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Exponential-Inverse Exponential[Weibull]: A New Distribution

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Abstract Statistical distributions play a major role in analyzing experimental data, and finding an appropriate one for the data at hand is not an easy task. Extending a known family of distribution to construct a new one is a long honored technique in this regard. The T-X[Y] methodology is utilized to construct a new distribution as described in this study. The T-inverse exponential family of distributions, which was previously introduced by the same authors, is used to examine the exponential-inverse exponential[Weibull] distribution (Exp-IE[Weibull]). Several fundamental properties are explored, including survival function, hazard function, quantile function, median, skewness, kurtosis, moments, Shannon's entropy, and order statistics. Our distribution exhibits a wide range of shapes with varying skewness and assume most possible forms of hazard rate function. The unknown parameters of the Exp-IE [Weibull] distribution are estimated via the maximum likelihood method for a complete and type II censored samples. We performed two applications on real data. The first one is vinyle chloride data, which is explained by [1] and the second is cancer patients data, which is explained by [2]. The significance of the Exp-IE[Weibull] model in relation to alternative distributions (Fréchet, Weibull-exponential, logistic-exponential, logistic modified Weibull, Weibull-Lomax [log-logistic] and inverse power logistic exponential) is demonstrated. When using the applied real data, the new distribution (Exp-IE[Weibull]) achieved better results for the AIC and BIC criterion compared to other listed distributions.

Keywords Quantile Function, Shannon's Entropy, T-X[Y] Framework, T-IE Family

1 Introduction

To accommodate the massive increase in the variety of data created in real life, there has been a rise in interest in constructing new and more flexible statistical distributions. Recently, numerous academicians have indicated an interest in expanding the generating family in an effort to improve data analysis.

A number of well-known generating families are; beta-G [3] used the beta distribution as a generator function. The beta generated distribution's cumulative function is represented by

$$G(x) = \int_0^{F(x)} b(\tau)\, d\tau, \tag{1}$$

where F is the CDF of any random variable, say X, and $b(\tau)$ is the beta distribution pdf. The beta-generated distribution's PDF is provided by;

$$g(x) = \frac{f(x)}{B(\alpha,\beta)} F^{\alpha-1}(x) \\ (1-F(x))^{\beta-1},\ \alpha;\beta > 0, \tag{2}$$

where B(α, β) denotes the beta function. Several studies employed various F in (2) to generate beta distributions, Kumaraswamy-G [4]-[5] replaced the beta distribution with the kumaraswamy distribution as a generator function, The transformed transformers family (T-X) family [6] presented a generalized method of producing families of distributions, allowing the use of any continuous pdf as a generator as an alternative to the beta or Kumaraswamy distribution. This method is built on three functions ($F_T(x)$, $F_X(x)$, and W), with $F_T(x)$ and $F_X(x)$ acting as the cdfs of two random variables (T and X). $W(.)$ is a real value function from [0,1] onto the support of T, where T is a random variable with support from [a,b]. The cdf and pdf of T-X family of distributions is given as, respectively;

$$G(x) = \int_a^{W(F_X(x))} f_T(x)\, dT \\ = F_T(W(F_X(x))), \quad (3)$$

where, $F_T(x)$ is the cdf of the generated random variable T and $f_T(x)$ is the pdf of T.

$$g(x) = [\frac{d}{dx}W(F_X(x))][f_T(x)(W(F_X(x)))]. \quad (4)$$

, and [7] introduced the T-$X[Y]$ family of distributions. replacing $W(.)$ in the T-X family with the quantile function of a random variable Y. The T-X[Y] approach is based on 3 functions $F(T)$, $F(X)$ and $Q(Y)$, with $F(T)$ and $F(X)$ serving as the cdfs of two random variables T and X, Q_Y is the quantile function of some variable Y. The cdf and pdf of T-X[Y] family of distributions is provided respectively as;

$$G(x) = \int_a^{Q_Y(F_X(x))} f_T(t)\, dt = F_T(Q_Y(F_X(x))), \quad (5)$$

and

$$g(x) = f_T[Q_Y(F_X(x))] \cdot Q'_Y(F_X(x)) \cdot f_X(x). \quad (6)$$

This equation can be written as

$$g(x) = f_X(x) \cdot \frac{f_T(Q_Y(F_X(x)))}{f_Y\,(Q_Y(F_X(x)))}.$$

Mahmoud et al. [8] proposed the T-IE family of distributions applying the T-X[Y] framework, with X following the inverse exponential distribution. The cdf and pdf of T-IE family of distributions are given as follows:

$$G(x) = \int_a^{Q_Y(e^{-\frac{\theta}{x}})} f_T(t)\, dt = F_T[Q_Y(e^{-\frac{\theta}{x}})], \quad (7)$$

and

$$g(x) = \frac{\theta}{x^2}\, e^{-\frac{\theta}{x}}\, \frac{f_T[Q_Y(e^{-\frac{\theta}{x}})]}{f_Y[Q_Y(e^{-\frac{\theta}{x}})]}. \quad (8)$$

In this article, we will look at a new four-parameter distribution based on the T-IE [Y] family. The paper is structured as follows: in Section 2, the new distribution is presented. In Section 3, some basic characteristics of the Exp-IE[Weibull] distribution are investigated. Parameter estimates are provided in Section 4. Some applications are considered in Section 5. Section 6 concludes with some final comments on our study .

2 Exponential-Inverse Exponential[Weibull] Distribution (Exp-IE[Weibull])

Mahmoud et al used T-X[Y] Method to generate T-IE[Y] family of distributions and introduced many sub families letting random variable Y follows different distributions. So with Y following the Weibull distribution and T following the exponential distribution, the exponential-inverse exponential [Weibull] distribution is generated, and its cdf and pdf are presented respectively as;

$$G(x) = 1 - \exp\left[-\lambda\alpha\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}}\right], \quad (9)$$

and

$$g(x) = \frac{\lambda\alpha\theta}{kx^2}\frac{e^{-\theta/x}}{1-e^{-\theta/x}}\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}-1} \\ \exp\left[-\lambda\alpha\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}}\right]. \quad (10)$$

There is no reason to deffreniate between α and λ in the new distribution, so we shall put $\gamma = \lambda\alpha$. The new formed Exp-IE[Weibull] cdf and pdf takes the forms as follows respectively;

$$G(x) = 1 - \exp\left[-\gamma\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}}\right], \quad (11)$$

and

$$g(x) = \frac{\gamma\theta}{kx^2}\frac{e^{-\theta/x}}{1-e^{-\theta/x}}\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}-1} \\ \exp\left[-\gamma\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}}\right]. \quad (12)$$

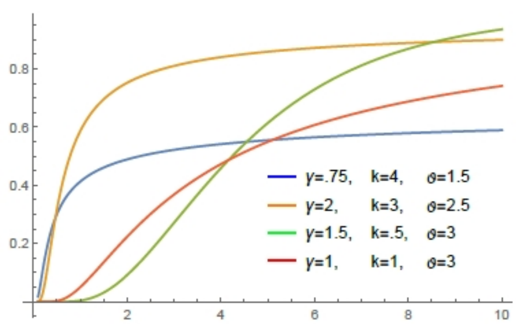

Figure 1. CDFs of Exp-IE[Weibull] distribution for different parameter values.

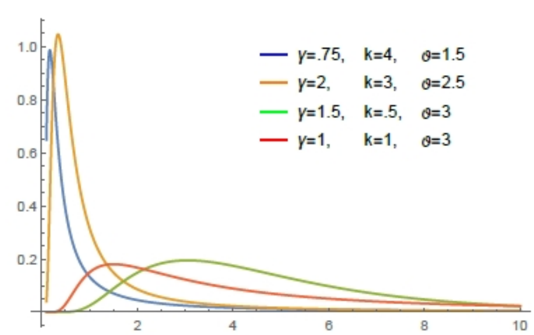

Figure 2. PDFs of Exp-IE[Weibull] distribution for different parameter values.

Reliability function and hazard function of Exp-IE[Weibull] distribution are provided respectively as;

$$\bar{G}(x) = \exp\left[-\gamma\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}}\right], \qquad (13)$$

and

$$h(x) = \frac{\gamma\,\theta}{kx^2}\,\frac{e^{-\theta/x}}{1-e^{-\theta/x}}\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}-1}. \qquad (14)$$

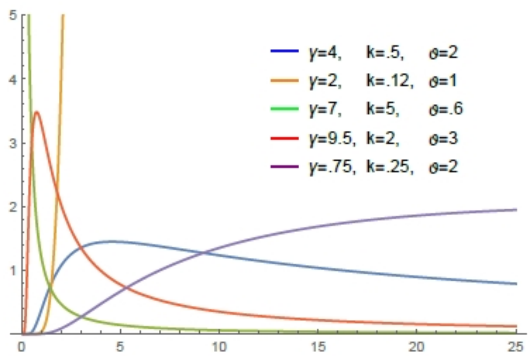

Figure 3. Hazard function of Exp-IE[Weibull] distribution for different parameters values.

Plots of the cdf, pdf and hazard function for some values of γ, k and θ are given in Figures 1–3 respectivly. The hazard function can be monotonically decreasing, increasing and an upside-down bathtub depending on the values of its parameters.

3 Exp-IE[Weibull] Distribution Properities

In this section a number of basic properities of Exp-IE[Weibull] distribution are computed, like moments, Shannon's entropy, mode, quantile function, median, skewness, kurtosis and order statistics.

3.1 Quantile Function and Median

Quantile function of any random variable's is simply the inverse of its distribution function. Exp-IE[Weibull] distribution quantile function can be obtained from the distribution cdf as follows:

$$Q_x(u) = \frac{-\theta}{\ln\left[1-e^{-\left(\frac{-\ln[1-u]}{\gamma}\right)^k}\right]}. \qquad (15)$$

Special quartiles and octiles of interest are given by;

$$Q_x(1/8) = \frac{-\theta}{\ln\left[1-e^{-\left(\frac{-\ln[.875]}{\gamma}\right)^k}\right]}, \qquad (16)$$

$$Q_x(1/4) = \frac{-\theta}{\ln\left[1-e^{-\left(\frac{-\ln[.75]}{\gamma}\right)^k}\right]}, \qquad (17)$$

$$Q_x(3/8) = \frac{-\theta}{\ln\left[1-e^{-\left(\frac{-\ln[.625]}{\gamma}\right)^k}\right]}, \qquad (18)$$

$$Q_x(1/2) = \frac{-\theta}{\ln\left[1-e^{-\left(\frac{-\ln[.5]}{\gamma}\right)^k}\right]}, \qquad (19)$$

where, $Q_x(1/2)$ is the median of Exp-IE[Weibull] distribution.

$$Q_x(5/8) = \frac{-\theta}{\ln\left[1-e^{-\left(\frac{-\ln[.375]}{\gamma}\right)^k}\right]}. \qquad (20)$$

$$Q_x(3/4) = \frac{-\theta}{\ln\left[1-e^{-\left(\frac{-\ln[.25]}{\gamma}\right)^k}\right]}. \qquad (21)$$

$$Q_x(7/8) = \frac{-\theta}{\ln\left[1-e^{-\left(\frac{-\ln[.125]}{\gamma}\right)^k}\right]}. \qquad (22)$$

3.2 The Skewness and Kurtosis

Quartile function can be used as an alternative to moments if one does not have enough information about the mean, mode, and standard deviation to compute skewness and kurtosis (see [9]). The Bowley skewness (S_B) and Moors kurtosis (K_M) definitions are given consecutively as;

$$S_B = \frac{Q(3/4) - 2Q(1/2) + Q(1/4)}{Q(3/4) - Q(1/4)},$$

and

$$K_M = \frac{Q(7/8) - Q(5/8) + Q(3/8) - Q(1/8)}{Q(3/4) - Q(1/4)}.$$

Skweness and kurtosis are calculated for γ= 1.5, k=3 and θ= 4 and the results are given as; S_B= 0.695 and K_M= 7.424.

3.3 The Mode

Mode of Exp-IE[Weibull] distribution can be obtained from the solution of this equation;

$$x = \frac{\theta(1-e^{-\theta/x}) + \theta e^{-\theta/x}}{2(1-e^{-\theta/x})} + \frac{\left(\frac{1}{k}-1\right)\left(\frac{\theta e^{-\theta/x}}{-\ln[1-e^{-\theta/x}]}\right)}{2(1-e^{-\theta/x})} - \frac{\frac{\gamma\theta e^{-\theta/x}}{k}\left(\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}-1}}{2(1-e^{-\theta/x})}. \qquad (23)$$

3.4 Moments

Moments of Exp-IE[Weibull] distribution is computed using [8] proposition 1 (i,iii) $i.\, X \overset{d}{=} Q_X(F_Y(T))$, $iii.\, E(X^r) =$

$E[(Q_X(F_Y(T)))^r]$, so for Exp-IE[Weibull] X and $E(X^r)$ are given by;

$$X = \frac{-\theta}{\ln[1-e^{-T^k}]} \tag{24}$$

$$E(X^r) = \theta^r E\left[\left(-\ln[1-e^{-T^k}]\right)^{-r}\right] \tag{25}$$

Applying the expansion of the expression $\left(-ln[1-e^{-T^k}]\right)^{-r}$ can be given by using the formula see ([10]);

$$(-\ln[1-z])^a = a\sum_{j=0}^{\infty}\binom{j-a}{j}\sum_{i=0}^{i}\frac{(-1)^{j+i}}{a-j}\binom{j}{i}P_{i,j}Z^{a+j}, \tag{26}$$

where $P_{i,j}$ is a constant and can be computed like that;

$P_{i,j} = \frac{1}{j}\sum_{m=1}^{j}\frac{(im-j+m)(-1)^m}{m+1}P_{i,j-m}$, for $j = 1,2,3,...,$ and $P_{i,0} = 1$.

$$E(X^r) = -\theta^r r\sum_{j=0}^{\infty}\binom{j+r}{j}\sum_{i=0}^{j}\frac{(-1)^{j+i}}{-r-i}\binom{j}{i}\cdot P_{i,j}\frac{\gamma}{kr-kj-\gamma}, \tag{27}$$

3.5 Shannon's Entropy

In economics and signal processing, entropy is a commonly used term as a measure of uncertainty. Shannon's entropy with pdf $f(z)$ is defined as $\eta_Z = E\left(-ln\left[f(z)\right]\right)$. Exp-IE-Weibull distribution Shannon's entropy is formed as follows:

$$\begin{aligned}\eta_X =& E(-\ln[\theta x]) + E(\ln[k]) + \\ & 2E(\ln[x]) + \theta E(\frac{1}{x}) + E\left(\ln[1-e^{-\theta/x}]\right) - \\ & -(\frac{1}{k}-1)E\left(\ln[-\ln[1-e^{-\theta/x}]]\right) + \\ & \gamma E\left((-\ln[1-e^{-\theta/x}])^{1/k}\right).\end{aligned} \tag{28}$$

3.6 Order Statistics

For $v = 1,...,n$ from iid random variables $X_1, X_2, ..., X_n$. the density of the r^{th} order statistics is indicated by;

$$g_{x_r}(x) = \frac{1}{\beta(r, n-r+1)}\, g(x)\, [G(x)]^{r-1}\, [1-G(x)]^{n-r}. \tag{29}$$

Applying Equation 9, and Equation 10 in Equation 29 , then we have the PDF of Exp-IE[Weibull] distribution order statistics as follows:

$$\begin{aligned}g_{x_r(x)} =& \frac{1}{\beta(r,n-r+1)}\,\frac{\gamma\theta}{kx^2}\,\frac{e^{-\theta/x}}{1-e^{-\theta/x}} \times \\ & \left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}-1}\exp\left[-\gamma\left(-\ln\left[1-e^{-\theta/x}\right]\right)^{\frac{1}{k}}\right] \times \\ & \left(1-\exp\left[-\gamma\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}}\right]\right)^{r-1} \times \\ & \left(\exp\left[-\gamma\left(-\ln[1-e^{-\theta/x}]\right)^{\frac{1}{k}}\right]\right)^{n-r}.\end{aligned} \tag{30}$$

4 Exp-IE[Weibull] Parameters Estimation

4.1 Complete Sample

The maximum likelihood method is employed to estimate the unknown parameters of the Exp-IE[Weibull] distribution.

Let $x_1, x_2, ..., x_n$ be a random sample from Exp-IE[Weibull] distribution. The likelihood function it will be as follows:

$$\begin{aligned}L(x) =& \prod_{i=1}^{n} g(x) \\ =& \frac{\gamma^n\theta^n}{k^n\prod_{i=1}^{n}x_i^2}\,\frac{e^{-\theta/\sum_{i=1}^{n}x_i}}{1-e^{-\theta/\sum_{i=1}^{n}x_i}} \times \\ & \left(-\ln\left[1-e^{-\theta/\sum_{i=1}^{n}x_i}\right]\right)^{\frac{1}{k}-1} \times \\ & \exp\left[-\gamma\left(-\ln[1-e^{-\theta/\sum_{i=1}^{n}x_i}]\right)^{\frac{1}{k}}\right].\end{aligned} \tag{31}$$

The log likelihood function $\ell(x)$, which is the natural logarithm of the likelihood function takes the form

$$\begin{aligned}\ell(x) =& n\ln[\gamma\theta] - n\ln[k] - \sum_{i=1}^{n}\ln[x_i^2] - \frac{\theta}{\sum_{i=1}^{n}x_i} - \\ & \ln\left[1-e^{-\theta/\sum_{i=1}^{n}x_i}\right] + \left(\frac{1}{k}-1\right) \times \\ & \ln\left[-\ln[1-e^{-\theta/\sum_{i=1}^{n}x_i}]\right] - \\ & \gamma\left(-\ln[1-e^{-\theta/\sum_{i=1}^{n}x_i}]\right)^{\frac{1}{k}}.\end{aligned} \tag{32}$$

Equation 32 can be used to obtain maximum likelihood estimates of the parameters γ, k and θ by solving the following equations:

$$\frac{\partial\ell(x)}{\partial\gamma} = \frac{n}{\gamma} - \left(-\ln[1-e^{-\theta/\sum_{i=1}^{n}}]\right)^{1/k}, \tag{33}$$

$$\frac{\partial \ell(x)}{\partial k} = -\frac{n}{k} - \frac{1}{k^2}\ln[-\ln[1-e^{-\theta/\sum_{i=1}^{n} x_i}]]+ \frac{\gamma}{k^2}\ln\left[-\ln[1-e^{-\theta/\sum_{i=1}^{n} x_i}]\right] \times \left[-\ln[1-e^{-\theta/\sum_{i=1}^{n} x_i}]\right]^{1/k}, \tag{34}$$

and

$$\frac{\partial \ell(x)}{\partial \theta} = \frac{n}{\theta} - \frac{1}{\sum_{i=1}^{n} x_i} - \frac{e^{-\theta/\sum_{i=1}^{n} x_i}}{(1-e^{-\theta/\sum_{i=1}^{n} x_i})\sum_{i=1}^{n} x_i} [1 - \frac{1}{k}\frac{1}{\ln[1-e^{-\theta/\sum_{i=1}^{n} x_i}]} + \frac{1}{\ln[1-e^{-\theta/\sum_{i=1}^{n} x_i}]} - \frac{\gamma}{k}\left(-\ln[1-e^{-\theta/\sum_{i=1}^{n} x_i}]\right)^{\frac{1}{k}-1}] = 0. \tag{35}$$

4.2 Type II Censoring Sample

Suppose now, a total of n items is placed on test, but we did not wait until all n items have failed. The experiment will be terminated after obtain r^{th} item failure. If $X_1, X_{2,...,X_n}$ are iid and have pdf (12) and survival function $\bar{G}(x)$ (13), the joint pdf of $X_1, X_2, ..., X_r$ is;

$$g(x) = \frac{n!}{(n-r)!} \times \prod_{i=1}^{r} g(x_i) \times (\bar{G}(x_r))^{n-r}. \tag{36}$$

The log likelihood function $\ell(x)$ can be written as;

$$\ell(x) = \frac{n!}{(n-r)!} + r\ln[\gamma\theta] - r\ln[k] - \sum_{i=1}^{r}\ln[x_i^2] - \frac{\theta}{\sum_{i=1}^{r} x_i} - \ln\left[1-e^{-\theta/\sum_{i=1}^{r} x_i}\right] + \left(\frac{1}{k}-1\right) \times \ln\left[-\ln[1-e^{-\theta/\sum_{i=1}^{r} x_i}]\right] - \gamma \times \left(-\ln[1-e^{-\theta/\sum_{i=1}^{n} x_i}]\right)^{\frac{1}{k}} - \gamma\,(n-r) \times -\ln\left[1-e^{-\theta/\sum_{i=1}^{r} x_i}\right] \tag{37}$$

By solving the following equations, one can derive maximum likelihood estimates of the parameters γ, k, and θ using equation 37:

$$\frac{\partial \ell(x)}{\partial \gamma} = \frac{r}{\gamma} - \left(-\ln[1-e^{-\theta/\sum_{i=1}^{r} x_i}]\right)^{1/k} - (n-r)\left(-\ln[1-e^{-\theta/\sum_{i=1}^{r} x_i}]\right), \tag{38}$$

$$\frac{\partial \ell(x)}{\partial k} = -\frac{r}{k} + \frac{1}{k^2}\ln\left[-\ln[1-e^{-\theta/\sum_{i=1}^{r}}]\right] + \frac{\gamma}{k^2}\left(-\ln[1-e^{-\theta/\sum_{i=1}^{r}}]\right)^{1/k} \times \ln\left[-\ln[1-e^{-\theta/\sum_{i=1}^{r}}]\right] \tag{39}$$

$$\frac{\partial \ell(x)}{\partial \theta} = \frac{r}{\theta} - \frac{1}{\sum_{i=1}^{r} x_i} - \frac{e^{-\theta/\sum_{i=1}^{r} x_i}}{(1-e^{-\theta/\sum_{i=1}^{r} x_i})\sum_{i=1}^{r} x_i} [1 - \frac{1}{k}\frac{1}{\ln[1-e^{-\theta/\sum_{i=1}^{r} x_i}]} + \frac{1}{\ln[1-e^{-\theta/\sum_{i=1}^{r} x_i}]} - \frac{\gamma}{k}\left(-\ln[1-e^{-\theta/\sum_{i=1}^{r} x_i}]\right)^{\frac{1}{k}-1} + \gamma(n-r)]. \tag{40}$$

5 Applications

Using real data sets to illustrate the flexibility of Exp-IE[Weibull] distribution, we applied the new distribution to vinyl chloride data set and cancer patient data.

The Akaike information criterion (AIC), the Bayesian information criterion (BIC), and the log-likelihood value were used to distinguish the Exp-IE[Weibull] distribution from a variety of other distributions. Maximum likelihood estimation method is applied to estimate distribution parameters. The best model among them has the lowest log-likelihood, AIC, and BIC values.

5.1 Vinyl Chloride Data

A sample of 34 observations of vinyl chloride data provided by [1] in mg/L gathered from clean up gradient ground-water monitoring wells is used. Parameters estimates for γ, k and θ are provided as follows: $\hat{\gamma} = 1.39$, $\hat{k} = 1.26$ and $\hat{\theta} = 1.15$.

Exp-IE[Weibull] distribution is compared to some distributions including (Fréchet, Weibull-exponential [11], logistic-exponential [12] and Weibull-Lomax [log-logistic]) [13]).

AIC, BIC, and log-likelihood values are presented in Table 1. The Exp-IE[Weibull] distribution best fits the data, according to Table 1's results, out of all the models mentioned.

Table 1. AIC, BIC and Log-Likelihood Measures for The Vinyl Chloride Data

Distribution	AIC	BIC	Log-likelihood
Exp-IE[Weibull]	18.486	23.066	-6.243
Fréchet	450.407	458.223	-222.204
Logistic-exponential	528.280	533.490	-262.140
Weibull-exponential	532.280	542.701	-262.140
Weibull-Lomax[log-logistic]	530.280	538.096	-262.140

Table 2. AIC, BIC and Log-Likelihood Measures for The Cancer Patients Data

Distribution	AIC	BIC	Log-likelihood
Exp-IE[Weibull]	903.768	912.324	-448.884
logistic modified Weibull	1155.040	1166.450	-573.522
Weibull- exponential	2405.600	2417.010	-1198.800
IPLE	1006.800	1015.350	-500.398
logistic-exponential	2401.600	2407.300	-1198.800

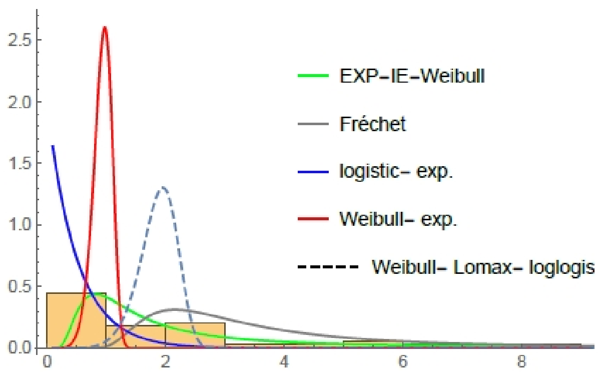

Figure 4. The vinyl chloride dataset's histogram and fitted PDFs

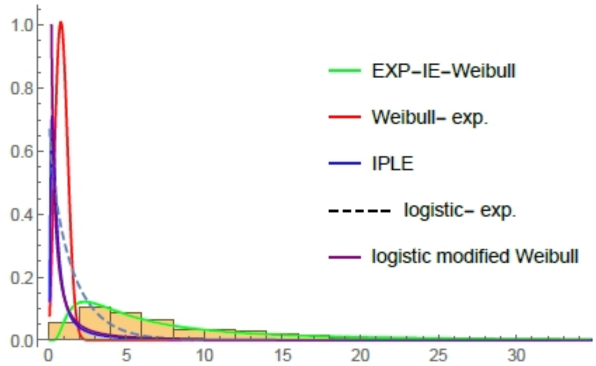

Figure 5. The cancer patients dataset's histogram and fitted PDFs

5.2 Cancer Patients Data

A sample of 128 bladder cancer patients' remission durations (measured in months) are included in the data set provided by [2]. Parameters estimates for γ, k and θ are provided as follows: $\hat{\gamma} = 2.39$, $\hat{k} = 0.89$ and $\hat{\theta} = 4.08$.

Exp-IE[Weibull] distribution is compared to some distributios including (logistic modified Weibull [14], Weibull-exponential [11], inverse power logistic exponential (IPLE) [15] and logistic-exponential [12]). AIC, BIC, and log-likelihood values are presented in Table 2.

The Exp-IE[Weibull] distribution best fits the data, according to Table 2's results, out of all the models mentioned. This section's results were generated using the *Mathematica* 12 softweare backage on my personal dell computer.

6 Summary and Conclusion

The three-parameter exponential-IE [Weibull] distribution is defined in this paper as a member of the T-IE family of distributions. A number of properties are introduced, such as mode, quantile function, median, hazard function, survival function, moments, order statistics, and Shannon's entropy. The parameters of the new distribution were estimated using the maximum likelihood method.

The exponential-IE [Weibull] distribution proved to be useful in analyzing some types of data and gives a better fit to such data than some other common distributions.

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