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A New Type of Single Server Queue Operating in A Multi-level Environment with Customer Impatience

Akshaya Ramesh, **S. Udayabaskaran***

Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, India

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Abstract A new type of single server queue is considered. In this type, the server asks for an assignment in a multi-level environment and the customer develops impatience during the assignment process. The environment has N levels and the server is assigned to operate in one of these levels with level dependent arrival and service rates. Customers arrive at the system all the time and there is an infinite buffer with the system. The assignment is done by a random switch which can initiate an assignment process only if at least one customer is in the system. The server working in any level of the environment reports to the random switch after serving the last customer in that level. Customers are not flushed out at any time. The random switch initiates an assignment process immediately at the epoch of arrival of a customer to the system. Assignment time is random and during the assignment period, customers are permitted to join the system. Once the assignment process starts, each customer waiting in the buffer clicks on a random impatience timer with him/her and leaves the system in case his/her timer ends before the assignment to the server is made. For this model, steady-state probabilities are found and a performance analysis is also made.

Keywords Single Server Queue, Multi-level Environment, Assignment, Random Switch, Customer Impatience

1. Introduction

Customer impatience is an inevitable phenomenon which arises in studies of queueing systems with buffer. It arises in the mind of an arriving customer who can not tolerate prolonged waiting in the queue. A huge amount of research on stochastic models of queues with customer impatience is already available in the literature. In his research on congestion in telephone traffic, Palm [1] developed the study of queueing systems subject to customer impatience. He used an $M/M/c/\infty$ queueing model where each one of the customers in the buffer switches on an exponentially distributed impatience timer immediately upon arrival and abandons the queueing system once for all at the epoch when the timer expires before getting service. Palm's paper triggered a huge variety of problems on queues with customer impatience. Customer impatience in an $M/M/1/N$ queueing model was studied by Ancker and Gafarian [2, 3] by assuming exponentially distributed impatience timer. Daley [4] studied a $GI/G/1$ queue with general customer impatience. Takacs [5] analysed an $M/G/1$ queue where customers in the buffer adopt fixed deadlines for waiting in the queue. Baccelli et al. [6] considered $GI/G/1$ queues with independent and identically distributed impatience times. Boxma and deWall [7] studied multi-server queues with customer impatience.

Customer impatience in queueing systems may also arise when the server is unavailable due to several reasons such as vacation or failure. In many situations, customers are allowed to join the buffer even though the server is on vacation or undergoing repair due to failure. These customers develop impatience as before which results in customer loss. Altman and Yechiali [8] analysed customer impatience in queues with server going on vacation. Yechiali [9] studied queues with system disasters and customer getting impatient when the server is undergoing repair. Perel and Yechiali [10] analysed an $M/M/c$ queueing system in a two-phase (fast and slow) Markovian environment where customers in the buffer become impatient in slow phase. Sudhesh [11] provided a transient analysis of an $M/M/1$ queue with system disasters and customer impatience. Ammar [12] provided a transient analysis of a two-heterogeneous servers queue with customer impatience. Ammar [13] studied the transient analysis of impatient customers in an $M/M/1$ disasters queue in random environment. Dora Pravina et al. [19] analysed an $M/M/1$ Queue with Server in Differentiated Phase Subject to Customer Impatience. Akshaya Ramesh and Udayabaskaran [14] studied by a new approach a single server queueing system operating in a random environment subject to disaster, repair and customer impatience.

There is a new class of queueing systems with customer impatience. Consider a single server queueing system where the server after finishing the service of the last customer reports immediately to a random control switch. The switch waits for an arrival of a customer to the system and takes a random time to put the server into service in any one of the levels of a multi-level random environment. During the random time to give fresh allotment to the server, the customers may join the queue and these customers exhibit impatience resulting in a loss of customers. Such queueing systems with customer impatience controlled by a random switch have not been studied so far in the literature. The objective of the present paper is to fill the gap. To be specific, we consider a single server queueing system with server asking for a new assignment in a multi-level environment and customer getting impatience during assignment interval. The assignment is done by a random switch which can initiate an assignment process only if at least one customer is in the system. The server working in a level of the environment reports to the random switch after serving the last customer in that level. Customers are not flushed out at any time. Assignment time is random and during the assignment period customers are permitted to join the system. Once the assignment process starts, each customer waiting in the buffer clicks on a random impatience timer with him/her and leaves the system in case his/her timer ends before the assignment to the server is made. For this model, we obtain steady-state probabilities and some performance measures. The present queueing model does not include flushing out the customers and therefore it differs from the papers of Paz and Yechiali [15], Udayabaskaran and Dora Pravina [16], Vinodhini and Vidhya [17], Ammar [13] and Akshaya Ramesh and Udayabaskaran [18].

The paper is organized as follows: Section 2 describes the model. The governing equations of the model are derived in Section 3. In Section 4, explicit results for the steady-state probabilities are obtained. Section 5 is devoted to explore some new performance measures of the system. A numerical illustration is provided in Section 6. Section 7 provides concluding remarks.

2. Model Description & Notation

Consider a single server queueing system operating in a random environment. Let there be N levels of the random medium. We label the levels as $1, 2, 3, \cdots, N$. When the system is in level $k, k = 1, 2, \cdots, N$, it behaves like $M(\lambda_k)/M(\mu_k)/1$ queue. The server after finishing the service of the last customer in a level of the environment reports immediately to a random switch. When the server is under control of the switch, we say that the queueing system is in level 0. The switch waits for an arrival of a customer and then takes a random time to put the server into service in any one of the $N-$ levels of a random environment with a positive probability. Let $q_r, r = 1, 2, \cdots, N$ be the probability that the server is assigned to work in the $r-$level of the environment so that $\sum_{r=1}^{N} q_r = 1$. Let customers arrive in Poisson fashion with rate λ_0 in level 0. During the random time taken by the random switch to give fresh allotment to the server, customers may join the queue and these customers exhibit impatience resulting into loss of customers. We assume that the random time of assignment is exponentially distributed with parameter γ. We also assume that all impatience times are independent and exponentially distributed with parameter ξ.

The following notations are used in the paper:

λ_k	: Arrival rate in level k
μ_k	: Service rate in level k
γ	: Assignment rate
q_k	: Probability that $k-$ th level is assigned to the server.
$f(t)©g(t)$	: $\int_0^t f(\tau)g(t-\tau)d\tau$
$f^*(s)$	: Laplace transform of $f(t)$
$E(X)$	: Mathematical Expectation of X

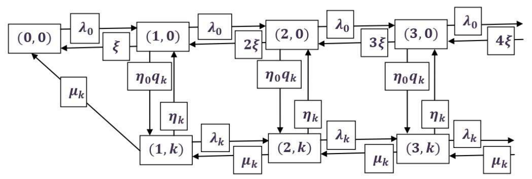
3. Governing Equations

Let $X(t)$ be the number of customers in the system and $S(t)$ be the level of the random environment at time t. As all random times are exponential, the two-dimensional stochastic process $\{(X(t), S(t))|t \geq 0\}$ is Markov. The state space is given by

$$\Omega = \{(0,0), (1,0), \cdots\} \cup \left(\cup_{k=1}^{N}\{(1,k), (2,k), \cdots\}\right).$$

The state transition diagram is given below: Let

Figure 1. Transition diagram



$p(j,k,t), (j,k) \in \Omega$ be the transient probabilities of the process $\{(X(t), S(t))|t \geq 0\}$. Using probability laws, we derive a system of governing equations for $p(j,k,t), (j,k) \in \Omega$ as follows:

For the level 0:

$$p^{'}(0,0,t) = -\lambda_0 p(0,0,t) + \sum_{k=1}^{N} p(1,k,t)\mu_k + p(1,0,t)\xi, \quad (1)$$

$$p^{'}(j,0,t) = -(\lambda_0 + j\xi + \gamma)p(j,0,t) + p(j-1,0,t)\lambda_0 + p(j+1,0,t)(j+1)\xi, j \geq 1, \quad (2)$$

For the level $k, k = 1, 2, \cdots, N$**:**

$$p^{'}(1,k,t) = -(\lambda_k + \mu_k)p(1,k,t) + p(1,0,t)\gamma q_k$$

$$+p(2,k,t)\mu_k, \quad (3)$$

$$p'(j,k,t) = -(\lambda_k+\mu_k)p(j,k,t) + p(j,0,t)\gamma q_k + p(j-1,k,t)\lambda_k + p(j+1,k,t)\mu_k, j \geq 2. \quad (4)$$

4. Steady State Probabilities

Let $\pi(j,k)$ be the steady state probability which is defined by $\pi(j,k) = \lim_{t\to\infty} p(j,k,t), (j,k) \in \Omega$. Equations (1)-(4) lead to the following system of equations:

For the state $(0,0)$:

$$\lambda_0\pi(0,0) = \sum_{k=1}^{N} \pi(1,k)\mu_k + \pi(1,0)\xi; \quad (5)$$

For the states $(j,0), j \geq 1$:

$$(\lambda_0+j\xi+\gamma)\pi(j,0) = \pi(j-1,0)\lambda_0 + \pi(j+1,0)(j+1)\xi, j \geq 1; \quad (6)$$

For the states $(1,k), k = 1,2,\cdots,N$:

$$(\lambda_k+\mu_k)\pi(1,k) = \pi(1,0)\gamma q_k + \pi(2,k)\mu_k; \quad (7)$$

For the states $(j,k), j \geq 2, k = 1,2,\cdots,N$:

$$(\lambda_k+\mu_k)\pi(j,k) = \pi(j,0)\gamma q_k + \pi(j-1,k)\lambda_k + \pi(j+1,k)\mu_k. \quad (8)$$

Define the partial generating functions

$$G_k(u) = \sum_{j=0}^{\infty} \pi(j,k)u^j, k = 0,1,2,\cdots,N.$$

By using (6), we obtain

$$\frac{\partial}{\partial u}G_0(u) - \frac{[\lambda_0(1-u)+\gamma]}{\xi(1-u)}G_0(u) = \frac{K}{\xi(1-u)}, \quad (9)$$

where

$$K = \xi\pi(1,0) - (\lambda_0+\gamma)\pi(0,0). \quad (10)$$

Integrating (9), we get

$$G_0(u) = \frac{\pi(0,0) + \frac{K}{\xi}\int_0^u e^{-\frac{\lambda_0}{\xi}v}(1-v)^{\frac{\gamma}{\xi}-1}dv}{e^{-\frac{\lambda_0}{\xi}u}(1-u)^{\frac{\gamma}{\xi}}}. \quad (11)$$

Since $|G_0(u)| \leq 1$ in $|u| \leq 1$, $G_0(1)$ is well-defined. Consequently, (11) yields

$$\pi(0,0) + \frac{K}{\xi}\int_0^1 e^{-\frac{\lambda_0}{\xi}v}(1-v)^{\frac{\gamma}{\xi}-1}dv = 0. \quad (12)$$

Eliminating K in (11) by using (12), we get

$$G_0(u) = \frac{e^{\frac{\lambda_0}{\xi}u}}{(1-u)^{\frac{\gamma}{\xi}}}\left[1 - \frac{\int_0^u e^{-\frac{\lambda_0}{\xi}v}(1-v)^{\frac{\gamma}{\xi}-1}dv}{\int_0^1 e^{-\frac{\lambda_0}{\xi}v}(1-v)^{\frac{\gamma}{\xi}-1}dv}\right]\pi(0,0). \quad (13)$$

From (13), we can obtain $\pi(j,0), j = 1,2,\cdots$ in terms of $\pi(0,0)$. For this, we define

$$\psi(u) = \int_0^u e^{-\frac{\lambda_0}{\xi}v}(1-v)^{\frac{\gamma}{\xi}-1}dv. \quad (14)$$

Substituting $u = 1$ in (14) and expanding, we have

$$\psi(1) = \frac{\xi}{\gamma} + \frac{\xi}{\gamma}\sum_{m=1}^{\infty}(-1)^m \Pi_{j=1}^{m}\left(\frac{\lambda_0}{\gamma+j\xi}\right). \quad (15)$$

By expansion, (13) gives

$$G_0(u) = \left[\sum_{n=0}^{\infty}\sum_{i=0}^{n}(-1)^{n-i}\binom{-\frac{\gamma}{\xi}}{n-i}\left(\frac{\lambda_0}{\xi}\right)^i\frac{u^n}{i!} - \frac{1}{\psi(1)}\sum_{j=1}^{\infty}\sum_{n=0}^{j-1}\sum_{i=0}^{n}\sum_{m=0}^{j-n-1}\frac{(-1)^{j-i-1}}{i!m!}\left(\frac{\lambda_0}{\xi}\right)^{m+i}\times \binom{-\frac{\gamma}{\xi}}{n-i}\binom{\frac{\gamma}{\xi}-1}{j-n-m-1}\frac{u^j}{j-n}\right]\pi(0,0). \quad (16)$$

Equating the coefficients of u^j on both sides of (16), we get

$$\pi(j,0) = D_j\pi(0,0), j = 1,2,\cdots, \quad (17)$$

where

$$D_j = \sum_{i=0}^{j}(-1)^{j-i}\binom{-\frac{\gamma}{\xi}}{j-i}\left(\frac{\lambda_0}{\xi}\right)^i\frac{1}{i!} - \frac{1}{\psi(1)}\sum_{n=0}^{j-1}\sum_{i=0}^{n}\sum_{m=0}^{j-n-1}\frac{(-1)^{j-i-1}}{i!m!}\left(\frac{\lambda_0}{\xi}\right)^{m+i}\times \binom{-\frac{\gamma}{\xi}}{n-i}\binom{\frac{\gamma}{\xi}-1}{j-n-m-1}\frac{1}{j-n}, j = 1,2,\cdots.$$

By (17), we have found $\pi(j,0), j = 1,2,\cdots$ in terms of $\pi(0,0)$. It remains to obtain $\pi(j,k), j = 1,2,\cdots; k = 1,2,\cdots,N$ and $\pi(0,0)$. For this, we assume $\lambda_k < \mu_k, k = 1,2,\cdots,N$. By using (7) and (8), we get

$$G_k(u) = \frac{\gamma q_k G_0(u)u - \mu_k\pi(1,k)u - \gamma q_k\pi(0,0)u}{(u-1)(\mu_k-\lambda_k u)}. \quad (18)$$

For each $k = 1,2,\cdots,N$, $G_k(1)$ is well-defined and this forces the numerator of $G_k(u)$ in (18) to vanish at $u = 1$. Consequently, we get

$$\mu_k\pi(1,k) + \gamma q_k\pi(0,0) = \gamma q_k G_0(1), k = 1,2,\cdots,N. \quad (19)$$

By using (18) and (19), we get

$$G_k(u) = \frac{\gamma q_k u\{G_0(u) - G_0(1)\}}{(u-1)(\mu_k-\lambda_k u)}, k = 1,2,\cdots,N. \quad (20)$$

From (20), we obtain

$$G_k(u) = \frac{\gamma q_k}{\mu_k}\sum_{n=1}^{\infty}\left(\sum_{r=1}^{n}\sum_{i=r}^{\infty}\pi(i,0)\frac{\lambda_k^{n-r}}{\mu_k^{n-r}}\right)u^n. \quad (21)$$

From (21), we obtain

$$\pi(j,k) = \frac{\gamma q_k}{\mu_k}\left(\sum_{r=1}^{j}\sum_{i=r}^{\infty}\pi(i,0)\frac{\lambda_k^{j-r}}{\mu_k^{j-r}}\right), j = 1,2,\cdots. \quad (22)$$

By total probability axiom, we have

$$G_0(1)+\sum_{k=1}^{N}G_k(1)=1. \tag{23}$$

By using L'Hospital rule in (13), we get

$$G_0(1)=\frac{\xi}{\gamma\psi(1)}\pi(0,0). \tag{24}$$

Using (20) and applying L'Hospital rule, we get

$$G_k(1)=\frac{\gamma q_k}{(\mu_k-\lambda_k)}\sum_{n=1}^{\infty}n\pi(n,0), k=1,2,\cdots,N. \tag{25}$$

Using (17) and (25), we get

$$\sum_{k=1}^{N}G_k(1)=\Delta\sum_{n=1}^{\infty}nD_n\pi(0,0), \tag{26}$$

where

$$\Delta=\gamma\sum_{k=1}^{N}\frac{q_k}{(\mu_k-\lambda_k)}. \tag{27}$$

Using (24) and (26) in (23), and solving for $\pi(0,0)$, we get

$$\pi(0,0)=\left[\frac{\xi}{\gamma\psi(1)}+\Delta\sum_{n=1}^{\infty}nD_n\right]^{-1}. \tag{28}$$

5. A Particular Case

When waiting customers do not develop impatience, it is the case that $\xi=0$. Consequently, from (9), we get

$$G_0(u)=\frac{(\lambda_0+\gamma)}{[\lambda_0(1-u)+\gamma]}\pi(0,0). \tag{29}$$

From (29), we obtain

$$\pi(j,0)=\left(\frac{\lambda_0}{\lambda_0+\gamma}\right)^j\pi(0,0), j=1,2,\cdots. \tag{30}$$

From (29), we obtain

$$G_0(1)=\frac{(\lambda_0+\gamma)}{\gamma}\pi(0,0). \tag{31}$$

Using (30) in (22), we get

$$\pi(j,k)=\frac{q_k(\lambda_0+\gamma)}{\mu_k}\left[\sum_{m=0}^{j-1}\left(\frac{\lambda_0}{\lambda_0+\gamma}\right)^{j-m}\frac{\lambda_k^m}{\mu_k^m}\right]\pi(0,0), \quad j=1,2,\cdots;k=1,2,\cdots,N. \tag{32}$$

Using (32), we get

$$G_k(1)=\frac{q_k\lambda_0}{(\mu_k-\lambda_k)}\frac{(\lambda_0+\gamma)}{\gamma}\pi(0,0), k=1,2,\cdots,N. \tag{33}$$

By using (23) and solving for $\pi(0,0)$, we get

$$\pi(0,0)=\frac{\gamma}{(\lambda_0+\gamma)}\left[1+\lambda_0\sum_{k=1}^{N}\frac{q_k}{(\mu_k-\lambda_k)}\right]^{-1}. \tag{34}$$

6. Performance Measures

In this section, we derive a few important measures of system performance such as

(i) stationary mean rate of occurrences of epochs of server entering into idle state (level 0 of the environment),

(ii) stationary mean rate of occurrences of epochs of server entering into busy state (level k of the environment, $k=1,2,\cdots,N$),

(iii) stationary mean number of customers in the idle state of the server,

(iv) stationary mean number of customers in the active state of the server,

6.1. Stationary Mean Rate of Occurrence of Entries into Idle State of the Server

Let $\mathcal{A}_1(t)$ be the number of times the server enters into idle state up to time t. Since the entry into idle state occurs from any one of the states $(1,k), k=1,2,\cdots,N$ upon the service completion of the customer in that state, we get

$$E[\mathcal{A}_1(t)]=\sum_{k=1}^{N}\int_0^t p(1,k,u)\mu_k du \tag{35}$$

Consequently, the stationary mean rate of occurrences of entries of the server into idle state is given by

$$a_1=\lim_{t\to\infty}\frac{E[\mathcal{A}_1(t)]}{t}=\sum_{k=1}^{N}\pi(1,k)\mu_k. \tag{36}$$

6.2. Stationary Mean Rate of Occurrence of Entries into Busy State of the Server

Let $\mathcal{A}_2(t)$ be the number of times the server enters into active state up to time t. Since the entry into active state occurs from any one of the states $(j,0), j=1,2,\cdots$ upon the completion of assignment time, we get

$$E[\mathcal{A}_2(t)]=\sum_{k=1}^{N}\sum_{j=1}^{\infty}\int_0^t p(j,0,u)\gamma q_k du. \tag{37}$$

Consequently, the stationary mean rate of occurrences of entries into busy state of the server is given by

$$a_2=\lim_{t\to\infty}\frac{E[\mathcal{A}_2(t)]}{t}=\gamma\sum_{j=1}^{\infty}\pi(j,0). \tag{38}$$

6.3. Stationary Mean Number of Customers in the Idle State of the Server

Let $\mathcal{A}_3(t)$ be the number of customers in the idle state of the server up to time t. Since $p(j,0,t)$ gives the probability that j customers are in the system with idle server at time t, we get

$$E[\mathcal{A}_3(t)]=\sum_{j=1}^{\infty}j\int_0^t p(j,0,u)du. \tag{39}$$

Consequently, the stationary mean number of customers in the idle state of the server is given by

$$a_3 = \lim_{t\to\infty} \frac{E[\mathcal{A}_3(t)]}{t} = \sum_{j=1}^{\infty} j\pi(j,0). \qquad (40)$$

6.4. Stationary Mean Number of Customers in the Active State of the Server

Let $\mathcal{A}_4(t)$ be the number of customers in the active state of the server up to time t. Since $p(j,k,t)$ gives the probability that j customers are in the system with server active in level k of the environment at time t, we get

$$E[\mathcal{A}_4(t)] = \sum_{k=1}^{N}\sum_{j=1}^{\infty} j \int_0^t p(j,k,u)du. \qquad (41)$$

Consequently, the stationary mean number of customers in the active state of the server is given by

$$a_4 = \lim_{t\to\infty} \frac{E[\mathcal{A}_4(t)]}{t} = \sum_{k=1}^{N}\sum_{j=1}^{\infty} j\pi(j,k). \qquad (42)$$

7. A Numerical Illustration

For the purpose of testing how the stationary probabilities are generated by the infinite series representations of the probabilities, we assume the following values for the parameters of the system subject to the constraints $\lambda_k < \mu_k, k = 1, 2, \cdots, N$:

$$N = 5; \lambda_0 = 10.0; 1 \le \gamma \le 30.0; \xi = 9.0;$$

$$\lambda_1 = 11.0; \mu_1 = 13.0; \lambda_2 = 12.0; \mu_2 = 14.0;$$

$$\lambda_3 = 13.0; \mu_3 = 15; \lambda_4 = 14.0; \mu_4 = 16;$$

$$\lambda_5 = 15.0; \mu_5 = 17;$$

$$q_1 = 0.2; q_2 = 0.1; q_3 = 0.3; q_4 = 0.1; q_5 = 0.3.$$

Now, we analyse how the performance measures indicated in Section 6 behave when the assignment rate increases from 1 to 30.

We have obtained the values of stationary mean rate a_1 of occurrence of entries into idle state of the server by varying γ from 1 to 30. Figure 2 portrays that a_1 increases as γ increases. This is quite natural since the mean time of stay in idle state decreases and hence the server moves to busy state.

Next, we have computed the values of stationary mean rate a_2 of occurrence of entries into busy state of the server by varying γ from 1 to 30. Figure 3 portrays that a_2 increases as γ increases. This is quite natural since the mean time of stay in idle state decreases and hence the server moves to busy state. Further, we observe that a_1 and a_2 are identical. This justifies that flows into idle state and busy state are well balanced. We have obtained the values of stationary mean number a_3 of customers in the idle state of the server by varying γ from 1 to

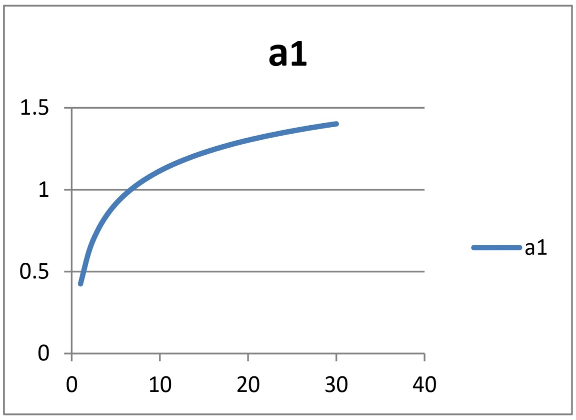


Figure 2. Variation of a_1 against γ

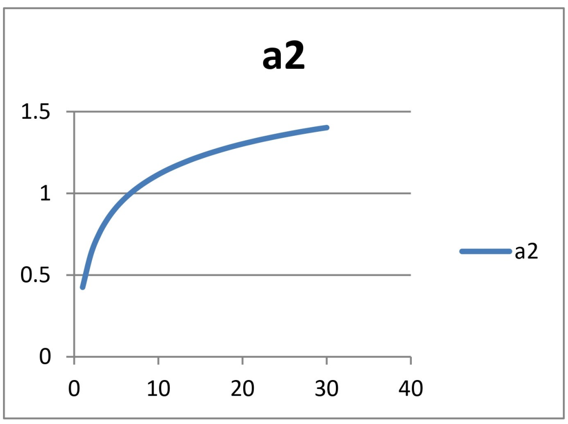


Figure 3. Variation of a_2 against γ

30. Figure 4 portrays that a_3 decreases as γ increases. This is quite natural since the mean time of stay in idle state decreases and hence the server moves to busy state with lesser number of customers in buffer. We have got the values of stationary

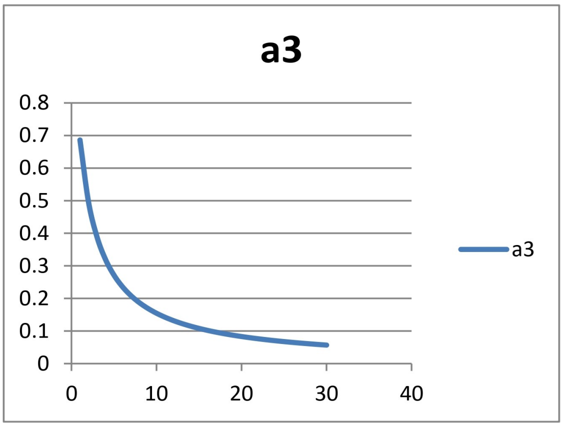


Figure 4. Variation of a_3 against γ

mean number a_4 of customers in the busy state of the server by varying γ from 1 to 30. Figure 5 exhibits that a_4 increases as γ increases. This is quite natural since the mean time of stay in idle state decreases and hence the server moves to busy state more often.

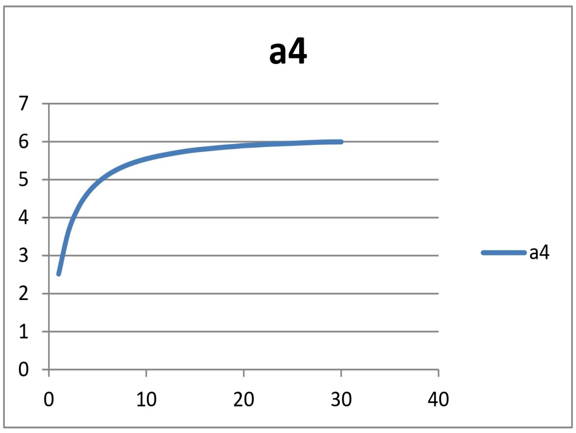


Figure 5. Variation of a_4 against γ

8. Conclusions

In the present paper, we studied a single server queueing model, in which the server is commissioned to work in any one of multiple levels of a stochastic environment. We introduced a novelty in the model by directing the control switch to initiate the assignment process only after a customer arrives. Based on the model, we derived explicit expressions for the steady-state probabilities. We deduced a particular case, in which impatience was absent. The behaviour of the model and its performance measures were tested numerically. It is noteworthy that this model has quite a lot of applications in managerial systems where the server is directed to work with distinguishable service rates depending upon the market conditions. Further the present research may be extended in future to study queueing problems with multi-servers arising in manufacturing systems where the service rates are chosen depending upon the market demands.

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