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Tachyons, the Four-Momentum Formalism and Simultaneity

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Abstract Since the dawn of relativity in the first two decades of the twentieth century, it has been maintained that faster-than-light (superluminal) motion could produce time travel into the past with its accompanying causality-violating paradoxes; hence, it was concluded that it is impossible to exceed the speed of light. Tachyons, hypothetical particles that always move faster than the speed of light, seem to present serious challenges to our understanding of reality and the mathematics used by physicists to explain it, if they exist. This paper addresses these problems from a classical perspective and presents solutions to the perplexities posed by superluminal motion. Specifically, classical analyses ruling out superluminal motion are shown to be flawed. Thus there are no classically-valid objections to such phenomena. To the contrary, correct analysis leads to the conclusion that tachyons could send signals faster that the speed of light yet *not* violate causality. There are, however, some limitations to how fast such signals can propagate with respect to the receiver; that is, for one point, they cannot be *infinitely* fast. For another, they cannot violate the relativity of simultaneity. These are important points since neglecting them has led to the claim that tachyons cannot exist because they produce phenomena that are irreconcilable with reality.

Keywords Tachyons, Special Relativity, Simultaneity, Causality, Communication

1 Introduction

1.1 Arguments against superluminal phenomena

In 1907 Albert Einstein considered it to be "sufficiently proven" that any velocity greater than that of light is an impossibility [1] by analysis of the Lorentz transformation equations (LTE). Given an inertial frame moving at velocity v with respect to a "stationary" frame, the time differential in the moving frame over a distance Δx in the stationary frame is

$$\Delta t' = \gamma(\Delta t - \frac{v\Delta x}{c^2}) \quad (1)$$

where Δt refers to the time differential in the "stationary" frame, c is the speed of light and $\gamma = 1/\sqrt{1 - v^2/c^2}$. He concluded that for an object moving a distance Δx in a time Δt less than $v\Delta x/c^2$, $\Delta t'$ would be negative, implying that any such speedy object would arrive at its destination before it departed from its origination point, according to a moving observer. Similarly, Richard Tolman pointed out in 1917 that velocities greater than the speed of light (specifically, greater than c^2/v where v is the velocity of a moving observer) presented the possibility that effect could precede cause [2].

The assertion that causality can be violated by superluminal travel is also a mainstream thought in this century. David Mermin wrote [3],

"In the [moving] frame ... the object is in two different places at the same time! This is such a bizarre situation that one's suspicion is strengthened that the difficulty we have already encountered in producing an object moving faster than light must be a reflection of the impossibility of such motion."

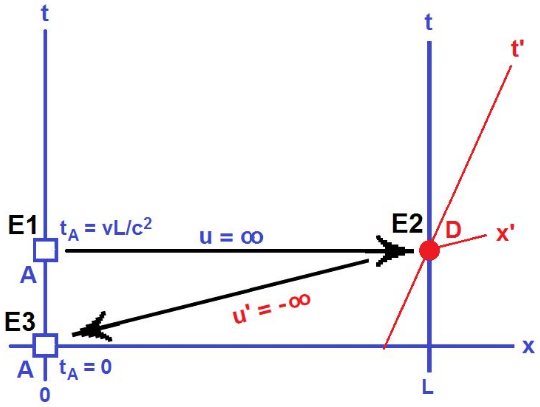

Figure 1. Typical Minkowski diagram showing purported causality violation. A and D are assumed to have some technology that allows superluminal communication.

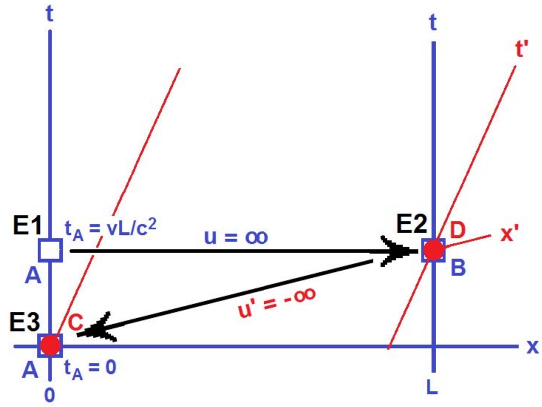

Figure 2. Minkowski diagram with additional participants. Superluminal communication occurs solely between observers at rest with each other (A to B and D to C). B receives a superluminal message from A, who has originated it at E1, and passes it to D by light-speed signals when they are momentarily adjacent, and C passes the message to A when *they* are momentarily adjacent. A presumably receives the message before it has been originated.

Figure 1 is a Minkowski diagram depicting the conventional view that superluminal communication results in causality violation [4]. The horizontal axis is distance and the vertical axis is the time axis in the "stationary" frame (labeled t), and the axes in the moving frame are labeled x' and t'. What is considered "stationary" and what is considered "moving" are, of course, arbitrary; however, the viewpoint in Figure 1 is from the frame in which A is at rest, as indicated by the fact that the t and x axes are perpendicular to each other. A and D are observers that have the hypothetical capability of sending signals to each other faster than the speed of light. The word "observer" means a conscious entity or a device that can detect local objects, indicate positions and local time, and relay that data to a conscious entity. Observer D is moving at some positive velocity, v, with respect to A, where v is less than c. In accord with the LTE, the axes of the moving frame, x′ and t′, are tilted with respect to the stationary frame, the t′ axis of the moving frame being defined by $t = x/v$ and the x′axis being defined by $t = vx/c^2$, where t and x are coordinates of the stationary frame. D is at x = L when the superluminal signal is received, and its clock reads $t_D{}' = 0$ at that point.

According to this view, A originates a signal at event E1, at time $t = vL/c^2$, and transmits it to D faster than the speed of light. The speed of the signal is represented as being infinitely-fast in Figure 1 by the horizontal black arrow. D receives the signal at time $t_D{}' = 0$, at event E2 and then transmits it instantaneously (in D's frame) back to A at time $t_A = 0$, at event E3. The downward-sloping, leftward-going black arrow follows the x' axis, indicating that the speed of the signal is infinitely-fast in the moving frame ($\Delta t' = 0$). Thus A at event E3 receives the signal from his future self at event E1, which apparently allows the earlier A to receive messages from the future. If such is possible, then causality can be violated. The speed of the signals need not be infinitely fast to produce this apparent problem, but it does need to be faster than the speed of light, and significantly faster in most cases.

Figure 1 depicts direct tachyon communication between observers in relative motion and will be referred to as Method I in this paper. Alternatively, a message can be passed to momentarily-adjacent observers in relative motion, and then sent superluminally to partners at rest with respect to each other, as depicted in Figure 2. This will be referred to as Method II.

1.2 Arguments for superluminal phenomena

The scenarios described in Figure 1 and Figure 2 represent particular subsets of the parameter space. For example, if either u or v in Equation (1) changes sign (that is, u and v are in opposite directions), then $\Delta t'$will always be greater than zero and there will be no causality violation. It seems quite peculiar that simply reversing direction of a variable can change such a benign outcome into one that has the catastrophic effect of violating causality. Perhaps there are physical restrictions to superluminal signalling that prevent the causality catastrophe.

Bilaniuk, Deshpande and Sudarshan considered superluminal particles in the context of and consistent with special relativity [5]. G. Feinberg later coined the name "tachyon" for a particle that always travels faster than the speed of light, satisfies the principle of relativity and is Lorentz-invariant [6]. The limiting value is c, but, as Feinberg points out, a limit has two sides. Thus in this context there are basically three types of particles: bradyons (those that always travel *slower* than the speed of light), luxons (those that always travel *at* the speed of light) and tachyons (those that always travel *faster* than the speed of light).

The possibility of "backward in time" phenomena with tachyons is reminiscent of Wheeler's concept of antimatter particles being normal matter particles moving backward in time [7]. Alternatively, a "reinterpretation principle" has been proposed wherein, although such tachyons would have "negative" energy moving backward in time, they would have positive energy moving forward in time [5], thus preserving causality. Others call this the "switching procedure" wherein the source of the tachyons becomes the receiver and the receiver becomes the source as perceived by certain observers.

Solutions of general relativity include closed timelike curves which present the possibilty of violating causality, but limitations have been proposed to preclude such action [8,9]. Since localized closed timelike curves appear to be very similar to tachyon physics [10], tachyons, if they exist, would not violate causality by the hypothetical limitations proposed for general relativity.

The existence of tachyons has neither been verified nor refuted; however, many papers have been published about the possibility of neutrinos being tachyons [11,12,13,14]. A lot of hyperbole has been generated, based on the four-vector momentum and the LTE, which asserts that both Method I and Method II can violate causality. This paper will demonstrate that superluminal communication by either method does *not* present the bizarre possibilities of sending messages backward in time when appropriate physics is applied.

2 Method I (Tachyon Dynamics)

The Lorentz transform and the Minkowski diagram are kinematic representations of reality, concerned with geometrically possible motion, but does not address dynamics, which considers the effects of momentum and energy. The 4-vector momentum for an object with mass, m, moving with velocity, v, is [15]

$$\mathbf{P} = (\gamma mc, \gamma m v_x, \gamma m v_y, \gamma m v_z) \tag{2}$$

and the inner product of **P** with itself is the well-known relativistic energy equation,

$$E^2 - p^2c^2 = m^2c^4. \tag{3}$$

E and p are real numbers for all three types of particles. For bradyons, because m^2c^2 is greater than zero, E^2 is always greater than p^2c^2. For tachyons, however, m is imaginary, so p^2c^2 is always greater than E^2. With m imaginary, p is real because the 3-vector velocity of the tachyon, u, is greater than c, so the imaginary unit in m cancels the imaginary unit in the denominator created by reversing the sign of $(1 - u^2/c^2)^{1/2}$: $p = mu\sqrt{u^2/c^2 - 1}$, where m is now the absolute value of the tachyon mass. Rewriting (3) for tachyons leads to

$$E^2 = \frac{m^2u^2c^2}{u^2/c^2 - 1} - m^2c^4 = \frac{m^2c^4}{u^2/c^2 - 1}. \tag{4}$$

Again, the m in (4) is the absolute value of the tachyon invariant mass. This shows that E, the energy of a tachyon, approaches zero as the tachyon velocity, u, approaches infinity. The energy of bradyons is *never* less than the invariant mc^2 energy, in contrast with tachyons. This presents a bit of a problem with tachyon dynamics. From the perspective of a different inertial frame, the four-momentum is

$$\mathbf{P}' = \eta \mathbf{P} \tag{5}$$

where, for an observer in motion along the x-axis,

$$\eta = \begin{bmatrix} \gamma & (-\gamma v/c) & 0 & 0 \\ (-\gamma v/c) & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

For motion solely along the x-axis, the y and z axes are superfluous, so they can be ignored. Thus

$$\mathbf{P} = [mc/(u^2/c^2 - 1)^{1/2}, mu/(u^2/c^2 - 1)^{1/2}]. \tag{7}$$

So

$$\mathbf{P}' = \frac{\gamma m}{(u^2/c^2 - 1)^{1/2}}[(c - uv/c), (u - v)]. \tag{8}$$

The energy component of $\mathbf{P}'$ is

$$E'/c = \gamma m(c - uv/c)/(u^2/c^2 - 1)^{1/2}. \tag{9}$$

For any value of $u > c^2/v$, E' will be negative. The "reinterpretation" principle proposed by Bilaniuk *et al* reverses the direction of the tachyon, making it go forward in time with positive energy [5]; however, note that the three-vector momentum of $\mathbf{P}'$ is

$$p_x{}' = \gamma m(u - v)/(u^2/c^2 - 1)^{1/2}. \tag{10}$$

thus $p_x{}'$ *never* changes sign with p_x since u is always greater than c and v is always less than c, indicating that the tachyon *never* reverses direction. Thus by the four-vector momentum formalism, if a tachyon is moving in a positive direction in one inertial frame, it's moving in the positive x' direction in all frames (the coordinate system is assumed to be oriented so the tachyon moves along the x-axis in the "stationary" frame). This contradicts the reinterpretation principle.

So what is wrong? What does negative energy mean? Could a tachyon draw energy from a bradyon with which it collides? Is there something wrong with tachyon four-momentum mathematics? Relativistic dynamics has a kinematics basis. Equation (9) can be derived from

$$E'/c = mc/(u'^2/c^2 - 1)^{1/2} \tag{11}$$

by noting from the kinematic Lorentz transform that

$$u' = \frac{u - v}{1 - uv/c^2} \tag{12}$$

Thus,

$$E'/c = mc/[(u - v)^2/(1 - uv/c^2)^2 - 1]^{1/2}. \tag{13}$$

It is customary to move the $(1 - uv/c^2)$ term into the numerator:

$$E'/c = \gamma mc(1 - uv/c^2)/(u^2/c^2 - 1)^{1/2}, \tag{14}$$

thus agreeing with (8) and (9). This works for bradyons since uv is always less than c^2, so $(1 - uv/c^2)$ is always positive; thus squaring a positive number and then taking its square root presents no problem, and this is the result obtained by the four-momentum method. However, $(1 - uv/c^2)$ can be negative for tachyons, and squaring a negative number and then taking its square root is problematic. The result is dependent upon the order in which the mathematical operations are performed.

There are rules for this, but it is important to look at the physics, too.

The problem is in the term from (11):

$$u'^2/c^2 = (\frac{u-v}{1-uv/c^2})^2/c^2 \quad (15)$$

Because u'/c is squared, (15) *never* becomes negative (in fact, it's always greater than unity), and therefore E', by (11), is never negative, either. The four-momentum matrix, (6), is invalid for tachyons! The inconsistency between (9) and (11) for tachyons has been pointed out by A. Sfarti, wherein he concluded that tachyons don't exist [16]; however, as shown here, (9) is the result of a problematic mathematical operation whereas (11) is correct from fundamental physics (specifically, the Principle of Relativity).

It is still possible, however, to force negative energy as a solution since $([(\frac{u-v}{1-uv/c^2})^2 - 1]^{1/2}$ is a square root, and square roots have *two* solutions. If one chooses the negative root, then E' would be negative, and that's what the four-momentum formalism does; however, that solution is rather contrived. The real problem is that (12) has an anomaly at $u = c^2/v$ in that u' becomes infinite. This is equivalent to a singularity, which typically indicates that the physics model fails at that point (and beyond), so accepting *either* root at $u = c^2/v$ and beyond is invalid. Since E' approaches zero as u approaches c^2/v, and since any successful signal transmission requires at least *some* expenditure of energy, it is reasonable to postulate that it is not physically possible to send or receive a tachyon signal at infinite speed relative to the receiver. When this paper refers to infinite speed it is to be understood as an idealization with the awareness that it will signify some speed that merely approaches infinity. This is reasonable because one can *approach* infinity but infinity cannot be reached physically.

Harking back to (1), note that if v is negative, Δx can be any positive value up to infinity and $\Delta t'$ will be positive; that is, there is no causality violation. However, a signal sent at infinite speed will be received at speed c^2/v, resulting in a time delay of $v\Delta x/c^2$ in the stationary frame (but *no* time delay in the moving frame). Similarly, if v is positive and Δx is negative, there is no causality violation. The limitation of sending speed, $|u| < c^2/v$, where Δx and Δt are the same sign, brings balance to the operation since the round-trip time delay is $\Delta t = v\Delta x/c^2$ and $\Delta t' = \gamma v \Delta x/c^2$.

Thus the correct sequence of events is shown in Figure 3. A initiates a message at Event E1 (x = 0, t = 0) and sends it to D at velocity $u = c^2/v$. D receives it at velocity $u' = \infty$ and sends it back to A at velocity $u' = -c^2/v$. A receives the message back at $u = -\infty$ at $t = vL/c^2$, *after* it was originated. Consequently, Method I does *not* violate causality.

Consideration of tachyons with nearly zero energy (but *not* negative energy) implies the perception that they can be detected only in certain inertial frames but not in others; that is, they would be "ghost particles" in some inertial frames. This is not so, however, if we suppose that tachyons can be created by a tachyon transmitter consisting of bradyons, then tachyons can interact with normal matter. A tachyon receiver would also consist of normal matter interacting with tachyons. If the receiver is at rest relative to an observer, and both are moving

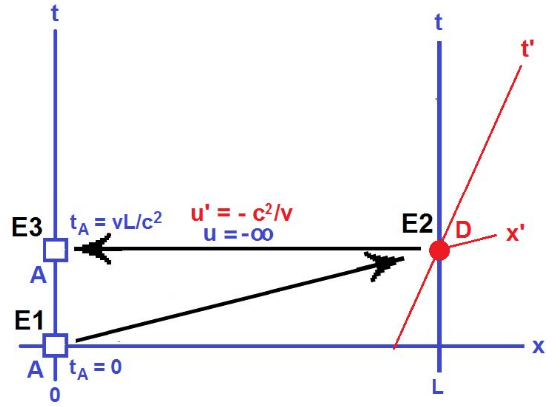

Figure 3. Minkowski diagram showing correct sequence of events governed by energy considerations. Causality violation is avoided.

away at velocity, v, from the tachyon source sending a signal at $u = \infty$, the observer can't receive the signal directly, but the observer could allow the receiver to move *toward* the source at speed, v. Thus the speed of the tachyon relative to the receiver could be nearly infinite and its energy, relative to the receiver, would be greater than zero. Therefore, the propagation time according to the receiver would be zero, opening up the possibility of a causality violation. This scenario is effectively Method II, which is discussed next.

3 Method II (Tachyon Kinematics)

Assuming the existence of tachyons, Method I provides a practical means of faster-than-light communication without causality violation. Contrarily, Method II has no pragmatic usefulness. It's sole purpose is to confirm or refute causality violation when relatively-moving participants are nearly adjacent and pass information one to another using bradyons or luxons, as depicted in Figure 2. It requires nanosecond timing and careful initial placement of the participants to pass this information because the two observers are not in close proximity for long, but (nearly) infinitely-fast tachyon signals should be allowed between participants which are at relative rest.

Method II appears to overcome the limitation on tachyon communication speed invoked by dynamical considerations, but there are other factors at work. The concept of the Minkowski diagram has been described as instantaneous layers of constant time [17]:

"We build a spacetime by taking instantaneous snapshots of space at successive instants of time and stacking them up."

In each snapshot, *all* participants appear. Figure 2 does not show B at t = 0 nor does it show C at $t = vL/c^2$. This deficiency is overcome in Figure 4 wherein three instantaneous snapshots are shown: at t = 0, at $t = vL/2c^2$ and at $t = vL/c^2$. Participant A initiates a message at $t = vL/c^2$ (event E1) and sends it to B (nearly) infinitely fast (since B is stationary with respect to A), then B passes it to D as D goes by (event E2), and D then sends it to C ... but *where* is C, or rather, *when* is C? A and B can verify that C is *not* at x = 0 when D sends the signal, whereas D can verify that C is! Does the signal sent by D go to C at event E3, or does it go to C at $t = vL/c^2$?

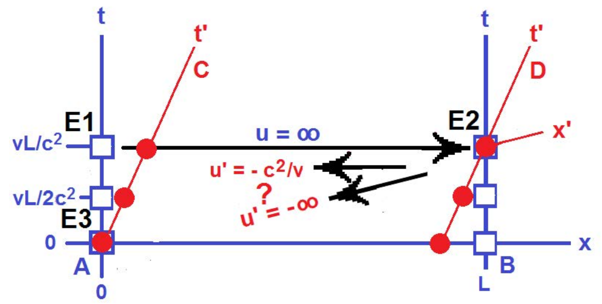

Figure 4. Minkowski diagram with additional participants. Modification of Figure 2 consistent with instantaneous snapshots of space. From the perspective of A and B, does C exist at $t = vL/c^2$ ($u' = -c^2/v$) or at t = 0 ($u' = -\infty$) when D launches the signal to C? (Figure 5 shows this problem from the perspective from C and D).

The Minkowski diagram in Figure 4 depicts the perspective of A and B. The perspective of C and D is in a different frame. David Morin has a procedure for dealing with this problem [18], and it becomes fundamental when tachyons are concerned:

"An extremely important strategy in solving relativity problems is to plant yourself in a frame and stay there. The only thoughts running through your head should be what you observe. That is, don't try to use reasoning along the lines of, 'Well, the person I'm looking at in this other frame sees such-and-such.' This will almost certainly cause an error somewhere along the way, because you will inevitably end up writing down an equation that combines quantities that are measured in different frames, which is a no-no."

According to A and B, D cannot send the signal faster than $u' = -c^2/v$ because $t_C{}' = \gamma vL/c^2$ and $t_D{}' = 0$. Thus a signal cannot be sent round-trip in this configuration since A isn't adjacent to C at $t = vL/c^2$. Figure 5 shows the perspective from C and D.

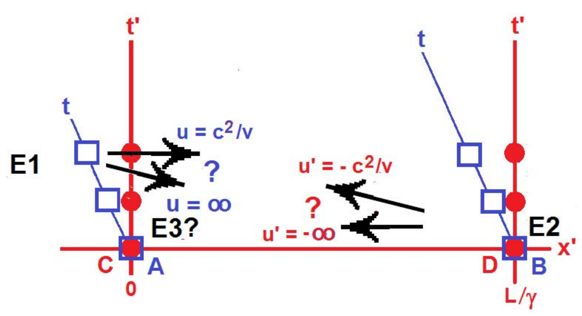

Figure 5. Minkowski diagram of the problem depicted in Figure 4 from the perspective of participants C and D. From their perspective, *when* does B exist?

From this perspective, D can send an infinitely-fast signal to C, but A (at $t_A = vL/c^2$) cannot send an infinitely-fast signal to B (at $t_B = vL/c^2$) since B is not at $t_B = vL/c^2$ when A attempts to send it, according to C and D. So, according to C and D, A can send the signal only slower than $u = c^2/v$; and according to A and B, D can send the signal to C only for $u' > -c^2/v$. Each pair of observers claims the *other* pair can't send signals faster than $|c^2/v|$.

Note that in Figure 5, in the layer of space at $t' = 0$, A and C are adjacent and B and D are adjacent, but B and D are *not* adjacent at t = 0 in Figure 4. This is because of the relativity of simultaneity (RoS) inherent in the Lorentz transform equations. Since RoS is a basic characteristic of spacetime modeled by the LTE, any thought experiment that purports to escape its consequences is more than suspect: it is wrong. That's why Figure 2 fails to complete a message loop and why causality is not violated.

The view from A and B in Figure 4 and the view from C and D in Figure 5 are inconsistent for completing a message loop since each pair claims the other pair has restricted signal speed. The solution is to restrict the speed in *both* frames. To complete a round-trip message loop consistently and successfully by Method II, the maximum signal speed according to the sender is exactly the same as determined for Method I. The participants would be configured as shown in Figure 6.

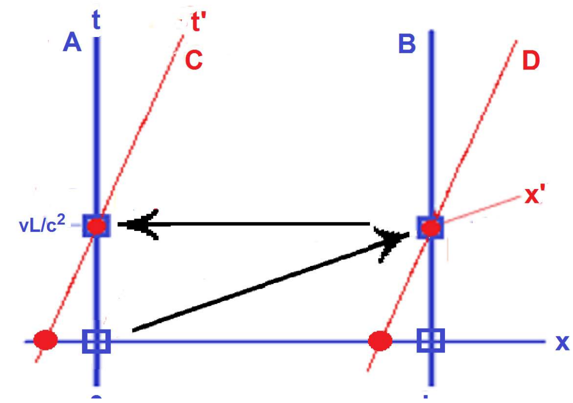

Figure 6. Minkowski diagram for Method II that affirms relativity of simultaneity and preserves causality.

A initiates a message at t = 0 and sends it to B at $u = c^2/v$. It arrives at $t = vL/c^2$ and B passes it immediately to D, whose clock reads $t_D{}' = 0$. D then sends it to C at $t_C{}' = \gamma vL/c^2$, and C immediately passes it to A at $t_A = vL/c^2$. RoS is obeyed and causality is preserved.

What about other observers traveling at some velocity v_2? If $v_2 < v$, the other observer can receive the signal and even participate (if he is in the proper initial position) without violating causality; and if $v_2 > v$, that observer won't be able to close the message loop unless the observers are properly positioned initially, in which case, causality will be preserved.

4 Conclusions

The Principle of Relativity, that is, that the laws of physics are the same in all inertial reference frames, is a fundamental postulate of the theory of relativity. Thus given that the energy of a superluminal particle with velocity, u, is

$$E = mc/(u^2/c^2 - 1)^{1/2}, \quad (16)$$

as observed from one inertial frame, then its energy in any other inertial frame is

$$E' = mc^2/(u'^2/c^2 - 1)^{1/2}, \quad (17)$$

where u' is the velocity of the particle in the new frame. For any value of u' over the range $-\infty < u' < +\infty$, $E' > 0$, hence,

Conclusion Number 1: *The four-momentum formalism is incorrect for tachyons since it predicts the possibility of negative energies, which is impossible by the Principle of Relativity.*

This is a fact which has been unrecognized since Bilaniuk *et al.* [5] first considered superluminal motion consistent with special relativity.

Conclusion Number 2: *Using the Principle of Relativity, it has been demonstrated that there is no valid argument leading to violation of causality for direct tachyon communication in a loop between transmitters and receivers in relative motion (Method I). On the contrary, it is adduced that this Method I will obey causality.*

This is an important point because textbooks and other sources have claimed that this method *does* violate causality [1,2,3,4,17,19].

Conclusion Number 3: *Since tachyon energy cannot be negative, the "reinterpretation principle" is not only unnecessary, but it is incorrect.*

Conclusion Number 4: *It has also been shown that supplying additional participants (Method II) most likely won't violate causality either, provided that tachyons and spacetime aren't imbued with fictitious properties.*

The limitations of rationality and relativity of simultaneity apply particularly to this approach, infringement of which has been responsible for assertions of causality violation. Relativity of simultaneity (RoS) is a fundamental consequence of relativity theory, and is basic to our real world, too. Method I obeys causality because energy is frame-dependent, and energy is frame-dependent because velocity is fundamentally frame-dependent for bradyons and tachyons. Thus Method I inherently obeys RoS, but claims that Method II violates causality rest on the supposition that RoS can be defeated, which is incorrect.

Conclusion Number 5: *Since the principle of relativity is a fundamental postulate and RoS is a fundamental consequence of relativity, assertions that tachyons would violate causality are unfounded.*

The Lorentz transform equations have been an excellent model of reality (in the absence of significant gravitational effects) for particles which travel slower or at the speed of light, but they place time and space on an equal footing, which presents problems when dealing with tachyons. At the "classical" level, where a large collection of particles (including observers) are involved, time is *never* observed to go backward; and there is good reason to believe that there is a basic asymmetry between time and space at the lowest levels of the real world [20]. Approaches to placing time and space back on an equal footing without suffering violation of causality include adding additional dimensions of time [21,22]. These may point toward future investigation but are beyond the scope of this paper.

5 Acknowledgments

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