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Three-way Analysis of Variance for Functional Data

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Abstract As the number of observation points examined in studies increases, it is assumed that these points come from an underlying real function. Statistical methods that have been developed for this type of data are called functional data analysis. Therefore, there has been a great interest in adapting some traditional statistical methods into the functional data sets like hypothesis testing, regression or analysis of variance (ANOVA). The analysis of variance (ANOVA) is used for comparing the means of more than three groups. Therefore, If the data comes as a functional data form, the functional ANOVA methodologies must be used to compare the mean function of the groups. In literature, one-way ANOVA and two-way ANOVA methodologies have been developed for functional data set. In this paper, we expand the functional one-way and two-way analysis of variance methodologies to three-way analysis of variance methodology or so-called factorial experimental design. We obtain the parameter estimations of the cell mean function and common variance-covariance function. We also propose sum of squares and test statistics by using these estimations. The test statistics and sum of squares are obtained via pointwise test methodology which has been used for ANOVA for functional data sets. A real data set about the measure of depression levels of health employees of Turkey during the Covid-19 pandemic is conducted for the illustration. The data set is collected between January 2021 and December 2021 for monthly and we have three factors that affect the depression levels of health workers as gender, profession and age. Therefore, in order to test the effects of the treatments, we use the proposed methodology for three-way ANOVA.

Keywords Functional Data Analysis, Three-way ANOVA, Pointwise Test

1 Introduction

One-way ANOVA test or one-way analysis of variance is a process used to test whether there is a statistically significant difference between the means of independent treatments. An experiment is with two or more independent variables. If it contains a factorial pattern, this is called a factorial pattern. If we have three independent variabales, the factorial design is called three-way ANOVA. However, all analysis is proposed based on one dimensional data.

Functional data are the conceptualization of multivariate data from finite and infinite dimensional space. In practice, functional data are obtained by observing a number of subjects over time, space, or other continua. In functional data analysis (FDA), such data are considered as random functions so-called functional data. The first step of FDA is converting the data observed at discrete points into continuous functions. The statistical analysis that has been developed to analyze functional data were first introduced by Ramsay and Dalzell [15]. In literature, a wide scope is given in [16], [17], [21] and [6].

FDA has also a strong link between longitudinal data analysis (LDA). LDA and FDA techniques are also intrinsically different. On the other hand, classical multivariate data analysis (MDA) tools can be applied directly. However, MDA treatment may encounter difficulties in many other situations. For example, the sampling time points of the observed functional data are not the same across various subjects, the sampling time points are not equally spaced and the number of sampling time points is larger than the number of subjects in a sample of functional data.

Many traditional statistical methods have been applied for functional data. These include, for example, canonical correlation analysis [13], [12], clustering analysis [19], [20], discriminant analysis [11], [14], [9], principal component analysis [3], [2], logistic regression [18], general linear models [10], regression [8], [4], [1], [5] and the analysis of variance (ANOVA) [8],

[21]. To the best of our knowledge, there is no previous work dealing with three-way ANOVA for functional data.

Consider the following three-way functional ANOVA model,

$$y_{ijkl} = \mu_0(t) + \alpha_i(t) + \beta_j(t) + \theta_k(t) + \alpha\beta_{ij}(t) + \alpha\theta_{ik}(t) + \beta\theta_{jk}(t) + \alpha\beta\theta_{ijk}(t) + \epsilon_{ijkl}(t), \quad t \in T, i = 1,2,...,a; j = 1,2,...,b; k = 1,2,...,c; l = 1,2,...,n_{ijk} \tag{1}$$

where $\mu_0(t)$ is the mean function, $\alpha_i(t)$, $\beta_j(t)$ and $\theta_k(t)$ are the i^{th}, j^{th} and k^{th} main-effect functions of factors A, B and C, respectively. $\alpha\beta_{ij}(t)$, $\alpha\theta_{ik}(t)$ and $\beta\theta_{jk}(t)$ are the interaction effect functions between the factors (A,B), (A,C) and (B,C). $\alpha\beta\theta_{ijk}(t)$ is the $(i,j,k)^{th}$ interaction effect function between the factors A, B and C.

$$\epsilon_{ijkl}(t) \sim (iid)SP(0,\gamma) \quad t \in T, i = 1,2,...,a; j = 1,2,...,b; k = 1,2,...,c; l = 1,2,...,n_{ijk} \tag{2}$$

where $\gamma(s,t)$ is the common covariance function for all the samples. The model can be further written as the following form

$$y_{ijkl} = \eta_{ijk}(t) + \epsilon_{ijkl}(t), \quad t \in T, i = 1,2,...,a; j = 1,2,...,b; k = 1,2,...,c; l = 1,2,...,n_{ijk} \tag{3}$$

where $\eta_{ijk}(t)$ is called cell function and obtained by

$$\eta_{ijk}(t) = \mu_0(t) + \alpha_i(t) + \beta_j(t) + \theta_k(t) + \alpha\beta_{ij}(t) + \alpha\theta_{ik}(t) + \beta\theta_{jk}(t) + \alpha\beta\theta_{ijk}(t). \tag{4}$$

The rest of the paper is organized as follows. In the next section, the methodology of three-way ANOVA for functional data is evaluated. The estimation of parameters, sum of squares and the test statistics are obtained. In Section 3, the real-life data is given for illustration of the proposed methodology. Conclusion is given at the end of the paper.

2 Methodology

Consider the ANOVA model given in equation (1), For this model, the following null hypotheses are tested:

$$\begin{aligned} H_{0A} &: \alpha_i(t) = 0; \\ H_{0B} &: \beta_j(t) = 0; \\ H_{0C} &: \theta_i(t) = 0; \\ H_{0AB} &: \alpha\beta_{ij}(t) = 0; \\ H_{0AC} &: \alpha\theta_{ik}(t) = 0; \\ H_{0BC} &: \beta\theta_{jk}(t) = 0; \\ H_{0ABC} &: \alpha\beta\theta_{ijk}(t) = 0. \end{aligned} \tag{5}$$

where $= 1,2,...,a; j = 1,2,...,b$ and $k = 1,2,...,c$.

The first three null hypothesis aim to test if the main-effects of Factor A, B and C are significant. The latter three one aims to test if there is an interaction-effect between the two factors, and the last hypothesis aims to test if there is an interaction-effect among all factors.

For three-way ANOVA, the cell mean functions $\eta_{ijk}(t),\ i = 1,2,...,a;\ j = 1,2,...,b;\ k = 1,2,...,c$ can be defined provided $\eta_{ijk} > 1, ; i = 1,2,...,a;\ j = 1,2,...,b;\ k = 1,2,...,c$. Their unbiased estimators of the cell functions are given by

$$\hat{\eta_{ijk}}(t) = \bar{y_{ijk.}}(t) = n_{ijk}^{-1} \sum_{k=1}^{n_{ijk}} y_{ijkl}(t), \quad i = 1,2,...,a, \quad j = 1,2,...,b, k = 1,2,...,c. \tag{6}$$

Therefore, the coomon covariance function can be estimated unbiasedly by the following pooled sample covariance function

$$\hat{\gamma(s,t)} = (n - abc)^{-1} \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n_{ijk}} \Big[y_{ijkl}(s) - \bar{y_{ijk.}}(s)\Big]\Big[y_{ijkl}(t) - \bar{y_{ijk.}}(t)\Big] \tag{7}$$

where $n = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} n_{ijk.}$. As all the abc cell samples are independent and have the common covariance function $\gamma(s,t)$, we have

$$\hat{\eta(t)} \sim SP(\eta, \gamma D), \tag{8}$$

where $D = diag(1/n_{111}, 1/n_{112}, ..., 1/n_{abc})$. Therefore, under the following assumptions:

- $\eta_{ijk}(t) \in \mathcal{L}^2(\mathcal{T}), i = 1,2,...,a,\ j = 1,2,...,b,\ k = 1,2,...,c$ and $tr(\gamma) < \infty$
- The abc samples are distributed normally.

We have the following outcomes:

$$\begin{aligned} D^{-1/2}[\hat{\eta(t)} - \eta(t)] &\sim NP(0, \gamma I_{abc}) \\ (n - abc)\hat{\gamma}(s,t) &\sim WP(n - ab, \gamma) \end{aligned} \tag{9}$$

where NP means normal process and WP is Wishart process.

It can be noticed that the three-way ANOVA model given in (1) is not identifiable since the functional parameters are not uniquely defined. For this reason, the following side constraints are strained with positive weights $w_{ijk}, i = 1,2,...,a,\ j = 1,2,...,b,\ k = 1,2,...,c$:

$$\sum_{i=1}^{a} w_{i..}\alpha_i(t) = 0, \sum_{j=1}^{b} w_{.j.}\beta_j(t) = 0, \quad (10)$$
$$\sum_{k=1}^{c} w_{..k} = 0\theta_k(t), \sum_{i=1}^{a}\sum_{j=1}^{b} w_{ij.}\alpha\beta_{ij}(t) = 0,$$
$$\sum_{i=1}^{a}\sum_{k=1}^{c} w_{i.k}\alpha\theta_{ik}(t) = 0, \sum_{j=1}^{b}\sum_{k=1}^{c} w_{.jk}\beta\theta_{jk}(t) = 0,$$
$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} w_{ijk}\alpha\beta\theta_{ijk}(t) = 0$$

where

$$w_{i..} = \sum_{j=1}^{b}\sum_{k=1}^{c} w_{ijk}, w_{.j.} = \sum_{i=1}^{a}\sum_{k=1}^{c} w_{ijk}, \quad (11)$$
$$w_{..k} = \sum_{i=1}^{a}\sum_{j=1}^{b} w_{ijk}, w_{ij.} = \sum_{k=1}^{c} w_{ijk},$$
$$w_{i.k} = \sum_{j=1}^{b} w_{ijk}, w_{.jk} = \sum_{i=1}^{a} w_{ijk}]$$

and w_{ijk} can be written as

$$w_{ijk} = g_i h_j v_t, i = 1, 2, ..., a; j = 1, 2, ..., b; \quad (12)$$
$$k = 1, 2, ..., c; l = 1, 2, ..., n_{ijk}$$

such that $g_i > 0, \sum g_i = 1$, $h_j > 0, \sum h_j = 1$ and $v_k > 0, \sum v_k = 1$, therefore, we can easily identify the parameters as

$$\eta_0(t) = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} g_i h_j v_k \eta_{ijk}(t), \quad (13)$$
$$\alpha_i(t) = \sum_{j=1}^{b}\sum_{k=1}^{c} h_j v_k \eta_{ijk}(t) - \eta_0(t),$$
$$\beta_j(t) = \sum_{i=1}^{a}\sum_{k=1}^{c} g_i v_k \eta_{ijk}(t) - \eta_0(t),$$
$$\theta_k(t) = \sum_{i=1}^{a}\sum_{j=1}^{b} g_i h_j \eta_{ijk}(t) - \eta_0(t),$$
$$\alpha\beta_{ij}(t) = \sum_{k=1}^{c} v_k \eta_{ijk}(t) - \eta_0(t),$$
$$\alpha\theta_{ik}(t) = \sum_{j=1}^{b} h_j \eta_{ijk}(t) - \eta_0(t),$$
$$\beta\theta_{jk}(t) = \sum_{i=1}^{a} g_i \eta_{ijk}(t) - \eta_0(t),$$

and

$$\alpha\beta\theta_{ijk}(t) = \eta_{ijk}(t) - \alpha_i(t) - \beta_j(t) - \theta_k(t) - \alpha\beta_{ij}(t) \quad (14)$$
$$-\alpha\theta_{ik}(t) - \beta\theta_{jk}(t) - \eta_0(t).$$

where

$i = 1, 2, ..., a; j = 1, 2, ..., b; g_i = 1/a,\ h_j = 1/b$ and $v_k = 1/c$

or

$$g_i = \sum_{j=1}^{b}\sum_{k=1}^{c} n_{ijk}/n,\ h_j = \sum_{i=1}^{a}\sum_{k=1}^{c} n_{ijk}/n \quad (15)$$
$$\text{and } v_k = \sum_{i=1}^{a}\sum_{j=1}^{b} n_{ijk}/n$$

Let $\mathbf{g}, \mathbf{h}$ and $\mathbf{v}$ be the vector representations and $\mathbf{e}_{r,p}$ be p-dimensional unit vector whose rth component is 1 and others are zero, then we have

$$\eta_0(t) = \left[\mathbf{g}^T \otimes \mathbf{h}^T \otimes \mathbf{v}^T\right]\eta(t), \quad (16)$$
$$\alpha_i(t) = \left[(\mathbf{e}_{i,a} - \mathbf{g})^T \otimes \mathbf{h}^T \otimes \mathbf{v}^T\right]\eta(t)$$
$$\beta_j(t) = \left[\mathbf{g}^T \otimes (\mathbf{e}_{i,b} - \mathbf{h})^T \otimes \mathbf{v}^T\right]\eta(t),$$
$$\theta_k(t) = \left[\mathbf{g}^T \otimes \mathbf{h}^T \otimes (\mathbf{e}_{i,c} - \mathbf{v})^T\right]\eta(t)$$
$$\alpha\beta_{ij}(t) = \left[(\mathbf{e}_{i,a} - \mathbf{g})^T \otimes (\mathbf{e}_{i,b} - \mathbf{h})^T \otimes \mathbf{v}^T\right]\eta(t),$$
$$\alpha\theta_{ik}(t) = \left[(\mathbf{e}_{i,a} - \mathbf{g})^T \otimes \mathbf{h}^T \otimes (\mathbf{e}_{i,c} - \mathbf{v})^T\right]\eta(t)$$
$$\beta\theta_{jk}(t) = \left[\mathbf{g}^T \otimes (\mathbf{e}_{i,b} - \mathbf{h})^T \otimes (\mathbf{e}_{i,c} - \mathbf{v})^T\right]\eta(t)$$

and

$$\alpha\beta\theta_{ijk}(t) = \left[(\mathbf{e}_{i,a} - \mathbf{g})^T \otimes (\mathbf{e}_{i,b} - \mathbf{h})^T \otimes (\mathbf{e}_{i,c} - \mathbf{v})^T\right]\eta(t) \quad (17)$$

Here, $\otimes$ means tensor product. By using these equations, the following matrices can be defined

$$A_a = [\mathbf{I}_a - \mathbf{1}_a\mathbf{g}^T] \otimes \mathbf{h}^T \otimes \mathbf{v}^T, \quad (18)$$
$$A_b = \mathbf{g}^T \otimes [\mathbf{I}_b - \mathbf{1}_b\mathbf{h}^T] \otimes \mathbf{v}^T,$$
$$A_c = \mathbf{g}^T \otimes \mathbf{h}^T \otimes [\mathbf{I}_c - \mathbf{1}_c\mathbf{v}^T],$$
$$A_{ab} = [\mathbf{I}_a - \mathbf{1}_a\mathbf{g}^T] \otimes [\mathbf{I}_b - \mathbf{1}_b\mathbf{h}^T] \otimes \mathbf{v}^T$$
$$A_{ac} = [\mathbf{I}_a - \mathbf{1}_a\mathbf{g}^T] \otimes \mathbf{h}^T \otimes [\mathbf{I}_c - \mathbf{1}_c\mathbf{v}^T]$$
$$A_{bc} = \mathbf{g}^T \otimes [\mathbf{I}_b - \mathbf{1}_b\mathbf{h}^T] \otimes [\mathbf{I}_c - \mathbf{1}_c\mathbf{v}^T]$$

and

$$A_{abc} = [\mathbf{I}_a - \mathbf{1}_a\mathbf{g}^T] \otimes [\mathbf{I}_b - \mathbf{1}_b\mathbf{h}^T] \otimes [\mathbf{I}_c - \mathbf{1}_c\mathbf{v}^T]$$

It can be noticed that $A_a, A_b, A_c, A_{ab}, A_{ac}, A_{bc}$ and A_{abc} are not full-rank matrices and have $(a-1), (b-1), (c-1), (a-1)(b-1), (a-1)(c-1), (b-1)(c-1)$ and $(a-1)(b-1)(c-1)$ ranks respectively. Define the following matrices as

$$\begin{aligned} \mathbf{H}_a = (\mathbf{I}_{a-1}, -\mathbf{1}_{a-1}),\ \mathbf{H}_b = (\mathbf{I}_{b-1}, -\mathbf{1}_{b-1}), \\ \mathbf{H}_c = (\mathbf{I}_{c-1}, -\mathbf{1}_{c-1}), \\ \mathbf{H}_{ab} = \mathbf{H}_a \otimes \mathbf{H}_b, \\ \mathbf{H}_{ac} = \mathbf{H}_a \otimes \mathbf{H}_c,\ \mathbf{H}_{bc} = \mathbf{H}_b \otimes \mathbf{H}_c,\ \text{and} \\ \mathbf{H}_{abc} = \mathbf{H}_a \otimes \mathbf{H}_b \otimes \mathbf{H}_c \end{aligned} \tag{19}$$

and

$$\begin{aligned} \mathbf{S}_a = \mathbf{H}_a\mathbf{A}_a,\ \mathbf{S}_b = \mathbf{H}_b\mathbf{A}_b,\ \mathbf{S}_c = \mathbf{H}_c\mathbf{A}_c, \\ \mathbf{S}_{ab} = \mathbf{H}_{ab}\mathbf{A}_{ab},\ \mathbf{S}_{ac} = \mathbf{H}_{ac}\mathbf{A}_{ac}, \\ \mathbf{S}_{bc} = \mathbf{H}_{bc}\mathbf{A}_{bc},\ \text{and}\ \mathbf{S}_{abc} = \mathbf{H}_{abc}\mathbf{A}_{abc}, \end{aligned} \tag{20}$$

where $\mathbf{S}$ becomes a full-rank matrix with q. Therefore, the sum of squares of the corresponding null hypothesis and the sum of squares are obtained as

$$SSH_n(t) = [\mathbf{S}\hat{\eta}(t)]^T(\mathbf{S}\mathbf{D}\mathbf{S}^T)^{-1}[\mathbf{S}\hat{\eta}(t)] \tag{21}$$

and

$$\begin{aligned} SSE_n(t) = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n_{ijk}}\left[y_{ijkl}(t) - \bar{y_{ijk\cdot}}(t)\right]^2 \\ = (n-abc)\hat{\gamma}(t,t). \end{aligned} \tag{22}$$

Using pointwise testing method, we define the following test statistics for each null hypothesis as,

$$F_n(t) = \frac{SSH_n(t)/q}{SSE_n(t)/(n-abc)} \tag{23}$$

It can be noticed that the F statistic is obtained by using Pointwise test methodology under the normality assumption. If the normality assumption is not satisfied, $\mathcal{L}^2$-norm-based test, Welch-Satterthwaite χ^2 approximation method or F-Type Test can be used.

3 Application

The hospital anxiety and depression scale were proposed by Zigmond and Snaith [22]. The scale is used to determine the risk of anxiety and depression in individuals and to measure its level and change in severity. It is a self-report tool which the individual evaluates himself. The scale consists of 14 items. 7 of these items are anxiety, the other 7 measure depression symptoms. Items are in the scale, 4-point. It is evaluated with a Likert scale and has a scoring system between 0-3. According to the scoring, 0-1 is non-patient, 2 borderline patients, 2-3 are considered as severely ill. Also, from the scale, it is seen that the scores are not affected by physical diseases.

In this application, the scale is applied to 43 health employees in Turkey between January 2021 and December 2021 for monthly. The main purpose of the project is to determine the anxiety and depression levels of health employees during Covid-19 pandemic. Every month, the same employees answer the same scale. For this reason, the variations in the anxiety and the depression levels can be determined during the pandemic according to the factor levels. In this paper, we use only the anxiety point of the employees. Figure 1 shows the data in functional form.

In this data, there are three treatments: the gender (23 female, 20 male), the profession (20 doctor, 23 nurse) and the age (13 25<, 16 25-40 and 14 40>). There are two levels of gender, two levels of the profession and three levels of the age also this data have 12 different time of measurement. Our motivation is to determine is there any significant difference between the treatment in different time points. Figure 2 shows the sample cell mean functions

Figure 3 shows the pooled sample covariance function obtained by using Equation (7).

By using w_{ijk} values obtained by using $g_i = 1/2,\ h_j = 1/2,$ and $v_k = 1/3$, we obtained the following estimations of the unknown parameters as given in Table 1. We give only the first estimations for the sake of brevity.

Therefore, the following hypothesis is constructed.

$$\begin{aligned} H_{0A} : \alpha_i(t) = 0; \\ H_{0B} : \beta_j(t) = 0; \\ H_{0C} : \theta_i(t) = 0; \\ H_{0AB} : \alpha\beta_{ij}(t) = 0; \\ H_{0AC} : \alpha\theta_{ik}(t) = 0; \\ H_{0BC} : \beta\theta_{jk}(t) = 0; \\ H_{0ABC} : \alpha\beta\theta_{ijk}(t) = 0. \end{aligned} \tag{24}$$

where $i = 1, 2; j = 1, 2; k = 1, 2, 3$ and $t \in [1, 12]$.

By using the matrices defined by Equations (20), (21) and (23), we obtain $SSH_n(t)$ and $SSE_n(t)$ values. Finally, we reach the test statistics defined in Equation (25). Figures 4-10 show the p-value of the test statistics according to the corresponding hypothesis testing. It can be noticed from Figures 4-10, the p-values for testing gender and the profession affects are greater than 0.05 for all t values, therefore, we can conclude that there is no significant difference in terms of anxieties between the genders and the professions during the pandemic. On the other hand, while comparing the anxieties among the age groups, it can be seen that at time 5 to 8, the p-values are less than 0.05. It can be concluded that there is a significant difference between the age groups in terms of anxiety points. These can be resulted by the fact that; as the vaccination numbers increase and the time continues, the younger age groups have less anxiety about Covid 19. After the time point 9, again the p values are greater than 0.05. In the interaction effects, all interaction effects except gender & age interaction are not significant for all time points. The interaction affect between age and gender are significant between the time points 1 to 10.

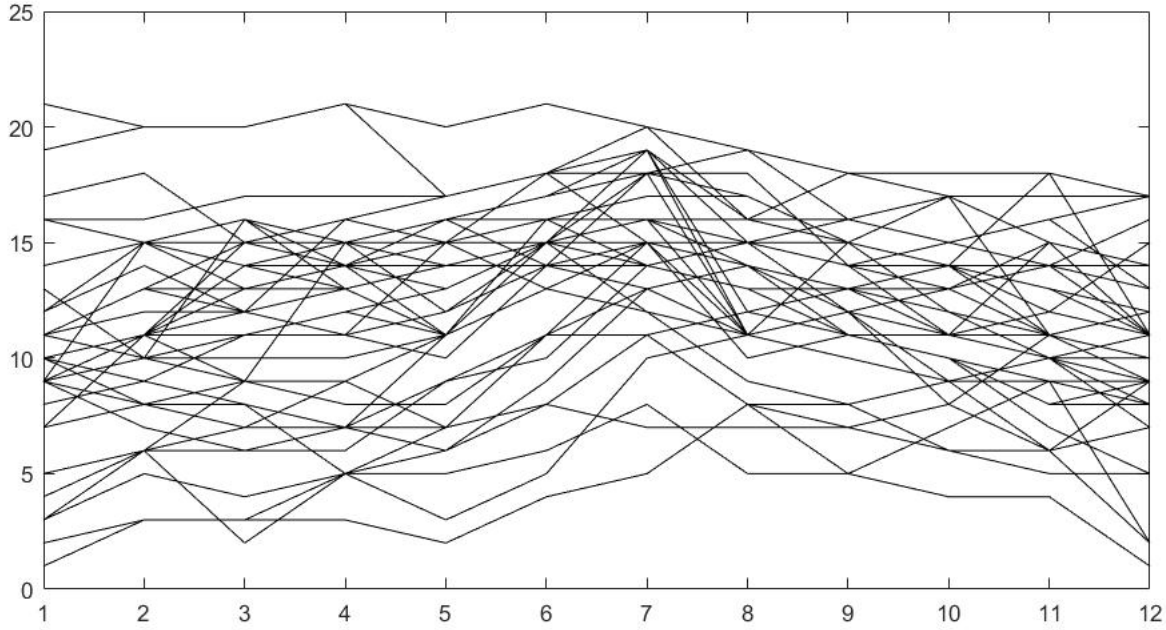

Figure 1. The anxiety data

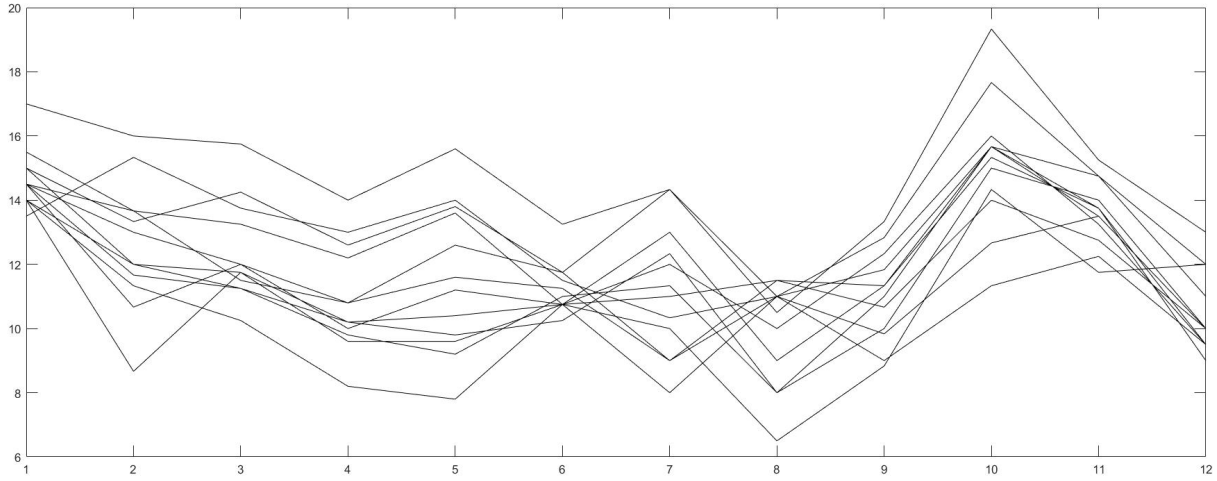

Figure 2. The sample cell mean functions

Table 1. The estimation of the unknown model parameters

t	1	2	3	4	5	6	7	8	9	10	11	12
$\alpha_1(t)$	-0,090	-0,007	-0,215	-0,193	-0,136	-0,154	0,487	0,542	0,497	0,575	0,401	0,274
$\beta_1(t)$	-0,326	-0,118	-0,035	0,138	0,261	-0,151	-0,293	-0,347	0,044	-0,089	-0,235	-0,051
$\theta_1(t)$	1,154	1,401	1,682	1,424	1,081	1,290	1,388	0,153	0,928	0,181	0,340	0,279
$\alpha\beta_{11}(t)$	1,382	1,368	0,979	0,874	1,122	0,585	1,471	1,222	1,475	1,328	0,988	1,018
$\alpha\theta_{11}(t)$	0,621	1,018	0,857	0,390	0,206	0,190	0,721	0,278	1,186	0,756	1,249	1,346
$\beta\theta_{11}(t)$	1,521	1,785	1,924	1,790	1,206	1,057	0,888	-1,056	0,586	-0,144	0,349	0,546
$\alpha\beta\theta_{111}(t)$	-0,740	-1,829	-2,185	-2,132	-1,283	-1,426	-2,440	-0,264	-1,881	0,000	0,007	0,135

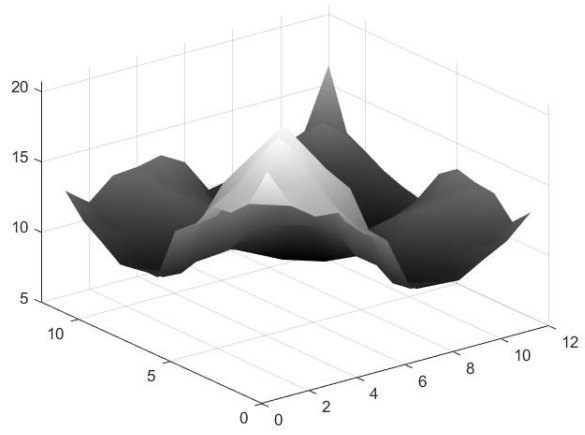

Figure 3. The pooled sample covariance function

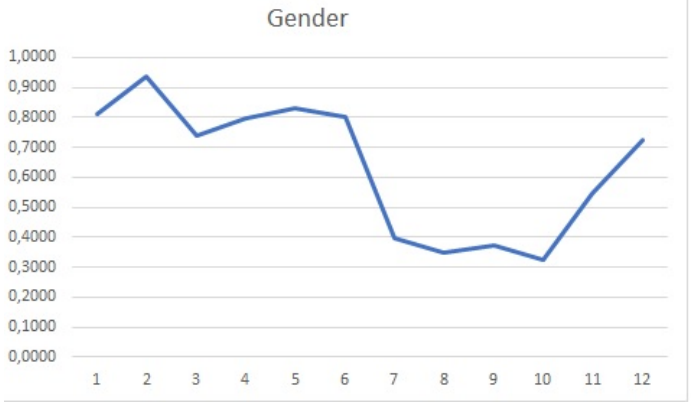

Figure 4. The p-values of the test statistics (Gender)

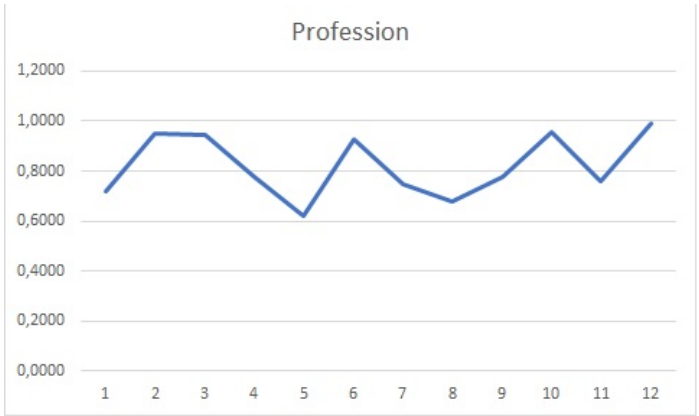

Figure 5. The p-values of the test statistics (Profession)

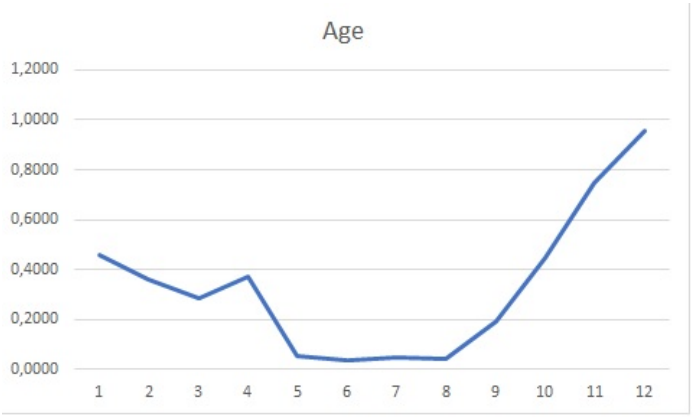

Figure 6. The p-values of the test statistics (Age)

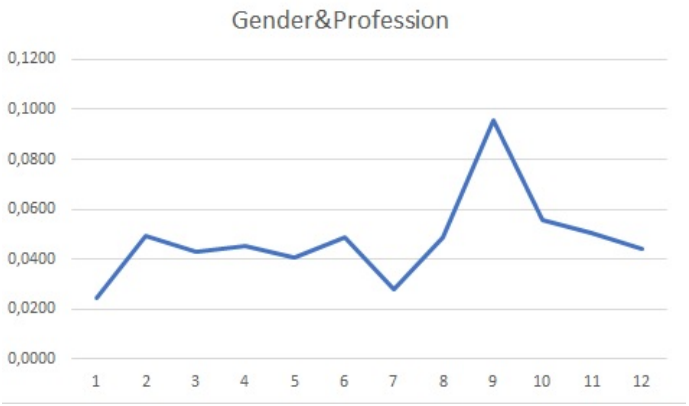

Figure 7. The p-values of the test statistics (Gender and Profession)

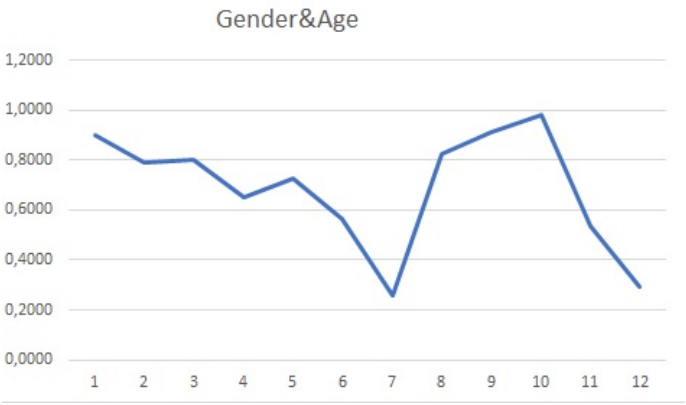

Figure 8. The p-values of the test statistics (Gender and Age)

Turkey during Covid-19 pandemic.

4 Conclusion

In this paper, we extend the methodology for one-way ANOVA to three-way ANOVA for functional data. We obtain the estimators of the unknown model parameters, the sum of squares and the F test statistics for main effects and the interaction effects. We illustrate the methodology by real life data analysis dealing with the anxieties of health employees in

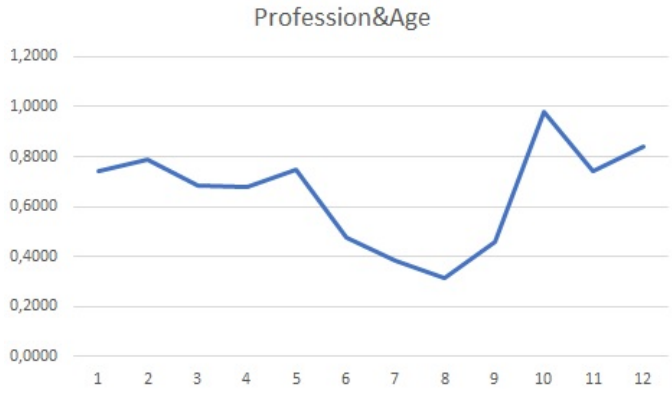

Figure 9. The p-values of the test statistics (Profession and Age)

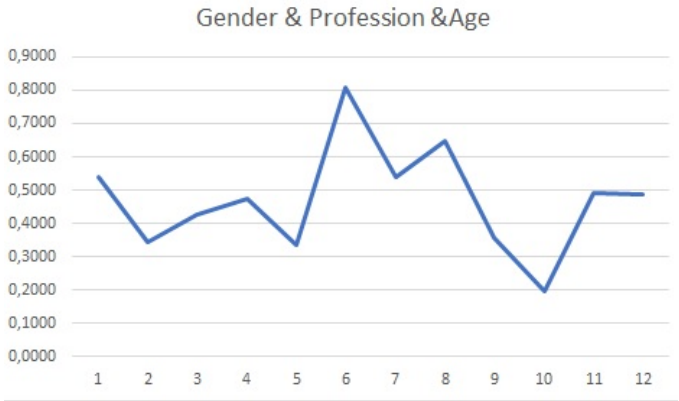

Figure 10. The p-values of the test statistics (Gender, Profession and Age

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