big\rangle$	$\big\langle 53.90, 2.30 \big\rangle$
1.0	$\big\langle 26, 4 \big\rangle$	$\big\langle 49, 2 \big\rangle$

The converted interval linear programming problem is

Table 5. Comparison: Optimal solution based on new de-neutrosophication technique vs Existing method (example 2)

$\alpha, \beta, \gamma,$ r & s	Using proposed de-neutrosophication technique		Existing Method	
	Simplex	RTSM [48]	Khatter [47]	Tamilarasi [49]
0	[148493,148507]	[145750,154500]	141345	149372.5
0.1	[141068.3,141081.7]	[138550,146700]	131312.50	141795
0.2	[133643.6,133656.4]	[131350,138900]	121390	134235
0.3	[126218.9,126231.1]	[124150,131100]	111485	126657.5
0.4	[118794.2,118805.8]	[116950,123300]	101562.50	120597.5
0.5	[111369.5,111380.5]	[110000,115800]	91657.5	111520
0.6	[103944.8,103955.2]	[102550,107700]	81735	103947.5
0.7	[96520.1,96529.9]	[95350,99900]	71830	96382.5
0.8	[89095.4,89104.6]	[88150,92100]	61920	88810
0.9	[81670.7,81679.3]	[80950,84300]	52002	81250
1.0	[74246,74254]	[73750,76500]	42092.5	73672.5

then solved by using simplex method and RTSM [48] in interval version. The interval form (mid-point and width (half-width)) of optimal solution are obtained same for all parameters α, β, γ, r & s. For simplex method, we get the optimal solution as $\widetilde{x_1}^{in} = \langle 500, 0 \rangle$ and $\widetilde{x_2}^{in} = \langle 1250, 0 \rangle$. For RTSM [48], we acquired the optimal solution as $\widetilde{x_1}^{in} = \langle 500, 0 \rangle$ and $\widetilde{x_2}^{in} = \langle 1375, 125 \rangle$. And the objective values interms of mid-point and width (half-width) are displayed in table 6.

Table 6. Optimal solution interms of mid-point and width (half-width) for different parameters of α, β, γ, r & s (example 2)

$\alpha, \beta, \gamma,$ r & s	$\widetilde{Z}^{in}$	
	Simplex	RTSM [48]
0	$\langle 148500, 7 \rangle$	$\langle 149825, 4375 \rangle$
0.1	$\langle 141075, 6.70 \rangle$	$\langle 142625, 4075 \rangle$
0.2	$\langle 133650, 6.40 \rangle$	$\langle 135125, 3775 \rangle$
0.3	$\langle 126225, 6.10 \rangle$	$\langle 127625, 3475 \rangle$
0.4	$\langle 118800, 5.80 \rangle$	$\langle 120125, 3175 \rangle$
0.5	$\langle 111375, 5.50 \rangle$	$\langle 112900, 2900 \rangle$
0.6	$\langle 103950, 5.20 \rangle$	$\langle 105125, 2575 \rangle$
0.7	$\langle 96525, 4.90 \rangle$	$\langle 97625, 2275 \rangle$
0.8	$\langle 89100, 4.60 \rangle$	$\langle 90125, 1975 \rangle$
0.9	$\langle 81675, 4.30 \rangle$	$\langle 82625, 1675 \rangle$
1.0	$\langle 74250, 4 \rangle$	$\langle 75125, 1375 \rangle$

Table 5 represents the contrast between the optimal solution of the anticipated technique with the prevailing method [47] and [49].

In that, for the parameter value 0, we obtain the optimal solution for simplex method as [148493,148507] and for RTSM [48] as [145750,154500] but Khatter [47] got 141345 and Tamilarasi [49] acquire 149372.5.

But for the parameter value 1, we got optimal solution for simplex as [74246,74254] and for RTSM [48] as [73750,76500] which is better than both of the existing approach [47, 49] and the optimal solution they obtained are 42092.5 & 73672.5 respectively.

By the observation, it is clearly visible by using simplex method the objective values for all degrees between [0,1] are efficient with Khatter [47], whereas it is efficient between [0.6,1] with Tamilarasi [49]. And for RTSM, the objective values are efficient for all the values between [0,1].

Figure 3 displays the pictorial representation of example 2 between the solutions attained by our proposed method with Khatter [47] and Tamilarasi [49].

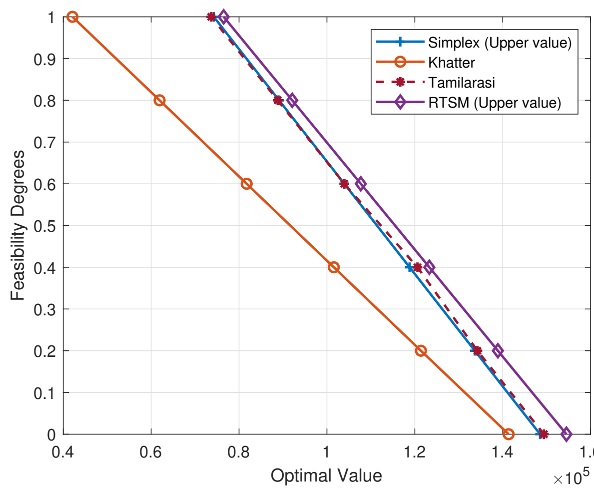

Figure 3. Comparison of example 2 with the existing methods

8 Advantages and Limitations of the proposed de-neutrosophication technique

An interval number is the best way to represent the uncertainty and ambiguity. It aids in problem solving where data is contradictory, imprecise, or indeterminate and falls within a particular range of permissible behaviour. Combining the interval-valued trapezoidal neutrosophic numbers into single interval number is more effective to analyze the interval-valued trapezoidal neutrosophic numbers rather than crisp numbers. Trapezoidal neutrosophic fuzzy parameters are considered to have known interval truth, interval indeterminacy and interval falsity membership degrees respectively in research on optimization approaches in interval neutrosophic environments. Eventhough the optimal solutions obtained by our proposed approach is efficient than Khatter [47] for all different parameters, there is a quite deviation with Tamilarasi [49] for some parameters while solving using simplex method. But using RTSM [48] all the optimal solutions are better than the existing. It seems that our proposed de-neutrosophication technique is more efficient in solving with the interval methods that designed for linear programming problems.

9 Conclusions

A new de-neutrosophication technique is proposed in this paper using interval numbers. Based on this approach, two maximization linear programming problems has been solved, where the parameters appears as interval-valued trapezoidal neutrosophic numbers. Such parameters have flexibility in expressing the neutrosophic function in a convenient way than the single-valued. And the efficiency is shown with illustrative examples in section 7 and it is clearly tabulated in table 3 & 5, and graphically represented in figure 2 & 3. Our forthcoming effort would deal on showing the efficacy of interval numbers in all categories of optimization methods. Additionally, we'll look into how our approaches might be applied to other, more complicated optimization variants.

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