Mathematics and Statistics 11(2): 335-339, 2023 http://www.hrpub.org
DOI: 10.13189/ms.2023.110212

On T-coloring and ST-coloring of Windmill Graph

Rubul Moran[1], **Niranjan Bora**[2,*], **Surashmi Bhattacharyya**[3]

[1]Department of Mathematics, Dibrugarh University, India

[2]Department of Mathematics, Dibrugarh University Institute of Engineering & Technology, Dibrugarh University, India

[3]Department of Agricultural Statistics, Biswanath College of Agriculture, India

Received December 6, 2022; Revised January 20, 2023; Accepted February 10, 2023

Cite This Paper in the following Citation Styles

(a)*: [1] Rubul Moran, Niranjan Bora, Surashmi Bhattachar, "On T-coloring and ST-coloring of Windmill Graph," Mathematics and Statistics, Vol.11, No.2, pp. 335-339, 2023. DOI: 10.13189/ms.2023.110212*

(b)*: Rubul Moran, Niranjan Bora, Surashmi Bhattachar, (2023). On T-coloring and ST-coloring of Windmill Graph. Mathematics and Statistics, 11(2), 335-339. DOI: 10.13189/ms.2023.110212*

Abstract The windmill graph $W(r,m)$; $m \geq 3, r \geq 2$ is the graph formed by joining a common vertex to every vertex of m copies of the complete graph K_r. T-coloring of a graph is a map h defined on the set of vertices in such a way that for any edge (w_1, w_2), $\mid h(w_1) - h(w_2) \mid$ does not belong to a finite set T of non-negative integers. Strong T-Coloring (ST-coloring) is a particular case of T-coloring and is defined as the map: $c : V(G) \rightarrow \mathbb{Z}^+$, for which $|c(w_1) - c(w_2)| \notin T$ for all $w_1 \neq w_2$ and $(w_1, w_2) \in E(G)$ and $|c(w_1) - c(w_2)| \neq |c(w_3) - c(w_4)|$ for any two distinct edges (w_1, w_2), $(w_3, w_4) \in E(G)$. Application of T and ST-coloring of graph naturally arises in the modeling of different scientific problems. Frequency assignment problem (FAP) is one of the well known problems in the field of telecommunication, which can be modeled using the concept of T and ST-coloring of graphs. In this paper, we will consider two special types of T-sets. The first one is λ-initial set, introduced by Cozzens and Roberts, which is of the form $\{0, 1, 2, \ldots, \lambda\} \cup S$ where S is any arbitrary set that doesn't contain any multiple of $(\lambda + 1)$. The second one is λ-multiple of q set, introduced by Raychaudhuri, which is of the form $\{0, q, 2q, \ldots, \lambda q\} \cup S$, where S is a subset of the set $\{q+1, q+2, q+3, \ldots, \lambda q\}$. We will discuss some parameters related to these two types of colorings viz. T-chromatic number, T-span, T-edge span on the basis of the two T-sets. We will also deduce some generalized results of ST-coloring of any graph based on any T-set, and with the help of these results we will obtain ST-chromatic number and bounds for the ST -span and ST-edge span of windmill graphs.

Keywords Graph Coloring, Chromatic Number, Span, Edge Span, Windmill Graph

1 Introduction

Let the graph $G = (V, E)$ be given and T be a finite set of non negative integers. The $T-$coloring and ST-coloring of G are defined in the works of [1, 16]. Application of T and $ST-$coloring of graph naturally arises in the modeling of different scientific problems. FAP is one of the well known problems in the field of telecommunication, which can be modeled using T-coloring. The growth of wireless communication systems all over the world has made a problem while assigning frequencies in an efficient manner, so called the FAP [13]. The FAP has been used to plan models for permanent frequency allocation, licensing, and regulation which maximize the efficient utilization of all radio spectra. It has been used extensively by engineers, mathematicians and scientists. It is inherently connected with the graph coloring problem, where transmitters are considered as vertices and if there is any interference between transmitters, then those interferences are considered as an edge. It was Hale [4], who introduced $T-$coloring theory to model many FAP. Cozzens et. al., [3] mapped out some standard results for efficient T-colorings which are based on particular $T-$set of the form $T = \{0, 1, 2, \ldots, \lambda\} \cup S$, where S is any arbitrary set that doesn't contain any multiple of $(\lambda + 1)$ which is termed as $\lambda-$initial set. A $T-$set of the form $T = \{0, q, 2q, ..., \lambda q\} \cup S$, where S is a subset of the set $\{q+1, q+2, q+3, ..., \lambda q\}, \lambda q \geq 1$ is termed as a $\lambda-$multiple of q set and this set was studied by Raychaudhuri [14]. Roberts [15] also discussed some recent results of T-coloring and some open problems of $T-$coloring in 1991. For more details on the literature on T-coloring one can see [2, 8, 9, 14, 17, 18, 20], and the references therein. While, the set T is permitted to differ for each interfering transmitters then ST-coloring becomes more suitable than $T-$coloring of graphs. In 2019, ST-coloring of some particular graphs viz., path graphs, cycle graphs, Wheel graphs, star graphs etc. has been studied by Roselin *et al.* [16]. They discussed the parameters namely, $ST-$chromatic number and $ST-$span of these graphs. Moran *et al.,* [10, 11] have studied $ST-$colorings of various graph operations. Based on these works they also computed $ST-$chromatic numbers of some non perfect graphs viz., Petersen graph, Wheel graph, Helm graph, closed Helm Flower graph and Sun Flower graph [12].

The Windmill graph is well investigated by the researchers. As windmill graph has $m(r-1)+1$ vertices and $\frac{mr(r-1)}{2}$ edges. Kooij [6] studied the characteristics of some generalized windmill graphs. Kulli [7] worked on leap Gourava indices of some certain windmill graphs. The eigenvalues and the corresponding eigenvectors of the distance matrix, the distance Laplacian matrix and the distance signless Laplacian matrix of windmill graphs are reported in [21]. Recently, in [19] Singh computed the metric and edge metric dimension of two classes of windmill graphs such as French windmill graph and Dutch windmill graph. Generalizations of these graphs motivate us to take up the present study on T and $ST-$coloring of windmill graphs.

The current paper is designed as follows: Section 2 contains some of the basic preliminaries which are used throughout the paper. In section 3, the main results based on T-coloring of windmill graphs are discussed. Some generalized results of $ST-$coloring of graphs are also included and by using these results, an inequality related to ST-coloring of windmill graphs is discussed. Finally, in section 4 an overall conclusion is drawn.

2 Preliminaries

Following definitions and previous results will be used throughout the paper. The symbols $\chi(G)$, $\chi_T(G)$ and $\chi_{ST}(G)$ respectively, represent Chromatic number, T-chromatic number and ST-chromatic number of the graph G.

Definition 2.1. *[21] The windmill graph denoted by $W(r,m), m \geq 3, r \geq 2$ is the graph obtained by taking m copies of the complete graph K_r joined at a shared universal vertex.*

Definition 2.2. *[16] For a $ST-coloring$ of G, $sp_{ST}(G)$ is the minimum (over all $ST-$coloring h of G) of the maximum value of $\mid h(u)-h(w) \mid$ overall the vertices of G.*

Definition 2.3. *[16] For a $ST-coloring$ of G, $esp_{ST}(G)$ is the minimum (over all $ST-$coloring h of G) of the maximum value of $\mid h(u)-h(w) \mid$ overall the edges (u,w) of G.*

Theorem 2.1. *[3] For all graphs G all T-sets, $\chi(G) = \chi_T(G) \leq \chi_{ST}(G)$.*

Theorem 2.2. *[3] If T is a λ-initial set, then for any graph G $sp_T(G) = sp_T(K_{\chi(G)}) = (\lambda+1)(\chi(G)-1)$.*

Theorem 2.3. *[14] If T is a λ-multiple of q set, then*

$$sp_T(G) = sp_T(K_{\chi(G)}) = \begin{cases} qt+\lambda qt - q\lambda - 1, & if \chi(G) = qt \\ qt + \lambda qt + n - 1, & if\ \chi(G) = qt+n \end{cases}$$

for any graph G, where $1 \leq n \leq q-1$ and $t\epsilon\{1,2,3,...\}$.

Theorem 2.4. *[5] If T is a λ-multiple of q set and $\chi(G) \leq q$, then for all graphs G, $sp_T(G) = esp_T(G) = \chi(G)-1$.*

Theorem 2.5. *[3] For any graph G,*
$sp_T(K_{\omega(G)}) \leq esp_{ST}(G) \leq sp_{ST}(G) \leq sp_T(K_{\chi(G)})$

Theorem 2.6. *[16] For all graphs G,*

1. $sp_T(G) \leq sp_{ST}(G)$
2. $esp_T(G) \leq esp_{ST}(G)$.

3 Main Result

Let $W(r,m)$ be the windmill graph. In $W(r,m)$, we denote the central vertex w_o, and the rest are $w_{i,j}$, where $w_{i,j}$ represents the j^{th} vertex of the i^{th} copy of K_r in which $i=1,\ldots,r$; $j=1,\ldots,m-1$.

3.1 $T-$coloring of windmill graph

Theorem 3.1. *The $T-$chromatic number for a windmill graph $W(r,m)$ is r, i.e., $\chi_T(w(r,m)) = r$*

Proof. By using Theorem (2.1), for all graphs G and all T-sets, $\chi_T(G) = \chi(G)$. Which gives, $\chi_T(W\ (r,m)) = \chi(W(r,m))$. Since, $\chi(W(r,m)) = r$. Therefore, $\chi_T(W(r,m)) = r$. ☐

Theorem 3.2. *Let $W(r,m)$ be the windmill graph. For a $\lambda-$initial set T,*

$$sp_T(W(r,m)) = esp_T(W(r,m)) = \lambda r - \lambda + r - 1$$

Proof. Let T be any λ-initial set. Let us assign the colors of every vertice as the function

$$g(w_{i,j}) = \begin{cases} 0, & if\ i=0 \\ \lambda(m+j), & if\ i \neq 0 \end{cases}$$

Then, there are two types of edges in $W(r,m)$. The first type consists of all edges that are adjacent to w_0. The second type consists of all edges that are adjacent to $w_{i,l}$s for some $l=1,\ldots,m-1$. For the first kind of edges, $|g(w_{i,j}) - g(w_0)| = (m+j)\lambda \notin T$. For the second kind and for any edge $(w_{i,j}, w_{i,l})$,

$$\begin{aligned} |g(w_{i,j}) - g(w_{i,l})| &= |\lambda(m+j) - \lambda(m+l)| \\ &= |\lambda(j-l)| \\ &= \lambda|j-l| \end{aligned}$$

which does not belong to the fixed set T. Hence, g is a $T-$coloring for the $\lambda-$initial set T

Since, for a windmill graph $W(r,m)$, $\chi(W(r,m)) = r$ and $\omega(W(r,m)) = r$. Using Theorem (2.2), we have

$$sp_T(K_{\omega(W(r,m))}) \leq esp_T(W(r,m)) \leq sp_T(W(r,m)) \leq sp_T(K_{\chi(W(r,m))})$$

$$\Rightarrow (\lambda+1)(r-1) \leq esp_T(W(r,m)) \leq sp_T(W(r,m)) \leq (\lambda+1)(r-1)$$

$$\Rightarrow \lambda r - \lambda + r - 1 \leq esp_T(W(r,m)) \leq sp_T(W(r,m)) \leq \lambda r - \lambda + r - 1$$

$$\Rightarrow sp_T(W(r,m)) = esp_T(W(r,m)) = \lambda r - \lambda + r - 1$$ □

Theorem 3.3. *Let $W(r,m)$ be the windmill graph. For a $\lambda-$multiple of q set T,*

$$sp_T(W(r,m)) = esp_T(W(r,m)) = \begin{cases} r+\lambda r - \lambda q - 1, & if\ r = qt \\ r + \lambda qt - 1, & if\ r = qt+n \end{cases}$$

where $1 \leq n \leq q-1; 2 \leq q \leq r$ and $t \in \{1,2,3,\ldots\}$.

Proof. Let T be any $\lambda-$multiple of q set. Let us assign the colors of every vertice as the function

$$g(w_{i,j}) = \begin{cases} 0, & if\ i = 0 \\ \lambda(m+j)q, & if\ i \neq 0 \end{cases}$$

Then, in $Wd(r,m)$, only two types of edges are there. The first type consists of all edges that are adjacent to w_0. The second type consists of all edges that are adjacent to $w_{i,l}$s for some $l = 1,\ldots,m-1$. For the first kind of edges, $|g(w_{i,j}) - g(w_0)| = \lambda(m+j)q \notin T$. For the second kind and for any edge $(w_{i,j}, w_{i,l})$,

$$\begin{aligned} |g(w_{i,j}) - g(w_{i,l})| &= |\lambda(m+j)q - \lambda(m+l)q| \\ &= |\lambda(j-l)q| \\ &= \lambda|j-l|q \end{aligned}$$

which does not belong to the fixed set T. Hence, g is a $T-$coloring for the $\lambda-$multiple of q-set T.
Since, for a $W(r,m)$, we have $\chi(W(r,m)) = r$ and $\omega(W(r,m)) = r$. Therefore by invoking Theorem (2.3),

$$sp_T(W(r,m)) = sp_T(K_r) = \begin{cases} qt+\lambda qt - \lambda q - 1, & if\ r = qt \\ qt+\lambda qt + n - 1, & if\ r = qt+n \end{cases}$$

$$\Rightarrow sp_T(W(r,m)) = \begin{cases} r+\lambda r - \lambda q - 1, & if\ r = qt \\ r+\lambda qt - 1, & if r = qt+n \end{cases} \quad (1)$$

where $1 \leq n \leq q-1, t\epsilon\{1,2,3,...\}$.
Since, $\chi(W(r,m)) = \omega(W(r,m))$ for any windmill graph. Using Theorem (2.2), we have

$$sp_T(W(r,m)) = esp_T(W(r,m)) = sp_T(K_{\chi(W(r,m))})$$

$$\Rightarrow sp_T(W(r,m)) = esp_T(W(r,m)) = \begin{cases} r+\lambda r - \lambda q - 1, & if r = qt \\ r+\lambda qt - 1, & if r = qt+n \end{cases}$$

where $1 \leq n \leq q-1$ and $t \in \{1,2,3,...\}$. □

Corollary 3.1. *Let $Wd(r,m)$ be the windmill graph. For a $\lambda-$multiple of q set T, where $q \geq r$, then $sp_T(W(r,m)) = esp_T(W(r,m)) = r-1$.*

Proof. By invoking Theorem (2.4),if $r \leq q$, then either $t = 0$ or $t = 1$, which gives $r = q$ or $r = n$, where $n \in \{1,2,3,\ldots,s-1\}$. Hence,

$$\Rightarrow esp_T(W(r,m)) = sp_T(W(r,m)) = \begin{cases} q+\lambda q - \lambda q - 1, & if\ r = q \\ q+\lambda q.0 - 1, & if r = n \end{cases}$$

where $1 \leq n \leq q-1$.
$\Rightarrow esp_T(W(r,m)) = sp_T(W(r,m)) = r-1$
Hence, for $q \geq r$, $sp_T(W(r,m)) = esp_T(W(r,m)) = r-1$. □

Corollary 3.2. *For a friendship graph F_m, and for a $\lambda-$ multiple of q set T,*

$$esp_T(F_m) = sp_T(F_m) = \left\{ \begin{array}{lll} 2(\lambda+1) & , & q = 1,2 \\ 2 & , & q > 3 \end{array} \right\}$$

Proof. Since a friendship graph F_m can be constructed by joining m copies of K_3 with a common vertex. Hence, the friendship graph F_m is isomorphic to the windmill graph $W(3,m)$.

1. **If** $q = 1$, and using Theorem (3.2), $sp_T(F_m) = esp_T(F_m) = \lambda r - \lambda + r - 1 = 2\lambda + 2$
   If $q = 2$, then by using Theorem (3.3), $r = 3 \Rightarrow t = 1$, $sp_T(F_m) = esp_T(F_m) = 2\lambda + 2$.
   Hence,
   $$esp_T(F_m) = sp_T(F_m) = 2(\lambda+1)\ if\ q = 1,2$$
2. Since, for a friendship graph F_n, $\chi(F_m) = 3$. Therefore, by using Theorem (3.1), $sp_T(F_m) = esp_T(F_m) = \chi(F_m) - 1 = 2$ for $q \geq 3$.

□

Corollary 3.3. *For a Star graph S_m, and for a $\lambda-$multiple of q set T,*

$$sp_T(S_m) = esp_T(S_m) = \left\{ \begin{array}{lll} 3 & , & q = 1 \\ 1 & , & q > 2 \end{array} \right\}$$

Proof. Since a star graph, S_m is a particular type of windmill graphs, which can be expressed as the windmill graph $W(2,m)$.

1. Using Theorem (3.2), $sp_T(S_m) = esp_T(S_m) = 3$.
2. Since, $\chi(S_m) = 2$ for a star graph S_m. Hence using Theorem (3.1), $sp_T(S_m) = esp_T(S_m) = \chi(S_m) - 1 = 1$, for $q \geq 2$.

□

3.2 $ST-$coloring of Graphs

In this section, we discuss some new results on ST-coloring of graphs.

Theorem 3.4. *For all graphs G and all $T-sets$,*
$sp_{ST}(K_{\omega(G)}) \leq esp_{ST}(G) \leq sp_{ST}(G) \leq sp_{ST}(K_{\chi(G)})$
$\leq sp_{ST}(K_{\chi_{ST}(G)})$

Proof. In case of $T-$coloring, by invoking theorem 2.5, for all graphs G and all $T-sets$,

$$sp_T(K_{\omega(G)}) \leq esp_T(G) \leq sp_T(G) \leq sp_T(K_{\chi(G)}) \quad (2)$$

Since, every $ST-$coloring of a graph G is also a $T-$coloring of G. Hence, $sp_T(G) \leq sp_{ST}(G)$, $esp_T(G) \leq esp_{ST}(G)$ and $\chi(G) = \chi_T(G) \leq \chi_{ST}(G)$, which implies $sp_{ST}(K_{\chi(G)}) \leq sp_{ST}(K_{\chi_{ST}(G)})$. Thus, equation 2 can be expressed as
$sp_{ST}(K_{\omega(G)}) \leq esp_{ST}(G) \leq sp_{ST}(G) \leq sp_{ST}(K_{\chi(G)})$
$\leq sp_{ST}(K_{\chi_{ST}(G)})$
which completes the proof. □

Theorem 3.5. *For a weakly $\gamma-$perfect graph G and for all $T-$sets,*

$$sp_{ST}(G) = esp_{ST}(G) = sp_{ST}(K_{\chi(G)})$$

Proof. If the graph G is weakly γ-perfect, then $\chi(G) = \omega(G)$. By theorem 3.4, for all $T-sets$, we have
$sp_{ST}(K_{\omega(G)}) \leq esp_{ST}(G) \leq sp_{ST}(G) \leq sp_{ST}(K_{\chi(G)})$. Then $sp_{ST}(K_{\chi(G)}) \leq esp_{ST}(G) \leq sp_{ST}(G) \leq sp_{ST}(K_{\chi(G)})$. Thus,

$$esp_{ST}(G) = sp_{ST}(G) = sp_{ST}(K_{\chi(G)})$$

□

Theorem 3.6. *For a $\lambda-$initial T-set, then*

$$sp_{ST}(G) \leq (\lambda+1)(\chi(G)-1)$$

Proof. In case of T-coloring, for a $\lambda-$initial T-set, $sp_T(G) = sp_T(K_{\chi(G)}) = (\lambda+1)(\chi(G)-1)$. Since, every $ST-$coloring of a graph is a $T-$coloring of G. Hence it follows then
$sp_T(G) \leq sp_{ST}(G)$ and $\chi(G) = \chi_T(G) \leq \chi_{ST}(G)$.
Hence,

$$\begin{aligned} sp_{ST}(G) \leq sp_{ST}(K_{\chi_{ST}(G)}) &\leq (\lambda+1)(\chi(G)-1) \\ &\leq (\lambda+1)(\chi_{ST}(G)-1) \end{aligned}$$

However, $(\lambda+1)(\chi(G)-1)$ is a closer upper than $(\lambda+1)(\chi_{ST}(G)-1)$. Thus,

$$sp_{ST}(G) \leq (\lambda+1)(\chi(G)-1)$$

□

3.2.1 ST-coloring of Windmill Graphs

Consider the graph $W(r,m)$ and let T be any λ-multiple of q set. Let us assign the colors of every vertice as the function

$$h(w_{i,j}) = \begin{cases} 0, & if\ w_{i,j} = w_0 \\ (\lambda q)^{ni+j}, & otherwise \end{cases}$$

Then, there are two types of edges in $W(r,m)$. The first type consists of all the edges that are adjacent to w_0. The second type consists of all the edges that are adjacent to $w_{i,l}$s for some $l = 1, \ldots, m-1$. For the first kind of edges, $|h(w_{i,j}) - h(w_0)| = (\lambda s)^{ni+j} \notin T$. For the second kind, and for any edge $(w_{i,j}, w_{i,l})$,

$$\begin{aligned} |h(w_{i,j}) - h(w_{i,l})| &= \mid (\lambda q)^{mi+j} - (\lambda q)^{mi+l} \mid \\ &= (\lambda q)^{mi} \mid (\lambda q)^j - (\lambda q)^l \mid \geq \lambda q \end{aligned}$$

which does not belong to the fixed set T. Hence, h is a $T-$coloring for the $\lambda-$multiple of q set T.
Now for any edge of the first kind, $(w_{i,j}, w_0)$,
$\mid h(w_{i,j}) - h(w_0) \mid = (\lambda q)^{mi+j}$,
which is distinct for all the edges. Similarly, for any two edges of the second kind $(w_{i,j}, w_{i,l})$ and $(w_{i,n}, w_{i,p})$, we need to show that,

$$\mid h((w_{i,j}) - h(w_{i,l}) \mid \neq \mid h((w_{i,n}) - h(w_{i,p}) \mid \quad (3)$$

If, $(w_{i,j}, w_{i,l})$ and $(w_{i,n}, w_{i,p})$ are adjacent, then Equation (3) holds. Let $(w_{i,j}, w_{i,l})$ and $(w_{i,n}, w_{i,p})$ be non adjacent edges. Then, j, l, n, p are all distinct. If possible, let equation (3) not hold. Then,

$$\begin{aligned} &\mid h((w_{i,j}) - h(w_{i,l}) \mid = \mid h((w_{i,n}) - h(w_{i,p}) \mid \\ &\mid (\lambda q)^{mi+j} - (\lambda q)^{mi+l} \mid = \mid (\lambda q)^{mi+l} - (\lambda q)^{mi+p} \mid \\ &\mid (\lambda q)^j - (\lambda q)^l \mid = \mid (\lambda q)^j - (\lambda q)^l \mid \end{aligned}$$

which is a not true for any distinct j, l, n, and p. Hence,

$$\mid h((w_{i,j}) - h(w_{i,l}) \mid \neq \mid h((w_{i,n}) - h(w_{i,p}) \mid$$

which shows that h is a ST-coloring.

Theorem 3.7. *The ST-chromatic number for any $W(r,m)$ is $m(r-1)+1$, i.e.*

$$\chi_{ST}(W(r,m)) = m(r-1)+1$$

Proof. In $W(r,m)$, every $w_{i,j}$ is adjacent to the central vertex w_0. Then, all the vertices of $W(r,m)$ must be of distinct colors. In other words, $h(w_{i,j}) \neq h(w_0)$ and $h(w_{i,j} \neq h(w_{l,n}))$ for all i, j, l and n. Hence, $\chi_{ST}(W(r,m)) = \mid V(W(r,m)) \mid = m(r-1)+1$. □

Theorem 3.8. *Let $W(r,m)$ be the windmill graph. For a $\lambda-$initial set T, $sp_{ST}(W(r,m)) = sp_{ST}(W(r,m)) \leq sp_{ST}K_{\chi(W(r,m))} \leq (\lambda+1)(r-1)$*

Proof. The proof of the theorem follows from the theorems 3.4, 3.5, 3.6. □

4 Conclusions

In this paper, we studied $T-$coloring of windmill graph. We obtained some results related to T-chromatic number, T-span and T-edge span. We deduced some generalized results of $ST-$coloring of any graph and by using these results we derived a bound for ST-span and ST-edge span of windmill graph. We also computed ST-chromatic number of windmill graph.

REFERENCES

[1] Moran R., Bora N., Pegu A., Chamua M., "Some results on T-coloring and ST-coloring of generalized butterfly graphs," Advances and Applications in Discrete Mathematics, Vol. 31, pp. 1-12, 2022. DOI:10.17654/0974165822022.

[2] Bonias I., "T-colorings of complete graphs," Ph.D. Thesis, Northeastern University, Boston, 1991.

[3] Cozzens M.B., Roberts F.S., "T-colorings of graphs and the channel assignment problem," Congr. Numer. Vol. 35, pp. 191-208, 1982.

[4] Hale W.K., "Frequency assignment: theory and applications," IEEE Proceedings, Vol. 68, pp. 1497-1514, 1980.

[5] Juan S-T J., Sun I-F., Wu P-X., "$T-$coloring on Folded Hypercubes," Taiwanese Journal of Mathematics, Vol. 13, No. 4, pp. 1331-1341, 2009.

[6] Kooij R., "On generalized windmill graphs," Linear Algebra and its Applications, Vol. 565, pp.25-46, 2019. DOI:10.1016/j.laa.2018.11.025

[7] Kulli V. R., Leap gourava indices of certain windmil graphs. International Journal of Mathematical Archive, Vol. 10, No. 11, pp. 7-14, 2019.

[8] Liu D. D. F., "Graph homomorphism and the channel assignment problem," Ph. D. Thesis, University of South Carolina, Colombia, SC, 1991.

[9] Liu D. D. F., "T-colorings of graphs," Discrete Mathematics, Vol. 101, pp. 203-212, 1992.

[10] R. Moran, N. Bora, A. K. Baruah and A. Bharali. "ST-coloring of join and disjoint union of graphs," Advances in mathematics: Scientific journal, Vol. 9, No. 11, pp. 9393-9399, 2020. DOI:10.37418/amsj.9.11.44

[11] R. Moran, N. Bora, A. K. Baruah and A. Bharali. "ST-coloring of some products of graphs," Journal of mathematical and computational science, Vol. 11, No. 1, pp. 337-347, 2020. DOI:10.28919/jmcs/5099.

[12] Moran R., Pegu A., Gogoi I., Bharali A., "A Note on ST-Coloring of Some Non Perfect Graphs," Theory and Practice of Mathematics and Computer Science Vol 11, Book Publisher International, 2021 , pp. 112-11. DOI:10.9734/bpi/tpmcs/v11/1498A

[13] Murphey R., Pardalos P., Resende M.G.G., "Frequency Assignment Problems," Handbook of Combinatorial Optimization, Kluwer Academic Publishers, 1999.

[14] Raychaudhuri A., "Further results on T-Coloring and Frequency assignment problems," SIAM Journal on Discrete Mathematics, Vol. 7, No. 4, pp. 605-613, 1994. DOI:10.1137/S0895480189171746

[15] Roberts F.S., "T-colorings of graphs: recent results and open problems," Discrete Mathematics, Vol. 93, No. 2-3, pp. 229-245, 1991. DOI.org/10.1016/0012-365X(91)90258-4

[16] S. J. Roselin, Raj L. B. M., Germina K.A., "Strong T-Coloring of Graphs," International Journal of Innovative Technology and Exploring Engineering, Vol. 8, No. 12, pp. 4677-4681, 2019.

[17] Roselin S.J., Raj L. B. M., "T-Coloring of certain non perfect Graphs," Journal of Applied Science and Computations, Vol. 6, No. 2, pp. 1456-1468, 2019.

[18] Roselin S.J., Raj L.B.M., "T-Coloring of wheel graphs," International Journal of Information and Computing Science, Vol. 6, No. 3, pp. 11-18, 2019.

[19] Singh P., Sharma S., Sharma S.K., Bhat V.K., "Metric dimension and edge metric dimension of windmill graphs," AIMS Mathematics, Vol. 6, No. 9, pp. 9138-9153, 2021. DOI: 10.3934/math.2021531

[20] Tesman B.A., "List T-Colorings of Graphs," Discrete Applied Mathematics, Vol. 45, No. 3, pp. 277-289, 1993. DOI: 10.1016/0166-218X(93)90015-G .

[21] Wang Z., Xu J., Xu. X., "On the distance spectrum of windmill graphs," Journal of Physics: Conference Series, Vol. 1592, The Third International Conference on Physics, Mathematics and Statistics, Kunming, China, May 20-22, 2020. DOI: 10.1088/1742-6596/1592/1/012056