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Solution Analysis of Riccati's Fractional Differential Equations Using the ADM-Laplace Transformation and the ADM-Kashuri-Fundo Transformation

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Abstract Fractional differential equations (FDEs) are differential equations that involve fractional derivatives. Unlike ordinary derivatives, fractional derivatives are defined by fractional powers of the differentiation operator. FDEs can arise in a variety of contexts, including physics, engineering, biology, and finance. They are typically more complex than ordinary differential equations, and their solutions may exhibit unusual properties such as long-range memory, non-locality, and power-law behavior. The solution of the Riccati Fractional Differential Equation (RFDE) is generally challenging due to its nonlinearity and the presence of the fractional power term. The fractional derivative operators in the RFDE are non-local and involve an integral over a certain range of the independent variable. The non-local nature of the fractional derivatives can make the RFDE harder to handle compared to ordinary differential equations. In this paper, we have examined the Riccati Fractional Differential Equation (RFDE) using the combined theorem of the Adomian Decomposition Method and Laplace Transform (ADM-LT). Furthermore, we have compared with Adomian Decomposition Method and Kashuri-Fundo Transformation (ADM-KFT). It is shown that the ADM-LT is equivalent to the ADM-KFT algorithm for solving the Riccati equation. In addition, we have added new theorem of the relationship between the Kashuri Fundo inverse and the Laplace Transform inverse. The main finding of our study shows that the Adomian Decomposition Method and Laplace Transform (ADM-LT) have a good agreement between numerical simulation and exact solution.

Keywords Fractional Calculus, Laplace Transform, Kashuri-Fundo Transformation, Adomian Decomposition Method

1. Introduction

Fractional Calculus has become an interesting study to describe the phenomena of system behavior in various application fields such as continuous-time IS–LM model with memory [1], the Fisher model's tax version, which includes inflation rate and stock price memory [2], fractional generalizations of the Gross domestic product (GDP) [3], logistic growth model with memory [4], Productivity with long memory and fatigue [5], electronic circuits [6], Field-Programmable Gate Array [7], wind turbine system [8], robotic manipulators [9], magnetic levitation system [10], active magnetic bearing systems [11] and turbo engine sensors [12].

Studies related to the solution of the Riccati differential equation have been intensively researched by scientists such as Adomian Decomposition Method (ADM) [13], Homotopy Perturbation Method [14], He Laplace Method [15], Mixing Sumudu transform [16], optimal homotopy asymptotic method [17], Taylor matrix method [18], Haar wavelet method [19], laplace transforms [20], sequential quadratic programming (SQP) and artificial neural networks (ANNs) [21].

Riccati differential equation has been widely studied in several engineering and applied science problems. Tsai [22] presented the nonlinear Riccati differential equations using ADM, the Laplace transform algorithm and the Padé approximant. They show three examples to demonstrate fast convergence and good accuracy when compared with the exact solution. In the economic growth model (EGM), Johansyah et al. [23] established a quadratic non-linear cost function and examined the suggested EGM to determine the exact solution. Additionally, they have assessed the numerical solution of the RFDEs on the EGM with memory effects utilizing the Kashuri-Fundo Transformation and Combined Theorem of ADM. In the same years, Johansyah et al. [24] applied Banach's fixed-point theorem to examine the existence and uniqueness of solutions to RFDE with constant coefficients. Additionally, the solution of the FDE is transformed into an infinite polynomial series using the combined theorem of the ADM and KIT. Abbasbandy [25] examined the Riccati differential problem using the Homotopy perturbation technique and the Adomian's decomposition strategy. His variational iteration method and ADM were also used by him to resolve the cubic Riccati differential problem [26]. Finally, in order to solve Riccati equations, by matching the near-field approximation, Biazar & Didgar [27] combined the ADM and Adomian's asymptotic decomposition method (AADM) to investigate the approximative global solution.

Based on the above research problem, we did not find any discussion regarding the RFDE solution using the ADM-Laplace Transformation and ADM-Kashuri-Fundo Trans-formation comparisons. Therefore, utilizing the combined theorem of the ADM-LT and ADM-KFT, we have proposed the Riccati fractional differential equation (ADM-KFT).

This paper is organized as follows: Background information about the development and modification of the ADM-LT and ADM-KFT theorem is presented in Section 2. The key findings about the theorem and solutions to the RFDE technique employing the ADM-LT and ADM-KFT are presented in Section 3. Our conclusions are presented in Section 4.

2. Background Theory

Definition 1. The Laplace transform $f(t)$ is defined as follows [28].

$$F(s)=\mathcal{L}[f(t)]=\int_0^\infty f(t)e^{-st}dt.$$

In practice, to solve differential equations using the Laplace transform, we have to find the inverse of the Laplace transform by finding a function $f(t)$ corresponding to some specified $F(s)$.

Definition 2. The inverse Laplace transform $F(s)$ is defined as follows.

$$f(t)=\mathcal{L}^{-1}[F(s)],$$

with $t>0$ and $\mathcal{L}^{-1}$ is called the inverse operator of the Laplace transform.

The Laplace transform of an ordinary function using elementary calculus can be shown that for all $\mu>-1$, and $\alpha\in\mathbb{R}$.

$$\mathcal{L}[t^\mu]=\frac{\Gamma(\mu+1)}{s^{\mu+1}}\ \text{dan}\ \mathcal{L}[e^{at}]=\frac{1}{s-a}\ .$$

Lemma 1. The Laplace transform of the Riemann-Liouville fractional integral operator with order $\alpha>0$ is

$$\mathcal{L}[D^{-\alpha}f(t)]=\frac{1}{\Gamma(\alpha)}\mathcal{L}\left[t^{\alpha-1}\right]\mathcal{L}[f(t)]=s^{-\alpha}F(s),v>0.$$

Lemma 2. The Laplace transform of the Caputo fraction derivative (CFD) with $n-1<\alpha<n$ is

$$\mathcal{L}\left[\frac{d^\alpha f}{dt^\alpha}f(t)\right]=s^\alpha F(s)-\sum_{m=0}^{n-1}s^{\alpha-m-1}\quad f^{(m)}(0).$$

Definition 3. Consider the following functions in the set F defined [29].

$$F=\left\{f(t):\exists M,k_1,k_2>0.\,|f(t)|<Me^{\frac{|x|}{k_j^2}},t\in(-1)^j\times[0,\infty)\right\}$$

For a function belonging to the set F, the constant M must be finite number. The k_1,k_2 may be finite or infinite. Kashuri Fundo transform is denoted by the operator $K(.)$ The KFT is defined as,

$$K[f(t)]=A(v)=\frac{1}{v}\int_0^\infty e^{-\frac{t}{v^2}}f(t)dt\,,t\geq0,-k_1<v<k_2.$$

Furthermore, the inverse is

$$K^{-1}[A(v)]=y(t),t\geq0. \tag{1}$$

However, for fractional numbers, it is

$$K[t^\alpha]=\Gamma(\alpha+1)v^{2\alpha+1}, \tag{2}$$

So, we get

$$K^{-1}[v^{2\alpha+1}] = \frac{t^{\alpha}}{\Gamma(\alpha+1)} \ .$$

Theorem 1. The Kashuri-Fundo transform of the CFD for $\alpha = 0$ is defined as

$$K[D_x^{\alpha} y(x)] = \frac{A(v)}{v^{2\alpha}} - \sum_{k=0}^{n-1} \frac{f^k(0)}{v^{2(\alpha-k)-1}}, n-1 < \alpha \leq n.$$

2.1. The Combined Theorem of the ADM-LT

Theorem 2. The Combined Laplace Transformation and ADM Theorem are

$$y_0 = y_0 + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[P]\right],$$

$$y_{n+1} = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Qy_n]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[RA_n]\right], n = 0,1,2, \ldots$$

Proof

Given the RFDE.

$$D_t^{\alpha} y(t) = P + Qy(t) + Ry^2(t), \qquad t > 0. \tag{3}$$

With the initial value of $y(0) = c$ and P, Q, R is a constant coefficient and $D_t^{\alpha} y(t)$ is CFD of the function $y(\text{t})$ with respect to t of order α, where $0 < \alpha \leq 1$.

Transform equation (3) with the Laplace transform, we get.

$$\mathcal{L}[D_t^{\alpha} y(t)] = \mathcal{L}[P] + \mathcal{L}[Qy(t)] + \mathcal{L}[Ry^2(\text{t})]. \tag{4}$$

Furthermore, by using the Laplace transform property and substituting the initial condition values, equation (4) becomes.

$$s^{\alpha}\mathcal{L}[y(t)] - s^{\alpha-1}y(0) = \mathcal{L}[P] + \mathcal{L}[Qy(t)] + \mathcal{L}[Ry^2(t)],$$

$$s^{\alpha}\mathcal{L}[y(t)] - s^{\alpha-1}\text{y}_0 = \mathcal{L}[P] + \mathcal{L}[Qy(t)] + \mathcal{L}[Ry^2(t)].$$

Divide both sides by s^{α}, so we get

$$\mathcal{L}[y(t)] = \frac{\text{y}_0}{s} + \frac{1}{s^{\alpha}}\mathcal{L}[P] + \frac{1}{s^{\alpha}}\mathcal{L}[Qy(t)] + \frac{1}{s^{\alpha}} \mathcal{L}[Ry^2(t)]. \tag{5}$$

Additionally, ADM makes the following assumption regarding how to decompose the function y into an infinite polynomial series:

$$y(t) = y_0 + y_1 + y_2 + \cdots = \sum_{n=0}^{\infty} y_n(t), \tag{6}$$

Where y_n can be calculated recursively. This approach also presupposes that the infinite polynomial series of the non-linear operator y^2 can be decomposed to give:

$$N(y) = y^2 = \sum_{n=0}^{\infty} A_n \tag{7}$$

Where $A_n = A_n(y_0, y_1, y_2, y_3, \ldots, y_n)$ is an adomian polynomial which can be described as follows:

$$A_0 = N(y_0) = y_0^2,$$

$$A_1 = yN'(y_0) = 2y_0y_1,$$

$$A_2 = y_2N'(y_0) + \frac{y_1^2}{2!}N''(y_0) = 2y_0y_1 + y_1^2,$$

$$A_3 = y_3N'(y_0) + y_1y_2N''(y_0) + \frac{y_1}{3!}N'''(y_0) = 2y_0y_3 + 2y_1y_2,$$

Furthermore, substitute equations (6) and (7) into equation (5), to obtain.

$$\mathcal{L}\left[\sum_{n=0}^{\infty} y_n(t)\right] = \frac{\text{y}_0}{s} + \frac{1}{s^{\alpha}}\mathcal{L}[P] + \frac{1}{s^{\alpha}}\mathcal{L}\left[Q\sum_{n=0}^{\infty} y_n(t)\right] + \frac{1}{s^{\alpha}} \mathcal{L}\left[R\sum_{n=0}^{\infty} A_n(t)\right]. \tag{8}$$

From equation (8), the iteration with recursion is obtained as follows:

$$\mathcal{L}[y_0(t)] = \frac{y_0}{s} + \frac{1}{s^{\alpha}}\mathcal{L}[P],$$

$$\mathcal{L}[y_1(t)] = \frac{1}{s^{\alpha}}\mathcal{L}[RA_0] + \frac{1}{s^{\alpha}}\mathcal{L}[Qy_0],$$

$$\mathcal{L}[y_2(t))] = \frac{1}{s^{\alpha}}\mathcal{L}[RA_1] + \frac{1}{s^{\alpha}}\mathcal{L}[Qy_1],$$

$$\mathcal{L}[y_n(t)] = \frac{1}{s^{\alpha}}\mathcal{L}[RA_{n-1}] + \frac{1}{s^{\alpha}}\mathcal{L}[Qy_{n-1}].$$

Thus, the Combined Laplace Transformation and ADM Theorem are

$$y_0 = y_0 + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[P]\right],$$

$$y_{n+1} = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Qy_n]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[RA_n]\right], n = 0,1,2, \ldots$$

2.2. The Combined Theorem of the ADM-KFT

Theorem 3. The combined Kashuri-Fundo and ADM Theorem are

$$y_0(t) = y(0) + K^{-1}[v^{2\alpha}K[P]],$$

$$y_{n+1}(t) = K^{-1}[v^{2\alpha}K[Qy_n]] + K^{-1}[v^{2\alpha}K[RA_n]], n = 0,1,2, \ldots$$

Proof

Given the RFDE.

$$D_t^{\alpha} y(t) = P + Qy(t) + Ry^2(t), \qquad t > 0 \tag{9}$$

With the initial value of $y(0) = c$ and P, Q, R is a constant coefficient and $D_t^{\alpha} y(t)$ is CFD of the function $y(\mathrm{t})$ with respect to t of order α, where $0 < \alpha \leq 1$.

Transform equation (9) with the Kashuri-Fundo transformation, so we get

$$K[D_t^{\alpha} y(t)] = K[P + Qy(t) + Ry^2(t)],$$

$$\frac{y(v)}{v^{2\alpha}} - \sum_{k=0}^{n-1} \frac{y^k(0)}{v^{2(\alpha-k)-1}} = K[P] + K[Qy(t)] + K[Ry^2(t)],$$

$$\frac{y(v)}{v^{2\alpha}} - \frac{y(0)}{v^{2\alpha-1}} = K[P] + K[Qy(t)] + K[Ry^2(t)],$$

Multiply both sides by $v^{2\alpha}$ to get

$$y(v) - vy(0) = v^{2\alpha}K[P] + v^{2\alpha}K[Qy(t)] + v^{2\alpha}K[Ry^2(t)],$$

$$y(v) = vy(0) + v^{2\alpha}K[P] + v^{2\alpha}K[Qy(t)] + v^{2\alpha}K[Ry^2(t)]. \quad (10)$$

Then, by using the inverse of KFT (1), then equation (10) becomes

$$K^{-1}[y(v)] = K^{-1}\left[vy(0) + v^{2\alpha}K[P] + v^{2\alpha}K[Qy(t)] + v^{2\alpha}K[Ry^2(t)]\right],$$

$$y(t) = y(0) + K^{-1}\left[v^{2\alpha}K[P]\right] + K^{-1}\left[v^{2\alpha}K[Qy(t)]\right] + K^{-1}\left[v^{2\alpha}K[Ry^2(t)]\right]. \quad (11)$$

Additionally, ADM makes the following assumption regarding how to decompose the function y into an infinite polynomial series:

$$y(t) = \sum_{n=0}^{\infty} y_n(t), \quad (12)$$

Where y_n is recursively specifiable. This approach also presupposes that the infinite polynomial series formed by the decomposition of the nonlinear operator y^2 can be used to obtain:

$$N(y) = y^2 = \sum_{n=0}^{\infty} A_n, \quad (13)$$

Where A_n is an adomian polynomial with the following structure:

$$A_0 = N(y_0) = y_0^2,$$

$$A_1 = yN'(y_0) = 2y_0y_1,$$

$$A_2 = y_2N'(y_0) + \frac{y_1^2}{2!}N''(y_0) = 2y_0y_1 + y_1^2,$$

$$A_3 = y_3N'(y_0) + y_1y_2N''(y_0) + \frac{y_1}{3!}N'''(y_0) = 2y_0y_3 + 2y_1y_2,$$

Furthermore, substitute equations (12) and (13) into equation (11), we obtained

$$\sum_{n=0}^{\infty} y_n(t) = y(0) + K^{-1}\left[v^{2\alpha}K[P]\right] + K^{-1}\left[v^{2\alpha}K\left[Q\sum_{n=0}^{\infty} y_n(t)\right]\right] + K^{-1}\left[v^{2\alpha}K\left[R\sum_{n=0}^{\infty} A_n\right]\right],$$

$$y_0(t) + y_1(t) + y_2(t) + \cdots = y(0) + K^{-1}\left[v^{2\alpha}K[P]\right] + K^{-1}\left[v^{2\alpha}K[Q(y_0 + y_1 + y_2 + \cdots)]\right] + K^{-1}\left[v^{2\alpha}K[R(A_1 + A_2 + A_3 + \cdots)]\right].$$

By combining the ADM with the Kashuri-Fundo Transformation, the following combined theorem may be found from the recursive relation of the fractional differential equation solution:

$$y_0(t) = y(0) + K^{-1}\left[v^{2\alpha}K[P]\right],$$

$$y_1(t) = K^{-1}\left[v^{2\alpha}K[Qy_0]\right] + K^{-1}\left[v^{2\alpha}K[RA_0]\right],$$

$$y_2(t) = K^{-1}\left[v^{2\alpha}K[Qy_1]\right] + K^{-1}\left[v^{2\alpha}K[RA_1]\right],$$

$$y_3(t) = K^{-1}\left[v^{2\alpha}K[Qy_2]\right] + K^{-1}\left[v^{2\alpha}K[RA_2]\right],$$

$$y_{n+1}(x) = K^{-1}\left[v^{2\alpha}K[Qy_n]\right] + K^{-1}\left[v^{2\alpha}K[RA_n]\right], n = 0,1,2, \ldots$$

Thus, the Combined Kashuri-Fundo and ADM Theorem are

$$y_0(t) = y(0) + K^{-1}\left[v^{2\alpha}K[P]\right],$$

$$y_{n+1}(t) = K^{-1}\left[v^{2\alpha}K[Qy_n]\right] + K^{-1}\left[v^{2\alpha}K[RA_n]\right], n = 0,1,2, \ldots$$

So that the Kashuri-Fundo transformation has a relationship with the Laplace transform which is stated as follows

$$K[f(t); v] = \frac{1}{v} \mathcal{L}\left[f(t); \frac{1}{v^2}\right] \quad (14)$$

Theorem 3 which is a development of equation (14), namely the relationship between the inverse of KFT and the inverse of LT as follows

$$K^{-1}[F(v); t] = \mathcal{L}^{-1}\left[v.F(v), v = \frac{1}{\sqrt{s}}; t\right] \quad (15)$$

Proof

Based on **Definitions** 1, we obtain

$$K[f(t)] = \frac{1}{v}\int_0^{\infty} e^{-\frac{t}{v^2}} f(t)dt, \quad (16)$$

$$\mathcal{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}dt, \quad (17)$$

Thus, from equations (16) and (17) it is obtained

$$\frac{1}{v}.\mathcal{L}[f(t)] = K[f(t)], \tag{18}$$

with

$$-st = -\frac{t}{v^2},$$

$$v = \frac{1}{\sqrt{s}},$$

Use the formula $f(t) = \mathcal{L}^{-1}[F(s)]$, so that equation (18) becomes

$$\frac{1}{v}.\mathcal{L}\left[\mathcal{L}^{-1}[F(s)]\right] = K\left[\mathcal{L}^{-1}[F(s)]\right] \ with \ v = \frac{1}{\sqrt{s}},$$

$$\frac{1}{v}.[F(s)] = K\left[\mathcal{L}^{-1}[F(s)]\right] \ with \ v = \frac{1}{\sqrt{s}},$$

$$\frac{1}{v}.K^{-1}[F(s)] = \mathcal{L}^{-1}[F(s)] \ with \ v = \frac{1}{\sqrt{s}},$$

$$K^{-1}[F(s)] = \mathcal{L}^{-1}[v.F(s)] \ with \ v = \frac{1}{\sqrt{s}},$$

$$K^{-1}\left[F\left(\frac{1}{v^2}\right)\right] = \mathcal{L}^{-1}\left[v.F\left(\frac{1}{v^2}\right)\right] \ with \ v = \frac{1}{\sqrt{s}},$$

$$K^{-1}\left[F\left(\frac{1}{v^3}.v\right)\right] = \mathcal{L}^{-1}\left[v.F\left(\frac{1}{v^3}.v\right)\right] with \ v = \frac{1}{\sqrt{s}},$$

$$\frac{1}{v^3}.K^{-1}[F(v)] = \frac{1}{v^3}.\mathcal{L}^{-1}[v.F(v)] \ with \ v = \frac{1}{\sqrt{s}},$$

$$K^{-1}[F(v)] = \mathcal{L}^{-1}[v.F(v)] \ with \ v = \frac{1}{\sqrt{s}}.$$

Thus, it is obtained

$$K^{-1}[F(v);t] = \mathcal{L}^{-1}\left[v.F(v), v = \frac{1}{\sqrt{s}};t\right]$$

3. Solution of the RFDE Using the Combined Theorem

Given RFDE as follows

$$D_t^{\alpha}(y) = 3 + 2y(t) - 0{,}5\ y^2(t), \qquad 0 < \alpha \leq 1, \tag{19}$$

with initial conditions y (0) = 0 and the exact solution based on Maple 18 Software is

$$f(t) = -\frac{1}{5}\left(-\sqrt{10} \quad 5\,tanh\left(\frac{1}{10}\left(\sqrt{10}\,arctanh\left(\frac{1}{5}\sqrt{10}\right) - 5t\right)\sqrt{10}\right)\ \right)\sqrt{10}$$

3.1. Combined Theorem of the ADM-LT

The approximate answer to the RFDE (19) is found using the combined Theorem ADM-LT:

$$\begin{aligned} y_0 &= 0 + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[P]\right], \\ y_0 &= \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[3]\right], \\ y_{n+1} &= \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Qy_n]\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[RA_n]\right], \\ y_{n+1} &= \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[2y_n]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[0{,}5A_n]\right], \end{aligned} \tag{20}$$

Where A_n represent the Adomian polynomial of the non-linear operator $Ny = y^2$, which can be described as follows:

$$A_0 = y_0^2,$$
$$A_1 = 2y_0y_1,$$
$$A_2 = 2y_0y_1 + y_1^2,$$

The solution approach (19) is described as follows:

$$y_0 = y_0(t) = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[3]\right] = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}.\frac{3}{s}\right]$$
$$= \mathcal{L}^{-1}\left[\frac{3}{s^{\alpha+1}}\right] = \frac{3t^\alpha}{\alpha!} = \frac{3t^\alpha}{\Gamma(\alpha+1)},$$

$$y_1 = y_1(t) = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[2y_0]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[0{,}5A_0]\right]$$
$$= \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[2\frac{3t^\alpha}{\Gamma(\alpha+1)}\right]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[0{,}5y_0^2]\right]$$
$$= \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[2\frac{3t^\alpha}{\Gamma(\alpha+1)}\right]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[0{,}5\frac{9t^{2\alpha}}{\Gamma^2(\alpha+1)}\right]\right]$$
$$= \mathcal{L}^{-1}\left[\frac{6}{\Gamma(\alpha+1)s^\alpha}\mathcal{L}[t^\alpha]\right] - \mathcal{L}^{-1}\left[\frac{4{,}5}{\Gamma^2(\alpha+1)s^\alpha}\mathcal{L}[t^{2\alpha}]\right]$$
$$= \mathcal{L}^{-1}\left[\frac{6}{\Gamma(\alpha+1)s^\alpha}.\frac{\alpha!}{s^{\alpha+1}}\right] - \mathcal{L}^{-1}\left[\frac{4{,}5}{\Gamma^2(\alpha+1)s^\alpha}.\frac{2\alpha!}{s^{2\alpha+1}}\right]$$
$$= \mathcal{L}^{-1}\left[\frac{6}{\Gamma(\alpha+1)}.\frac{\Gamma(\alpha+1)}{s^{2\alpha+1}}\right] - \mathcal{L}^{-1}\left[\frac{4{,}5}{\Gamma^2(\alpha+1)}.\frac{\Gamma(2\alpha+1)}{s^{3\alpha+1}}\right]$$
$$= \mathcal{L}^{-1}\left[\frac{6}{s^{2\alpha+1}}\right] - \frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)}\mathcal{L}^{-1}\left[\frac{1}{s^{3\alpha+1}}\right]$$
$$= \frac{6t^{2\alpha}}{2\alpha!} - \frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)}.\frac{t^{3\alpha}}{3\alpha!}$$
$$= \frac{6t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{4{,}5\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)},$$

$$y_2 = y_2(t) = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[2y_1]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[0{,}5A_1]\right]$$
$$= \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[2y_1]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[0{,}5(2y_0y_1)]\right]$$
$$= \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[2\left(\frac{6t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{4{,}5\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right)\right]\right]$$
$$-\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[0{,}5(2)\left(\frac{3t^\alpha}{\Gamma(\alpha+1)}\right)\left(\frac{6t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{4{,}5\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right)\right]\right]$$
$$= \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[\frac{12t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{9\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right]\right]$$
$$-\mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left[\frac{18t^{3\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} - \frac{13{,}5\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}\right]\right]$$
$$= \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}.\frac{12}{\Gamma(2\alpha+1)}.\frac{2\alpha!}{s^{2\alpha+1}} - \frac{1}{s^\alpha}.\frac{9\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}.\frac{3\alpha!}{s^{3\alpha+1}}\right]$$

$$-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}.\frac{18}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}.\frac{3\alpha!}{s^{3\alpha+1}} - \frac{1}{s^{\alpha}}.\frac{13{,}5\Gamma(2\alpha+1)}{\Gamma^{3}(\alpha+1)\Gamma(3\alpha+1)}.\frac{4\alpha!}{s^{4\alpha+1}}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{12}{\Gamma(2\alpha+1)}.\frac{\Gamma(2\alpha+1)}{s^{3\alpha+1}} - \frac{9\Gamma(2\alpha+1)}{\Gamma^{2}(\alpha+1)\Gamma(3\alpha+1)}.\frac{\Gamma(3\alpha+1)}{s^{4\alpha+1}}\right]$$

$$-\mathcal{L}^{-1}\left[\frac{18}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}.\frac{\Gamma(3\alpha+1)}{s^{4\alpha+1}} - \frac{13{,}5\Gamma(2\alpha+1)}{\Gamma^{3}(\alpha+1)\Gamma(3\alpha+1)}.\frac{\Gamma(4\alpha+1)}{s^{5\alpha+1}}\right]$$

$$= \frac{12t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{9\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma^{2}(\alpha+1)\Gamma(4\alpha+1)} - \frac{18\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)} + \frac{13{,}5\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma^{3}(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}.$$

So, the solution to equation (19) is

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots,$$

$$= \frac{3t^{\alpha}}{\Gamma(\alpha+1)} + \frac{6t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{4{,}5\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^{2}(\alpha+1)\Gamma(3\alpha+1)} + \frac{12t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{9\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma^{2}(\alpha+1)\Gamma(4\alpha+1)} - \frac{18\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)} + \frac{13{,}5\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma^{3}(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} + \cdots,$$

The RFDE solution in equation (19) for $\alpha = 0.7$ is

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots$$

$$= \frac{3t^{0{,}7}}{\Gamma(0{,}7+1)} + \frac{6t^{1{,}4}}{\Gamma(1{,}4+1)} - \frac{4{,}5\Gamma(1{,}4+1)t^{2{,}1}}{\Gamma^{2}(0{,}7+1)\Gamma(2{,}1+1)} + \frac{12t^{2{,}1}}{\Gamma(2{,}1+1)} - \frac{9\Gamma(1{,}4+1)t^{2{,}8}}{\Gamma^{2}(0{,}7+1)\Gamma(2{,}8+1)} - \frac{18\Gamma(2{,}1+1)t^{2{,}8}}{\Gamma(0{,}7+1)\Gamma(1{,}4+1)\Gamma(2{,}8+1)} + \frac{13{,}5\Gamma(1{,}4+1)\Gamma(2{,}8+1)t^{3{,}5}}{\Gamma^{3}(0{,}7+1)\Gamma(2{,}1+1)\Gamma(3{,}5+1)} + \cdots$$

$$= \frac{3t^{0{,}7}}{\Gamma(1{,}7)} + \frac{6t^{1{,}4}}{\Gamma(2{,}4)} - \frac{4{,}5\Gamma(2{,}4)t^{2{,}1}}{\Gamma^{2}(1{,}7)\Gamma(3{,}1)} + \frac{12t^{2{,}1}}{\Gamma(3{,}1)} - \frac{9\Gamma(2{,}4)t^{2{,}8}}{\Gamma^{2}(1{,}7)\Gamma(3{,}8)} - \frac{18\Gamma(3{,}1)t^{2{,}8}}{\Gamma(1{,}7)\Gamma(2{,}4)\Gamma(3{,}8)} + \frac{13{,}5\Gamma(2{,}4)\Gamma(3{,}8)t^{3{,}5}}{\Gamma^{3}(1{,}7)\Gamma(3{,}1)\Gamma(4{,}5)} + \cdots$$

$$= \frac{3t^{0{,}7}}{0{,}908639} + \frac{6t^{1{,}4}}{1{,}242169} - \frac{4{,}5(1{,}242169)t^{2{,}1}}{(0{,}908639)^{2}(2{,}19762)} + \frac{12t^{2{,}1}}{(2{,}19762)} - \frac{9(1{,}242169)t^{2{,}8}}{(0{,}908639)^{2}(4{,}694174)} - \frac{18(2{,}19762)t^{2{,}8}}{(0{,}908639)(1{,}242169)(4{,}694174)} + \frac{13{,}5(1{,}242169)(4{,}694174)t^{3{,}5}}{(0{,}908639)^{3}(2{,}19762)(11{,}63173)} + \cdots$$

$$= 3{,}301642\ t^{0{,}7} + 4{,}830259\ t^{1{,}4} - 3{,}080762\ t^{2{,}1} + 5{,}460452\ t^{2{,}1} - 2{,}884574\ t^{2{,}8} - 7{,}466101736\ t^{2{,}8} + 4{,}104905\ t^{3{,}5} + \cdots,$$

$$y(t) = 3{,}301642\ t^{0{,}7} + 4{,}830259\ t^{1{,}4} + 2{,}37969\ t^{2{,}1} - 10{,}35068\ t^{2{,}8} + 4{,}104905\ t^{3{,}5} + \cdots.$$

The RFDE solution in equation (19) for $\alpha = 0.8$ is

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots$$

$$= \frac{3t^{0{,}8}}{\Gamma(0{,}8+1)} + \frac{6t^{1{,}6}}{\Gamma(1{,}6+1)} - \frac{4{,}5\Gamma(1{,}6+1)t^{2{,}4}}{\Gamma^2(0{,}8+1)\Gamma(2{,}4+1)} + \frac{12t^{2{,}4}}{\Gamma(2{,}4+1)} - \frac{9\Gamma(1{,}6+1)t^{3{,}2}}{\Gamma^2(0{,}8+1)\Gamma(3{,}2+1)} - \frac{18\Gamma(2{,}4+1)t^{3{,}2}}{\Gamma(0{,}8+1)\Gamma(1{,}6+1)\Gamma(3{,}2+1)} + \frac{13{,}5\Gamma(1{,}6+1)\Gamma(3{,}2+1)t^{4}}{\Gamma^3(0{,}8+1)\Gamma(2{,}4+1)\Gamma(4+1)} + \cdots$$

$$= \frac{3t^{0{,}8}}{\Gamma(1{,}8)} + \frac{6t^{1{,}6}}{\Gamma(2{,}6)} - \frac{4{,}5\Gamma(2{,}6)t^{2{,}4}}{\Gamma^2(1{,}8)\Gamma(3{,}4)} + \frac{12t^{2{,}4}}{\Gamma(3{,}4)} - \frac{9\Gamma(2{,}6)t^{3{,}2}}{\Gamma^2(1{,}8)\Gamma(4{,}2)} - \frac{18\Gamma(3{,}4)t^{3{,}2}}{\Gamma(1{,}8)\Gamma(2{,}6)\Gamma(4{,}2)} + \frac{13{,}5\Gamma(2{,}6)\Gamma(4{,}2)t^{4}}{\Gamma^3(1{,}8)\Gamma(3{,}4)\Gamma(5)} + \cdots$$

$$= \frac{3t^{0{,}8}}{0{,}931384} + \frac{6t^{1{,}6}}{1{,}429625} - \frac{4{,}5(1{,}242169)t^{2{,}4}}{(0{,}931384)^2(2{,}981206)} + \frac{12t^{2{,}4}}{(2{,}981206)} - \frac{9(1{,}429625)t^{3{,}2}}{(0{,}931384)^2(7{,}75669)} - \frac{18(2{,}981206)t^{3{,}2}}{(0{,}931384)(1{,}429625)(7{,}75669)} + \frac{13{,}5(1{,}429625)(7{,}75669)t^{4}}{(0{,}931384)^3(2{,}981206)(24)} + \cdots$$

$$= 3{,}22104\ t^{0{,}8} + 4{,}196906\ t^{1{,}6} - 2{,}487626\ t^{2{,}4} + 4{,}025216\ t^{2{,}4} - 1{,}912189\ t^{3{,}2} - 5{,}195621\ t^{3{,}2} + 2{,}589661\ t^{4} + \cdots,$$

$$y(t) = 3{,}22104\ t^{0{,}8} + 4{,}196906\ t^{1{,}6} + 1{,}537589\ t^{2{,}4} - 7{,}10781\ t^{3{,}2} + 2{,}589661\ t^{4} + \cdots.$$

The RFDE solution in equation (19) for $\alpha = 0.9$ is

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots$$

$$= \frac{3t^{0{,}9}}{\Gamma(0{,}9+1)} + \frac{6t^{1{,}8}}{\Gamma(1{,}8+1)} - \frac{4{,}5\Gamma(1{,}8+1)t^{2{,}7}}{\Gamma^2(0{,}9+1)\Gamma(2{,}7+1)} + \frac{12t^{2{,}7}}{\Gamma(2{,}7+1)} - \frac{9\Gamma(1{,}8+1)t^{3{,}6}}{\Gamma^2(0{,}9+1)\Gamma(3{,}6+1)} - \frac{18\Gamma(2{,}7+1)t^{3{,}6}}{\Gamma(0{,}9+1)\Gamma(1{,}8+1)\Gamma(3{,}6+1)} + \frac{13{,}5\Gamma(1{,}8+1)\Gamma(3{,}6+1)t^{4{,}5}}{\Gamma^3(0{,}9+1)\Gamma(2{,}7+1)\Gamma(4{,}5+1)} + \cdots$$

$$= \frac{3t^{0{,}9}}{\Gamma(1{,}9)} + \frac{6t^{1{,}8}}{\Gamma(2{,}8)} - \frac{4{,}5\Gamma(2{,}8)t^{2{,}7}}{\Gamma^2(1{,}9)\Gamma(3{,}7)} + \frac{12t^{2{,}7}}{\Gamma(3{,}7)}$$

$$-\frac{9\Gamma(2{,}8)t^{3{,}6}}{\Gamma^2(1{,}9)\Gamma(4{,}6)}-\frac{18\Gamma(3{,}7)t^{3{,}6}}{\Gamma(1{,}9)\Gamma(2{,}8)\Gamma(4{,}6)}$$
$$+\frac{13{,}5\Gamma(2{,}8)\Gamma(4{,}6)t^{4{,}5}}{\Gamma^3(1{,}9)\Gamma(3{,}7)\Gamma(5{,}5)}+\cdots$$
$$=\frac{3t^{0{,}9}}{0{,}961766}+\frac{6t^{1{,}8}}{1{,}676491}$$
$$-\frac{4{,}5(1{,}242169)t^{2{,}7}}{(0{,}961766)^2(4{,}170625)}+\frac{12t^{2{,}7}}{(4{,}170625)}-\frac{9(1{,}429625)t^{3{,}6}}{(0{,}961766)^2(13{,}38129)}$$
$$-\frac{18(2{,}981206)t^{3{,}6}}{(0{,}961766)(1{,}676491)(13{,}38129)}$$
$$+\frac{13{,}5(1{,}676491)(13{,}38129)t^{4{,}5}}{(0{,}961766)^3(4{,}170625)(52{,}34278)}+\cdots$$
$$=3{,}119262t^{0{,}9}+3{,}578904\ t^{1{,}8}-1{,}95556\ t^{2{,}7}$$
$$+\ 2{,}877248\ t^{2{,}7}-1{,}21901\ t^{3{,}6}-3{,}47943\ t^{3{,}6}+1{,}559423\ t^{4{,}5}+\cdots,$$
$$y(t)=3{,}119262t^{0{,}9}+3{,}578904\ t^{1{,}8}$$
$$+\ 0{,}921688\ t^{2{,}7}-4{,}698439\ t^{3{,}6}+\ 1{,}559423\ t^{4{,}5}+\cdots.$$

The RFDE solution in equation (19) for $\alpha = 0.95$ is

$$y(t)=y_0(t)+y_1(t)+y_2(t)+\cdots$$
$$=\frac{3t^{0{,}95}}{\Gamma(0{,}95+1)}+\frac{6t^{1{,}9}}{\Gamma(1{,}9+1)}$$
$$-\frac{4{,}5\Gamma(1{,}9+1)t^{2{,}85}}{\Gamma^2(0{,}95+1)\Gamma(2{,}85+1)}+\frac{12t^{2{,}85}}{\Gamma(2{,}85+1)}$$
$$-\frac{9\Gamma(1{,}9+1)t^{3{,}8}}{\Gamma^2(0{,}95+1)\Gamma(3{,}8+1)}-\frac{18\Gamma(2{,}85+1)t^{3{,}8}}{\Gamma(0{,}95+1)\Gamma(1{,}9+1)\Gamma(3{,}8+1)}$$
$$+\frac{13{,}5\Gamma(1{,}9+1)\Gamma(3{,}8+1)t^{4{,}75}}{\Gamma^3(0{,}95+1)\Gamma(2{,}85+1)\Gamma(4{,}75+1)}+\cdots$$
$$=\frac{3t^{0{,}95}}{\Gamma(1{,}95)}+\frac{6t^{1{,}9}}{\Gamma(2{,}9)}-\frac{4{,}5\Gamma(2{,}9)t^{2{,}85}}{\Gamma^2(1{,}95)\Gamma(3{,}85)}+\frac{12t^{2{,}85}}{\Gamma(3{,}85)}$$
$$-\frac{9\Gamma(2{,}9)t^{3{,}8}}{\Gamma^2(1{,}95)\Gamma(4{,}8)}-\frac{18\Gamma(3{,}85)t^{3{,}8}}{\Gamma(1{,}95)\Gamma(2{,}9)\Gamma(4{,}8)}$$
$$+\frac{13{,}5\Gamma(2{,}9)\Gamma(4{,}8)t^{4{,}75}}{\Gamma^3(1{,}95)\Gamma(3{,}85)\Gamma(5{,}75)}+\cdots$$
$$=\frac{3t^{0{,}95}}{0{,}979881}+\frac{6t^{1{,}9}}{1{,}827355}$$
$$-\frac{4{,}5(1{,}827355)t^{2{,}85}}{(0{,}979881)^2(4{,}985735)}+\frac{12t^{2{,}85}}{(4{,}985735)}$$
$$-\frac{9(1{,}827355)t^{3{,}8}}{(0{,}979881)^2(17{,}83786)}-\frac{18(4{,}985735)t^{3{,}8}}{(0{,}979881)(1{,}827355)(17{,}83786)}$$
$$+\frac{13{,}5(1{,}827355)(17{,}83786)t^{4{,}75}}{(0{,}979881)^3(4{,}985735)(78{,}78448)}+\cdots$$
$$=3{,}061597\ t^{0{,}95}+3{,}283434\ t^{1{,}9}$$
$$-\ 1{,}71775\ t^{2{,}85}+2{,}406867\ t^{2{,}85}-0{,}960232\ t^{3{,}8}-2{,}809718\ t^{3{,}8}$$
$$+1{,}190721\ t^{4{,}75}+\cdots,$$
$$y(t)=3{,}061597\ t^{0{,}95}+3{,}283434\ t^{1{,}9}$$
$$+\ 0{,}689117\ t^{2{,}85}-3{,}76995\ t^{3{,}8}+1{,}190721\ t^{4{,}75}+\cdots.$$

The RFDE solution in equation (19) for $\alpha = 1$ is

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots$$
$$= \frac{3t^1}{\Gamma(1+1)} + \frac{6t^2}{\Gamma(2+1)} - \frac{4{,}5\Gamma(2+1)t^3}{\Gamma^2(1+1)\Gamma(3+1)}$$
$$+ \frac{12t^3}{\Gamma(3+1)} - \frac{9\Gamma(2+1)t^4}{\Gamma^2(1+1)\Gamma(4+1)}$$
$$- \frac{18\Gamma(3+1)t^4}{\Gamma(1+1)\Gamma(2+1)\Gamma(4+1)}$$
$$+ \frac{13{,}5\Gamma(2+1)\Gamma(4+1)t^5}{\Gamma^3(1+1)\Gamma(3+1)\Gamma(5+1)} + \cdots$$
$$= \frac{3t}{\Gamma(2)} + \frac{6t^2}{\Gamma(3)} - \frac{4{,}5\Gamma(3)t^3}{\Gamma^2(2)\Gamma(4)} + \frac{12t^3}{\Gamma(4)} - \frac{9\Gamma(3)t^4}{\Gamma^2(2)\Gamma(5)}$$
$$- \frac{18\Gamma(4)t^4}{\Gamma(2)\Gamma(3)\Gamma(5)} + \frac{13{,}5\Gamma(3)\Gamma(5)t^5}{\Gamma^3(2)\Gamma(4)\Gamma(6)} + \cdots$$
$$= \frac{3t}{1} + \frac{6t^2}{2} - \frac{4{,}5(2)t^3}{6} + \frac{12t^3}{6} - \frac{9(2)t^4}{24} - \frac{18(6)t^4}{(2)(24)}$$
$$+ \frac{13{,}5(2)(24)t^5}{(6)(120)} + \cdots$$
$$= 3t + 3t^2 - 1{,}5t^3 + 2t^3 - 0{,}75t^4 - 2{,}25t^4$$
$$+ \ 0{,}9t^5 + \cdots,$$
$$y(t) = 3t + 3t^2 + 0{,}5t^3 - 3t^4 + 0{,}9t^5 + \cdots.$$

Furthermore, we have used Maple to graph the RFDE solution to equation (19) using the ADM-LT for values $\alpha = 0.7; 0.8; 0.9; 0.95; 1$ and $0 \le t \le 1$, up to iteration k = 10. The numerical simulation for $0 \le t \le 1$ is presented in Figure 1.

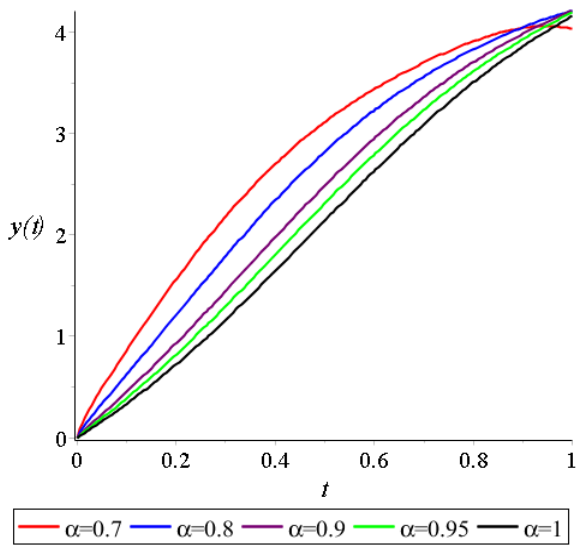


Figure 1. Approximate results RFDE using ADM-LT

3.2. Combined Theorem of ADM-KFT

Based on the combined Theorem ADM-KFT, the approximate solution to the RFDE (19) is obtained:

$$y_0(t) = 0 + K^{-1}\left[v^{2\alpha}K[P]\right],$$
$$y_0(t) = K^{-1}\left[v^{2\alpha}K[3]\right], \tag{21}$$

$$y_{n+1}(t) = K^{-1}\left[v^{2\alpha}K[2y_n]\right] + K^{-1}\left[v^{2\alpha}K[-0{,}5A_n]\right],$$
$$y_{n+1}(t) = 2K^{-1}\left[v^{2\alpha}K[y_n]\right] - K^{-1}\left[v^{2\alpha}K[0{,}5A_n]\right].$$

where A_n is the Adomian polynomial of the non-linear operator $Ny = y^2$, it is defined as:

$$A_0 = y_0^2,$$
$$A_1 = 2y_0y_1,$$
$$A_2 = 2y_0y_1 + y_1^2,$$

The solution approach (21) is as follows:

$$y_0 = y_0(t) = K^{-1}\left[v^{2\alpha}K[3]\right] = K^{-1}\left[v^{2\alpha}K[3t^0]\right]$$
$$= K^{-1}\left[3v^{2\alpha}\Gamma(0+1)v^{2(0)+1}\right] = K^{-1}[v^{2\alpha}3v]$$
$$= K^{-1}[3v^{2\alpha+1}] = \frac{3t^\alpha}{\Gamma(\alpha+1)},$$
$$y_1 = y_1(t) = 2K^{-1}\left[v^{2\alpha}K[y_0]\right] - K^{-1}\left[v^{2\alpha}K[0{,}5A_0]\right]$$
$$= 2K^{-1}\left[v^{2\alpha}K[y_0]\right] - K^{-1}\left[v^{2\alpha}K[0{,}5y_0^2]\right],$$
$$= 2K^{-1}\left[v^{2\alpha}K\left[\frac{3t^\alpha}{\Gamma(\alpha+1)}\right]\right] - K^{-1}\left[v^{2\alpha}K\left[0{,}5\left(\frac{9t^{2\alpha}}{\Gamma^2(\alpha+1)}\right)\right]\right]$$
$$= 2K^{-1}\left[v^{2\alpha}\frac{3\Gamma(\alpha+1)}{\Gamma(\alpha+1)}v^{2\alpha+1}\right] - K^{-1}\left[v^{2\alpha}\frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)}v^{4\alpha+1}\right]$$
$$= 2K^{-1}[3v^{4\alpha+1}] - K^{-1}\left[\frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)}v^{6\alpha+1}\right]$$
$$= \frac{6t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha},$$
$$y_2 = y_2(t) = 2K^{-1}\left[v^{2\alpha}K[y_1]\right] - K^{-1}\left[v^{2\alpha}K[0{,}5A_1]\right]$$
$$= 2K^{-1}\left[v^{2\alpha}K[y_1]\right] - K^{-1}\left[v^{2\alpha}K[0{,}5(2y_0y_1)]\right]$$
$$= 2K^{-1}\left[v^{2\alpha}K\left[\frac{6t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha}\right]\right]$$
$$- K^{-1}\left[v^{2\alpha}K\left[0{,}5(2)\left(\frac{3t^\alpha}{\Gamma(\alpha+1)}\right)\left(\frac{6t^{2\alpha}}{\Gamma(2\alpha+1)}\right.\right.\right.$$
$$\left.\left.\left.- \frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha}\right)\right]\right]$$
$$= 2K^{-1}\left[v^{2\alpha}K\left[\frac{6t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha}\right]\right]$$
$$- K^{-1}\left[v^{2\alpha}K\left[\frac{18t^{3\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} - \frac{13{,}5\Gamma(2\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}t^{4\alpha}\right]\right]$$
$$= 2K^{-1}\left[v^{2\alpha}\left(\frac{6\Gamma(2\alpha+1)}{\Gamma(2\alpha+1)}v^{4\alpha+1} - \frac{4{,}5\Gamma(2\alpha+1)\Gamma(3\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}v^{6\alpha+1}\right)\right]$$
$$- K^{-1}\left[v^{2\alpha}\left(\frac{18\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}v^{6\alpha+1}\right.\right.$$
$$\left.\left.- \frac{13{,}5\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}x^{8\alpha+1}\right)\right]$$
$$= 2K^{-1}\left[6v^{6\alpha+1} - \frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1\ )}v^{8\alpha+1}\right]$$

$$- K^{-1}\left[\frac{18\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}v^{8\alpha+1} - \frac{13{,}5\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)}t^{10\alpha+1}\right]$$

$$= 2\left(\frac{6t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}\right)$$

$$-\frac{18\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}$$

$$+\frac{13{,}5\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}t^{5\alpha} \ ,$$

$$= \frac{12t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{9\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}$$

$$-\frac{18\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}$$

$$+\frac{13{,}5\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}t^{5\alpha},$$

So, the solution to the above equation is

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots,$$

$$= \frac{3t^{\alpha}}{\Gamma(\alpha+1)} + \frac{6t^{2\alpha}}{\Gamma(2\alpha+1)}$$

$$-\frac{4{,}5\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}t^{3\alpha} + \frac{12t^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$-\frac{9\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha} - \frac{18\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)}t^{4\alpha}$$

$$+\frac{13{,}5\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}t^{5\alpha} + \cdots.$$

The RFDE solution in equation (19) for $\alpha = 0.7$ is

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots$$

$$= \frac{3t^{0,7}}{\Gamma(0{,}7+1)} + \frac{6t^{1,4}}{\Gamma(1{,}4+1)}$$

$$-\frac{4{,}5\Gamma(1{,}4+1)t^{2,1}}{\Gamma^2(0{,}7+1)\Gamma(2{,}1+1)} + \frac{12t^{2,1}}{\Gamma(2{,}1+1)}$$

$$-\frac{9\Gamma(1{,}4+1)t^{2,8}}{\Gamma^2(0{,}7+1)\Gamma(2{,}8+1)} - \frac{18\Gamma(2{,}1+1)t^{2,8}}{\Gamma(0{,}7+1)\Gamma(1{,}4+1)\Gamma(2{,}8+1)}$$

$$+\frac{13{,}5\Gamma(1{,}4+1)\Gamma(2{,}8+1)t^{3,5}}{\Gamma^3(0{,}7+1)\Gamma(2{,}1+1)\Gamma(3{,}5+1)} + \cdots$$

$$= \frac{3t^{0,7}}{\Gamma(1{,}7)} + \frac{6t^{1,4}}{\Gamma(2{,}4)} - \frac{4{,}5\Gamma(2{,}4)t^{2,1}}{\Gamma^2(1{,}7)\Gamma(3{,}1)} + \frac{12t^{2,1}}{\Gamma(3{,}1)}$$

$$-\frac{9\Gamma(2{,}4)t^{2,8}}{\Gamma^2(1{,}7)\Gamma(3{,}8)} - \frac{18\Gamma(3{,}1)t^{2,8}}{\Gamma(1{,}7)\Gamma(2{,}4)\Gamma(3{,}8)}$$

$$+\frac{13{,}5\Gamma(2{,}4)\Gamma(3{,}8)t^{3,5}}{\Gamma^3(1{,}7)\Gamma(3{,}1)\Gamma(4{,}5)} + \cdots$$

$$= \frac{3t^{0,7}}{0{,}908639} + \frac{6t^{1,4}}{1{,}242169} - \frac{4{,}5(1{,}242169)t^{2,1}}{(0{,}908639)^2(2{,}19762)}$$

$$+\frac{12t^{2,1}}{(2,19762)}-\frac{9(1,242169)t^{2,8}}{(0,908639)^2(4,694174)}$$
$$-\frac{18(2,19762)t^{2,8}}{(0,908639)(1,242169)(4,694174)}$$
$$+\frac{13,5(1,242169)(4,694174)t^{3,5}}{(0,908639)^3(2,19762)(11,63173)}+\cdots$$
$$=3,301642\ \ t^{0,7}+4,830259\ \ t^{1,4}-3,080762\ \ t^{2,1}$$
$$+\ 5,460452\ \ t^{2,1}-2,884574\ \ t^{2,8}-7,466101736\ \ t^{2,8}$$
$$+\ 4,104905\ \ t^{3,5}+\cdots,$$
$$y(t)=3,301642\ \ t^{0,7}+4,830259\ \ t^{1,4}$$
$$+\ 2,37969\ \ t^{2,1}-10,35068\ \ t^{2,8}+\ \ 4,104905\ \ t^{3,5}+\cdots.$$

The RFDE solution in equation (19) for $\alpha = 0.8$ is

$$y(t)=y_0(t)+y_1(t)+y_2(t)+\cdots$$
$$=\frac{3t^{0,8}}{\Gamma(0,8+1)}+\frac{6t^{1,6}}{\Gamma(1,6+1)}$$
$$-\frac{4,5\Gamma(1,6+1)t^{2,4}}{\Gamma^2(0,8+1)\Gamma(2,4+1)}+\frac{12t^{2,4}}{\Gamma(2,4+1)}$$
$$-\frac{9\Gamma(1,6+1)t^{3,2}}{\Gamma^2(0,8+1)\Gamma(3,2+1)}-\frac{18\Gamma(2,4+1)t^{3,2}}{\Gamma(0,8+1)\Gamma(1,6+1)\Gamma(3,2+1)}$$
$$+\frac{13,5\Gamma(1,6+1)\Gamma(3,2+1)t^{4}}{\Gamma^3(0,8+1)\Gamma(2,4+1)\Gamma(4+1)}+\cdots$$
$$=\frac{3t^{0,8}}{\Gamma(1,8)}+\frac{6t^{1,6}}{\Gamma(2,6)}-\frac{4,5\Gamma(2,6)t^{2,4}}{\Gamma^2(1,8)\Gamma(3,4)}+\frac{12t^{2,4}}{\Gamma(3,4)}$$
$$-\frac{9\Gamma(2,6)t^{3,2}}{\Gamma^2(1,8)\Gamma(4,2)}-\frac{18\Gamma(3,4)t^{3,2}}{\Gamma(1,8)\Gamma(2,6)\Gamma(4,2)}$$
$$+\frac{13,5\Gamma(2,6)\Gamma(4,2)t^{4}}{\Gamma^3(1,8)\Gamma(3,4)\Gamma(5)}+\cdots$$
$$=\frac{3t^{0,8}}{0,931384}+\frac{6t^{1,6}}{1,429625}$$
$$-\frac{4,5(1,242169)t^{2,4}}{(0,931384)^2(2,981206)}+\frac{12t^{2,4}}{(2,981206)}$$
$$-\frac{9(1,429625)t^{3,2}}{(0,931384)^2(7,75669)}-\frac{18(2,981206)t^{3,2}}{(0,931384)(1,429625)(7,75669)}$$
$$+\frac{13,5(1,429625)(7,75669)t^{4}}{(0,931384)^3(2,981206)(24)}+\cdots$$
$$=3,22104\ \ t^{0,8}+4,196906\ \ t^{1,6}-2,487626\ \ t^{2,4}+$$
$$4,025216\ \ t^{2,4}-1,912189\ \ t^{3,2}-5,195621\ \ t^{3,2}+2,589661\ \ t^{4}+\cdots,$$
$$y(t)=3,22104\ \ t^{0,8}+4,196906\ \ t^{1,6}$$
$$+\ 1,537589\ \ t^{2,4}-7,10781\ \ t^{3,2}+\ \ 2,589661\ \ t^{4}+\cdots.$$

The RFDE solution in equation (19) for $\alpha = 0.9$ is

$$y(t)=y_0(t)+y_1(t)+y_2(t)+\cdots$$
$$=\frac{3t^{0,9}}{\Gamma(0,9+1)}+\frac{6t^{1,8}}{\Gamma(1,8+1)}$$

$$-\frac{4{,}5\Gamma(1{,}8+1)t^{2{,}7}}{\Gamma^2(0{,}9+1)\Gamma(2{,}7+1)}+\frac{12t^{2{,}7}}{\Gamma(2{,}7+1)}$$
$$-\frac{9\Gamma(1{,}8+1)t^{3{,}6}}{\Gamma^2(0{,}9+1)\Gamma(3{,}6+1)}-\frac{18\Gamma(2{,}7+1)t^{3{,}6}}{\Gamma(0{,}9+1)\Gamma(1{,}8+1)\Gamma(3{,}6+1)}$$
$$+\frac{13{,}5\Gamma(1{,}8+1)\Gamma(3{,}6+1)t^{4{,}5}}{\Gamma^3(0{,}9+1)\Gamma(2{,}7+1)\Gamma(4{,}5+1)}+\cdots$$
$$=\frac{3t^{0{,}9}}{\Gamma(1{,}9)}+\frac{6t^{1{,}8}}{\Gamma(2{,}8)}-\frac{4{,}5\Gamma(2{,}8)t^{2{,}7}}{\Gamma^2(1{,}9)\Gamma(3{,}7)}+\frac{12t^{2{,}7}}{\Gamma(3{,}7)}$$
$$-\frac{9\Gamma(2{,}8)t^{3{,}6}}{\Gamma^2(1{,}9)\Gamma(4{,}6)}-\frac{18\Gamma(3{,}7)t^{3{,}6}}{\Gamma(1{,}9)\Gamma(2{,}8)\Gamma(4{,}6)}$$
$$+\frac{13{,}5\Gamma(2{,}8)\Gamma(4{,}6)t^{4{,}5}}{\Gamma^3(1{,}9)\Gamma(3{,}7)\Gamma(5{,}5)}+\cdots$$
$$=\frac{3t^{0{,}9}}{0{,}961766}+\frac{6t^{1{,}8}}{1{,}676491}$$
$$-\frac{4{,}5(1{,}242169)t^{2{,}7}}{(0{,}961766)^2(4{,}170625)}+\frac{12t^{2{,}7}}{(4{,}170625)}$$
$$-\frac{9(1{,}429625)t^{3{,}6}}{(0{,}961766)^2(13{,}38129)}-\frac{18(2{,}981206)t^{3{,}6}}{(0{,}961766)(1{,}676491)(13{,}38129)}$$
$$+\frac{13{,}5(1{,}676491)(13{,}38129)t^{4{,}5}}{(0{,}961766)^3(4{,}170625)(52{,}34278)}+\cdots$$
$$=3{,}119262t^{0{,}9}+3{,}578904\ \ t^{1{,}8}-1{,}95556\ \ t^{2{,}7}+$$
$$2{,}877248\ \ t^{2{,}7}-1{,}21901\ \ t^{3{,}6}-3{,}47943\ \ t^{3{,}6}$$
$$+1{,}559423\ \ t^{4{,}5}+\cdots,$$
$$y(t)=3{,}119262t^{0{,}9}+3{,}578904\ \ t^{1{,}8}+0{,}921688\ \ t^{2{,}7}-4{,}698439\ \ t^{3{,}6}+\ 1{,}559423\ \ t^{4{,}5}+\cdots.$$

The RFDE solution in equation (19) for $\alpha = 0.95$ is

$$y(t)=y_0(t)+y_1(t)+y_2(t)+\cdots$$
$$=\frac{3t^{0{,}95}}{\Gamma(0{,}95+1)}+\frac{6t^{1{,}9}}{\Gamma(1{,}9+1)}$$
$$-\frac{4{,}5\Gamma(1{,}9+1)t^{2{,}85}}{\Gamma^2(0{,}95+1)\Gamma(2{,}85+1)}+\frac{12t^{2{,}85}}{\Gamma(2{,}85+1)}$$
$$-\frac{9\Gamma(1{,}9+1)t^{3{,}8}}{\Gamma^2(0{,}95+1)\Gamma(3{,}8+1)}-\frac{18\Gamma(2{,}85+1)t^{3{,}8}}{\Gamma(0{,}95+1)\Gamma(1{,}9+1)\Gamma(3{,}8+1)}$$
$$+\frac{13{,}5\Gamma(1{,}9+1)\Gamma(3{,}8+1)t^{4{,}75}}{\Gamma^3(0{,}95+1)\Gamma(2{,}85+1)\Gamma(4{,}75+1)}+\cdots$$
$$=\frac{3t^{0{,}95}}{\Gamma(1{,}95)}+\frac{6t^{1{,}9}}{\Gamma(2{,}9)}-\frac{4{,}5\Gamma(2{,}9)t^{2{,}85}}{\Gamma^2(1{,}95)\Gamma(3{,}85)}+\frac{12t^{2{,}85}}{\Gamma(3{,}85)}$$
$$-\frac{9\Gamma(2{,}9)t^{3{,}8}}{\Gamma^2(1{,}95)\Gamma(4{,}8)}-\frac{18\Gamma(3{,}85)t^{3{,}8}}{\Gamma(1{,}95)\Gamma(2{,}9)\Gamma(4{,}8)}$$
$$+\frac{13{,}5\Gamma(2{,}9)\Gamma(4{,}8)t^{4{,}75}}{\Gamma^3(1{,}95)\Gamma(3{,}85)\Gamma(5{,}75)}+\cdots$$
$$=\frac{3t^{0{,}95}}{0{,}979881}+\frac{6t^{1{,}9}}{1{,}827355}$$

$$-\frac{4{,}5(1{,}827355)t^{2{,}85}}{(0{,}979881)^2(4{,}985735)}+\frac{12t^{2{,}85}}{(4{,}985735)}$$
$$-\frac{9(1{,}827355)t^{3{,}8}}{(0{,}979881)^2(17{,}83786)}-\frac{18(4{,}985735)t^{3{,}8}}{(0{,}979881)(1{,}827355)(17{,}83786)}$$
$$+\frac{13{,}5(1{,}827355)(17{,}83786)t^{4{,}75}}{(0{,}979881)^3(4{,}985735)(78{,}78448)}+\cdots$$
$$=3{,}061597\ \ t^{0{,}95}+3{,}283434\ \ t^{1{,}9}-$$
$$1{,}71775\ \ t^{2{,}85}+2{,}406867\ \ t^{2{,}85}-0{,}960232\ \ t^{3{,}8}-2{,}809718\ \ t^{3{,}8}+1{,}190721\ \ t^{4{,}75}+\cdots,$$
$$y(t)=3{,}061597\ \ t^{0{,}95}+3{,}283434\ \ t^{1{,}9}$$
$$+0{,}689117\ \ t^{2{,}85}-3{,}76995\ \ t^{3{,}8}+1{,}190721\ \ t^{4{,}75}+\cdots.$$

The RFDE solution in equation (19) for $\alpha = 0.1$ is

$$y(t)=y_0(t)+y_1(t)+y_2(t)+\cdots,$$
$$=\frac{3t^1}{\Gamma(1+1)}+\frac{6t^2}{\Gamma(2+1)}-\frac{4{,}5\Gamma(2+1)t^3}{\Gamma^2(1+1)\Gamma(3+1)}$$
$$+\frac{12t^3}{\Gamma(3+1)}-\frac{9\Gamma(2+1)t^4}{\Gamma^2(1+1)\Gamma(4+1)}$$
$$-\frac{18\Gamma(3+1)t^4}{\Gamma(1+1)\Gamma(2+1)\Gamma(4+1)}$$
$$+\frac{13{,}5\Gamma(2+1)\Gamma(4+1)t^5}{\Gamma^3(1+1)\Gamma(3+1)\Gamma(5+1)}+\cdots$$
$$=\frac{3t}{\Gamma(2)}+\frac{6t^2}{\Gamma(3)}-\frac{4{,}5\Gamma(3)t^3}{\Gamma^2(2)\Gamma(4)}+\frac{12t^3}{\Gamma(4)}-\frac{9\Gamma(3)t^4}{\Gamma^2(2)\Gamma(5)}$$
$$-\frac{18\Gamma(4)t^4}{\Gamma(2)\Gamma(3)\Gamma(5)}+\frac{13{,}5\Gamma(3)\Gamma(5)t^5}{\Gamma^3(2)\Gamma(4)\Gamma(6)}+\cdots$$
$$=\frac{3t}{1}+\frac{6t^2}{2}-\frac{4{,}5(2)t^3}{6}+\frac{12t^3}{6}-\frac{9(2)t^4}{24}-\frac{18(6)t^4}{(2)(24)}$$
$$+\frac{13{,}5(2)(24)t^5}{(6)(120)}+\cdots$$
$$=3t+3t^2-1{,}5t^3+2t^3-0{,}75t^4-2{,}25t^4+0{,}9t^5+\cdots,$$
$$y(t)=3t+3t^2+0{,}5t^3-3t^4+0{,}9t^5+\cdots.$$

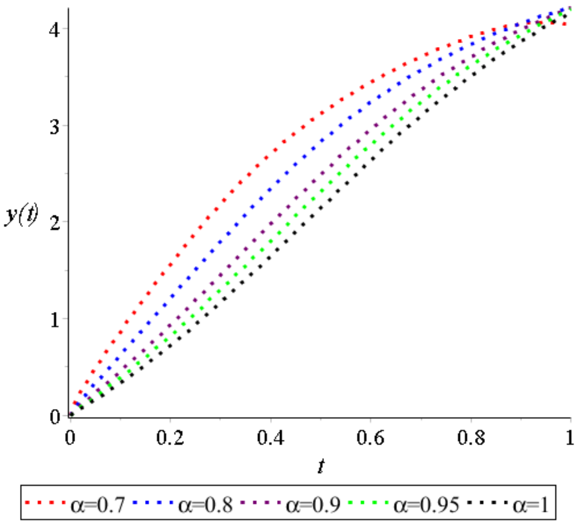


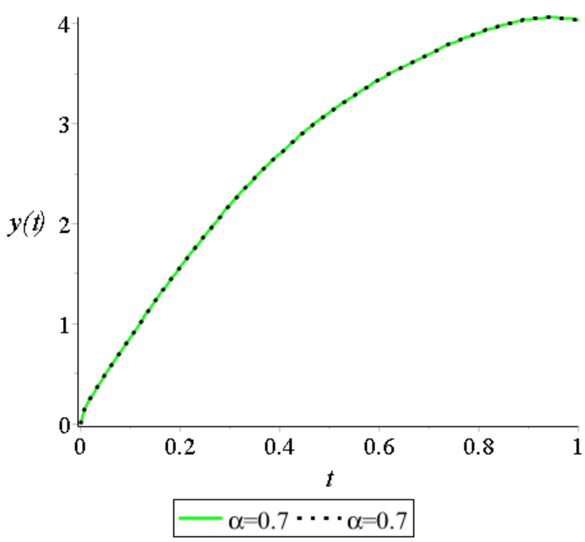
Figure 2. Approximate results RFDE using ADM-KFT

Furthermore, we have used Maple to graph the RFDE solution to equation (19) using the ADM-KFT for values $\alpha = 0.7; 0.8; 0.9; 0.95; 1$ and $0 \leq t \leq 1$, up to iteration k = 10. The numerical simulation for $0 \leq t \leq 1$ is presented in Figure 2.

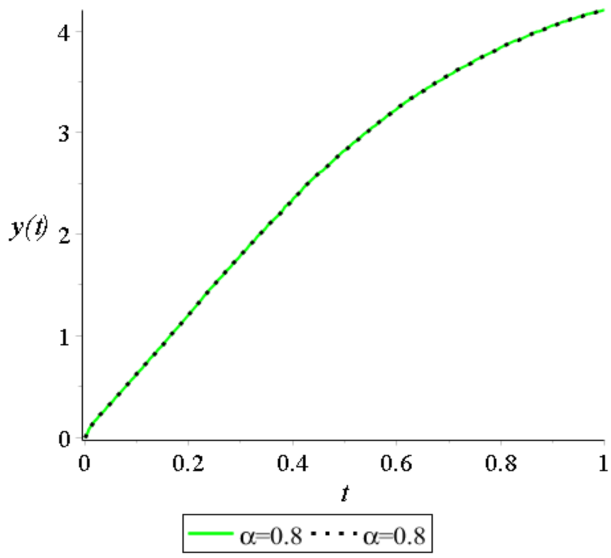
3.3. Comparative analysis ADM-LT and ADM-KFT

Figure 3 shows a graph comparison of the exact solution and the RFDE approach solution using ADM-LT (green smooth curve) and ADM-KFT (black dot) with parameters $\alpha = 0.7; 0.8; 0.9; 0.95; 1$ for $0 \leq t \leq 1$. It shows that the graphic is coincide.

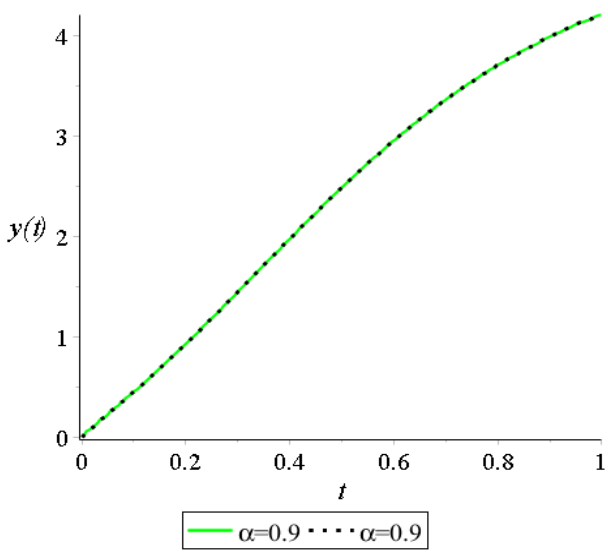
Furthermore, Figure 4 shows a comparison of the exact solution (red dotted curve) and the RFDE approximation solution using ADM-LT (green smooth curve) and ADM-KFT (black dot) for parameters $\alpha = 1$ and $0 \leq t \leq 1$. It shows that the graphic is coincide.



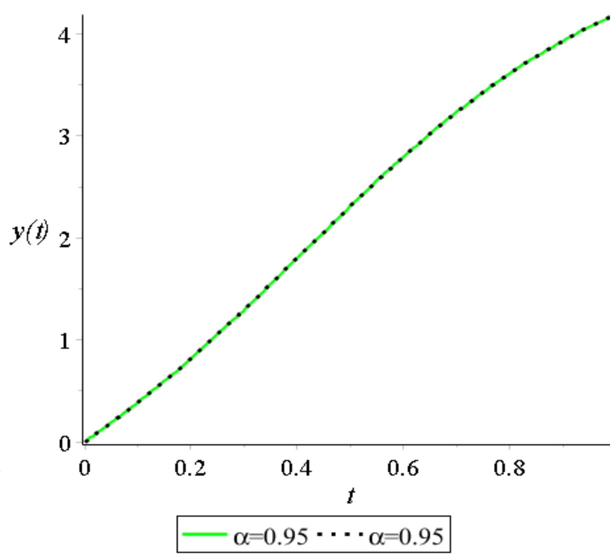
(a)



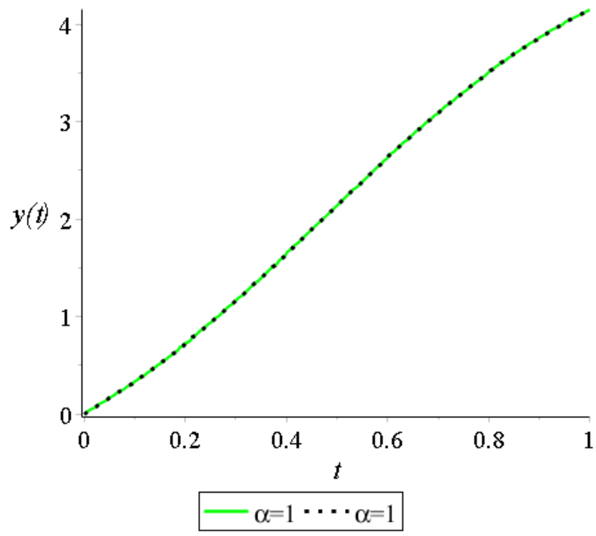
(b)



(c)



(d)



(e)

Figure 3. Comparison of approximate results RFDE using ADM-LT (yellow smooth curve) with ADM-KFT (black dots) for (a) $\alpha = 0.7$, (b) $\alpha = 0.8$, (c) $\alpha = 0.9$, (d) $\alpha = 0.95$, and (e) $\alpha = 1$

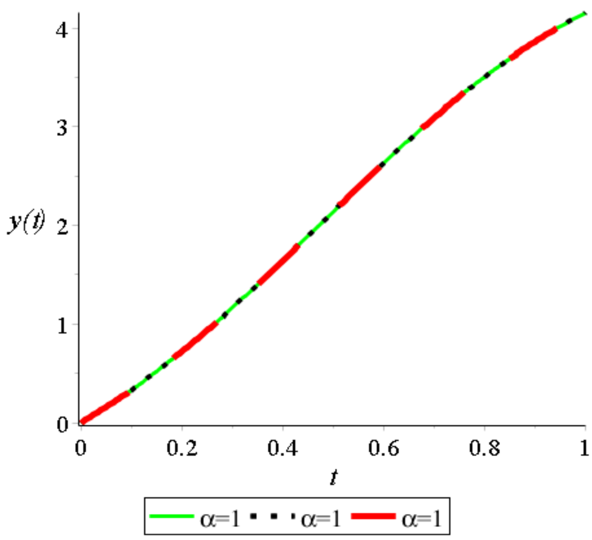


Figure 4. Comparison of exact solutions and RFDE approximate solutions

4. Conclusions

This paper examined the RFDE using the combined theorem of the ADM-LT and ADM-KFT. It is shown that the ADM-LT is consistence to the ADM-KFT algorithm for solving the RFDE. This has been proven by comparison of exact solutions and RFDE approximate solutions using the ADM-LT and ADM-KFT. The special solutions of these both methods have the same form, graphics, and results.

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Conflicts of Interest

The authors declare no conflict of interest.

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