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Strong Form of Nano Ideal Set in Nano Ideal Topological Spaces

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Abstract The purpose of this article is to define and analyse certain new types of a strongly open set namely $NI\mathcal{M}_{\mathcal{V}} - open$ ($NI\mathcal{M}_{\mathcal{V}} - closed$) in nano ideal topological space and compare it with the other existing sets in $nano\ ideal\ topology$. Here the author uses the lower approximation, upper approximation and boundary region to define nano topology. To emphasize the inclusive relationship of this particular $nano\ ideal$ set with other existing familiar nano ideal sets like $NI\alpha - open$, $NIpre - open$, $NIsemi - open$ and $NI\beta - open$, some counter examples are provided. We have also established the independence of this $NI\mathcal{M}_{\mathcal{V}} - open$ set with both $NI - regular\ open$ set and $NI - open$ set in nano ideal topological spaces. In addition, $NI\mathcal{M}_{\mathcal{V}} - interior$, $NI\mathcal{M}_{\mathcal{V}} - closure$, $NI\mathcal{M}_{\mathcal{V}} - derived$, $NI\mathcal{M}_{\mathcal{V}} - neighborhood$ are introduced, investigated with its basic results and fundamental properties. The Exterior operator plays a vital role in topological spaces. Unless like the interior operator, the exterior operator varies in some cases, for example it reverses inclusions when it comes to the subset property in topological spaces. In the next section, we have defined $NI\mathcal{M}_{\mathcal{V}} - Exterior$ and analysed some of its basic properties. We have also introduced $NI\mathcal{M}_{\mathcal{V}} - Border$ and discussed its correlation between $NI\mathcal{M}_{\mathcal{V}} - Border$ and $NI\mathcal{M}_{\mathcal{V}} - interior$. The paper finally concludes with the definition of $NI\mathcal{M}_{\mathcal{V}} - Frontier$ and describes the relationship of $NI\mathcal{M}_{\mathcal{V}} - Frontier$ with $NI\mathcal{M}_{\mathcal{V}} - closure$, $NI\mathcal{M}_{\mathcal{V}} - interior$ and $NI\mathcal{M}_{\mathcal{V}} - Border$.

Keywords $NI\mathcal{M}_{\mathcal{V}} - open$, $NI\mathcal{M}_{\mathcal{V}} - interior$, $NI\mathcal{M}_{\mathcal{V}} - closure$, $NI\mathcal{M}_{\mathcal{V}} - derived$, $NI\mathcal{M}_{\mathcal{V}} - neighborhood$

1. Introduction

An ideal $\mathcal{I}$ [3] on a non-empty collection of subsets of X which satisfies (i) $H \in \mathcal{I},\ K \subseteq H\ \ implies\ \ K \in \mathcal{I}$ and (ii) $H \in \mathcal{I}, K \in \mathcal{I}\ implies\ H \bigcup K \in \mathcal{I}$. Given a topological space (X, τ) and an ideal $\mathcal{I}$ on X, let the set operator $\{x \in X: V \bigcap A \notin \mathcal{I}\ \ for\ \ every\ \ V \in \tau(x)\}$ be denoted by A^*, for each $A \subseteq X$, where $\tau(x) = \{\ V \in \tau : x \in V\}$. Then the operator $(.)^*$: $\mathcal{P}(X) \to \mathcal{P}(X)$ defined by $cl^*(A) = A\ \cup\ A^*$ is a $Kuratowski\ \ closure\ operator$. So, the family $\tau^*(\mathcal{I}) = \{T \subseteq X :\ \ cl^*(X \backslash T) = X \backslash T\}$ is a topology on the set X. $Jankovic\ and\ \ Hamlett$ have examined various properties of the topological space $(X, \tau^*(\mathcal{I}))$ in [2]. Also given a non-empty finite set $\mathcal{U}$, an equivalence relation $\mathcal{R}$ on $\mathcal{U}$ and a subset X on $\mathcal{U}$, the sets, the *Lower(upper) approximation* of X with respect to $\mathcal{R}$ denoted by $L_{\mathcal{R}}(X)(U_{\mathcal{R}}(X))$ were defined by Z. Pawlak in [13] and then the five-element topology, called

$nano\ \ topology, \tau_{\mathcal{R}}(X) = \{\emptyset, \mathcal{U}, L_{\mathcal{R}}(X),\ U_{\mathcal{R}}(X), U_{\mathcal{R}}(X)\backslash L_{\mathcal{R}}(X)\}$ on the finite set $\mathcal{U}$ has been defined by $Lellis\ Thivagar\ M\ \ and\ \ Richard\ \ C$ [5] and it is defined as: $L_{\mathcal{R}}(X) = \cup\{\mathcal{R}(u) \subseteq X, u \in \mathcal{U}\}$ where $\mathcal{R}(u)$ denotes the equivalence class determined by $u \in \mathcal{U}$ $(U_{\mathcal{R}}(Z) = \cup\{\mathcal{R}(u): \mathcal{R}(u) \cap X \neq \emptyset, u \in \mathcal{U}\}$. Given a Nano Topological Space (briefly, NTS) $(\mathcal{U}, \tau_{\mathcal{R}}(X))$ and an ideal $\mathcal{J}$ on the set $\mathcal{U}$, the triple $(\mathcal{U}, \tau_{\mathcal{R}}(X), \mathcal{J})$ is called Nano Ideal Topological Spaces (briefly, $NITS$). In 2006, Parimala, Noiri and Jafari have examined some basic properties of the topological space $(\mathcal{U}, \tau^*(\mathcal{J}))$ where $(\mathcal{U}, \tau_{\mathcal{R}}(X))$ is a nano topological space and $\mathcal{J}$ is an ideal on the set $\mathcal{U}$. M. Parimala introduced nano ideal topological spaces in [7], $\mathcal{N}-$*local function* in [8] and *closure operator* $cl_N{}^*(.)$ in [9] which is defined as follows: $H_N{}^*(\mathcal{J}, \mathcal{N}) = \{u \in \mathcal{U} : V\ \cap\ H \notin \mathcal{J}\ \ for\ \ every\ \ V \in V_N(u)\}$ where $V_N(u) = \{\ V :\ \ u \in V\ \ and\ \ V \in \mathcal{N}\}$, $cl_N{}^*(H) = H \cup H_N{}^*$, for $H \subseteq\ \ \mathcal{U}\}$ and discussed some of its properties. In [1] Abd El-Fattah, A. El-Atik and Hanan Z. Hassan introduced new forms of nano ideal continuous functions and analysed their properties.

2. Preliminaries

Property 2.1[4]: If $(\mathcal{U}, \mathcal{R})$ is an approximation space and $M, T \subseteq \mathcal{U}$, then

(i) $L_{\mathcal{R}}(M) \subseteq M \subseteq U_{\mathcal{R}}(M)$.

(ii) $L_{\mathcal{R}}(\emptyset) = U_{\mathcal{R}}(\emptyset) = \emptyset$.

(iii) $L_{\mathcal{R}}(\mathcal{U}) = U_{\mathcal{R}}(\mathcal{U}) = \mathcal{U}$.

(iv) $U_{\mathcal{R}}(M \cup T) = U_{\mathcal{R}}(M)\ \cup\ U_{\mathcal{R}}(T)$.

(v) $U_{\mathcal{R}}(M \cap T) \subseteq U_{\mathcal{R}}(M)\ \cap\ U_{\mathcal{R}}(T)$.

(vi) $L_{\mathcal{R}}(M \cup T) \supseteq L_{\mathcal{R}}(M) \cup L_{\mathcal{R}}(T)$.

(vii) $L_{\mathcal{R}}(M \cap T) = L_{\mathcal{R}}(M)\ \cap\ L_{\mathcal{R}}(T)$.

(viii) $L_{\mathcal{R}}(M) \subseteq L_{\mathcal{R}}(T)$ and $U_{\mathcal{R}}(M) \subseteq U_{\mathcal{R}}(T)$ whenever $M \subseteq T$.

(ix) $U_{\mathcal{R}}(M^c) = [L_{\mathcal{R}}(M)]^c$ and $L_{\mathcal{R}}(M^c) = [U_{\mathcal{R}}(M)]^c$.

(x) $U_{\mathcal{R}}[U_{\mathcal{R}}(M)] = L[U_{\mathcal{R}}(M)] = U_{\mathcal{R}}(M)$.

(xi) $L_{\mathcal{R}}[L_{\mathcal{R}}(M)] = U_{\mathcal{R}}[L_{\mathcal{R}}(M)] = L_{\mathcal{R}}(M)$.

Property 2.2[4]: Let $\mathcal{U}$ be the $universe$, $\mathcal{R}$ be the $equivalence\ relation$ on $\mathcal{U}$ and $\tau_{\mathcal{R}}(Z) = \{\emptyset, \mathcal{U}\ ,\ \ L_{\mathcal{R}}(Z), U_{\mathcal{R}}(Z), U_{\mathcal{R}}(Z)\backslash L_{\mathcal{R}}(Z)\}$ where $Z \subseteq \mathcal{U}$. Then by Property 2.1, $\tau_{\mathcal{R}}(Z)$ satisfies the following axioms:

(i) $\mathcal{U}$ and $\emptyset \in \tau_{\mathcal{R}}(Z)$.

(ii) The $union$ of elements of any sub-collection of $\tau_{\mathcal{R}}(Z)$ is in $\tau_{\mathcal{R}}(Z)$.

(iii) The $intersection$ of elements of any finite sub-collection of $\tau_{\mathcal{R}}(Z)$ is in $\tau_{\mathcal{R}}(Z)$.

Then $\tau_{\mathcal{R}}(Z)$ is a topology on $\mathcal{U}$ called the $nano\ \ topology$ on $\mathcal{U}$ with respect to Z and $(\mathcal{U}, \tau_{\mathcal{R}}(Z))$ as a Nano Topological Space (briefly, NTS). The members of $\tau_{\mathcal{R}}(Z)$ are known as nano-open sets. We denote $\tau_{\mathcal{R}}(Z)$ by $\mathcal{N}$ and so $(\mathcal{U}, \mathcal{N})$ is the NTS.

The $nano\ \ topology$ $(\mathcal{U}, \mathcal{N})$ with ideal $\mathcal{J}$ forms the triplet $(\mathcal{U}, \mathcal{N}, \mathcal{J})$ called the $Nano\ \ Ideal\ \ Topological\ \ Space$ [7] and it is denoted by $NITS$.

Also, for $H \subseteq\ \ \mathcal{U}$ the nano-interior and nano-closure (nano*-closure) of H are denoted by $Nint(H)$ and $cl_N(H)$ $(cl_N{}^*(H))$.

Definition 2.3[6, 8]: A subset H of a $NITS(\mathcal{U}, \mathcal{N}, \mathcal{J})$ is said to be,

(i) $NI-open$ if $H \subseteq Nint(H_N{}^*)$ and its complement is $NI-closed$.

(ii) $NIpre-open$ if $H \subseteq Nint(cl_N{}^*(H))$ and $NIpre-closed$ if $cl_N{}^*\big(Nint(H)\big) \subseteq H$.

(iii) $NIsemi-open$ if $H \subseteq cl_N{}^*(Nint(H))$ and $NIsemi-closed$ if $Nint(cl_N{}^*(H)) \subseteq H$.

(iv) $NI\alpha-open$ if $H \subseteq Nint(cl_N{}^*(Nint(H)))$ and $NI\alpha-closed$ if $cl_N{}^*(Nint(cl_N{}^*(H))) \subseteq H$.

(v) $NI\beta-open$ if $H \subseteq cl_N{}^*(Nint(cl_N{}^*(H)))$ and $NI\beta-closed$ if $Nint(cl_N{}^*(Nint(H))) \subseteq H$

(vi) $NIregular-open$ if $H = Nint(cl_N{}^*(H))$ and $NIregular-closed$ if $cl_N{}^*\big(Nint(H)\big) = H$.

The family of all $NI-open$ (resp. $NIpre-open$, $NIsemi-open$, $NI\alpha-open$, $NI\beta-open$, $NIregular-opens$) sets of a $NITS$ is denoted by $NIO(\mathcal{U}, Z)$ $(resp. NIPO(\mathcal{U}, Z)$, $NISO(\mathcal{U}, Z)$, $NI\alpha O(\mathcal{U}, Z)$, $NI\beta O(\mathcal{U}, Z)$, $NIRO(\mathcal{U}, Z))$.

3. $NI\mathcal{M}_{\mathcal{V}}-Open\ \ Set$

Throughout this paper $(\mathcal{U}, \mathcal{N}, \mathcal{J})$ and $(\mathcal{U}, \tau_{\mathcal{R}}(Z), \mathcal{J})$ represents $NITS$.

Definition 3.1: Let $(\mathcal{U}, \mathcal{N}, \mathcal{J})$ be a $NITS$ and $H \subseteq \mathcal{U}$. Then $NIcl_{\mathcal{M}_{\mathcal{V}}}(H) = \{u \in\ \ \mathcal{U} :\ int^*_{\tau_N}\big(cl_N{}^*(V)\big)\ \cap\ H \neq \emptyset$, for each $V \in V_N(u)\}$, where $V_N(u) = \{V :\ \ u \in V\ \ and\ \ V \in \mathcal{N}\}$ and $\tau_N{}^*(\mathcal{J}) = \{G \subseteq \mathcal{U} :\ \ cl_N{}^*(\mathcal{U} - G) = \mathcal{U} - G\}$. Here $int^*_{\tau_N}(V)$ denotes the $interior$ of V in $\tau_N{}^*(\mathcal{J})$. If $NIcl_{\mathcal{M}_{\mathcal{V}}}(H) = H$ then H is a $NI\mathcal{M}_{\mathcal{V}}-closed\ \ set$ and its complement is $NI\mathcal{M}_{\mathcal{V}}-open$.

The family of all $NI\mathcal{M}_{\mathcal{V}}-open$ sets of a $NITS$ is denoted by $NI\mathcal{M}_{\mathcal{V}}O(\mathcal{U}, Z)$.

Remark 3.2: The following example shows that

1) Finite intersection of $NI\mathcal{M}_{\mathcal{V}}-open$ sets is $NI\mathcal{M}_{\mathcal{V}}-open$.

2) Union of two $NI\mathcal{M}_{\mathcal{V}}-open$ sets is $NI\mathcal{M}_{\mathcal{V}}-open$.

Example 3.3: Let $\mathcal{U} = \{e, h, q, w\}$ $be\ \ the\ \ universe, Z = \{h, q\}$, $\mathcal{U}/\mathcal{R} = \{\{e\}, \{h, w\}, \{q\}\}$, $\tau_{\mathcal{R}}(Z) = \{\emptyset, \{q\},\ \{h, w\}, \{h, q, w\}, \mathcal{U}\}$ and the

$ideal\ \mathcal{I} = \{\emptyset, \{e\}\}$ .Then $NI\mathcal{M}_\mathcal{V} - open$ sets are $\{\emptyset, \{q\}, \{h, w\}, \{h, q, w\}, \mathcal{U}\}$
1) $\{h, q, w\} \cap \{h, w\} = \{h, w\} \in NI\mathcal{M}_\mathcal{V} - open$
2) $\{q\} \cup \{h, w\} = \{h, q, w\} \in NI\mathcal{M}_\mathcal{V} - open.$

Proposition 3.4: In a $S\ (\mathcal{U}, \tau_\mathcal{R}(Z), \mathcal{I})$,

1) Every $NI\mathcal{M}_\mathcal{V} - open$ set is a $NI\alpha - open$ set.

2) Every $NI\mathcal{M}_\mathcal{V} - open$ set is a $NIpre - open$ set.

Remark 3.5: The proposition 3.4 need not be true for the converse part as is evidenced below.

Example 3.6: Let $\mathcal{U} = \{e, h, q, w\}$ $be\ the\ universe, Z = \{e, h\}$, $\mathcal{U}/\mathcal{R} = \{\{e\}, \{h, q\}, \{w\}\}$, $\tau_\mathcal{R}(Z) = \{\emptyset, \{e\}, \{h, q\}, \{e, h, q\}, \mathcal{U}\}$ and the al $\mathcal{I} = \{\emptyset, \{e\}, \{h\}, \{e, h\}\}$. Here $S = \{h, q\}$ is a $NI\alpha - open$ set but S need not be $NI\mathcal{M}_\mathcal{V} - open.$

Example 3.7: Let $\mathcal{U} = \{e, h, q, w\}$ $be\ the\ universe, Z = \{h, q\}$, $\mathcal{U}/\mathcal{R} = \{\{e, h\}, \{q\}, \{w\}\}, \tau_\mathcal{R}(Z) = \{\emptyset, \{q\}, \{e, h\}, \{e, h, q\}, \mathcal{U}\}$ and the $ideal\ \mathcal{I} = \{\emptyset, \{e\}\}$. Here $Q = \{h, q, w\}$ which is $NIpre - open$ but not $NI\mathcal{M}_\mathcal{V} - open.$

Preposition 3.8: In a $S\ (\mathcal{U}, \tau_\mathcal{R}(Z), \mathcal{I})$,
1) Every $NI\mathcal{M}_\mathcal{V} - open$ set is a $NIsemi - open$ set.
2) Every $NI\mathcal{M}_\mathcal{V} - open$ set is a $NI\beta - open$ set.

Remark 3.9: The $preposition$ 3.8 need not be true for the converse part as is evidenced below.

Example 3.10: Let $= \{e, h, q, w\}$ $be\ the\ universe,$ $Z = \{e, h\}$, $\mathcal{U}/\mathcal{R} = \{\{e\}, \{h, q\}, \{w\}\}$, $\tau_\mathcal{R}(Z) = \{\emptyset, \{e\}, \{h, q\}, \{e, h, q\}, \mathcal{U}\}$ and the $ideal$ $\mathcal{I} = \{\emptyset, \{e\}, \{h\}, \{e, h\}\}$. Here $R = \{e, h, q\}$ is a $NIsemi - open$ set but R need not be $NI\mathcal{M}_\mathcal{V} - open$. Also, for $T = \{q, w\}$ which is $NI\beta - open$ but not $NI\mathcal{M}_\mathcal{V} - open.$

Preposition 3.11: In a $NITS\ (\mathcal{U}, \tau_\mathcal{R}(Z), \mathcal{I})$, $NI - open$ and $NI\mathcal{M}_\mathcal{V} - open$ are independent of each other.

Example 3.12: (i) Let $\mathcal{U} = \{e, h, q, w\}$ $be\ the\ universe, Z = \{h, w\}$, $\mathcal{U}/\mathcal{R} = \{\{e\}, \{h, q\}, \{w\}\}$, $\tau_\mathcal{R}(Z) = \{\emptyset, \{w\}, \{h, q\}, \{h, q, w\}, \mathcal{U}\}$ and the $ideal$ $\mathcal{I} = \{\emptyset, \{e\}\}$. Here $E = \{q\}$ is a $NI - open$ set but E need not be $NI\mathcal{M}_\mathcal{V} - open.$

(ii) Let $\mathcal{U} = \{e, h, q, w\}$ $be\ the\ universe$, $Z = \{e, h\}$, $\mathcal{U}/\mathcal{R} = \{\{e\}, \{h, q\}, \{w\}\}, \tau_\mathcal{R}(Z) = \{\emptyset, \{e\}, \{h, q\}, \{e, h, q\}, \mathcal{U}\}$ and the $ideal$ $\mathcal{I} = \{\emptyset, \{e\}, \{h\}, \{e, h\}\}$. Here $H = \{e\}$ is a $NI\mathcal{M}_\mathcal{V} - open$ set but H need not be $NI - open$ set.

Preposition 3.13: In a $NITS\ (\mathcal{U}, \tau_\mathcal{R}(Z), \mathcal{I})$, $NIregular - open$ and $NI\mathcal{M}_\mathcal{V} - open$ are independent of each other.

Example 3.14: (i) Let $\mathcal{U} = e, h, q, w\}be\ the\ universe, Z = \{h, w\}, \mathcal{U}/\mathcal{R} = \{\{e, h\}, \{q\}, \{w\}\}$, $\tau_\mathcal{R}(Z) = \{\emptyset, \{w\}, \{e, h\}, \{e, h, w\}, \mathcal{U}\}$ and the $ideal$ $\mathcal{I} = \{\emptyset, \{e\}, \{h\}, \{q\}, \{e, h\}, \{e, q\}, \{h, q\}, \{e, h, q\}\}$ and. Here $D = \{w\}$ is a $NIregular - open$ set but D need not be $NI\mathcal{M}_\mathcal{V} - open.$

(ii) Let $\mathcal{U} = \{e, h, q, w\}$ $be\ the\ universe, Z = \{h, q\}$, $\mathcal{U}/\mathcal{R} = \{\{e\}, \{h, w\}, \{q\}\}, \tau_\mathcal{R}(Z) = \{\emptyset, \{q\}, \{h, w\}, \{h, q, w\}, \mathcal{U}\}$ and the $ideal$ $\mathcal{I} = \{\emptyset, \{e\}, \{h\}, \{e, h\}\}$. Here $G = \{h, q, w\}$ is a $NI\mathcal{M}_\mathcal{V} - open$ set but G need not be $NIregular - open$ set.

The following Figure 1 depicts the relationship between $NI\mathcal{M}_\mathcal{V} - open$ set and other existing nano ideal open sets.

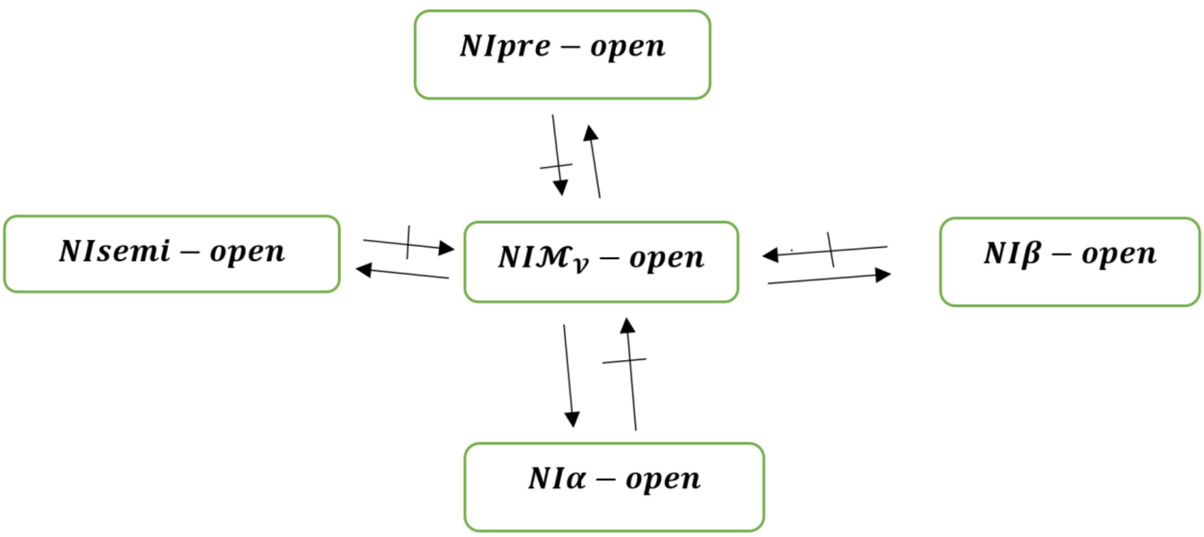

Figure 1. Implications

$NI-open$ and $NIregular-open$ sets are independent of $NI\mathcal{M}_{\mathcal{V}}-open$ set.

Definition 3.15: Let $(\mathcal{U},\mathcal{N},\mathcal{I})$ be a $NITS$ and $H\subseteq\mathcal{U}$. A point $h\in\mathcal{U}$ is said to be $NI\mathcal{M}_{\mathcal{V}}-Limit\ \ point$ of H if for every $NI\mathcal{M}_{\mathcal{V}}-open$ set $V\in V_N(h)$, $V\cap\{H-\{h\}\}\neq\emptyset$. The collection of all $NI\mathcal{M}_{\mathcal{V}}-Limit\ \ point$ of H is called a $NI\mathcal{M}_{\mathcal{V}}-Derived\ \ set$ of H and is denoted by $NID_{\mathcal{M}_{\mathcal{V}}}(H)$.

Theorem 3.16: Let $(\mathcal{U},\mathcal{N},\mathcal{I})$ be a $NITS$ and $Y\subseteq\mathcal{U}$. For $K,P\subseteq Y$,

(i) $NID_{\mathcal{M}_{\mathcal{V}}}(\emptyset)=\emptyset$

(ii) If $K\subseteq P$ then $NID_{\mathcal{M}_{\mathcal{V}}}(K)\subseteq NID_{\mathcal{M}_{\mathcal{V}}}(P)$

(iii) $NID_{\mathcal{M}_{\mathcal{V}}}(K)\cup NID_{\mathcal{M}_{\mathcal{V}}}(P)\subseteq NID_{\mathcal{M}_{\mathcal{V}}}(K\cup \mathrm{P})$

(iv) $NID_{\mathcal{M}_{\mathcal{V}}}(K\cap P)\subseteq NID_{\mathcal{M}_{\mathcal{V}}}(K)\cap NID_{\mathcal{M}_{\mathcal{V}}}(P)$

(v) $NID_{\mathcal{M}_{\mathcal{V}}}\left(NID_{\mathcal{M}_{\mathcal{V}}}(K)\right)-K\subseteq NID_{\mathcal{M}_{\mathcal{V}}}(K)$

(vi) $NID_{\mathcal{M}_{\mathcal{V}}}(K\cup NID_{\mathcal{M}_{\mathcal{V}}}(P))\subseteq K\cup NID_{\mathcal{M}_{\mathcal{V}}}(P)$

Proof: (i) Let $h\in\mathcal{U}$. Then for any $V\in V_N(h)$, $V\cap\{\emptyset\cap\{h\}^c\}=\emptyset$. Therefore, $NID_{\mathcal{M}_{\mathcal{V}}}(\emptyset)=\emptyset$.

(ii) Let $h\in NID_{\mathcal{M}_{\mathcal{V}}}(K)$. Then $V\cap K-\{h\}\neq\emptyset$ for every $V\in V_N(h)$. Since $K\subseteq P$, $W\cap(P-\{h\}\}\neq\emptyset$. This implies $h\in NID_{\mathcal{M}_{\mathcal{V}}}(P)$. Therefore, $NID_{\mathcal{M}_{\mathcal{V}}}(K)\subseteq NID_{\mathcal{M}_{\mathcal{V}}}(P)$ for any $K\subseteq P$.

(iii) Since K and P are the subsets of $K\cup P$, by (ii), $NID_{\mathcal{M}_{\mathcal{V}}}(K)\subseteq NID_{\mathcal{M}_{\mathcal{V}}}(K\cup P)$ and $NID_{\mathcal{M}_{\mathcal{V}}}(P)\subseteq NID_{\mathcal{M}_{\mathcal{V}}}(K\cup P)$. Hence, $NID_{\mathcal{M}_{\mathcal{V}}}(K)\cup NID_{\mathcal{M}_{\mathcal{V}}}(P)\subseteq NID_{\mathcal{M}_{\mathcal{V}}}(K\cup P)$.

(iv) Since $K\cap P$ is the subset of both K and P, by (ii) the proof is obvious.

(v) Let $h\in NID_{\mathcal{M}_{\mathcal{V}}}\left(NID_{\mathcal{M}_{\mathcal{V}}}(K)\right)-K$ and $V\in V_N(h)$. Then, $V\cap(NID_{\mathcal{M}_{\mathcal{V}}}(K)-\{h\})\neq\emptyset$. Let $v\in V\cap\left(NID_{\mathcal{M}_{\mathcal{V}}}(K)-\{h\}\right)$. Then $v\in V$ and $v\in NID_{\mathcal{M}_{\mathcal{V}}}(K)$. Then $V\cap\{K-\{v\}\}\neq\emptyset$. Let $w\in V\cap\{K-\{h\}\}$. Then $w\neq h$ for $w\in K$ and $v\notin K$. Therefore, $V\cap(K-\{h\})\neq\emptyset$ and so $h\in NID_{\mathcal{M}_{\mathcal{V}}}(K)$.

(vi) Let $h\in NID_{\mathcal{M}_{\mathcal{V}}}(K\cup NID_{\mathcal{M}_{\mathcal{V}}}(K))$. If $h\in K$ then the proof completes. Suppose $h\in NID_{\mathcal{M}_{\mathcal{V}}}\left(K\cup(NID_{\mathcal{M}_{\mathcal{V}}}(K)\right)-K$, then for $NI\mathcal{M}_{\mathcal{V}}-open$ set V containing h, $V\cap(K\cup(NID_{\mathcal{M}_{\mathcal{V}}}(K)-\{h\})\neq\emptyset$. Thus, $V\cap(K-\{h\})\neq\emptyset$ or $V\cap\left(NID_{\mathcal{M}_{\mathcal{V}}}(K)-\{h\}\right)\neq\emptyset$. Also, by (iv) $V\cap(K-\{h\})\neq\emptyset$. Therefore, $h\in NID_{\mathcal{M}_{\mathcal{V}}}(K)$. Hence, $D_{\mathcal{M}_{\mathcal{V}}}(K\cup NID_{\mathcal{M}_{\mathcal{V}}}(K))\subseteq K\cup NID_{\mathcal{M}_{\mathcal{V}}}(K)$.

Theorem 3.17: Let H be any subset of a $NITS(\mathcal{U},\mathcal{N},\mathcal{I})$. Then $NIcl_{\mathcal{M}_{\mathcal{V}}}(H)=H\cup NID_{\mathcal{M}_{\mathcal{V}}}(H)$.

Proof: Since $NID_{\mathcal{M}_{\mathcal{V}}}(H)\subseteq NIcl_{\mathcal{M}_{\mathcal{V}}}(H)$. $H\cup NID_{\mathcal{M}_{\mathcal{V}}}(H)\subseteq NIcl_{\mathcal{M}_{\mathcal{V}}}(H)$. Let $h\in H$, then the proof completes. Suppose $h\notin H$, then $V\cap(H-\{h\})\neq\emptyset$ for every $NI\mathcal{M}_{\mathcal{V}}-open$ set V and $v\in V$. Therefore, $h\in NID_{\mathcal{M}_{\mathcal{V}}}(H)$.

Definition 3.18: In a $NITS(\mathcal{U},\mathcal{N},\mathcal{I})$, a subset W of $\mathcal{U}$ is said to be $NI\mathcal{M}_{\mathcal{V}}-neighborhood$ (briefly, $NI\mathcal{M}_{\mathcal{V}}-nbd$) of a point $u\in\mathcal{U}$ if there exists a $NI\mathcal{M}_{\mathcal{V}}-open$ set V such that $u\in V\subseteq W$. The set of all $NI\mathcal{M}_{\mathcal{V}}-nbd$ of a point $u\in\mathcal{U}$ is said to be $NI\mathcal{M}_{\mathcal{V}}-neighborhood$ Collection of u and it is denoted by $NI\mathcal{M}_{\mathcal{V}}-nbdC(u)$.

Theorem 3.19: In a $NITS(\mathcal{U},\mathcal{N},\mathcal{I})$, arbitrary union of family of $NI\mathcal{M}_{\mathcal{V}}-nbd$ of a point $u\in\mathcal{U}$ is a $NI\mathcal{M}_{\mathcal{V}}-nbd$ of u.

Proof: Let $\{H_\alpha:\ \alpha\in J\}$ be an arbitrary family of $NI\mathcal{M}_{\mathcal{V}}-nbd$ of $u\in\mathcal{U}$. Then, H_α is $NI\mathcal{M}_{\mathcal{V}}-nbd$ of u for every α. Therefore, there is a $NI\mathcal{M}_{\mathcal{V}}-open$ set V with $u\in V\subseteq H$. But for every $\alpha\in J$, $H_\alpha\subseteq\cup H_\alpha$. Therefore, $u\in W\subseteq\cup H_\alpha$. Hence, $\cup H_\alpha$ is a $NI\mathcal{M}_{\mathcal{V}}-nbd$ of u.

Theorem 3.20: In a $NITS(\mathcal{U},\mathcal{N},\mathcal{I})$, for any $w\in\mathcal{U}$, $NI\mathcal{M}_{\mathcal{V}}-nbdC(u)$ satisfies,

(i) $NI\mathcal{M}_{\mathcal{V}}-nbdC(w)\neq\emptyset$.

(ii) If a subset $H\in NI\mathcal{M}_{\mathcal{V}}-nbdC(w)$ then $w\in H$.

(iii) If $H\in NI\mathcal{M}_{\mathcal{V}}-nbdC(w)$ and $H\subseteq K$ then $K\in NI\mathcal{M}_{\mathcal{V}}-nbdC(w)$.

Proof: (i) For each $w\in W$ and V is $NI\mathcal{M}_{\mathcal{V}}-open$, $w\in W\subseteq W$. Hence, W is a $NI\mathcal{M}_{\mathcal{V}}-nbd(w)$ and so $W\in NI\mathcal{M}_{\mathcal{V}}-nbdC(w)$. Therefore, $NI\mathcal{M}_{\mathcal{V}}-nbdC(w)\neq\emptyset$.

(i) Since $H\in NI\mathcal{M}_{\mathcal{V}}-nbdC(w)$, H is a $NI\mathcal{M}_{\mathcal{V}}-nbdC(w)$. Then there exists a $NI\mathcal{M}_{\mathcal{V}}-open$ set V such that $w\in W\subseteq H$.

(ii) Since $H\in NI\mathcal{M}_{\mathcal{V}}-nbdC(w)$, there exists a $NI\mathcal{M}_{\mathcal{V}}-open$ set V such that $w\in W\subseteq H\subseteq K$. Hence, $K\in NI\mathcal{M}_{\mathcal{V}}-nbdC(w)$.

Theorem 3.21: Let the arbitrary union of family of $NI\mathcal{M}_{\mathcal{V}}-open$ sets of a $NITS(\mathcal{U},\mathcal{N},\mathcal{I})$ be $NI\mathcal{M}_{\mathcal{V}}-open$ and $K\subseteq\mathcal{U}$. Then, K is $NI\mathcal{M}_{\mathcal{V}}-open$ if and only if K is $NI\mathcal{M}_{\mathcal{V}}-nbd$ of each of its points.

Proof: Let K be any $NI\mathcal{M}_{\mathcal{V}}-open$ subset of $\mathcal{U}$ and $w\in K$. Then K is $NI\mathcal{M}_{\mathcal{V}}-nbd(w)$. Since w is arbitrary, H is $NI\mathcal{M}_{\mathcal{V}}-nbd$ for each $w\in K$.

Conversely, suppose K is $NI\mathcal{M}_{\mathcal{V}}-nbd$ for each $w\in K$. Then for each $w\in K$, there exists a $NI\mathcal{M}_{\mathcal{V}}-open$ set V_w such that $w\in V_w\subseteq K$. If $w\in K$ then there exists at least one $NI\mathcal{M}_{\mathcal{V}}-open$ set V_w such that $w\in V_w\subseteq\cup_{w\in K}V_w$. Hence, $K\subseteq\cup_{w\in K}V_w$. Also, $v\in\cup_{w\in K}V_w$ implies $v\in V_w$ for some $w\in K$. Then $v\in K$. Therefore,

$\bigcup_{w \in K} V_w \subseteq K$. Therefore, $K = \bigcup_{w \in K} V_w$. Also each V_w is a $NI\mathcal{M}_{\mathcal{V}} - open$ set implying K is a $NI\mathcal{M}_{\mathcal{V}} - open$ set.

Theorem 3.22: Let the arbitrary union of family of $NI\mathcal{M}_{\mathcal{V}} - open$ subsets of a $NITS(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be $NI\mathcal{M}_{\mathcal{V}} - open$. If H is $NI\mathcal{M}_{\mathcal{V}} - closed$ and $w \in H^c$ then there exists a $NI\mathcal{M}_{\mathcal{V}} - nbd$ V of w such that $V \cap H = \emptyset$.

Proof: Let H be a $NI\mathcal{M}_{\mathcal{V}} - closed$ set. Then by Theorem 3.21, H^c is $NI\mathcal{M}_{\mathcal{V}} - nbd$ of each of its points. Let $w \in H^c$. Then there exists a $NI\mathcal{M}_{\mathcal{V}} - open$ set V such that $w \in V \subseteq H^c$. Therefore, $V \cap H = \emptyset$.

Definition 3.23: In a $NITS(\mathcal{U}, \mathcal{N}, \mathcal{I})$, a point $u \in \mathcal{U}$ is said to be a $NI\mathcal{M}_{\mathcal{V}} - interior$ point of H if there exists a $NI\mathcal{M}_{\mathcal{V}} - open$ set $V \in V_N(u)$ such that $V \subseteq H$. The set of all $NI\mathcal{M}_{\mathcal{V}} - interior$ points of H is said to be $NI\mathcal{M}_{\mathcal{V}} - interior$ of H and is denoted by $NIint_{\mathcal{M}_{\mathcal{V}}}(H)$.

Theorem 3.24: For any subset K of a $NITS(\mathcal{U}, \mathcal{N}, \mathcal{I})$.

$NIint_{\mathcal{M}_{\mathcal{V}}}(K) = \bigcup\{V : V \text{ is } NI\mathcal{M}_{\mathcal{V}} - open \text{ and } V \subseteq K\}$.

Proof: Let $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(K)$. Then w is the $NI\mathcal{M}_{\mathcal{V}} - interior$ of K. Therefore, there exists a $NI\mathcal{M}_{\mathcal{V}} - open$ set V and $w \in V \subseteq K$. Hence $w \in \bigcup\{V : V \text{ is } NI\mathcal{M}_{\mathcal{V}} - open \text{ and } V \subseteq K\}$. Conversely, suppose $w \in \bigcup\{V : V \text{ is } NI\mathcal{M}_{\mathcal{V}} - open \text{ and } V \subseteq K\}$. Then, $w \in V$ and $V \subseteq K$ for some $NI\mathcal{M}_{\mathcal{V}} - open$ sets V. Hence w is the $NI\mathcal{M}_{\mathcal{V}} - interior$ of K.

Theorem 3.25: Let H and K be two subsets of a $NITS(\mathcal{U}, \mathcal{N}, \mathcal{I})$. Then,

(i) $NI\mathcal{M}_{\mathcal{V}} - interior$ of H is the largest $NI\mathcal{M}_{\mathcal{V}} - open$ subset in H.

(ii) H is $NI\mathcal{M}_{\mathcal{V}} - open$ if and only if $H = NIint_{\mathcal{M}_{\mathcal{V}}}(H)$.

(iii) $NIint_{\mathcal{M}_{\mathcal{V}}}(\emptyset) = \emptyset$ and $NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U}) = \mathcal{U}$.

(iv) If $H \subseteq K$ then $NIint_{\mathcal{M}_{\mathcal{V}}}(H) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(K)$.

(v) $NIint_{\mathcal{M}_{\mathcal{V}}}(H) \cup NIint_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(H \cup K)$.

(vi) $NIint_{\mathcal{M}_{\mathcal{V}}}(H \cap K) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(H) \cap NIint_{\mathcal{M}_{\mathcal{V}}}(K)$

(vii) $NIint_{\mathcal{M}_{\mathcal{V}}}\left(NIint_{\mathcal{M}_{\mathcal{V}}}(H)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}(H)$.

(viii) $NIint_{\mathcal{M}_{\mathcal{V}}}(H) = H - NID_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H)$.

(ix) $\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(H) = NIcl_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H)$.

Proof: (i) Let V be any $NI\mathcal{M}_{\mathcal{V}} - open$ subset of H and $w \in V$. Then $w \in V \subseteq H$. Since V is $NI\mathcal{M}_{\mathcal{V}} - open$, u is the $NI\mathcal{M}_{\mathcal{V}} - interior$ of H. Therefore, for $w \in V$, it implies that $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(H)$. Thus, every $NI\mathcal{M}_{\mathcal{V}} - open$ subset of H is contained in $NIint_{\mathcal{M}_{\mathcal{V}}}(H)$. Therefore, $NIint_{\mathcal{M}_{\mathcal{V}}}(H)$ is the largest $NI\mathcal{M}_{\mathcal{V}} - open$ subset of H.

(ii) Let H be any $NI\mathcal{M}_{\mathcal{V}} - open$ set. Since $H \subseteq H, H$ is the largest $NI\mathcal{M}_{\mathcal{V}} - open$ subset of H. Hence by (i) the proof completes.

(iii) $\emptyset$ and $\mathcal{U}$ are always $NI\mathcal{M}_{\mathcal{V}} - open$ sets. Therefore, by (ii), $NIint_{\mathcal{M}_{\mathcal{V}}}(\emptyset) = \emptyset$ and $NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U}) = \mathcal{U}$.

(iv) Let $H \subseteq K$. Then $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(H)$ implies that there exists a $NI\mathcal{M}_{\mathcal{V}} - open$ set V such that $w \in V \subseteq H$. Therefore, $w \in V \subseteq H \subseteq K$ that is, $w \in V \subseteq K$ and hence $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(K)$.

(v) Since $H \subseteq H \cup K$ and $K \subseteq H \cup K$, $NIint_{\mathcal{M}_{\mathcal{V}}}(H) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(H \cup K)$ and $NIint_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(H \cup K)$ by (iv), $NIint_{\mathcal{M}_{\mathcal{V}}}(H) \cup NIint_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(H \cup K)$.

(vi) By (iv) the proof is obvious.

(vii) The proof is obvious by (ii) and (i).

(viii) Let $w \in H - NID_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H)$. Then $w \in H$ and $w \notin NID_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H)$. Therefore, w is not a $NI\mathcal{M}_{\mathcal{V}} - limit\ point$ of $\mathcal{U} - H$. Therefore, there exists a $NI\mathcal{M}_{\mathcal{V}} - open$ set V that contains w but not the points of $\mathcal{U} - H$. Hence, $V \cap (\mathcal{U} - H) = \emptyset$ implies $V \subseteq H$. Therefore, $w \in V \subseteq H$ implies $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(H)$. On the other hand, let $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(H)$. Then $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(H) \subseteq H$. But $NIint_{\mathcal{M}_{\mathcal{V}}}(H)$ is the largest $NI\mathcal{M}_{\mathcal{V}} - open$ set and $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(H)$ and does not contain the points of $\mathcal{U} - H$. Therefore, w is not a $NI\mathcal{M}_{\mathcal{V}} - limit\ point$ of $\mathcal{U} - H$ implying $w \in H - NID_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H)$. Hence, $NIint_{\mathcal{M}_{\mathcal{V}}}(H) = H - NID_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H)$.

(ix) $\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(H) = \mathcal{U} - \left(H - NID_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H)\right) = \mathcal{U} - (H \cap (\mathcal{U} - NID_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H) = \mathcal{U} - H \cup NID_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H) = NIcl_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - H)$.

Theorem 3.26: Let M and T be two subsets of a $NITS(\mathcal{U}, \mathcal{N}, \mathcal{I})$. Then, $(i) NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - M) \subseteq \mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(M)$.

(ii) $NIint_{\mathcal{M}_{\mathcal{V}}}(M - T) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(M) - NIint_{\mathcal{M}_{\mathcal{V}}}(T)$.

Proof: (i) Let $w \in NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - M)$. Then $w \notin M$ and hence $w \notin NIint_{\mathcal{M}_{\mathcal{V}}}(M)$. Therefore, $w \in \mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(M)$.

(ii) $NIint_{\mathcal{M}_{\mathcal{V}}}(M - T) = NIint_{\mathcal{M}_{\mathcal{V}}}(M \cap (\mathcal{U} - T)) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(M) \cap NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - T) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(M) \cap (\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(T)) = NIint_{\mathcal{M}_{\mathcal{V}}}(M) - NIint_{\mathcal{M}_{\mathcal{V}}}(T)$ by (i).

4. $NI\mathcal{M}_{\mathcal{V}} - Exterior$ (resp.Border and Frontier)

Definition 4.1: Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $NITS$ and $K \subseteq \mathcal{U}$. Then,

(i) The $NI\mathcal{M}_{\mathcal{V}} - Exterior$ of K (simply, $NIExt_{\mathcal{M}_{\mathcal{V}}}(K)$) is defined as $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) = NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K)$.

(ii) The $NI\mathcal{M}_{\mathcal{V}} - Border$ of K (simply, $NIBr_{\mathcal{M}_{\mathcal{V}}}(K)$) is defined as $NIBr_{\mathcal{M}_{\mathcal{V}}}(K) = K - NIint_{\mathcal{M}_{\mathcal{V}}}(K)$.

(iii) The $NI\mathcal{M}_{\mathcal{V}} - Frontier$ of K (simply, $NIFr_{\mathcal{M}_{\mathcal{V}}}(K)$) is defined as $NIFr_{\mathcal{M}_{\mathcal{V}}}(K) = NIcl_{\mathcal{M}_{\mathcal{V}}}(K) - NIint_{\mathcal{M}_{\mathcal{V}}}(K)$.

Theorem 4.2: Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $NITS$ and $K, P \subseteq \mathcal{U}$. Then,

(i) $NIExt_{\mathcal{M}_{\mathcal{V}}}(\emptyset) = \mathcal{U}$ and $NIExt_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U}) = \emptyset$.

(ii) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K)$ is a $NI\mathcal{M}_{\mathcal{V}} - open$ set.

(iii) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) = \mathcal{U} - NIcl_{\mathcal{M}_{\mathcal{V}}}(K)$.

(iv) If $K \subseteq P$ then, $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq NIExt_{\mathcal{M}_{\mathcal{V}}}(P)$.

(v) $NIExt_{\mathcal{M}_{\mathcal{V}}}\left(NIExt_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}(NIcl_{\mathcal{M}_{\mathcal{V}}}(K))$.

(vi) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K \cup P) \subseteq NIExt_{\mathcal{M}_{\mathcal{V}}}(K) \cup NIExt_{\mathcal{M}_{\mathcal{V}}}(P)$.

(vii) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) \cap NIExt_{\mathcal{M}_{\mathcal{V}}}(P) \subseteq NIExt_{\mathcal{M}_{\mathcal{V}}}(K \cap P)$.

(viii) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) = NIExt_{\mathcal{M}_{\mathcal{V}}}\left(\mathcal{U} - NIExt_{\mathcal{M}_{\mathcal{V}}}(K)\right)$.

(ix) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq NIExt_{\mathcal{M}_{\mathcal{V}}}(NIExt_{\mathcal{M}_{\mathcal{V}}}(K))$.

(x) $K \cap NIExt_{\mathcal{M}_{\mathcal{V}}}(K) = \emptyset$.

(xi) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq \mathcal{U} - K$.

Proof: (i) By Definition, the proof completes.

(ii) Obvious.

(iii) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) = NIExt_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K) = \mathcal{U} - NIcl_{\mathcal{M}_{\mathcal{V}}}(K)$.

(iv) Since $K \subseteq P, NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - P) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K)$. Hence, by Definition the proof completes.

(v) $NIExt_{\mathcal{M}_{\mathcal{V}}}\left(NIExt_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIExt_{\mathcal{M}_{\mathcal{V}}}\left(NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K)\right) = NIExt_{\mathcal{M}_{\mathcal{V}}}\left(\mathcal{U} - NIcl_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}\left(\mathcal{U} - (\mathcal{U} - NIcl_{\mathcal{M}_{\mathcal{V}}}(K))\right) = NIint_{\mathcal{M}_{\mathcal{V}}}(NIcl_{\mathcal{M}_{\mathcal{V}}}(K))$.

(vi) The Proof is clear by (iv).

(vii) The Proof completes by (iv).

(viii) $NIExt_{\mathcal{M}_{\mathcal{V}}}\left(\mathcal{U} - NIExt_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}\left(\mathcal{U} - \left(\mathcal{U} - NIExt_{\mathcal{M}_{\mathcal{V}}}(K)\right)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}\left(\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K) = NIExt_{\mathcal{M}_{\mathcal{V}}}(K)$.

(ix) Since $K \subseteq NIcl_{\mathcal{M}_{\mathcal{V}}}(K)$, $NIint_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}\left(NIcl_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}\left(\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K)\right) = NIExt_{\mathcal{M}_{\mathcal{V}}}\left(NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K)\right) = NIExt_{\mathcal{M}_{\mathcal{V}}}(NIExt_{\mathcal{M}_{\mathcal{V}}}(K))$.

(x) $K \cap NIExt_{\mathcal{M}_{\mathcal{V}}}(K) = K \cap NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K) \subseteq K \cap (\mathcal{U} - K) = \emptyset$.

(xi) $NIExt_{\mathcal{M}_{\mathcal{V}}}(K) = NIint_{\mathcal{M}_{\mathcal{V}}}(\mathcal{U} - K) \subseteq \mathcal{U} - K$.

Theorem 4.3: Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $NITS$ and $K \subseteq \mathcal{U}$. Then K is $NI\mathcal{M}_{\mathcal{V}} - open$ if and only if $NIBr_{\mathcal{M}_{\mathcal{V}}}(K) = \emptyset$.

Proof: Let K be a $NI\mathcal{M}_{\mathcal{V}} - open$ set. Then $NIint_{\mathcal{M}_{\mathcal{V}}}(K) = K$. Hence, $NIBr_{\mathcal{M}_{\mathcal{V}}}(K) = \emptyset$. Conversely, suppose $NIBr_{\mathcal{M}_{\mathcal{V}}}(K) = \emptyset$. Then, $K - NIint_{\mathcal{M}_{\mathcal{V}}}(K) = \emptyset$.

Theorem 4.4: Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $NITS$ and $K, P \subseteq \mathcal{U}$. Then the following statements hold true.

(i) $NIBr_{\mathcal{M}_{\mathcal{V}}}(NIint_{\mathcal{M}_{\mathcal{V}}}(K)) = \emptyset$.

(ii) $NIint_{\mathcal{M}_{\mathcal{V}}}\left(NIBr_{\mathcal{M}_{\mathcal{V}}}(K)\right) = \emptyset$.

(iii) $NIBr_{\mathcal{M}_{\mathcal{V}}}\left(NIBr_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIBr_{\mathcal{M}_{\mathcal{V}}}(K)$.

(iv) $NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cap NIBr_{\mathcal{M}_{\mathcal{V}}}(K) = \emptyset$.

Proof: (i) $NIBr_{\mathcal{M}_{\mathcal{V}}}\left(NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}(K) - NIint_{\mathcal{M}_{\mathcal{V}}}(NIint_{\mathcal{M}_{\mathcal{V}}}(K)) = NIint_{\mathcal{M}_{\mathcal{V}}}(K) - NIint_{\mathcal{M}_{\mathcal{V}}}(K) = \emptyset$.

(ii) $NIint_{\mathcal{M}_{\mathcal{V}}}\left(NIBr_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}\left(K - NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIint_{\mathcal{M}_{\mathcal{V}}}\left(K \cap \left(\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right)\right) \subseteq NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cap NIint_{\mathcal{M}_{\mathcal{V}}}\left(\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right) = \emptyset$, since, $NIint_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq K$.

(iii) $NIBr_{\mathcal{M}_{\mathcal{V}}}\left(NIBr_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIBr_{\mathcal{M}_{\mathcal{V}}}(K) - NIint_{\mathcal{M}_{\mathcal{V}}}\left(NIBr_{\mathcal{M}_{\mathcal{V}}}(K)\right) = NIBr_{\mathcal{M}_{\mathcal{V}}}(K)$.

(iv) $NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cap NIBr_{\mathcal{M}_{\mathcal{V}}}(K) = NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cap (K - NIint_{\mathcal{M}_{\mathcal{V}}}(K)) = NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cap (K \cap (\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(K))) = NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cap \emptyset = \emptyset$.

Theorem 4.5: Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $NITS$ and $K, P \subseteq \mathcal{U}$. Then,

(i) $NIcl_{\mathcal{M}_{\mathcal{V}}}(K) = NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cup NIFr_{\mathcal{M}_{\mathcal{V}}}(K)$.

(ii) $NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cap NIFr_{\mathcal{M}_{\mathcal{V}}}(K) = \emptyset$.

(iii) $NIFr_{\mathcal{M}_{\mathcal{V}}}\left(NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right) \subseteq NIFr_{\mathcal{M}_{\mathcal{V}}}(K)$.

(iv) $NIBr_{\mathcal{M}_{\mathcal{V}}}(K) \subseteq NIFr_{\mathcal{M}_{\mathcal{V}}}(K)$.

Proof: (i) $NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cup NIFr_{\mathcal{M}_{\mathcal{V}}}(K) = NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cup (NIcl_{\mathcal{M}_{\mathcal{V}}}(K) - NIint_{\mathcal{M}_{\mathcal{V}}}(K)) = NIcl_{\mathcal{M}_{\mathcal{V}}}(K)$.

(ii) $NIcl_{\mathcal{M}_{\mathcal{V}}}(K) \cap NIFr_{\mathcal{M}_{\mathcal{V}}}(K) = NIint_{\mathcal{M}_{\mathcal{V}}}(K) \cap (NIcl_{\mathcal{M}_{\mathcal{V}}}(K) - NIint_{\mathcal{M}_{\mathcal{V}}}(K)) = \emptyset$.

(iii) $NIFr_{\mathcal{M}_{\mathcal{V}}}\left(NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right) \subseteq NIcl_{\mathcal{M}_{\mathcal{V}}}\left(NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right) - NIint_{\mathcal{M}_{\mathcal{V}}}\left(NIint_{\mathcal{M}_{\mathcal{V}}}(K)\right) \subseteq NIcl_{\mathcal{M}_{\mathcal{V}}}(K) - NIint_{\mathcal{M}_{\mathcal{V}}}(K) = NIFr_{\mathcal{M}_{\mathcal{V}}}(K)$.

(iv) $NIBr_{\mathcal{M}_{\mathcal{V}}}(K) = K - NIint_{\mathcal{M}_{\mathcal{V}}}(K) = K \cap (\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(K)) \subseteq NIcl_{\mathcal{M}_{\mathcal{V}}}(K) \cap (\mathcal{U} - NIint_{\mathcal{M}_{\mathcal{V}}}(K)) = NIcl_{\mathcal{M}_{\mathcal{V}}}(K) - NIint_{\mathcal{M}_{\mathcal{V}}}(K) = NIFr_{\mathcal{M}_{\mathcal{V}}}(K)$.

5. Conclusions

This study focuses on demonstrating a strong type of a $nano\ ideal\ set$ in $nano\ ideal$ topological spaces. The inclusive relationship between the various types of nano ideal open sets is defined. Also we have investigated the characterizations of interior, closure and exterior of this strong variant nano ideal set. Further for future study, these results may be extended to $NI\mathcal{M}_{\mathcal{V}} - continuous\ functions$ [10], $NI\mathcal{M}_{\mathcal{V}} - connected$ spaces [11] and $NI\mathcal{M}_{\mathcal{V}} - compact\ spaces$ [12] in nano ideal topological spaces. Also it leads to certain new results of $NI\mathcal{M}_{\mathcal{V}} - open(closed)$ mappings and $NI\mathcal{M}_{\mathcal{V}} - separation\ axioms$ in nano ideal topological spaces.

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