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A Time Truncated New Group Chain Sampling Plan Based on Log-Logistic Distribution

Nazrina Aziz[1,2,*], Seu Wen Fei[1], Waqar Hafeez[1], Shazlyn Milleana Shaharudin[3], Javid Shabbir[4]

[1]Department of Mathematics & Statistics, School of Quantitative Sciences, Universiti Utara Malaysia, 06010 UUM Sintok, Kedah, Malaysia
[2]Institute of Strategic Industrial Decision Modelling (ISIDM), Universiti Utara Malaysia, 06010 UUM Sintok, Kedah, Malaysia
[3]Department of Mathematics, Faculty of Sciences and Mathematics, Universiti Pendidikan Sultan Idris, 35900, Malaysia
[4]Department of Statistics, University of Wah, 47040 Wah Cantt, Pakistan
[*]Corresponding Author: nazrina@uum.edu.my, seuwenfei@gmail.com, waqarhafeez78601@yahoo.com, shazlyn@fsmt.upsi.edu.my, javidshabbir@gmail.com

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Abstract The acceptance sampling is a technique for ensuring that both producers and consumers are satisfied with the product's quality. This paper proposes a new group chain sampling plan (NGChSP) using Log-logistic distribution when the life test is truncated at a predetermined time. The minimum number of groups, g and the probability of lot acceptance, $L(p)$ are determined through satisfying the consumer's risk, β under the specified design parameter. This paper shows that the minimum number of groups, g decreases when the value of design parameters such as β, a, i and r increases. With the same design parameters, the minimum g increases when the shape parameter increases. Moreover, the $L(p)$ increases as shape parameter and minimum g increases. An illustrative example for NGChSP is provided. The findings suggest that as the test time termination constant decreases, the minimum g increases. Furthermore, as the mean ratio, $\frac{\mu_0}{\mu_1}$ increases, the $L(p)$ increases as well. In comparison to GChSP, the NGChSP requires a smaller number of groups, indicating that using the NGChSP for inspection will contribute to lower inspection time and costs. The NGChSP provides a higher probability of lot acceptance than GChSP. This paper concludes that the NGChSP performed better than the GChSP. Therefore, the NGChSP is better equipped for lot inspection in the manufacturing industry.

Keywords Truncated Life Test, Log-Logistic Distribution, New Group Chain Sampling Plan, Probability of Lot Acceptance

1. Introduction

Acceptance sampling is a technique that falls between zero inspection and 100% inspection. Acceptance sampling is the process of inspecting the sample of items from a production lot. Meanwhile, items are accepted without being inspected at all in zero inspection, whereas each item is inspected before being accepted in 100% inspection [1]. Hence, acceptance sampling was constructed as an alternative for 100% inspection. This is because 100% inspection is impractical, particularly when the testing is destructive or expensive. The purpose of acceptance sampling is to assist the manufacturer in accepting or rejecting the batch, neither to estimate or improve the quality of the batch [2]. The acceptance sampling can indirectly encourage manufacturers to improve quality, thereby reducing the chance of batch rejection. Acceptance sampling is often employed as a quality control measure of raw materials and components, work-in-progress, or finished goods in various industries.

The development of several sampling plans that consider customer's risk, producer's risk, inspection time, and sample size can be used to track the evolution of acceptance sampling. The probability of rejecting a good lot is defined as the producer's risk, while the probability of accepting a bad lot is identified as the consumer's risk [3]. The development of a sampling plan has the purpose of determining the minimum number of samples that will be inspected. Many researchers have proposed various combinations of sampling plans and lifetime distributions to obtain the minimum sample size.

The single sampling plan (SSP) was first proposed [4], and then the double sampling plan (DSP) was further developed. Nevertheless, there are weaknesses in these two acceptance sampling plans, as these plans provide high protection for consumers with a zero acceptance number. This is due to the fact that these plans are unfavorable to the producer, because even with a slight increase in the percent defective, the probability of lot acceptance will decline dramatically [5].

Therefore, the chain sampling plan (ChSP-1) was proposed by [6], to address the weaknesses in SSP and DSP with zero acceptance number. Hence [7,8] developed ChSP-1 with the Generalized Rayleigh distribution and Weibull distribution. For ChSP-1, the current lot under inspection will be accepted if only one defective item is detected in the sample, and no further defective items are detected in following lots. While compared to the SSP and DSP, the ChSP-1 has been shown to have a higher probability of lot acceptance when the acceptance number is zero. However, the chain sampling plan only inspects one item at a time.

Consequently, the group acceptance sampling plan (GASP) is proposed for inspecting several items at the same time. Researchers such as [9-13] have developed GASP with different lifetime distributions Marshall-Olkin extended Lomax distribution, Generalized Exponential distribution, Weibull distribution, Generalized Pareto distribution and Pareto distribution of 2nd Kind. However, there is a weakness to the GASP, which is, the probability of lot acceptance drops rapidly as the acceptance number is zero or one.

Thus, the group chain sampling plan (GChSP) is introduced as a solution to the GASP's weakness. The GChSP lowers inspection costs and time while maintaining a low probability of lot acceptance. Researchers such as [14,15] have developed GChSP with different lifetime distributions, Such as: exponential, Pareto distribution of the second kind and Log-logistics distributions.

On the other hand, when the preceding lots, $i = 2$, GChSP has five acceptance criteria that lead to lot sentencing, and type-II errors may occur during this phase. Therefore, the NGChSP is proposed, which operates based on four acceptances criteria. For NGChSP, it has been applied to Marshall-Olkin's Extended Lomax (MOEL) distribution. So far, no author has applied the Log-logistic distribution for NGChSP. Hence, this paper intends to propose a time truncated new group chain sampling plan with Log-logistic distribution.

Glossary of Symbols

g: number of groups
i: number of preceding lots
r: group size (number of testers in a group)
n: sample size
μ/μ_0: mean ratio
t_0: test termination time
a: test termination time multiplier (constant)
d: number of defective items
D: cumulative number of defective items
α: producer's risk
β: consumer's risk
$L(p)$: probability of lot acceptance

2. Methodology

2.1. Operating Procedure

The operating procedure of NGChSP is described below:

1. The test termination time, t_0 is set.
2. The minimum number of groups is determined by complying to the condition that probability of lot acceptance is less than a predetermined consumer's risk.
3. From the production lot, a random sample is taken. The sample is distributed into g groups where a group comprises r items, with $g \times r$ being the sample size. The life test is then started.
4. When the inspection time reaches $t = t_0$, the test is terminated. All units are inspected simultaneously and the number of defectives, d is counted and recorded.
5. The production lot is rejected if $d > 1$.
6. The production lot is accepted if $d = 0$ in the current lot and that the preceding lots have at most 1 defective unit, $D_i \leq 1$.
7. The production lot is accepted if $d = 1$ in the current lot and the cumulative number of defectives in all the preceding lots is 0, $D_i = 0$.

2.2. Proportion Defective and Probability of Lot Acceptance

In the acceptance sampling plan, [15] applied the Log-logistics distribution for GChSP and new two-sided group chain sampling plan (NTSGChSP) respectively. The proportion defective of an item is computed using cumulative distribution function (CDF) of the involved lifetime distribution. By assuming that an item's lifetime follows a scaled log-logistic distribution whose CDF is:

$$F(t;\sigma,\gamma)=\frac{(\frac{t}{\sigma})^{\gamma}}{1+(\frac{t}{\sigma})^{\gamma}} \quad (1)$$

$t>0$, where σ is a scale parameter and γ is a shape parameter.

According to equation (1), the lifetime relies on 3 parameters, which are time, scale and shape. The maximum point of time for the experiment in a time truncated test is the test termination time. Therefore, proportion defective, p during test termination time, t_0 can be expressed as:

$$p=F(t_0;\sigma,\gamma) \quad (2)$$

The test termination time can be defined as a product of predetermined constant, a and predetermined mean life, μ_0. The following is an expression of the test termination time:

$$t_0=a\mu_0 \quad (3)$$

Meanwhile, the true mean life of the log-logistic distribution is given by the following formula:

$$\mu=\frac{\left(\frac{\pi\sigma}{\gamma}\right)}{sin\left(\frac{\pi}{\gamma}\right)} \quad (4)$$

By substituting equations (3) and (4) in equation (2), we can use the following formula to obtain the proportion defective for item that follows the log-logistic distribution:

$$p=\frac{\left(\frac{a\pi}{\gamma sin\left(\frac{\pi}{\gamma}\right)}\right)^{\gamma}}{\left(\frac{\mu}{\mu_0}\right)^{\gamma}+\left(\frac{a\pi}{\gamma sin\left(\frac{\pi}{\gamma}\right)}\right)^{\gamma}} \quad (5)$$

where μ/μ_0 and a are mean ratio and predetermined constant respectively. Both are experimental design parameters which are used to evaluate the performance of our proposed plan.

Suppose that the probability of detecting 0 and 1 defectives in the lot under inspection are denoted by P_0 and P_1 respectively, and that ${P_0}^i$ and ${P_1}^i$ are the probability of detecting 0 and 1 defectives in the preceding i samples respectively. Since NGChSP considers four acceptance criteria, the probability of acceptance, $L(p)$ of NGChSP is written as:

$$L(p)={P_0}^i[P_0+(1+i)P_1] \quad (6)$$

Due to the dual nature of lot sentencing, the Binomial model can be used. From equation (6), we let $n=g\times r$, the P_0 and P_1 are given by:

$$P_0=(1-p)^{gr} \quad (7)$$

$$P_1=grp(1-p)^{gr-1} \quad (8)$$

Based on equations (7) and (8), n, g, r and p represent the sample size, number of groups, group size and proportion defective respectively. Then, we substitute equations (7) and (8) into equation (6) to obtain the probability of lot acceptance equation. Therefore, the equation (6) is rewritten as:

$$\text{L}(p)=(1-p)^{gr(i+1)}+grp(i+1)(1-p)^{gr(i+1)-1} \quad (9)$$

where i is the number of preceding lots.

3. Results

In our proposed plan, the two performance parameters are minimum number of groups, g and probability of lot acceptance, $L(p)$. The values of design and shape parameters are predetermined where

$$\beta=0.01,0.05,0.10,0.25,$$

$$a=0.25,0.50,0.75,1.00,1.25,1.50,1.75,2.00;$$

$$\mu/\mu_0=1,2,4,6,8,10,12;$$

$i=1,2,3,4$; $r=2,3,4,5$ and $\gamma=2,3,4$. Since our proposed plan emphasizes consumer protection, see [16,17]. Hence, the equation is introduced as follows:

$$L(p_0)\leq\beta \quad (10)$$

where p_0 is the probability of defective at $\mu=\mu_0$. In order to comply with equation (10), a predetermined consumer's risk will not be surpassed.

By using equation (10), the minimum number of groups is obtained. The results of minimum g for our proposed plan with different shape parameters at four different levels of consumer's risk with numerous combinations of a, r and i are shown in Tables 1, 2 and 3.

Table 1. Minimum g for NGChSP following Log-logistic distribution when $\gamma=2$

β	i	r	a							
			0.25	0.5	0.75	1	1.25	1.5	1.75	2
0.01	1	2	12	4	3	2	2	2	1	1
	2	3	6	2	1	1	1	1	1	1
	3	4	3	1	1	1	1	1	1	1
	4	5	2	1	1	1	1	1	1	1
0.05	1	2	9	3	2	2	1	1	1	1
	2	3	4	2	1	1	1	1	1	1
	3	4	3	1	1	1	1	1	1	1
	4	5	2	1	1	1	1	1	1	1
0.10	1	2	7	3	2	1	1	1	1	1
	2	3	4	1	1	1	1	1	1	1
	3	4	2	1	1	1	1	1	1	1
	4	5	2	1	1	1	1	1	1	1
0.25	1	2	5	2	1	1	1	1	1	1
	2	3	3	1	1	1	1	1	1	1
	3	4	2	1	1	1	1	1	1	1
	4	5	1	1	1	1	1	1	1	1

Table 2. Minimum g for NGChSP following Log-logistic distribution when $\gamma = 3$

β	i	r	a							
			0.25	0.5	0.75	1	1.25	1.5	1.75	2
0.01	1	2	62	9	4	2	2	2	1	1
	2	3	28	4	2	1	1	1	1	1
	3	4	16	3	1	1	1	1	1	1
	4	5	10	2	1	1	1	1	1	1
0.05	1	2	44	7	3	2	1	1	1	1
	2	3	20	3	2	1	1	1	1	1
	3	4	11	2	1	1	1	1	1	1
	4	5	7	1	1	1	1	1	1	1
0.10	1	2	36	5	2	2	1	1	1	1
	2	3	16	3	1	1	1	1	1	1
	3	4	9	2	1	1	1	1	1	1
	4	5	6	1	1	1	1	1	1	1
0.25	1	2	25	4	2	1	1	1	1	1
	2	3	12	2	1	1	1	1	1	1
	3	4	7	1	1	1	1	1	1	1
	4	5	4	1	1	1	1	1	1	1

Table 3. Minimum g for NGChSP following Log-logistic distribution when $\gamma = 4$

β	i	r	a							
			0.25	0.5	0.75	1	1.25	1.5	1.75	2
0.01	1	2	281	19	5	2	2	1	1	1
	2	3	125	9	2	1	1	1	1	1
	3	4	71	5	2	1	1	1	1	1
	4	5	45	3	1	1	1	1	1	1
0.05	1	2	201	14	4	2	1	1	1	1
	2	3	89	6	2	1	1	1	1	1
	3	4	51	4	1	1	1	1	1	1
	4	5	33	3	1	1	1	1	1	1
0.10	1	2	165	11	3	2	1	1	1	1
	2	3	73	5	2	1	1	1	1	1
	3	4	42	3	1	1	1	1	1	1
	4	5	27	2	1	1	1	1	1	1
0.25	1	2	114	8	2	1	1	1	1	1
	2	3	51	4	1	1	1	1	1	1
	3	4	29	2	1	1	1	1	1	1
	4	5	19	2	1	1	1	1	1	1

Table 1 illustrates the minimum g for the NGChSP for log-logistic distribution with $\gamma = 2$, at different quality levels. For instance, the minimum g is 3 if the β is set as 0.05, group size is 2, number of preceding lot is 1, predetermined constant, a is 0.50. Moreover, Table 2 illustrates the minimum g for the NGChSP for log-logistic distribution with $\gamma = 3$, at different quality levels. For instance, the minimum g is 7 if the β is set as 0.05, group size is 2, number of preceding lot is 1, predetermined constant is 0.50. Furthermore, Table 3 illustrates the minimum g for the NGChSP for log-logistic distribution with $\gamma = 4$, at different quality levels. For instance, the minimum g is 14 if the β is set as 0.05, group size is 2, number of preceding lot is 1, predetermined constant is 0.50. Therefore, we can observe that the minimum g increases when the shape parameter increases with the same design parameters. For example, when $(\beta, a, i, r) = (0.05, 0.50, 1, 2)$, the minimum g for $\gamma = 2$ is 3 while the minimum g for $\gamma = 4$ is 14. Additionally, the minimum g decreases when the design parameters such as β, a, i and r increase, as shown in Table 1, 2 and 3. For example, for $\gamma = 2$, the minimum g is 12 when $(\beta, a, i, r) = (0.01, 0.25, 1, 2)$ whereas the minimum g is 1 when $(\beta, a, i, r) = (0.25, 2.00, 4, 5)$.

To compute the $L(p)$, the values of minimum g obtained in Tables 1, 2 and 3 when $i = 1$ and $r = 2$ are inserted into equation (9). The pair of design parameters (i $= 1$ and $r = 2$) is selected, since the group sampling technique needs minimum g must always be greater than 1. The results of $L(p)$ for our proposed plan with different shape parameters at seven different values of mean ratio with various combinations of β, a and g are shown in Tables 4, 5 and 6.

Table 4 shows the $L(p)$ when shape parameter $\gamma = 2$ and the pair design parameters $(i, r) = (1, 2)$. For instance, the $L(p)$ is 0.992269 when β is 0.05, mean ratio is 8, minimum g is 3 and predetermined constant is 0.50. Next, Table 5 illustrates the $L(p)$ when shape parameter $\gamma = 3$ and the pair design parameters $(i, r) = (1, 2)$. For example, the $L(p)$ is 0.999931 when β is 0.05, mean ratio is 8, minimum g is 7 and predetermined constant is 0.50. Besides, Table 6 shows the $L(p)$ when shape parameter $\gamma = 4$ and the pair design parameters $(i, r) = (1, 2)$. For instance, the $L(p)$ is 0.99999917 when β is 0.05, mean ratio is 8, minimum g is 14 and predetermined constant is 0.50. Hence, we can observe that the $L(p)$ increases as shape parameter and minimum g increases. For example, the $L(p)$ is 0.992269 when $\gamma = 2$ and $g = 3$ while the $L(p)$ is 0.99999917 when $\gamma = 4$ and $g = 14$. Additionally, the $L(p)$ increases as the μ/μ_0 increases, as shown in Table 4, 5 and 6. For instance, for $\gamma = 2$, when $\beta = 0.05$, $g = 9$, $a = 0.25$ and $\mu/\mu_0 = 1$, the $L(p)$ is 0.048794, whereas the $L(p)$ is 0.999204 when μ/μ_0 rises to 12.

Table 4. Probability of lot acceptance for NGChSP following Log-logistic distribution when $\gamma = 2$; $i = 1$ and $r = 2$

β	g	a	mean ratio						
			1	2	4	6	8	10	12
0.01	12	0.25	0.007930	0.376413	0.892397	0.973641	0.990989	0.996173	0.998118
	4	0.50	0.008604	0.325887	0.865953	0.965738	0.988093	0.994903	0.997482
	3	0.75	0.005636	0.225791	0.798870	0.943858	0.979797	0.991205	0.995614
	2	1.00	0.007390	0.199154	0.759287	0.928391	0.973521	0.988315	0.994128
	2	1.25	0.001849	0.084660	0.593042	0.855501	0.942285	0.973521	0.986398
	2	1.50	0.000549	0.036007	0.431856	0.759287	0.895305	0.949770	0.973521
	1	1.75	0.000188	0.015910	0.298407	0.649501	0.833315	0.915996	0.954408
	1	2.00	0.009878	0.117313	0.563046	0.823998	0.924416	0.963871	0.980978
0.05	9	0.25	0.048794	0.595630	0.948788	0.988330	0.996115	0.998370	0.999204
	3	0.50	0.023673	0.441823	0.907253	0.977366	0.992269	0.996718	0.998386
	2	0.75	0.036007	0.431856	0.895305	0.973521	0.990826	0.996078	0.998064
	2	1.00	0.007390	0.199154	0.759287	0.928391	0.973521	0.988315	0.994128
	1	1.25	0.001849	0.084660	0.593042	0.855501	0.942285	0.973521	0.986398
	1	1.50	0.029323	0.260744	0.761969	0.924416	0.971113	0.987026	0.993413
	1	1.75	0.016478	0.174033	0.662373	0.879030	0.951059	0.977353	0.988302
	1	2.00	0.009878	0.117313	0.563046	0.823998	0.924416	0.963871	0.980978
0.1	7	0.25	0.091546	0.683939	0.964254	0.992053	0.997377	0.998905	0.999466
	3	0.50	0.067568	0.588461	0.943858	0.986933	0.995614	0.998153	0.999096
	2	0.75	0.036007	0.431856	0.895305	0.973521	0.990826	0.996078	0.998064
	1	1.00	0.007390	0.199154	0.759287	0.928391	0.973521	0.988315	0.994128
	1	1.25	0.056294	0.388896	0.852569	0.958469	0.984939	0.993413	0.996706
	1	1.50	0.029323	0.260744	0.761969	0.924416	0.971113	0.987026	0.993413
	1	1.75	0.016478	0.174033	0.662373	0.879030	0.951059	0.977353	0.988302
	1	2.00	0.009878	0.117313	0.563046	0.823998	0.924416	0.963871	0.980978
0.25	5	0.25	0.238098	0.821735	0.983148	0.996391	0.998825	0.999512	0.999763
	2	0.50	0.199154	0.759287	0.973521	0.994128	0.998064	0.999191	0.999606
	1	0.75	0.036007	0.431856	0.895305	0.973521	0.990826	0.996078	0.998064
	1	1.00	0.117313	0.563046	0.924416	0.980978	0.993413	0.997184	0.998610
	1	1.25	0.056294	0.388896	0.852569	0.958469	0.984939	0.993413	0.996706
	1	1.50	0.029323	0.260744	0.761969	0.924416	0.971113	0.987026	0.993413
	1	1.75	0.016478	0.174033	0.662373	0.879030	0.951059	0.977353	0.988302
	1	2.00	0.009878	0.117313	0.563046	0.823998	0.924416	0.963871	0.980978

Table 5. Probability of lot acceptance for NGChSP following Log-logistic distribution when $\gamma = 3$; $i = 1$ and $r = 2$

$\boldsymbol{\beta}$	$\boldsymbol{g}$	$\boldsymbol{a}$	**mean ratio**						
			1	2	4	6	8	10	12
0.01	62	0.25	0.009616	0.734148	0.992461	0.999295	0.999873	0.999966	0.999989
	9	0.50	0.008659	0.699194	0.990834	0.999135	0.999843	0.999959	0.999986
	4	0.75	0.003908	0.566853	0.983027	0.998346	0.999698	0.999920	0.999973
	2	1.00	0.002276	0.422547	0.968683	0.996800	0.999409	0.999843	0.999947
	2	1.25	0.002632	0.335875	0.952168	0.994856	0.999036	0.999743	0.999913
	2	1.50	0.000429	0.142190	0.882854	0.985584	0.997200	0.999242	0.999743
	1	1.75	0.000083	0.053054	0.771871	0.966668	0.993201	0.998127	0.999359
	1	2.00	0.004898	0.190745	0.869623	0.982478	0.996502	0.999044	0.999674
0.05	44	0.25	0.048133	0.850430	0.996473	0.999677	0.999942	0.999985	0.999995
	7	0.50	0.046912	0.831412	0.995818	0.999614	0.999931	0.999982	0.999994
	3	0.75	0.040889	0.777694	0.993689	0.999409	0.999893	0.999972	0.999991
	2	1.00	0.019086	0.628859	0.985584	0.998592	0.999743	0.999932	0.999977
	1	1.25	0.002632	0.335875	0.952168	0.994856	0.999036	0.999743	0.999913
	1	1.50	0.025666	0.488627	0.967248	0.996502	0.999346	0.999825	0.999941
	1	1.75	0.010686	0.313597	0.929159	0.991609	0.998385	0.999564	0.999852
	1	2.00	0.004898	0.190745	0.869623	0.982478	0.996502	0.999044	0.999674
0.1	36	0.25	0.098388	0.896699	0.997755	0.999796	0.999963	0.999990	0.999997
	5	0.50	0.072633	0.863154	0.996781	0.999705	0.999947	0.999986	0.999995
	2	0.75	0.040889	0.777694	0.993689	0.999409	0.999893	0.999972	0.999991
	2	1.00	0.019086	0.628859	0.985584	0.998592	0.999743	0.999932	0.999977
	1	1.25	0.067987	0.693394	0.987687	0.998785	0.999777	0.999941	0.999980
	1	1.50	0.025666	0.488627	0.967248	0.996502	0.999346	0.999825	0.999941
	1	1.75	0.010686	0.313597	0.929159	0.991609	0.998385	0.999564	0.999852
	1	2.00	0.004898	0.190745	0.869623	0.982478	0.996502	0.999044	0.999674
0.25	25	0.25	0.242967	0.947194	0.998966	0.999907	0.999983	0.999996	0.999999
	4	0.50	0.176518	0.921648	0.998346	0.999850	0.999973	0.999993	0.999998
	2	0.75	0.142190	0.882854	0.997200	0.999743	0.999954	0.999988	0.999996
	1	1.00	0.190745	0.869623	0.996502	0.999674	0.999941	0.999984	0.999995
	1	1.25	0.067987	0.693394	0.987687	0.998785	0.999777	0.999941	0.999980
	1	1.50	0.025666	0.488627	0.967248	0.996502	0.999346	0.999825	0.999941
	1	1.75	0.010686	0.313597	0.929159	0.991609	0.998385	0.999564	0.999852
	1	2.00	0.004898	0.190745	0.869623	0.982478	0.996502	0.999044	0.999674

Table 6. Probability of lot acceptance for NGChSP following Log-logistic distribution when $\gamma = 4$; $i = 1$ and $r = 2$

β	g	a	mean ratio						
			1	2	4	6	8	10	12
0.01	281	0.25	0.009815	0.933723	0.999665	0.999987	0.999999	0.999999	0.999999
	19	0.50	0.008243	0.925267	0.999614	0.999985	0.999998	0.999999	0.999999
	5	0.75	0.004094	0.885277	0.999345	0.999974	0.999997	0.999999	0.999999
	2	1.00	0.00805	0.851233	0.999045	0.999962	0.999996	0.999999	0.999999
	2	1.25	0.000126	0.537632	0.994588	0.999774	0.999977	0.999996	0.999999
	1	1.50	0.005541	0.607329	0.995075	0.999792	0.999979	0.999996	0.999999
	1	1.75	0.001067	0.356406	0.984418	0.999298	0.999928	0.999988	0.999997
	1	2.00	0.000238	0.175201	0.959801	0.998009	0.999792	0.999965	0.999992
0.05	201	0.25	0.049217	0.963393	0.999828	0.999993	0.999999	0.999999	0.999999
	14	0.50	0.039013	0.956411	0.99979	0.999992	0.999999	0.999999	0.999999
	4	0.75	0.016151	0.921858	0.999584	0.999984	0.999998	0.999999	0.999999
	2	1.00	0.00805	0.851233	0.999045	0.999962	0.999996	0.999999	0.999999
	1	1.25	0.032074	0.836645	0.998795	0.999951	0.999995	0.999999	0.999999
	1	1.50	0.005541	0.607329	0.995075	0.999792	0.999979	0.999996	0.999999
	1	1.75	0.001067	0.356406	0.984418	0.999298	0.999928	0.999988	0.999997
	1	2.00	0.000238	0.175201	0.959801	0.998009	0.999792	0.999965	0.999992
0.1	165	0.25	0.098449	0.974464	0.999884	0.999995	0.999999	0.999999	0.999999
	11	0.50	0.095142	0.971957	0.999871	0.999995	0.999999	0.999999	0.999999
	3	0.75	0.0606	0.953611	0.99977	0.999991	0.999999	0.999999	0.999999
	2	1.00	0.00805	0.851233	0.999045	0.999962	0.999996	0.999999	0.999999
	1	1.25	0.032074	0.836645	0.998795	0.999951	0.999995	0.999999	0.999999
	1	1.50	0.005541	0.607329	0.995075	0.999792	0.999979	0.999996	0.999999
	1	1.75	0.001067	0.356406	0.984418	0.999298	0.999928	0.999988	0.999997
	1	2.00	0.000238	0.175201	0.959801	0.998009	0.999792	0.999965	0.999992
0.25	114	0.25	0.248643	0.987199	0.999944	0.999998	0.999999	0.999999	0.999999
	8	0.50	0.220782	0.984596	0.999932	0.999997	0.999999	0.999999	0.999999
	2	0.75	0.209018	0.97874	0.999902	0.999996	0.999999	0.999999	0.999999
	1	1.00	0.175201	0.959801	0.999792	0.999992	0.999999	0.999999	0.999999
	1	1.25	0.032074	0.836645	0.998795	0.999951	0.999995	0.999999	0.999999
	1	1.50	0.005541	0.607329	0.995075	0.999792	0.999979	0.999996	0.999999
	1	1.75	0.001067	0.356406	0.984418	0.999298	0.999928	0.999988	0.999997
	1	2.00	0.000238	0.175201	0.959801	0.998009	0.999792	0.999965	0.999992

Let us consider the true unknown mean life for an experiment is at least 1000 hours. To reduce time taken during inspection, the experiment is designed to truncate at 500 hours ($a = 0.50$) with a consumer risk of 0.05. The NGChSP is used for truncated life tests that determine the minimum g based on the time termination ratio and the consumer's risk. Assume that the lifetime of an item follows a log-logistic distribution with $\gamma = 2$.

If the test depended on the group size $r = 2$ and preceding lot $i = 1$, then the minimum g is 3. Hence the design parameters for the NGChSP are $(a, i, r, g) = (0.50, 1, 2, 3)$. This indicates the experimenter chooses a random sample of size 6 from the lot and assigns 2 items to each of the 3 groups. If no more than one defect is found within 500 hours and no defective items are detected in the preceding lot, the lot will be accepted. As the mean ratio increases from 1 to 12, as shown in Table 4, the $L(p)$ increases from 0.023673 to 0.998386 if the test with the same design parameters $(a, i, r, g) = (0.50, 1, 2, 3)$. The findings suggest that as the test time termination constant decreases, the minimum g increases. Furthermore, as the μ/μ_0 increases, the $L(p)$ increases as well.

4. Performance Comparison

In this section, the performance of the NGChSP and the GChSP with 3 different shape parameters are compared. The comparisons in the minimum g are shown in Tables 7, 8 and 9. The difference in the NGChSP and GChSP are especially distinctive when $\beta = 0.05$; $i = 1$; and $r = 2$.

Table 7. Minimum number of groups, g, required for NGChSP and GChSP under log-logistic distribution when $\gamma = 2$

β	i	r	a	NGChSP	GChSP
0.05	1	2	0.25	9	11
			0.50	3	4
			0.75	2	2
			1.00	2	2
			1.25	1	2
			1.50	1	1
			1.75	1	1
			2.00	1	1

Table 8. Minimum number of groups, g, required for NGChSP and GChSP under log-logistic distribution when $\gamma = 3$

β	i	r	a	NGChSP	GChSP
0.05	1	2	0.25	44	58
			0.50	7	8
			0.75	3	3
			1.00	2	2
			1.25	1	2
			1.50	1	1
			1.75	1	1
			2.00	1	1

The sample size, n, is usually related to the number of minimum groups, g. If the number of minimum groups is large, the sample size is also large. The cost and time of the inspection are determined by the sample size, n. If n is a large number, the cost and inspection time will increase. Tables 7, 8 and 9 compare the minimum number of groups required for NGChSP and GChSP, indicating that NGChSP requires a smaller number of groups. Hence, using the NGChSP for inspection will contribute to lower inspection time and costs. Tables 10, 11 and 12 show the comparisons in the probability of lot acceptance for NGChSP and GChSP at the shortest period of test termination time, $a = 0.25$ to compare the performance of both sampling plans.

Table 9. Minimum number of groups, g, required for NGChSP and GChSP under log-logistic distribution when $\gamma = 4$

β	i	r	a	NGChSP	GChSP
0.05	1	2	0.25	201	264
			0.50	14	18
			0.75	4	5
			1.00	2	2
			1.25	1	2
			1.50	1	1
			1.75	1	1
			2.00	1	1

Table 10. Probability of lot acceptance, $L(p)$ required for NGChSP and GChSP following Log-logistic distribution when $\gamma = 2$

β	i	r	a	mean ratio	$L(p)$	
					NGChSP	GChSP
					g= 9	g=11
0.05	1	2	0.25	1	0.037502	0.048794
				2	0.611763	0.595630
				4	0.953655	0.948788
				6	0.989590	0.988330
				8	0.996552	0.996115
				10	0.998558	0.998370
				12	0.999296	0.999204

Table 11. Probability of lot acceptance, $L(p)$ required for NGChSP and GChSP following Log-logistic distribution when $\gamma = 3$

β	i	r	a	mean ratio	$L(p)$	
					NGChSP	GChSP
					g=44	g=58
0.05	1	2	0.25	1	0.048428	0.048133
				2	0.876452	0.850430
				4	0.997273	0.996473
				6	0.999752	0.999677
				8	0.999955	0.999942
				10	0.999988	0.999985
				12	0.999996	0.999995

Based on Tables 10, 11 and 12, NGChSP not only operates with a lower cost, but also performs better when inspecting production lots with lower mean ratio. The NGChSP provides a higher probability of lot acceptance than GChSP at mean ratios 2, 4, 6, 10 and 12. This result is obtained when the design parameters for both plans are $(\beta, i, r, a) = (0.05, 1, 2, 0.25)$. In comparison to GChSP, NGChSP performed better overall at the same level of risk.

Table 12. Probability of lot acceptance, $L(p)$ required for NGChSP and GChSP following Log-logistic distribution when $\gamma = 4$

β	i	r	a	mean ratio	$L(p)$	
					NGChSP	**GChSP**
					g=201	g=264
				1	0.04921717	0.04972501
				2	0.96339337	0.95440861
				4	0.99982805	0.99977795
0.05	1	2	0.25	6	0.99999322	0.99999123
				8	0.99999932	0.99999912
				10	0.99999989	0.99999985
				12	0.99999997	0.99999997

5. Conclusions

The new group chain sampling plan for truncated life test when an item follows Log-Logistic distribution is developed in this paper. For NGChSP, the results have shown that the minimum g decreases when the value of design parameters such as β, a, i and r increases. With the same design parameters, the minimum g increases when the shape parameter increases. Besides, the $L(p)$ increases as shape parameter and minimum g increases. The findings suggest the test time termination constant decreases, the minimum g increases. Furthermore, as the μ/μ_0 increases, the $L(p)$ increases as well. Since the NGChSP operates based on four acceptance criteria, which is a more balanced plan in lot sentencing compared to the GChSP. The comparison has shown that NGChSP requires a smaller number of groups than the GChSP while still having a higher probability of lot acceptance. A smaller sample size can result in lower inspection operating costs and a lower risk of item damage due to mishandling. In conclusion, we suggest that NGChSP is a better alternative option for manufacturers to use for production lot inspection. In future the proposed plan can be extended to Bayesian sampling plans [18] and for economic reliability sampling plans by using different distributions [19].

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