Mathematics and Statistics 11(2): 252-262, 2023 http://www.hrpub.org
DOI: 10.13189/ms.2023.110203

Reliability Evaluation of Linear or Circular Consecutive k-out-of-n: F System Using Dynamic Bayesian Network

R Sakthivel[1,2], G Vijayalakshmi[1,*]

[1]Department of Mathematics, Faculty of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur – 603 203, India
[2]Department of Career Development Centre, SRM Institute of Science and Technology, Kattankulathur – 603 203, India

Received November 13, 2022; Revised January 7, 2023; Accepted January 28, 2023

Cite This Paper in the Following Citation Styles

(a): *[1] R Sakthivel, G Vijayalakshmi , "Reliability Evaluation of Linear or Circular Consecutive k-out-of-n: F System Using Dynamic Bayesian Network," Mathematics and Statistics, Vol. 11, No. 2, pp. 252 - 262, 2023. DOI: 10.13189/ms.2023.110203.*

(b): *R Sakthivel, G Vijayalakshmi (2023). Reliability Evaluation of Linear or Circular Consecutive k-out-of-n: F System Using Dynamic Bayesian Network. Mathematics and Statistics, 11(2), 252 - 262. DOI: 10.13189/ms.2023.110203.*

Abstract In the field of reliability theory, one of the most significant topics to discuss is the process of determining the reliability of a complex system based on the reliabilities of its individual components. The consecutive k-out-of-n:F system is used in telephone networks, photographing in nuclear accelerators, spacecraft relay stations, telecommunication system consisting of relay stations connecting transmitter and receiver, microwave relay stations, the design of integrated circuits, vacuum systems in accelerators, oil pipeline systems and computing networks. The reliability estimation of the consecutive k-out-of-n:F system is studied because it plays an important role in many physical systems. Dynamic Bayesian networks are graphical models for time-varying probabilistic inference and causal analysis under system uncertainty. The dynamic Bayesian network is built for the proposed system since time is continuously measured. The consecutive k-out-of-n:F system depends on the k components, because the system fails when the consecutive k components fail, otherwise the system works. The contributions are the dynamic Bayesian network construction of the proposed system and the reliability analysis of the linear and circular consecutive k-out-of-n:F system. Furthermore, Dynamic Bayesian network- based reliability is shown to be significantly higher than the reliability achieved by Malinowski, Preuss and Gao, Liu, Wang, Peng and Amirian, Khodadadi, Chatrabgoun. The Dynamic Bayesian network- based Reliability of linear and circular consecutive k-out-of-n:F system is also compared.

Keywords Bayesian Network, Consecutive k-out-of-n:F System, Dynamic Bayesian Network, System Reliability, Time Series

1. Introduction

The theory of the consecutive k-out-of-n:F (Cons.k/n:F) system was first proposed by Niu and Chiang [1]. If a system with n components fail whenever k successive components fail, it is called a Cons.k/n:F system [1,2]. If all n components are organized in a straight line, the system is linear. If all n components are organized in a circle, the system is circular. Bayesian network (BN) is a directed acyclic graph (DAG) made up of variables and directed edges, all connected by a table of conditional probabilities for each variable on all of its parents. As a consequence, it is a graphical representation of unknown quantities that demonstrates the probabilistic causal dependence between variables as well as the model's information flow.

In the early 1990 Dagum developed the dynamic Bayesian network [3,4]. The temporal relationship between nodes is defined using a dynamic Bayesian network. In general, natural events like throwing a coin do not change the current probabilistic distribution based on the previous outcome. On the other hand, some phenomenon like earthquakes and natural language processing have fundamental dependencies between the present state at time t and preceding state at time t_1, however, the regular Bayesian network avoids temporal causality [5]. There are many methods to evaluate the reliability of a system and some of the important research works are studied in the following literature.

Malinowski and Preuss [6] presented recursive algorithm for evaluating the exact reliability of a circular consecutive k-within-m-out-of-n:F system with independent components whose failure probabilities may be unequal. Kontoleon [7] introduced the reliability of a model consisting of n consecutive interrelated subsystems. At least one successive sub system has to fail in order for the model to fail. Ghahramani [8] has provided a brief tutorial of learning and inference methods in dynamic Bayesian networks. Friedman, Murphy, and Russell [9] examined numerous distinct types of models in their depiction, including DBNs, accurate and estimated inference output in DBNs, and sequential data explained by DBN models. Tong, Boland and Proschan [10] developed a model integrating positive dependency between adjacent components and showed that the system's reliability is a declining function on its dependency for $k \geq (n+1)/2$. Hwang, Lee and Cho [12] proposed a technique that uses genetic algorithms to produce and create a DBN structure to deal with improbability and dynamic property in the real world. Yuan and Cui [13] dealt with the repair phenomenon by believing that repairmen had several vacations in the system. The r repair men taking several vacations were integrated into the traditional Cons.k/n:F scheme using the pairs (i ,| j |) factor. Indexes for reliability were provided. Frohlich and Korf [14] were motivated to examine interactions in greater detail among epidermal growth factor receptor (EGFR) and sonic hedgehog (SHH) dependent signal. There was no arithmetic model illustrating the interaction in medulloblastoma between EGFR and SHH dependent signal. They have a completely probabilistic approach with dynamic Bayesian networks. Byun, Noh and Song [15] introduced a method of matrix based system reliability to extend k-out-of-n:F structures by changing event and likelihood vector formulations. The planned method can integrate statistical dependency for together non homogeneous and homogeneous k-out-of-n:F models. Gao, Liu, Wang, and Peng [16] proposed the iterative relationship between the failure probability of a consecutive k-out-of-n: F system. Pan, Lee and Sanchez [17] presented a model of a Bayesian network to test system reliability. The knowledge fusion for the development of BN models is also addressed. Elghamry, Eldamcesse and Nayel [18] discussed a new algorithm for calculating the fuzzy reliability with independent, un-repairable, and non-identical components of the fuzzy linear Cons.k/n:F systems.

Radwan [19] suggested bounds for multistate Cons.k/n:F system reliability. These bounds are suitable for all types of systems whether they are unequal or equal component possibilities. Zhang, Zhao and Du [20] created Birnbaum importance-based quantum genetic algorithm which boosts the efficacy and accuracy of component assignment problem resolution. The linear sequential k-out-of-n systems reliability optimization model is first developed. The Birnbaum importance- based quantum genetic algorithm, which is presented in the second place, solves the component assignment problem. Nashwan [21] has demonstrated that the conventional consecutive k-out-of-m-from-n:F linear and circular system with numerous failure criteria was crucial in obtaining the reliability and failure probability functions. Amirian, Khodadadi, and Chatrabgoun [22] developed a novel approach and software to determine the precise reliability for a wide range of sequential circular k-out-of-r-from-n:F systems, with a focus on probability of equal and unequal components. Madhumitha and Vijayalakshmi [23] calculated the mean time to failure and confidence interval using Bayesian methods for the Cons.k/n:F systems. Bibartiu, Durr and Grau [24] defined the memory growth for the k/n voting gate was lowered from exponential to polynomial in the range of input events due to a scalable Bayesian network model. Nashwan [25] provided formula to calculate the precise reliability and failure likelihood functions for the linear and circular r-gap successive k-out-of-m-from-n:F systems. Amirian and Khodadadi [26] developed a new algorithm, which can determine the exact reliability for a large class of sequential linear and circular systems. Jegatheesan, Gundala [27] produced an evaluation of the linear (circular) Cons.k/n:F system's fuzzy Bayesian reliability using the squared error loss function. Yin, Cui and Balakrishnan [28] proposed a method for describing F systems with common components that combines the theoretical study of linear and circular k-out-of-n with the finite Markov chain imbedding approach. In addition, there are MATLAB programs that provide accurate reliability for sequential linear and circular systems.

2. Materials and Methods

Notations and Terminologies

- $C_1(t), C_2(t), \ldots C_n(t)$ – Basic components of Linear or Circular system at time *t*
- $X_1(t+1), X_2(t+1), X_3(t+1) \ldots$ – Child components (Intermediate components) of consecutive *k* components at time *t+1*
- Cons.k/n : F – Consecutive k-out-of-n : F
- FS – Final system
- $Par(X_i)$ – Parents of node X_i
- k – Number of consecutive failure component
- R – Reliability
- BN– Bayesian Network
- DBN – Dynamic Bayesian Network
- DBN R – Dynamic Bayesian Network Reliability

- DBN RL – Dynamic Bayesian Network Reliability for Linear system
- DBN RC – Dynamic Bayesian Network Reliability for Circular system
- Root nodes – The nodes without any arrows directed into them
- Child nodes –The nodes that have arrows directed into them
- Parent nodes –The nodes that have arrows directed from them
- Evidence – The calculation of beliefs of events, given the observation of other events

Assumptions

- p – component reliability in a system with independent and identically distributed components
- q – component unreliability in a system with independent and identically distributed components
- The component is either in a working (1) state or in a failure (0) state
- In a Bayesian network, the root nodes have prior probability tables (discrete nodes) or functions (continuous nodes) associated with them
- Each child has a conditional probability table (or function) associated with it, given the state (or value) of the parent nodes

Dynamic Bayesian Network

The time slices are connected through temporal links to constitute a full model. If the structures of the time slices are identical, and if the temporal links are the same, we say that the model is a repetitive temporal model. If the conditional probabilities are also identical, we call the model a dynamic Bayesian network model.

There are several benefits of creating a DBN [11]. Once the network has been established between the time steps, a model can be developed based on this data. This model can then be used to predict future responses by the system. The ability to predict future responses can also be used to explore different alternatives for the system and determine which alternative gives the desired results. DBN's also provide a suitable environment for model predictive controllers and can be useful in creating the controller. Another advantage of DBN's is that they can be used to create a general network that does not depend on time. Once the DBN has been established for the different time steps, the network can be collapsed to remove the time component and show the general relationships between the variables.

A DBN is made up with interconnected time slices of static BN's. The nodes at certain time can affect the nodes at a future time slice, but the nodes in the future cannot affect the nodes in the previous time slice. The causal links across the time slices are referred to as temporal links. The advantage of this is that it gives DBN an unambiguous direction of causality.

3. System Reliability Evaluation by DBN

3.1. Linear Consecutive k-out-of-n:F System

All the basic components $C_1, C_2, \dots C_n$ (at time t) are placed in linear arrangement, and all components have two states, namely working (1) or failure (0). The purpose of intermediate components $X_1, X_2, \dots, X_{n-k+1}$ (at time $t + 1$) is to find final system (FS) component. In the beginning linear consecutive k-out-of-n:F system will function normally but it will fail only when consecutive k components fail. The intermediate component X_1 connects the first k parent components with temporal link in the linear arrangement, and this will fail after all k parent components have failed. Similarly, k consecutive parent components are connected with temporal link to corresponding intermediate components $X_2, X_3, \dots, X_{n-k+1}$ respectively. All the intermediate components depended on their respective parent components. This arrangement of components is shown in Figure 1. If any one of the intermediate components fails, then linear consecutive k-out-of-n:F system will fail.

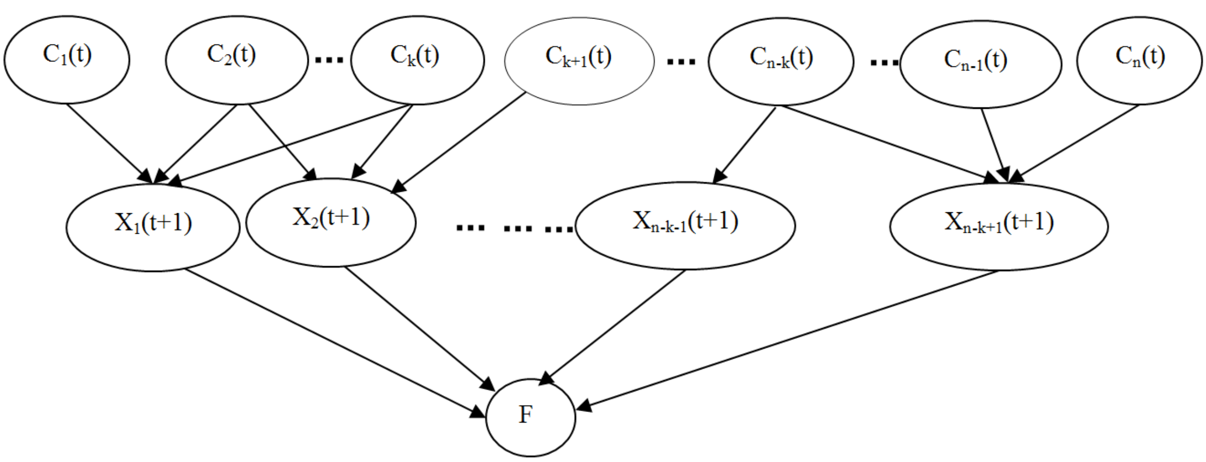

Figure 1. DBN for linear consecutive k-out-of-n:F system

Joint Probability function is

$$P(Z) = P(C_1, \dots C_n, X_1, \dots X_{n-k+1}, FS) = P(FS/Par(FS)) \prod_{i=1}^{n-k+1} P(X_i/Par(X_i)) \prod_{i=1}^{n} P(C_i) \quad (3)$$

$$P(Z) = P(FS/X_1, \dots . X_{n-k+1}) P(X_1/C_1, .. C_k) P(X_2/C_2, \dots C_{k+1}) \dots \dots$$
$$P(X_{n-K-1}/C_{n-k}, \dots C_{n-1}) \, P(X_{n-k+1}/C_{n-k}, \dots C_n) \, P(C_1) P(C_2) \dots \dots \; P(C_n) \quad (4)$$

Marginal Probability function is

$$P(FS) = \sum_{C_1=0,\dots C_n=0, X_1=0,\dots X_{n-k+1}=0}^{1} P(FS/Par(FS)) \prod_{i=1}^{n-k+1} P(X_i/Par(X_i)) \prod_{i=1}^{n} P(C_i) \quad (5)$$

$$P(FS) = \sum_{\substack{C_1=0,\dots C_n=0, \\ X_1=0,\dots X_{n-k+1}=0}}^{1} \begin{matrix} P(FS/X_1, \dots . X_{n-k+1}) P(X_1/C_1, .. C_k) P(X_2/C_2, \dots C_{k+1}) \dots \dots \\ P(X_{n-K-1}/C_{n-k}, \dots C_{n-1}) \, P(X_{n-k+1}/C_{n-k}, \dots C_n) \, P(C_1) P(C_2) \dots \dots \; P(C_n) \end{matrix} \quad (6)$$

Reliability of failure for final system (FS) is

$$R(FS = 0) = \sum_{\substack{No.\ of\ failure \\ components\ in\ Par(FS) \geq 1}} P(FS = 0/Par(FS)) \, P(Par(FS)) \quad (7)$$

Where

$$Par(FS) = X_1, X_2, X_3 \dots ., X_{n-k+1}$$
$$P(X_i = 0) = P(X_i = 0/Par(X_i = 0)) \, P(Par(X_i = 0))$$
$$Par(X_i) = Consecutive\ k\ components\ from$$
$$C_1, C_2, C_3 \dots ., C_n \ (Start\ with\ i^{th}\ component, i = 1,2,3, \dots ., n)$$

3.2. Circular Consecutive k-out of-n:F System

All the basic components $C_1, C_2, \dots C_n$ (at time t) are placed in a circular arrangement, and all components have two states, namely working (1) or failure (0). The purpose of intermediate components $X_1, X_2, \dots, X_n$ (at time $t + 1$) is to find final system (FS) component. In the beginning circular consecutive k-out-of-n:F system will function normally but it will fail only when consecutive k components fail. The intermediate component X_1 connects the first k parent components with temporal link in the circular arrangement, and this will fail after all k parent components have failed. Similarly, k consecutive parent components are connected with temporal link to corresponding intermediate components $X_2, X_3, \dots, X_{n-k+1}$ respectively. All the intermediate components depended on their respective parent components. This arrangement of components is shown in Figure 2. If any one of the intermediate components fails, then circular consecutive k-out-of-n:F system will fail.

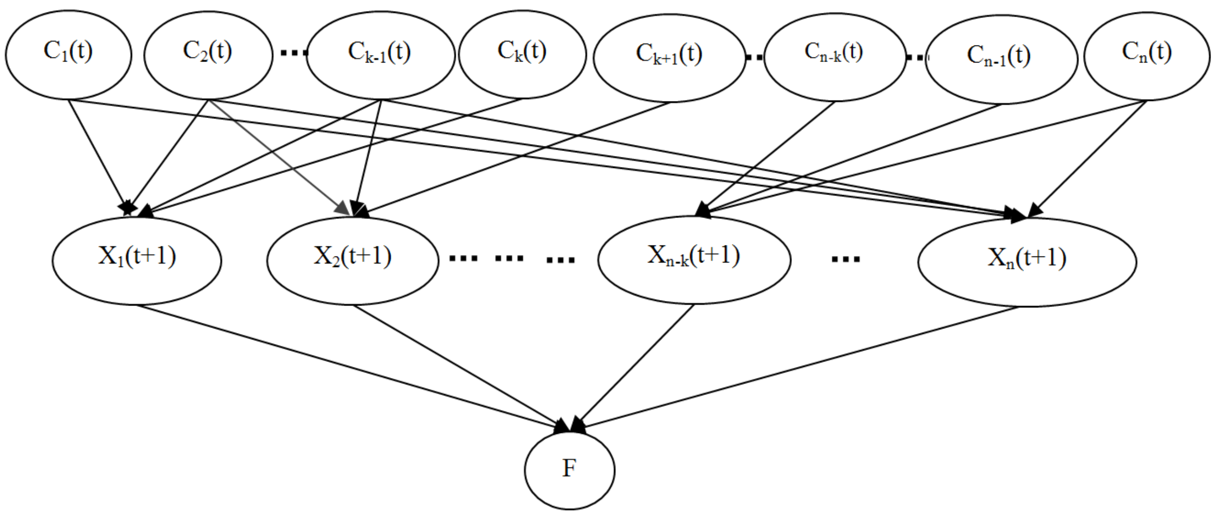

Figure 2. DBN for circular consecutive k-out-of-n:F system

Joint Probability function is

$$P(Z) = P(C_1, \dots C_n, X_1, \dots, X_n, FS) = P(FS/Par(FS)) \prod_{i=1}^{n} P(X_i/Par(X_i)) \prod_{i=1}^{n} P(C_i) \quad (8)$$

$$P(Z) = P(FS/X_1, \dots \dots, X_n) P(X_1/C_1, \dots, C_k) P(X_2/C_2, \dots, C_{k+1}) \dots "P(X_{n-k}/C_{n-k}, C_{n-k+1}, \dots, C_n) \dots$$
$$P(X_n/C_n, C_1, \dots, C_{k-1}) P(C_1) P(C_2) \dots \dots P(C_n)" \quad (9)$$

Marginal Probability function is

$$P(FS) = \sum_{C_1=0,\dots C_n=0, X_1=0,\dots X_n=0}^{1} P(FS/Par(FS)) \prod_{i=1}^{n} P(X_i/Par(X_i)) \prod_{i=1}^{n} P(C_i) \quad (10)$$

$$P(FS) = " \sum_{C_1=0,\dots C_n=0, X_1=0,\dots X_n=0}^{1} \begin{matrix} P(FS/X_1, \dots \dots, X_n) P(X_1/C_1, \dots, C_k) P(X_2/C_2, \dots, C_{k+1}) \\ \dots P(X_{n-k}/C_{n-k}, C_{n-k+1}, \dots, C_n) \dots \\ P(X_n/C_n, C_1, \dots, C_{k-1}) P(C_1) P(C_2) \dots \dots P(C_n)" \end{matrix} \quad (11)$$

Reliability of failure for final system is

$$R(FS = 0) = \sum_{\substack{No.\ of\ failure \\ components\ in\ Par(FS) \geq 1}} P(FS = 0/Par(FS))\, P(Par(FS)) \quad (12)$$

Where

$$Par(FS) = X_1, X_2, X_3 \dots., X_n$$
$$P(X_i = 0) = P(X_i = 0/Par(X_i = 0))\, P(Par(X_i = 0))$$
$$Par(X_i) = Consecutive\ k\ components\ from$$
$$C_1, C_2, C_3 \dots., C_n\ (Start\ with\ i^{th}\ component, i = 1,2,3, \dots., n)$$

4. Particular Cases

Case (i)

When k=1, the system behaves like a series system. If any one of system components fails, the entire system fails. In other words, if all of the components are operational, the system will function; otherwise, it will fail. Each component has failure probability $P(X_i)$, and then reliability formula is

$$R_s(S) = \prod_{i}^{n} P(X_i) \quad (13)$$

Case (ii)

When k = n, the system requires all components to fail, so the system behaves like a parallel system. Each component has failure probability $P(X_i)$, and then reliability formula is

$$R_P(S) = 1 - \prod_{i}^{n} (1 - P(X_i)) \quad (14)$$

5. Results and Discussion

Numerical illustrations of DBN reliability of linear consecutive k-out-of-n:F and circular consecutive k-out-of-n:F systems are given. The reliability values for the various failure probabilities are shown in Tables 1 and 2. Table 3 shows the comparison of reliability Amirian et al. (2020) and DBN reliability for both linear and circular systems.

Table 1. DBN Reliability for linear consecutive k-out-of-7:F system

q / DBN R	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
k=2	0.946	0.807	0.624	0.434	0.266	0.138	0.0559	0.0149	0.0015
k=3	0.995	0.966	0.898	0.785	0.633	0.457	0.281	0.131	0.0325
k=4	0.9996	0.995	0.975	0.928	0.844	0.715	0.544	0.345	0.147

Table 2. DBN Reliability for circular consecutive k-out-of-7:F system

q / DBN R	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
k=2	0.938	0.783	0.587	0.391	0.227	0.109	0.0399	0.009	0.00063
k=3	0.994	0.955	0.867	0.73	0.555	0.367	0.197	0.0735	0.0115
k=4	0.9994	0.991	0.96	0.896	0.773	0.617	0.411	0.217	0.0624

Table 3. Reliability versus DBN Reliability for linear and circular at n=7 and p=0.75

K	1	2	3	4	5	Average	Difference b/w R and DBN	Increase in Reliability by DBN
R	0.13348	0.44495	0.75641	0.94098	0.99481	0.65413	-	-
DBN RL	0.133	0.719	0.938	0.987	0.998	0.755	0.100874	10%
DBN RC	0.133	0.687	0.918	0.979	0.995	0.7424	0.088274	9%

6. Reliability Comparison

The DBN reliability graph of the linear consecutive k-out-of-7:F system and the circular consecutive k-out-of-7:F system are shown in Figure 3 and 5 respectively for values k = 2, 3, and 4. The DBN reliability of the system has been plotted for various probability values. It is observed that when the failure probability of component increases, reliability of the system decreases in both linear and circular systems. Table 4 shows that the DBN reliability of linear system is approximately 22% more than the reliability calculated by Malinowski and Preuss [6] and DBN reliability of circular system is approximately 21% more than the reliability calculated by Malinowski and Preuss [6]. Table 5 indicates that the suggested system's DBN reliability is 0.2% greater than the reliability computed by Gao, Liu, Wang and Peng [14]. Figures 4 and 6 show that the reliability evaluation for the proposed system using DBN is approximately 10% and 9% higher than the reliability (R) by Amirian, Khodadadi, and Chatrabgoun [20], since DBN is an advance model giving accurate results. Figure 7 reveals that the linear consecutive k-out-of-7:F system structure has approximately 2% higher reliability in comparison to the circular consecutive k-out-of-7:F system. This is because circular system has more failures compared to linear system. Figures 3-7 are drawn using MATLAB.

Table 4. Reliability Comparison for consecutive k-out-of-n:F system

n	k	p	R (Malinowski and Preuss)	DBN RL	DBN RC	Increase in DBN RL	Increase in DBN RC
15	2	0.9	0.641	0.869	0.858	0.228	0.217
15	3	0.75	0.441	0.661	0.641	0.22	0.2
					Average	0.224	0.21

Table 5. Reliability Comparision for consecutive k-out-of-5:F system at p=0.1

k	R (Gao et.al)	DBN RL	Increase in DBN RL
1	0.59	0.59	0
2	0.962	0.963	0.001
3	0.991	0.997	0.006
		Average	0.0023

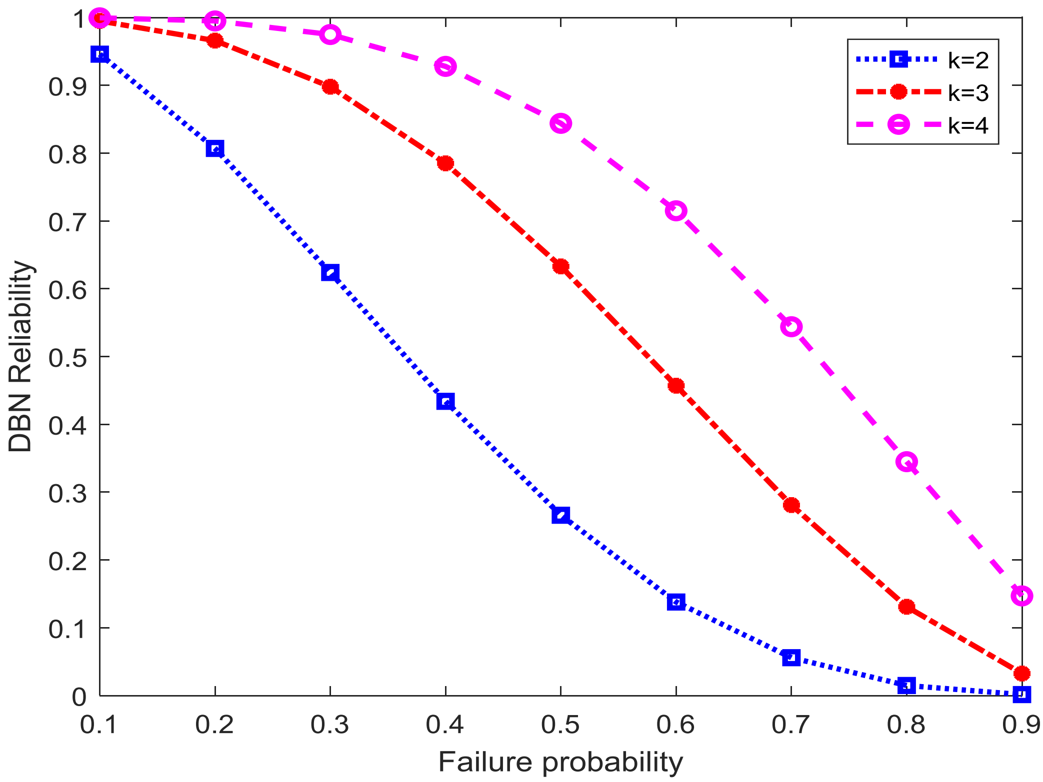

Figure 3. DBN Reliability for linear system

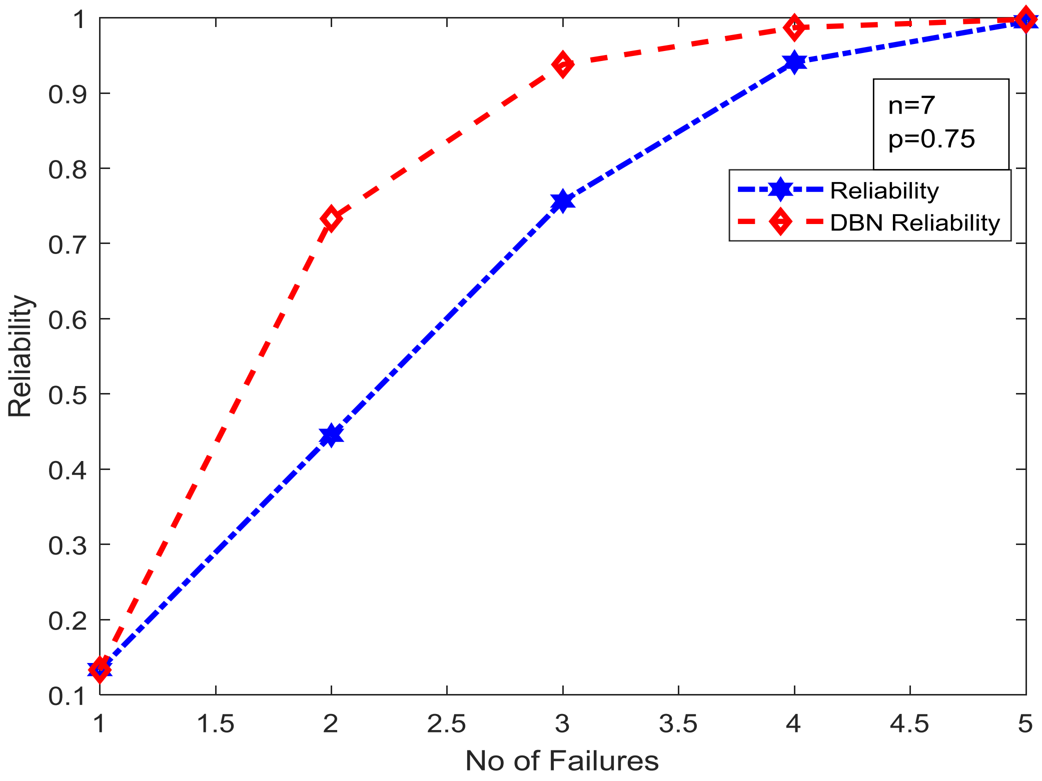

Figure 4. Reliability versus DBN Reliability for linear system

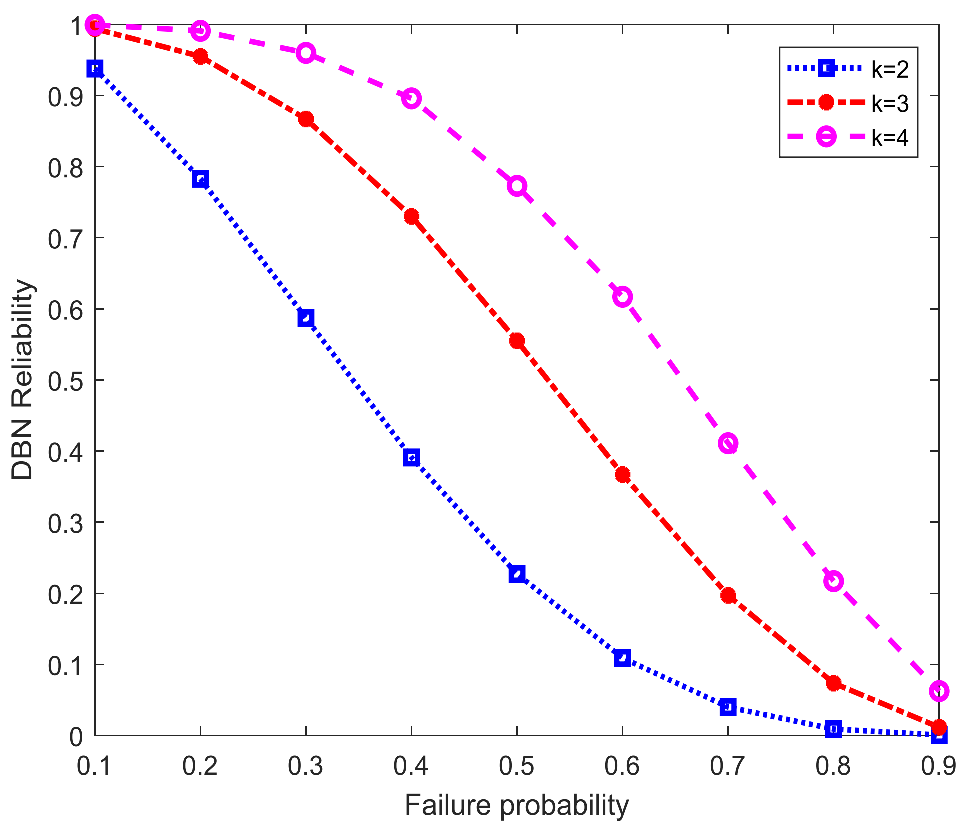

Figure 5. DBN Reliability for circular system

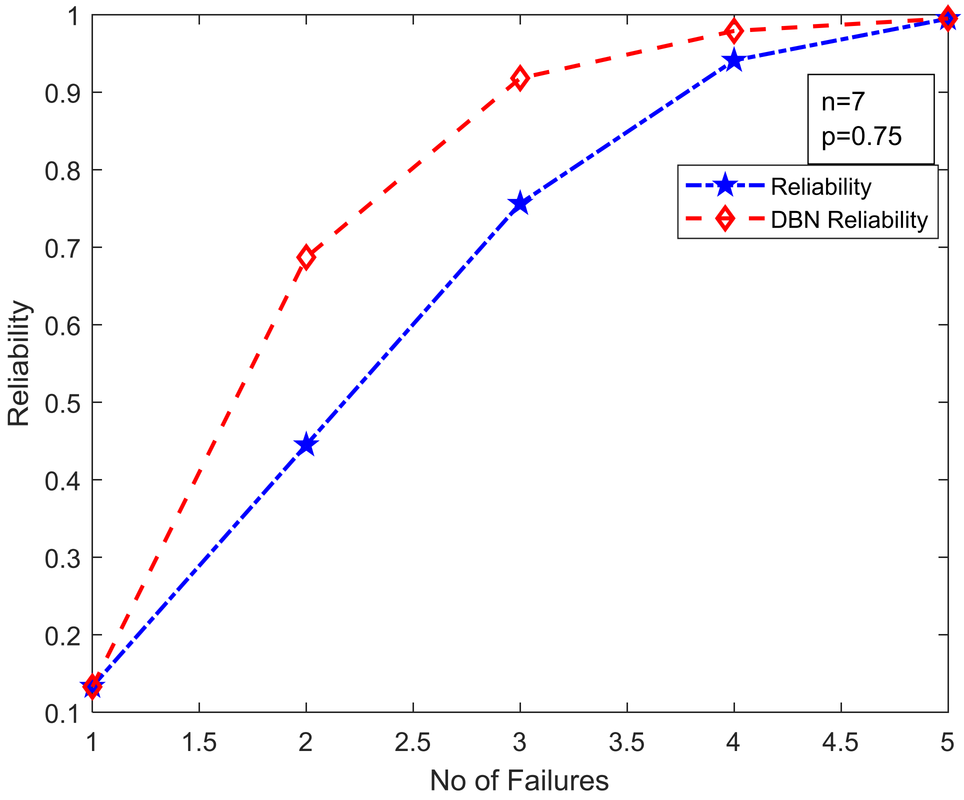

Figure 6. Reliability versus DBN Reliability for circular system

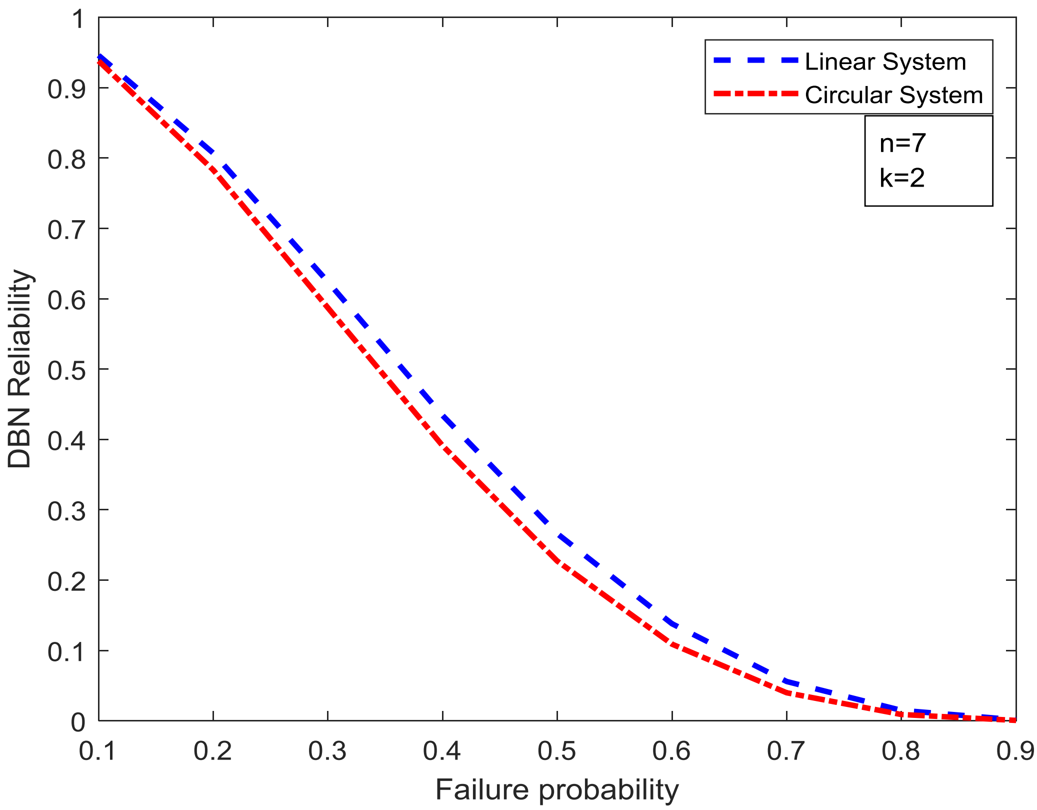

Figure 7. DBN Reliability of linear versus circular system

7. Conclusions

In this paper reliability of the proposed system is evaluated using the Dynamic Bayesian network. Dynamic Bayesian network is a popular probabilistic method that can be used to describe complex dependency and implement uncertainty reasoning between random variables. In order to investigate the reliability of the linear and circular Cons.k/n:F systems, we developed a dynamic Bayesian network for the recommended system and compared their performance. In comparison to the results obtained by Malinowski, Preuss and Gao, Liu, Wang, Peng and Amirian, Khodadadi, Chatrabgoun, it is proven that the reliability based on dynamic Bayesian networks is substantially greater. The DBN is an alternate method that can be used to determine the reliability of linear or circular Cons.k/n:F systems. For computing the reliability of the system, the proposed method is more efficient in very large Cons.k/n:F systems. In addition, it has been demonstrated that the reliability of linear Cons.k/n:F structure has greater reliability than the circular Cons.k/n:F structure.

REFERENCES

[1] Chiang, D. T., & Niu, S. C., “Reliability of consecutive k-out-of-n:F system”, IEEE Transactions on Reliability, vol. 30, pp. 87-89, 1981. doi: 10.1109/TR.1981.5220981.

[2] Kuo, W., & Zuo, M. J., “Optimal reliability modelling: principles and applications,” John Wiley & Sons, 2003.

[3] Dagum, P., Galper, A., Horvitz, E., & Seiver, A., “Uncertain reasoning and forecasting,” International Journal of Forecasting, vol. 11, pp. 73-87, 1995. doi:10.1016/0169- 2070(94)02009-E.

[4] Dagum, P., Galper, A., & Horvitz, E., “Dynamic network models for forecasting,” Uncertainty in artificial intelligence, pp. 41-48. Morgan Kaufmann, 1992. doi:10.1016/B978-1-4832-8287-9.50010-4.

[5] Jensen, F. V., & Nielsen, T. D., “Bayesian networks and decision graphs”, Vol. 2, New York: Springer, 2007.

[6] Malinowski, J., & Preuss, W., “A recursive algorithm evaluating the exact reliability of a circular consecutive k-within-m-out-of-n: F system,” Microelectronics Reliability, vol. 36(10), pp. 1389-1394, 1996. doi:10.1016/0026-2714(96)00015-7.

[7] Kontoleon, J. M., “Reliability determination of a r-successive-out-of-n:F system,” IEEE Transactions on Reliability, vol. 29, pp. 437-437, 1980. doi:10.1109/TR.1980.5220921.

[8] Ghahramani, Z., “Learning dynamic Bayesian networks,” International School on Neural Networks, Initiated by IIASS and EMFCSC, pp. 168-197, 1997. doi:10.1007/BFb0053992.

[9] Friedman, N., Murphy, K., & Russell, S., “Learning the structure of dynamic probabilistic networks,” arXiv preprint arXiv:1301.7374, 2013.

[10] Boland, P. J., Proschan, F., & Tong, Y. L., “Linear Dependence in Consecutive k-out-of-n:F Systems,” Probability in the Engineering and Informational Sciences, vol. 4, pp. 391-397, 1990. doi:10.1017/S0269964800001686.

[11] Woolf, P. J., "Chemical Process Dynamics and Controls." Libretexts, 2009.

[12] Hwang, J. W., Lee, Y. S., & Cho, S. B. (2011)., “Structure evolution of dynamic Bayesian network for traffic accident detection,” Congress of Evolutionary Computation, pp. 1655-1671, 2011. doi:10.1109/CEC.2011.5949815

[13] Yuan, L., & Cui, Z. D., “Reliability analysis for the consecutive k-out-of-n:F system with repairmen taking multiple vacations,” Applied mathematical modelling, vol. 37, pp. 4685-4697, 2013. doi:10.1016/j.apm.2012.09.008.

[14] Frohlich, H., Bahamondez, G., Gotschel, F., & Korf, U., “Dynamic Bayesian network modelling of the interplay between EGFR and Hedgehog signalling,” PloS one, vol. 10, pp. 1-9, 2015. doi:10.1371/journal.pone.0142646.

[15] Byun, J. E., Noh, H. M., & Song, J., “Reliability growth analysis of k-out-of-N systems using matrix based system reliability method,” Reliability Engineering & System Safety, vol. 165, pp. 410-421, 2017. doi:10.1016/j.ress.2017.05.001.

[16] Gao, K., Liu, X., Wang, H., Ma, X. & Peng, R., “A New Iterative Approach to Evaluate the Reliability of a Combined Consecutive k and v-Out-Of-n System,” International Journal of Performability Engineering, vol. 14(12), p.2983, 2018. DOI: 10.23940/ijpe.18.12.p8.29832993.

[17] Pan, R., Lee, D., Yontay, P., & Sanchez, L. M., “System reliability assessment through Bayesian network modelling,” Advances in System Reliability Engineering, pp. 227-241, 2019. doi:10.1016/B978-0-12-815906-4.00009-9.

[18] Elghamry, E., Eldamcesse, M. A., & Nayel, M. S., “Reliability Model of Fuzzy Consecutive k-out-of-n: F System,” International Journal of Reliability, Risk and Safety: Theory and Application, vol. 2, pp. 1-4, 2019. doi:10.30699/IJRRS.2.1.1.

[19] Radwan, T., “New Bounds for All Types of Multi-state Consecutive k -Out-of- r -From- n: F System Reliability,” IEEE Access, vol. 7, pp. 172562-172570, 2019. doi:10.1109/ACCESS.2019.2956292.

[20] Zhang, S., Zhao, J. B., Zhu, W. J., & Du, L., “Reliability optimization of linear consecutive k-out-of-n:F systems with Birnbaum importance based quantum genetic algorithm,” Advances in Mechanical Engineering, vol. 11, pp. 1-16, 2019. doi:10.1177/1687814019842996.

[21] Nashwan, I. I., “Reliability and Failure Probability Functions of the Consecutive-k-out-of-m-from-n: F System with Multiple Failure Criteria,” Palestinian Journal of Technology and Applied Sciences, vol. 3, pp. 47-56, 2020. doi:10.33977/2106-000-003-005.

[22] Amirian, Y., Khodadadi, A., & Chatrabgoun, O., “Exact Reliability for a Consecutive Circular k-out-of-r-from-n:F System with Equal and Unequal Component Probabilities,” International Journal of Reliability, Quality and Safety Engineering, vol. 27, pp. 1-26, 2020. doi:10.1142/S0218539320500035.

[23] Madhumitha, J., & Vijayalakshmi, G., "Bayesian Estimation of Linear/Circular Consecutive k-out-of-n: F System Reliability," International Journal of Performability Engineering, vol. 16, 2020.

[24] Bibartiu, O., Durr, F., Rothermel, K., Ottenw ̈alder, B., & Grau, A., "Scalable k-out-of-n models for dependability analysis with Bayesian networks," Reliability Engineering & System Safety, vol. 210, pp. 107533, 2021. doi: 10.1016/j.ress.2021.107533.

[25] Nashwan, I. I., "Linear And Circular R-Gap-Consecutive-K-Out-Of-M-From-N: F Systems," International Journal of Scientific & Technology Research, vol. 10, pp. 346-352, 2021.

[26] Amirian, Y., & Khodadadi, A., "Modeling and Exact Reliability for a Consecutive Linear (k1, k2, . . ., kn−r+1)−Out−of−r−from−n:F System and Consecutive Circular (k1, k2, . . ., kn)−Out−of−r− from−n:F System. International Journal of Reliability," Quality and Safety Engineering, vol.28, pp.2150029-2150029, 2021. doi:10.1142/S0218539321500297.

[27] Jegatheesan, M., & Gundala, V. "Fuzzy Bayesian Estimation of Linear (Circular) Consecutive k-out-of-n:F System Reliability," Trends in Sciences, vol.19, pp. 2706-2706, 2022. doi: https://doi.org/10.48048/tis.2022.2706.

[28] Yin, J., Cui, L., Sun, Y., & Balakrishnan, N., "Reliability modelling for linear and circular k-out-of-n:F systems with shared components," Reliability Engineering & System Safety, vol. 219, pp. 108172, 2022. doi:10.1016/j.ress.2021.108172.