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Steiner Antipodal Number of Graphs Obtained from Some Graph Operations

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Abstract The Steiner $p-$antipodal graph $SA_p(G)$ of a connected graph $G,$ has vertex set like G and p number of vertices are adjacent to each other in $SA_p(G)$ whenever they are $p-$antipodal in G. If G has more than one component, then p vertices are adjacent to each other in $SA_p(G)$ if at least one vertex from different components. Draw K_p related to $p-$antipodal vertices in $SA_p(G)$. The Steiner antipodal number $a_s(G)$ of a graph G is the smallest natural number p, so that the Steiner $p-$antipodal graph of G is complete. In this article, Steiner antipodal number has been determined for the generalized corona of graphs and for each natural number $p \geq 2$, we can construct many non-isomorphic graphs of order p having Steiner antipodal number p. Also for any pair of natural numbers $l, m \geq 3$ with $l \leq m$, there is a graph whose Steiner antipodal number is l and Steiner antipodal number of its line graph is m. For every natural number $p \geq 1$, there is a graph G whose complement $\bar{G}$ has Steiner antipodal number p.

Keywords Steiner n-eccentricity, Steiner Antipodal Number, Generalized Corona of Graphs, Line Graph, Complement Graph

2010 Mathematics Subject Classification: 05C12

1. Introduction

Finite simple graphs are considered throughout this paper. The subgraph induced by A in G is represented by $\langle A \rangle$, where the set A is the collection of vertices of G. In [14], the *Steiner distance* of the set A in G indicated by $d_G(A)$, is described as the minimum size among all connected subgraphs of G whose vertex set contains A. For every vertex $v \in G$, the Steiner n-eccentricity $e_k(v) = \max\{d_G(A): A \subseteq V(G)$ with $v \in A$ and $|A| = k\}$. The n*-radius* $rad_n(G)$ and n*-diameter* $diam_n(G)$ of G are least and biggest Steiner n-eccentricity among the vertices of G respectively. Further, the notion of Steiner distance is addressed in [5, 16, 18].

In [1], we initiated the idea of Steiner antipodal number of a graph G. Each p vertex of a graph G is said to be p-antipodal to each other if the Steiner distance between them is $diam_p(G)$ of G. The Steiner $p-$antipodal graph $SA_p(G)$ of a connected graph $G,$ has vertex sets like G and p number of vertices are adjacent to each other in $SA_p(G)$ whenever they are $p-$antipodal in G. If G has more than one component, then p vertices are adjacent to each other in $SA_p(G)$ if at least one vertex from different components. Draw K_p related to $p-$antipodal vertices in $SA_p(G)$. The Steiner *antipodal number* $a_s(G)$ of a graph G is the smallest natural number p, so that the Steiner $p-$antipodal graph of G is complete. When $p=2$, Steiner p-antipodal graph of G matches with antipodal graph of G. The concept of graph operator has found various applications in chemical research [10, 11]. Let H be any connected graph of order n and let $H_1, H_2, \dots, H_n$ be an arbitrary collection of simple graphs. The generalized corona, symbolized by $\tilde{o} \wedge_{i=1}^{n} H_i$, is obtained by taking one copy of graphs $H, H_1, H_2, \dots, H_n$

and joining the i^{th} vertex of H to every vertex of H_i [7]. The line graph $L(G)$ of a graph G has vertices corresponding to the edges of G and two vertices are adjacent in $L(G)$ if their equivalent edges of G have a common end vertex [7]. Line graph parameters are used to evaluate the complexity of molecular graphs and design of novel topological indices [3, 4]. The complement graph of a graph G denoted by $\overline{G}$ has vertex set $V(G)$ and two vertices are adjacent in $\bar{G}$ whenever they are not adjacent in G [2]. The Complete graph K_n is a graph on n vertices such that any two distinct vertices are adjacent. The empty graph is a graph whose vertex and edge sets are empty. The graph G attained from K_{1,p_1} and K_{1,p_2} by joining their centers by an edge is known as bistar and is symbolized by $B(p_1, p_2)$. The graph $K_{1,n-1}$ is identified as star. The Complete bipartite graph $K_{m,n}$ has two partitions with m and n number of vertices such that any two vertices are adjacent whenever they are in different partitions. The join $G = G_1 + G_2$ of graphs $G_1$and G_2has vertex set as vertices $G_1 \cup G_2$and edge set as edges of G_1, G_2 and each vertex in $G_1$is adjacent to all the vertices of G_2. We follow [5] for graph theoretic terminology.

We use the subsequent results for the proof of our results.

Lemma 1.1 [1, Proposition 2.5] *If a graph G is disconnected but not totally disconnected, then $a_S(G) = 3$.*

Lemma 1.2 [1, Theorem 2.2] *For a graph $G, a_S(G) = 2$ if and only if G is either complete or totally disconnected.*

Lemma 1.3 [1, Proposition 2.4] *Let S be the set of all full degree vertices of a graph G of order p. Then $a_S(G)$ is $p - |S| + 1$ when $G - S$ is disconnected and $p - |S|$ when $G - S$ is connected with at least one pendant vertex.*

Lemma 1.4 [1, Proposition 2.3] *For any tree T on m vertices with $p(\neq m-1)$ pendant vertices, $a_S(T) = p + 2$.*

Lemma 1.5 [1, Proposition 2.3] *For any complete bipartite graph $K_{m,n}$ with $m \leq n$ and $m \neq 1$, $a_s(K_{m,n}) = m + 1$.*

2. Main Results

2.1. Steiner Antipodal Number for Generalized Corona of Graphs

Theorem 2.1.1 *Let H be any connected graph with n vertices. If $H_1, H_2, \ldots, H_n$ be an arbitrary collection of simple graphs of order $t_1, t_2, \ldots, t_n$ $(t_i \geq 1)$ respectively, then $a_s(H\tilde{o} \wedge_{i=1}^{n} H_i) = m + 2$, where $m = t_1 + t_2 + \cdots + t_n$.*

Proof. Let $V = \{v_1, v_2, \ldots, v_n\}$ be the vertex set of H. For $1 \leq i \leq n$, let $Z_i = \{v_{i1}, v_{i2}, \ldots, v_{it_i}\}$ be the vertex set of each graph H_i. Let $Z = Z_1 \cup Z_2 \cup \ldots \cup Z_n$. Assume $G = H\tilde{o} \wedge_{i=1}^{n} H_i$.

Case.1 Structure of H is necessary to find k-eccentricity of any vertex $v \in G$ for all $3 \leq k \leq n-1$.

The adjacency between the vertices of H is not known, to find $e_{n-1}(v_i)$, first we fix a vertex $v_i \in H$ for $1 \leq i \leq n$ and then we have to choose the remaining $(n-2)$ vertices in G. Since $(n-1)$-eccentricity of v_i is the maximum Steiner distance of all $(n-1)$-element sets containing v_i, maximum Steiner distance of v_i is obtained only when we have to choose $(n-2)$-elements in different H_i other than H_i corresponding to v_i. Therefore, $(n-2)$-vertices in different $H_i's$ from v_i are connected by selecting $(n-2)$-vertices in H and hence these $(n-1)$-vertices have Steiner distance either $n-2$ or $n-1$ in H. That is, $e_{n-1}(v_i) = 2n-4$ or $2n-3$ and hence $(n-1)$-eccentricity of v_i is not unique for all $1 \leq i \leq n$. This proves Case.1.

But, to find k-eccentricity of any vertex $v \in G$ for all $k \geq n$, we do not need the structure of H. Now, $e_n(v_i) = 2n-2$ and $e_n(w) = 2n-1$ for all $v_i \in H$ and $w \in Z$ and hence $diam_n(G) = 2n-2$.

Case.2 $a_s(G) > n$.

Consider the set $\{v_i, v_j\} \cup A$, where $A \subseteq Z$ such that $|A| = n-2$ has Steiner distance less than or equal to $2n-3$ and hence $v_i$is not adjacent with $v_j$in $SA_n(G)$. Hence, $SA_n(G) \ncong K_{n+m}$.

Case.3 $a_s(G) > m + 1$.

Now, $e_\beta(v_i) = n + \beta - 2$ and $e_\beta(w) = n + \beta - 1$ for all $(n+1) \leq \beta \leq m$, $v_i \in H$ and $w \in Z$. Also $e_\beta(v_i) = e_\beta(w) = n + \beta - 2$ for $\beta = m + 1$. Therefore, $diam_\beta(G) = n + \beta - 2$ for all $(n+1) \leq \beta \leq (m+1)$. Consider the set $\{v_i, v_j\} \cup B$, where $B \subseteq Z$ with $|B| = \beta - 2$ has Steiner distance less than or equal to $n + \beta - 3$ and hence $v_i$is not adjacent with $v_j$in $SA_\beta(G)$ for all $(n+1) \leq \beta \leq (m+1)$. Therefore, $SA_\beta(G) \ncong K_{n+m}$ and hence $a_s(G) > m + 1$.

Case.4 $a_s(G) = m + 2$.

Now, $e_{m+2}(v_i) = e_{m+2}(w) = n + m - 1$ for all $v_i \in H$ and $w \in Z$. Hence $diam_{m+2}(G) = n + m - 1$. Take any $(m+2)$-element sets with exactly two elements from V always has Steiner distance $n + m - 1$. Hence in $SA_{m+2}(G)$, the subgraphs induced by $\langle V \rangle$ and $\langle Z \rangle$ are complete, and $\langle V \rangle$ and $\langle Z \rangle$ are mutually adjacent. Hence the result follows.

Theorem 2.1.2 For each natural number $p \geq 2$, there exists a graph G on p vertices whose $a_s(G) = p$.

Proof. Let $p \geq 2$ be any natural number. Lemma 1.2 demonstrates that the conclusion is valid for $p = 2$ when $G = K_2$. Lemma 1.1 reveals that the conclusion is valid for $p = 3$ when $G = K_2 \cup K_1$. Lemma 1.4 illustrates that conclusion is valid for $p = 4$ when $G = P_4$. Lemma 1.4 proves that the conclusion is valid for $p = 5$ by taking G as shown in Figure 1.

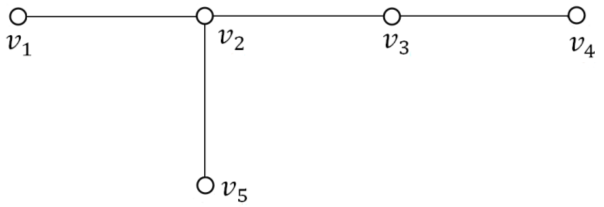

Figure 1. Graph G

Construct G_1 as given below: Let P_4 be the path on 4 vertices and let H_1, H_2, H_3 and H_4 be simple graphs such that H_1 and H_4 are empty, H_2 and H_3 are of order t_1 and t_2 respectively. Assume G_1 is given by the generalized corona product $P_4 \tilde{o} \bigwedge_{i=1}^{4} H_i$ as shown in Figure 2.

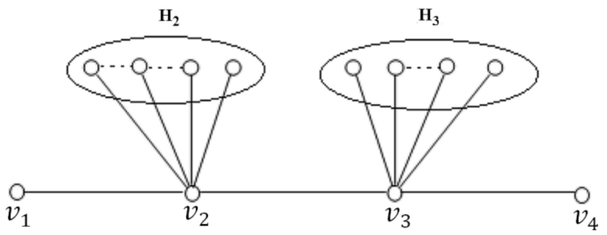

Figure 2. Graph G_1

Assume $p \geq 6$. Take $t_1 + t_2 = p - 4$. Consider, $e_{p-1}(w) = p - 1$ for all $w \in G_1$ and hence $diam_{p-1}(G_1) = p - 1$. Consider the set $\{v_2, v_3\} \cup C$ where $C \subseteq V(G_1)$ with $|C| = p - 3$ has Steiner distance equal to $p - 2$. Therefore, v_2 and v_3 are not adjacent in $SA_{p-1}(G_1)$ and hence $a_s(G_1) > p - 1$. Since $e_p(w) = p - 1$ for all $w \in G_1$, $diam_p(G_1) = p - 1$. Thus, $SA_p(G_1) \cong K_p$. This completes the proof.

Remark 2.1.3 *If we take 2 different graphs H_2 and H_3 of orders t_1 and t_2 respectively and connect every vertices of H_i to vertex $v_i, i = 2, 3$, then also we get the Steiner antipodal number as in Theorem 2.1.2, for the resultant graph. Thus, we can construct many non-isomorphic graphs of order k having Steiner antipodal number k.*

2.2. Steiner Antipodal Number for Line Graph of Some Graphs

Proposition 2.2.1 *For every star graph $K_{1,p-1}$ with p number of vertices, $a_S(L(K_{1,p-1})) = 2$.*

Proof. Line graph of star graph on p vertices is K_{p-1}. By Lemma 1.2, the result follows.

Proposition 2.2.2 *For any bistar graph* $\mathrm{B}(\mathrm{p}_1, \mathrm{p}_2)$, $a_S\left(L\left(\mathrm{B}(\mathrm{p}_1, \mathrm{p}_2)\right)\right) = p_1 + p_2 + 1$.

Proof. Line graph of bistar graph $B(p_1, p_2)$ on $p_1 + p_2 + 2$ vertices is a graph obtained by identifying a vertex of K_{p_1+1} with a vertex of K_{p_2+1}. This shows that $L\left(\mathrm{B}(\mathrm{p}_1, \mathrm{p}_2)\right)$ has a full degree vertex of degree $p_1 + p_2$ and hence whose removal disconnects the graph. Therefore, by Lemma 1.3 we get $a_S\left(L\left(\mathrm{B}(\mathrm{p}_1, \mathrm{p}_2)\right)\right) = (p_1 + p_2 + 1) - 1 + 1$. This concludes the proof.

Theorem 2.2.3 *If G is a tree of order m which is neither a star nor a bistar, then $a_S(L(G)) = a_S(G)$.*

Proof. Assume G is any tree with m vertices other than star and bistar with p pendant vertices. Let $v_1, v_2, \ldots, v_p$ be the vertices in $L(G)$ corresponding to those p pendant edges of G.

Since $L(G)$ has $m - 1$ vertices, $d(S) \leq m - 2$ for every subset S of $V(L(G))$. Now, $e_n(v) = m - 2$ for $n = p + 1$ and $v \in L(G)$. Let ξ_i and ξ_j be two non pendant edges of G. For any set $Y \subseteq \{v_1, v_2, \ldots, v_p\}$ with $|Y| = p - 1$, the Steiner distance of the set $\{\xi_i, \xi_j\} \cup X$ is less than $m - 2$ and hence ξ_i and ξ_j are non-adjacent in Steiner $(p+1)$-antipodal of $L(G)$. Since $(p+2)$ - diameter of $L(G)$ is $m - 2$ and $\{\xi_i, \xi_j, v_1, v_2, v_3, \ldots v_p\}$ has Steiner distance $m - 2$ in $L(G)$, the Steiner $(p+2)$-antipodal of $L(G)$ is K_m. Therefore, $a_S(L(G)) = p + 2$. By Lemma 1.4, the result follows.

Theorem 2.2.4 *For every $K_{\mathrm{m,n}}$ with* $\mathrm{m} \leq \mathrm{n}$ *and* $\mathrm{m} \neq 1$, $a_S(L(K_{\mathrm{m,n}})) = \mathrm{n} + 1$.

Proof. Consider $\{\alpha_1, \alpha_2, \ldots, \alpha_{\mathrm{m}}\}$ and $\{\gamma_1, \gamma_2, \ldots, \gamma_{\mathrm{n}}\}$ are the two partitions of $K_{\mathrm{m,n}}$. Let $\{e_{i,j}: 1 \leq i \leq \mathrm{n}, 0 \leq j \leq \mathrm{m} - 1\}$ be the vertices of $L(K_{\mathrm{m,n}})$. For each vertex $e_{i,j}$,

$$e_k(e_{i,j}) = \begin{cases} 2k - 2 \quad , & if \quad k \leq \mathrm{m} \\ k + \mathrm{m} - 2, & if \quad \mathrm{m} + 1 \leq k \leq \mathrm{n} \\ \mathrm{m} + \mathrm{n} - 1, & if \quad \mathrm{n} + 1 \leq k \leq \mathrm{mn} \end{cases}$$

Since any set of n vertices having the vertices $e_{1,0}$ and $e_{1,1}$ has Steiner distance less than k -radius, $r_S(L(K_{\mathrm{m,n}})) > \mathrm{n}$. By graph symmetry, if $e_{1,0}$ is adjacent to all the remaining vertices in $SA_{\mathrm{n}+1}(L(K_{\mathrm{m,n}}))$, then the result follows. By division algorithm, $\mathrm{n} = l\mathrm{m} + r$. The set $S = \{e_{i,j}: 1 \leq i \leq l\mathrm{m} + r$ and $j = (i-1) \, mod \, \mathrm{m}\}$ is a set of n vertices whose Steiner distance is $\mathrm{m} + \mathrm{n} - 2$. By adding any vertex $e_{i,j} \in V(L(K_{\mathrm{m,n}})) - S$, the Steiner distance of $S \cup e_{i,j}$ is $\mathrm{m} + \mathrm{n} - 1$. Thus $e_{1,0}$ is adjacent all the vertices of $SA_{\mathrm{n}+1}(L(K_{\mathrm{m,n}}))$. Hence the result follows.

Theorem 2.2.5 *For every pair of natural numbers $l, m \geq 3$ with $l \leq m$ there is a graph G whose $a_s(G) = l$ and $a_s(L(G)) = m$.*

Proof. By taking m = $l - 1$ and n = $m - 1$ in Lemma 1.5 and Theorem 2.2.4, the result follows.

2.3. Steiner Antipodal Number for Complement of Some Graphs

Observation 2.3.1 *For every complete graph, the Steiner antipodal number of its complement is 2.*

Proof. The observation is attained using Lemma 1.2.

Proposition 2.3.2 *For every complete k −partite graph other than complete graph, the Steiner antipodal number of its complement is 3.*

Proof. The Complement of complete k −partite graph is the union of k number complete graphs and hence by Lemma 1.1, the result follows.

Proposition 2.3.3 For every natural number n ,

$a_s(\overline{P_n}) = \begin{cases} n, & if\ n = 1,2,3 \\ 4, & if\ n \geq 4 \end{cases}$

Proof. When $n = 1$, the answer is clear. *When* $n = 2$, $\overline{P_n}$ *is totally disconnected and hence by lemma 1.2,* $a_S(\overline{P_2}) = 2$. When $n = 3$, $\overline{P_n} \cong P_2 \cup K_1$ and hence by lemma 1.1, $a_S(\overline{P_3}) = 3$. Assume $n \geq 4$. Consider $V(P_n) = \{v_1, v_2, \dots, v_n\}$ such that v_1 and v_n are vertices of degree one. In $\overline{P_n}$, $\{v_i, v_{i-1}, v_{i+1}\}$ are not adjacent for all $2 \leq i \leq n-1$. Also v_1 and v_n are non-adjacent with v_2 and v_{n-1} respectively. $e_3(v_i) = 3$ for all $1 \leq i \leq n$. Hence $diam_3(\overline{P_n}) = 3$. Steiner distance of the sets $\{v_1, v_n, v_i(i \neq 1, n)\}$ are 2 and hence v_1 and v_n are not adjacent in Steiner 3-antipodal of $\overline{P_n}$. Since 4-diameter of $\overline{P_n}$ is 3 , any set $\{v_i, v_j, v_k, v_l\}$ has Steiner distance 3. Therefore, $SA_4(\overline{P_n}) \cong K_n$ and hence the result follows.

Proposition 2.3.4 For every natural numbers m and n with $m \leq n, a_S(\overline{B_{m,n}}) = m + 3$

Proof. Let $V = \{\mu, \mu_1, \mu_2, \dots, \mu_m\} \cup \{\tau, \tau_1, \tau_2, \dots, \tau_n\}$ be the vertex set of $B_{m,n}$ such that μ and τ are non-pendant vertices. In $\overline{B_{m,n}}$, μ_i's are not adjacent with τ and τ_i's are not adjacent with μ, μ and τ are also non-adjacent. Therefore, $e_{m+2}(w) = m + 2$, for all $w \in \overline{B_{m,n}}$. Hence, $diam_{m+2}(\overline{B_{m,n}}) = m + 2$. Consider the set $\{\mu_i, \tau_j\} \cup A$ where $A \subseteq V(\overline{B_{m,n}})$ such that $|A| = m$ has Steiner distance $m + 1 < m + 2$ and hence μ_i and τ_j are non-adjacent in $SA_{m+2}(\overline{B_{m,n}})$. Since $(m + 3) -$ diameter of $\overline{B_{m,n}}$ is $m + 2$ and any $(m + 3)$-elements set has Steiner distance $(m + 2)$, the Steiner $(m + 3)$-antipodal graph of $\overline{B_{m,n}}$ is K_{m+n+2} and hence the result follows.

Theorem 2.3.5 For every natural number $p \geq 1$, there exists a graph G whose complement having Steiner antipodal number p.

Proof. By Propositions 2.3.3 and 2.3.4, the result follows.

Theorem 2.3.6 *If* G *is a* n *vertices graph and* $\bar{G}$ *is disconnected, then* $a_s(G + \bar{G}) = n + 1$.

Proof. Let $V = \{\alpha_1, \alpha_2, \dots, \alpha_n\} \cup \{\alpha_1', \alpha_2', \dots, \alpha_n'\}$ be the vertex set of $G + \bar{G}$ such that $\alpha_i \in G$ and $\alpha_i' \in \bar{G}$ for all $1 \leq i \leq n$. In $G + \bar{G}$, $e_n(\alpha_i) = n - 1$ for all $\alpha_i \in G$ and $e_n(\alpha_i') = n$ for all $\alpha_i' \in \bar{G}$. Hence, $diam_n(G + \bar{G}) = n$. Consider the set $\{\alpha_i, \alpha_i'\} \cup B$ where $B \subseteq V(G + \bar{G})$ such that $|B| = n - 2$ has Steiner distance $n - 1 < n$ and hence α_i and α_i' are not adjacent in $SA_n(G + \bar{G})$. Since $e_{n+1}(w) = n$ for all $w \in V(G + \bar{G})$, $diam_{n+1}(G + \bar{G}) = n$. Any $(n + 1)$ -elements set has Steiner distance n in $G + \bar{G}$. Therefore, $SA_{n+1}(G + \bar{G}) \cong K_{2n}$ and hence the result follows.

3. Conclusions

Generalized corona of graphs, Line graphs and Complement of graphs are useful in chemical graph theory. This research explores the Steiner antipodal number ($a_s(G)$) for generalized corona of graphs, line graph of any tree and we provided characterization of Steiner antipodal number of a graph in terms of line graph and complement of a graph.

4. Future Scope

In our further research, we intend to focus on following points: (i) Characterization for Steiner antipodal number of graphs using generalized edge corona operation (ii) Relation between Steiner radial number and Steiner antipodal number of graphs for lexicographic product operation. (iii) Steiner antipodal number of graphs for Mycielskians of a graph.

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