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New Results on Face Magic Mean Labeling of Graphs

S. Vani Shree[1], **S. Dhanalakshmi**[2,*]

[1]Research Scholar, Department of Mathematics, SRM IST-Ramapuram, Chennai, India
[2]Faculty of Engineering and Technology, Department of Mathematics, SRM IST-Ramapuram, Chennai, India

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Abstract In the midst of the 1960s, a theory by Kotzig-Ringel and a study by Rosa sparked curiosity in graph labeling. Our primary objective is to examine some types of graphs which admit Face Magic Mean Labeling (FMML). A bijection ϕ : V$(G) \rightarrow \{1,2,\ldots,|V(G)|\}$ is called a (1,0,0) F-Face magic mean labeling [FMML] of G if the induced face labeling

$$\phi^*(f_i) = \left\lfloor \frac{sum\ of\ the\ label\ of\ the\ vertices\ in\ the\ boundary\ of\ f_i}{\deg(f_i)} \right\rfloor = \left\lfloor \frac{\sum\limits_{v_j \in f_i} \phi(v_j)}{\deg(f_i)} \right\rfloor = k,\ constant.$$

A bijection ϕ :V(G) $\cup$ E(G) $\rightarrow$ $\{1,2,\ldots,$ V$|G|$ $\cup$ $|E(G)|\}$ is called a (1,1,0) F-Face magic mean labeling [FMML] of G if the induced face labeling

$$\phi^*(f_i) = \left\lfloor \frac{sum\ of\ the\ label\ of\ the\ vertices\ and\ edges\ in\ the\ boundary\ of\ f_i}{\deg(f_i)} \right\rfloor = \left\lfloor \frac{\sum\limits_{v_j \in f_i} \phi(v_j) + \sum\limits_{e_j \in f_i} \phi(e_j)}{\deg(f_i)} \right\rfloor = k,\ constant.$$

In this paper it is being investigated that the (1, 0, 0) – Face Magic Mean Labeling (F-FMML) of Ladder graphs, Tortoise graph and Middle graph of a path graph. Also (1,0,0) and (1,1,0) F-Face Magic Mean Labeling is verified for Ortho Chain Square Cactus graph, Para Chain Square Cactus graph and some snake related graphs like Triangular snake graphs and Quadrilateral snake graphs. For a wide range of applications, including the creation of good kind of codes, synch-set codes, missile guidance codes and convolutional codes with optimal auto correlation characteristics, labeled graphs serve as valuable mathematical models. They aid in the ability to develop the most efficient non-standard integer encodings; labeled graphs have also been used to identify ambiguities in the access protocol of communication networks; data base management to identify the best circuit layouts, etc.

Keywords Face Magic Mean Labeling, Ladder Graph, Ortho Chain Square Cactus Graph, Para Chain Square Cactus Graph, Tortoise Graph, Middle Graph of a Path, Triangular Snake, Quadrilateral Snake

1 Introduction

Graph labeling offers valuable mathematical models for a variety of high-tech applications (data security, astronomy, cryptography (secret sharing schemes), various coding theory, communication networks, and so on). In network engineering, for example, all systems connected in a network can be converted into a graph, and specific numeric labels assigned to the converted graph according to specified rules aid in data flow regulation. In particular, antimagic and magic graphs, used as models for surveillance and security systems in urban planning. Magic labeling has applications in communication networks. These networks are made up of devices and the communication links that connect them. Each communication line has its own identity to prevent collisions. The identity of a line must be deduced from the devices and it connects for security reasons. The use of radar impulses to determine the distance between objects is another application of magic labeling. Different antimagic labeled graphs have been used for encryption and decryption techniques for an enhancement of security in transferring data [1]. In this work, concentration given on the planar graphs which reduce the likelihood of various entities overlapping in many real situations and increase the source of efficiency in organizations. A finite and undirected simple connected planar graph is explored in this work. A graph labeling [2] is a map that connects graph elements to numbers (usually to the positive or non-negative integers). For a deeper understanding of graph theoretic concepts, we refer the reader to [3]. The notion of mean labeling of graph was introduced by Somasundaram and Ponraj [4]. Magic labeling [5] was introduced by Sedláček in 1963. S. Arockiaraj and A. MeenaKumari [6] introduced the concept of F- Face Magic Mean Labeling. In [7], (1,1,0)-F-FMML of $P_n + K_1$, Lattitude graph, Cyclic Ladder and some duplication of graphs were analyzed. Arockiaraj et al. [8, 9] explored some ideas of F-FMML of P_n graph, Butterfly graph, Middle graph ($M(C_n)$) and Duplication of graphs. A. MeenaKumari et al [10] investigated (0,1,0), (1,1,0)-F-Face magic mean labeling for ladder graph, globe graph and new graphs obtained by applying graph operations. In this work, we have proved that Ladder, Tortoise and Middle graph of a Path admits (1,0,0)-F-FMML. Also it is verified that Ortho Chain Square Cactus graph, Para Chain Square Cactus graph, triangular snake graph and quadrilateral snake graph admits (1,0,0), (1,1,0) – F-FMML.

2 Main Results

2.1 Results on (1, 0, 0) F-Face magic mean labeling

Theorem 2.1.1: Ladder graph L_n, $n \geq 2$, admits (1, 0, 0) F-Face magic mean labeling.

Proof: Let G (V, E, F) be a ladder graph L_n for ($n \geq 2$) for face magic mean labeling of graph which contains (2n) vertices, (3n – 2) edges and (n – 1) interior face and one exterior face of G. Let the vertex set be V, where V = $\{u_l ; 1 \leq l \leq n : v_l ; 1 \leq l \leq n\}$, the edge set be E, where E = $\{u_l u_{l+1}; 1 \leq l \leq n-1 : v_l v_{l+1}; 1 \leq l \leq n-1\}$ and the face set be F, where F = $\{u_l; 1 \leq l \leq n : v_l ; 1 \leq l \leq n : u_l u_{l+1} ; 1 \leq l \leq n-1 : v_l v_{l+1}; 1 \leq l \leq n-1\}$. For (1,0,0) - Face magic mean labeling the deg(f_l) = 4: $1 \leq l \leq n-1$; and deg (f_0) = 2n.

Type: (1, 0, 0) – FMML

Consider a mapping: V (G) $\to \{1, 2, 3 \ldots 2n\}$ as

Case(i): $n \equiv 1 \pmod 2$

$$\phi(u_l) = \frac{l+1}{2}; (1 \leq l \leq n)\, and\, l \equiv 1 \;(\mathrm{mod}\ 2)$$

$$\phi(u_l) = \frac{3n+1+l}{2}; (1 \leq l \leq n)\, and\, l \equiv 0 \;(\mathrm{mod}\ 2)$$

$$\phi(v_l) = \frac{3n+2-l}{2}; (1 \leq l \leq n)\, and\, l \equiv 1 \;(\mathrm{mod}\ 2)$$

$$\phi(v_l) = \frac{2n+2-l}{2}; (1 \leq l \leq n)\, and\, l \equiv (0 \;\mathrm{mod}\ 2)$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*(f_l) = \left\lfloor \frac{1}{4}[8n+6] \right\rfloor = \lfloor n \rfloor ; (1 \leq l \leq n-1)$$

$$\phi^*(f_0) = \left\lfloor \frac{1}{2n}[n(2n+1)] \right\rfloor = \lfloor n \rfloor ;$$

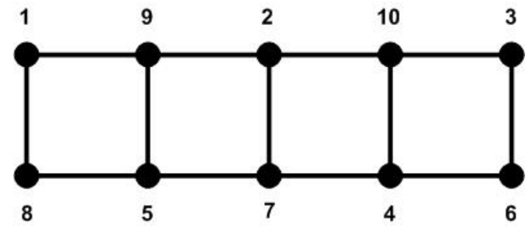

Figure 1. (1,0,0) - FMML of L_5 .

Case (ii): $n \equiv 0 \pmod 2$

$$\phi(u_l) = \frac{l+1}{2}; (1 \leq l \leq n)\, and\, l \equiv 1\, (mod\, 2)$$

$$\phi(u_l) = \frac{3n-l+2}{2}; (1 \leq l \leq n)\, and\, l \equiv 0\, (mod\, 2)$$

$$\phi(v_l) = \frac{4n+1-l}{2}; (1 \leq l \leq n)\, and\, l \equiv 1\, (mod\, 2)$$

$$\phi(v_l) = \frac{n+2l-2}{2}; (1 \leq l \leq n)\, and\, l \equiv 0\, (mod\, 2)$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*(f_l) = \left\lfloor \frac{1}{4}[8n+4] \right\rfloor = \lfloor n \rfloor ; (1 \leq l \leq n-1)$$

$$\phi^*(f_0) = \left\lfloor \frac{1}{2n}[n(2n+1)] \right\rfloor = \lfloor n \rfloor ;$$

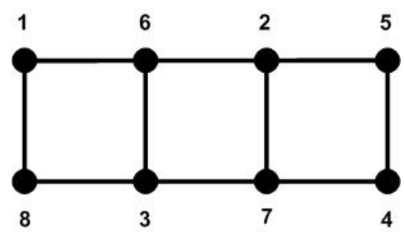

Figure 2. (1, 0, 0) - FMML of L_4 .

Theorem 2.1.2: Tortoise graph T_n for n≥4 admits (1, 0, 0) F-Face magic mean labeling.

Proof: Let G(V,E,F) be a tortoise graph T_n for (n≥4) for face magic mean labeling of graph which contains (n) vertices , $\left(\frac{3n-3}{2}\right)$ edges for ($n \equiv 1$ mod 2), $\left(\frac{3n}{2} - 1\right)$ edges for ($n \equiv 0$ (mod 2)) and (n–1) interior face and one exterior face. Let the vertex set be V, where V = $\{u_l ; 1 \leq l \leq n\}$ edge set be E, E = $\{u_l u_{l+1}; 1 \leq l \leq n-1\}$ and face set be F, where F = $\{f_l: u_l; 1 \leq l \leq n\} \cup \{f_0 : u_l ; 1 \leq l \leq n; u_l u_{l+1}; 1 \leq l \leq n-1\}$ where f_l is the interior face and f_0 is the exterior face of G. For (1, 0, 0) -Face magic mean labeling the deg (f_l) = $\frac{n}{2}$ and deg (f_0) = n.

Type: (1, 0, 0) - FMML

Consider a mapping: V (G) $\to \{1, 2, 3 ,\ldots, n\}$as

Case (i): n ≡ 0 (mod 2)

$$\phi\left(u_{l}\right)=\frac{2l}{2};(1\leq l\leq n)\,and\equiv 1\;(mod\;2\;)$$

$$\phi\left(u_{l}\right)=\frac{2l}{2};(1\leq l\leq n)\,and\,l\equiv 0\;(mod\;2\;)$$

Hence the induced face magic mean labeling ϕ^{*} is given by

$$\phi^{*}\left(f_{l}\right)=\left\lfloor\frac{2}{n-1}\left[\frac{4n+4}{2}\right]\right\rfloor=\left\lfloor\frac{n}{2}\right\rfloor;\left(2\leq l\leq\frac{n}{2}\right)$$

$$\phi^{*}\left(f_{1}\right)=\left\lfloor\frac{2}{n-1}\left[n+1\right]\right\rfloor=\left\lfloor\frac{n}{2}\right\rfloor;$$

$$\phi^{*}\left(f_{0}\right)=\left\lfloor\frac{n(n+1)}{2}\right\rfloor=\left\lfloor\frac{n}{2}\right\rfloor;$$

Case (ii): n ≡ 1 (mod 2)

$$\phi\left(u_{l}\right)=l;(1\leq l\leq n)$$

Hence the induced face magic mean labeling ϕ^{*} is given by

$$\phi^{*}\left(f_{1}\right)=\left\lfloor\frac{1}{n}\left[\frac{3n+3}{2}\right]\right\rfloor=\left\lfloor\frac{n+1}{2}\right\rfloor;$$

$$\phi^{*}\left(f_{0}\right)=\left\lfloor\frac{n(n+1)}{2}\right\rfloor=\left\lfloor\frac{n}{2}\right\rfloor;$$

$$\phi^{*}\left(f_{j}\right)=\left\lfloor\frac{1}{n}\left[\frac{4n+4}{2}\right]\right\rfloor=\left\lfloor\frac{n}{2}\right\rfloor;\left(2\leq l\leq\frac{n-1}{2}\right)$$

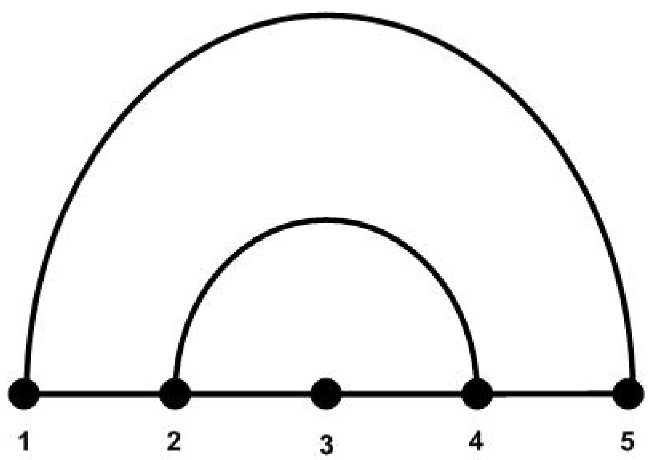

Figure 3. (1,0,0) - FMML of T_5 .

Theorem 2.1.3: Middle graph of a path graph M (P_n), n ≥ 2 admits (1,0,0) F-Face magic mean labeling .

Proof: Let G(V,E,F) be a middle graph of a path graph M(P_n) for (n ≥ 2) for face magic mean labeling of graph which contains (2n + 1) vertices, $(3n-1)$ edges and (n − 1) interior face and one exterior face. Let the vertex set be V, where V ={ u_l : $1\leq l\leq$ n}∪{ v_l : $1\leq l\leq$ n +1}, edge set be E, E = {u_lu_{l+1}, u_lv_{l+1} : $1\leq l\leq$ n − 1}∪{u_lv_l:$1\leq l\leq$ n} and face set be F, where F = {f_l: u_l : $1\leq l\leq$ n} ∪ {f_0 : u_l : $1\leq l\leq$ n; v_l; $1\leq l\leq$ n +1; u_lu_{l+1} u_lv_{l+1}; $1\leq l\leq$ n – 1; u_lv_l:$1\leq l\leq$ n} where f_l is the interior face and f_0 is the exterior face of G. For (1, 0, 0) – Face magic mean labeling the deg (f_l) = 3; $1\leq l\leq$ n; and deg (f_0) = 2n + 1.

Type: (1,0,0) - FMML

Consider a mapping: V (G) → {1,2,3,. . . ,2n + 1} as

Case (i): n ≡ 1 (mod 2)

$$\phi\left(u_{l}\right)=\frac{l+1}{2}\;;(1\leq l\leq n)\;and\,l\equiv\;1\;(\mathrm{mod}\;2)$$

$$\phi\left(u_{l+1}\right)=\frac{3n+l+4}{2}\;;(1\leq l\leq n)\;and\;\equiv\;1\;(\mathrm{mod}\;2\;)$$

$$\phi\left(v_{l}\right)=\frac{3n+5-2l}{2}\;;(1\leq l\leq n+1)$$

Hence the induced face magic mean labeling ϕ^{*} is given by

$$\phi^{*}\left(f_{l}\right)=\left\lfloor\frac{1}{3}\left[3n+4\right]\right\rfloor=\lfloor n+1\rfloor$$

$$\phi^{*}\left(f_{0}\right)=\lfloor n+1\rfloor$$

Case (ii): n ≡ (0 mod 2)

$$\phi\left(u_{l}\right)=\frac{l+1}{2}\;;(1\leq l\leq n)\;and\,l\equiv\;1\;(\mathrm{mod}\;2\;)$$

$$\phi\left(u_{l+1}\right)=\frac{3n+2l+2}{2}\;;(1\leq l\leq n-1)$$

$$\phi\left(v_{l}\right)=\frac{3n+4-2l}{2}\;;(1\leq l\leq n+1)$$

Hence the induced face magic mean labeling ϕ^{*} is given by

$$\phi^{*}\left(f_{l}\right)=\left\lfloor\frac{1}{3}\left[3n+3\right]\right\rfloor=\lfloor n+1\rfloor$$

$$\phi^{*}\left(f_{0}\right)=\lfloor n+1\rfloor$$

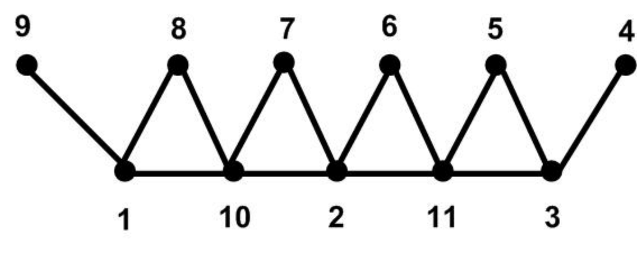

Figure 4. (1,0,0) - FMML of M(P_5).

2.2 Results on (1, 0, 0) , (1,1,0) F-Face Magic Mean Labeling

Theorem 2.2.1: Ortho chain square cactus graph O_n, $n\geq 2$ admits (1,0,0) and (1,1,0) F-Face magic mean labeling.

Proof: Let G (V, E, F) be a Ortho chain graph (O_n) for n ≥ 2 for mean magic labeling of graph which contains (3n– 2) vertices, (4n - 4) edges and (n – 1) interior faces and one exterior face. Let the vertex set be V where V = {u_l : $1\leq l\leq$ n ; v_l,y_l : $1\leq l\leq$ n – 1} and edge set be E where E = { u_lu_{l+1}, u$_l$v$_l$, v$_l$y$_l$y$_l$u$_{l+1}$: $1 \le l \le$ n – 1$\}$ and face set be F where F = $\{$f$_l$: u$_l$v$_l$y$_l$u$_{l+1}$;$1 \le l \le$ n – 1$\}$ $\cup$ $\{$f$_0$: u$_l$: $1 \le l \le$ n ; v$_l$,y$_l$: $1 \le l \le$ n – 1, u$_l$u$_{l+1}$, u$_l$v$_l$, v$_l$y$_l$y$_l$u$_{l+1}$: $1 \le l \le$ n – 1$\}$. Where f$_l$ is the interior face of G and f$_0$ is the exterior face of G. For (1,0,0) – face magic mean labeling the deg (f$_l$) = 4 : $1 \le l \le$ n – 1; deg (f$_0$) = 3n–2 and for (1,1,0) – face magic mean labeling the deg (f$_l$) = 8: $1 \le l \le$ n – 1; deg (f$_0$) = 7n–6.

Type (i): (1,0,0) – FMML
Consider a mapping: V (G) $\rightarrow \{1, 2, 3 \ldots 3\text{n - }2\}$ as

$$\phi\,(u_l) = l;\ (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(u'_l) = l;\ (1 \le l \le \mathrm{n})\ and\ l \equiv 0\ (\mathrm{mod}\ 2)$$
$$\phi\,(v_l) = 2n + l + 1;\ (1 \le l \le \mathrm{n})\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(v'_l) = 2n + l + 2;\ (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(y_l) = 2n - l + 2; (1 \le l \le \mathrm{n}+1)$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*\,(f_l) = \left\lfloor \frac{1}{4}\,[6n+4] \right\rfloor = \left\lfloor \frac{3n+2}{2} \right\rfloor; (1 \le l \le \mathrm{n})$$

$$\phi^*\,(f_0) = \left\lfloor \frac{1}{3n+1}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) + \left(\sum_{l=1}^{n-1} y_l\right)\right]\right\rfloor = \left\lfloor \frac{3n+2}{2} \right\rfloor$$

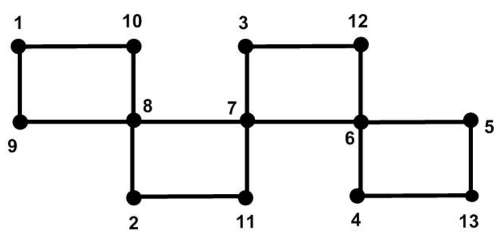

Figure 5. (1,0,0) - FMML of O_4.

Type (ii): (1, 1, 0) – FMML
Consider a mapping: V$\cup$E $\rightarrow \{1, 2, 3 \ldots 7\text{n - }6\}$ as

$$\phi\,(u_l) = n + l; (1 \le l \le \mathrm{n})\ and\ l \equiv 1\ (\mathrm{mod}\ 2\,)$$
$$\phi\,(v_l) = 7n - l + 2;\ (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(u'_l) = n + l + 1;\ (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(y_l) = 3n + l;\ (1 \le l \le \mathrm{n}+1)$$
$$\phi\,(v'_l) = 7n - l + 1;\ (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(u_l v_l) = 6n + 2 - l; (1 \le l \le \mathrm{n})\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(u'_l v'_l) = 6n - l + 1; (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(v_l y_l) = 5n - l + 2;\ (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\left(v'_l y'_{2l-1}\right) = 5n - l + 1; (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(y_{2l-1} u_l) = l; (1 \le l \le \mathrm{n}-1)\ and\ l \equiv 1\ (\mathrm{mod}\ 2)$$
$$\phi\,(u'_l y_{2l}) = 2l; (1 \le l \le \mathrm{n})\ and\ l \equiv 0\ (\mathrm{mod}\ 2)$$
$$\phi\left(y_l y_{l+1}\right) = 3n - l + 1; (1 \le l \le \mathrm{n})$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*\,(f_l) = \left\lfloor \frac{1}{8}\,[28n+8] \right\rfloor = \left\lfloor \frac{7n+2}{2} \right\rfloor (1 \le l \le \mathrm{n})$$

$$\phi^*\,(f_0) = \left\lfloor \frac{1}{7n-6}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) + \left(\sum_{l=1}^{n-1} y_l\right) + \left(\sum_{l=1}^{n} u'_l\right) + \left(\sum_{l=1}^{n} v'_l\right) + \left(\sum_{l=1}^{n} y'_l\right) + \left(\sum_{l=1}^{n-1} u_l v_l\right) + \left(\sum_{l=1}^{n-1} u_l y_{2l-1}\right) + \left(\sum_{l=1}^{n-1} y_l y_{l+1}\right) + \left(\sum_{l=1}^{n-1} v_l y_{2l-1}\right) + \left(\sum_{l=1}^{n-1} u'_l y_{2l}\right) + \left(\sum_{l=1}^{n-1} v'_l y_{2l-1}\right) + \left(\sum_{l=1}^{n-1} u'_l v'_l\right)\right]\right\rfloor = \left\lfloor \frac{7n+2}{2} \right\rfloor$$

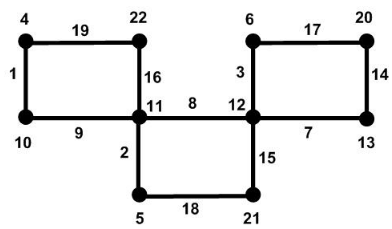

Figure 6. (1,1,0) - FMML of OO_3.

Theorem 2.2.2: Para chain square cactus graph q$_n$, $n \ge 2$ admits (1,0,0) and (1,1,0) F-Face magic mean labeling.

Proof: Let G (V, E, F) be a para chain square graph (q$_n$) for n $\ge$ 2 for mean magic labeling of graph which contains (3n – 2) vertices, (4n – 4) edges and (n – 1) interior faces and one exterior face. Let the vertex set be V where V = $\{$u$_l$: $1 \le l \le$ n ; v$_l$,y$_l$: $1 \le l \le$ n – 1$\}$ and edge set be E where E = $\{$ u$_l$u$_{l+1}$, u$_l$v$_l$, v$_l$y$_l$, y$_l$u$_{l+1}$: $1 \le l \le$ n – 1$\}$ and face set be F where F = $\{$f$_l$: u$_l$v$_l$y$_l$u$_{l+1}$;$1 \le l \le$ n – 1$\}$ $\cup$ $\{$f$_0$: u$_l$: $1 \le l \le$ n ; v$_l$,y$_l$: $1 \le l \le$ n – 1, u$_l$u$_{l+1}$, u$_l$v$_l$, v$_l$y$_l$, y$_l$u$_{l+1}$: $1 \le l \le$ n – 1$\}$. Where f$_l$is the interior face of G and f$_0$ is the exterior face of G. For (1,0,0) – face magic mean labeling the deg (f$_l$) = 4 : $1 \le l \le$ n – 1; deg (f$_0$) = 3n – 2 and for (1,1,0) – Face magic mean labeling the deg (f$_l$) = 8: $1 \le l \le$ n – 1; deg (f$_0$) = 7n – 6.

Type (i): (1,0,0) - FMML
Consider a mapping: V (G) $\rightarrow \{1, 2, 3 \ldots 3\text{n - }2\}$ as

$$\phi\,(u_l) = 2n - l + 2;\ (1 \le l \le\ \mathrm{n}+1\,)$$

$$\phi\left(v_l\right) = l;\ (1 \leq l \leq \text{n})$$

$$\phi\left(y_l\right) = \ 2n + l + 1; (1 \leq l \leq \text{n})$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*\left(f_l\right) = \left\lfloor \frac{1}{4}\left[6n+4\right] \right\rfloor = \left\lfloor \frac{3n+2}{2} \right\rfloor; (1 \leq l \leq \text{n})$$

$$\phi^*\left(f_0\right) = \left\lfloor \frac{1}{3n-2}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) + \left(\sum_{l=1}^{n-1} y_l\right) \right] \right\rfloor = \left\lfloor \frac{3n+2}{2} \right\rfloor$$

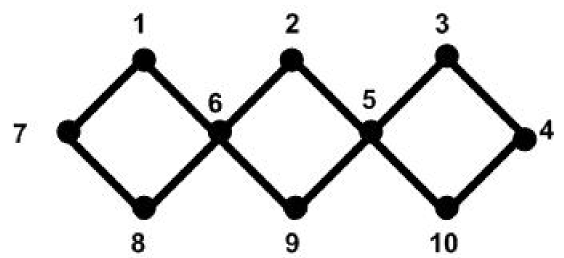

Figure 7. (1,0,0) - FMML of q_3.

Type (ii): (1,1,0) - FMML
Consider a mapping: V∪E → {1, 2, 3…7n - 6} as

$$\phi\left(u_l\right) = 3n + l;\ (1 \leq l \leq \ \text{n}+1)$$

$$\phi\left(v_l\right) = n + l;\ (1 \leq l \leq \text{n})$$

$$\phi\left(y_l\right) = 7n - l + 2;\ (1 \leq l \leq \text{n} - 1)$$

$$\phi\left(u_l v_l\right) = l;\ (1 \leq l \leq \text{n})$$

$$\phi\left(v_l u_{l+1}\right) = 6n - l + 2;\ (1 \leq l \leq \text{n})$$

$$\phi\left(u_l y_l\right) = 3n - l + 1;\ (1 \leq l \leq \text{n})$$

$$\phi\left(y_l u_{l+1}\right) = 5n - l + 2;\ (1 \leq l \leq \text{n})$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*\left(f_l\right) = \left\lfloor \frac{1}{8}\left[28n+8\right] \right\rfloor = \left\lfloor \frac{7n+2}{2} \right\rfloor;\ 1 \leq l \leq \text{n}$$

$$\phi^*\left(f_0\right) = \left\lfloor \frac{1}{7n-6}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) + \left(\sum_{l=1}^{n-1} y_l\right) + \left(\sum_{l=1}^{n-1} u_l v_l\right) + \left(\sum_{l=1}^{n-1} v_l u_{l+1}\right) + \left(\sum_{l=1}^{n-1} y_l u_{l+1}\right) + \left(\sum_{l=1}^{n-1} y_l u_l\right) \right] \right\rfloor = \left\lfloor \frac{7n+2}{2} \right\rfloor$$

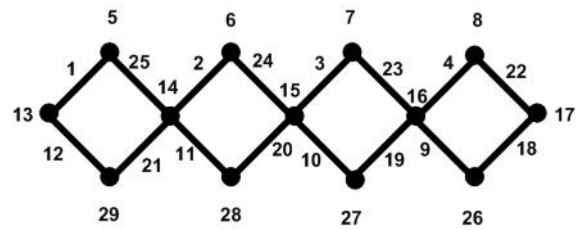

Figure 8. (1,1,0) - FMML of q_4.

Theorem 2.2.3: Triangular snake graph T(S_n), n ≥ 2 admits (1,0,0), (1,1,0) – F-Face Magic Mean labeling.

Proof: Let G (V,E,F) be a triangular snake graph T(S_n) for n ≥ 2 for mean magic labeling of graph which contains (2n – 1) vertices , (3n – 3) edges and (n – 1) interior faces and one exterior face. Let the vertex set be V where V = {u_l : $1 \leq l \leq$ n ; v_l $1 \leq l \leq$ n – 1} and edge set be E where E = {$u_l u_{l+1}$, $u_l v_l$, $v_l u_{l+1}$: $1 \leq l \leq$ n – 1} and face set be F where F = {f_l:$u_l v_l u_{l+1}$;$1 \leq l \leq$ n – 1} ∪{f_0: u_l :$1 \leq l \leq$ n , v_l:$1 \leq l \leq$ n – 1, $u_l u_{l+1}$: $1 \leq l \leq$ n – 1,$u_l v_l$: $1 \leq l \leq$ n – 1,$u_{l+1} v_l$:$1 \leq l \leq$ n – 1 }, where f_l is the interior face of G and f_0 is the exterior face of G. For (1,0,0) – face mean magic labeling the deg (f_l) = 3: $1 \leq l \leq$ n – 1; deg (f_0) = 2n – 1 and for (1,1,0) – face mean magic labeling the deg (f_l) = 6: $1 \leq l \leq$ n – 1; deg (f_0) = 5n – 4.

Type (i) :(1,0,0) – FMML
Consider a mapping: V(G) → {1,2,3…2n - 1} as

Case (i): n ≡ 1 (mod 2)

$$\phi\left(u_l\right) = \frac{l+1}{2};\ 1 \leq l \leq \ \text{n};\ and\ l \equiv 1(mod 2)$$

$$\phi\left(u_{\frac{2l+2}{2}}\right) = \frac{3n+l}{2};\ 1 \leq l \leq \ \text{n} - 2;\ for\ l \equiv 1(mod 2)$$

$$\phi\left(v_l\right) = \frac{3n-2l+1}{2};\ 1 \leq l \leq \text{n} - 1$$

Hence the induced face labeling ϕ^* in G is given by

$$\phi^*\left(f_l\right) = \left\lfloor \frac{1}{3}\left[3n+1\right] \right\rfloor = n$$

$$\phi^*\left(f_0\right) = \left\lfloor \frac{1}{2n-1}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) \right] \right\rfloor = n$$

Case (ii): n ≡ 0(mod 2)

$$\phi\left(u_l\right) = \frac{l+1}{2}; 1 \leq l \leq n\ ;\ and\ l \equiv \ 1\ (\text{mod } 2)$$

$$\phi\left(u_{\frac{2l+2}{2}}\right) = \frac{3n+l-1}{2}; 1 \leq l \leq n-1; and\ \ l \equiv 1\ (\text{mod } 2)$$

$$\phi\left(v_l\right) = \frac{3n-2l}{2}; 1 \leq l \leq \text{n} - 1$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*\left(f_l\right) = \left\lfloor \frac{1}{3}\left[3n\right] \right\rfloor = n$$

$$\phi^*\left(f_0\right) = \left\lfloor \frac{1}{2n-1}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) \right] \right\rfloor = n$$

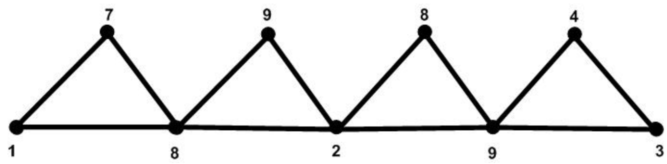

Figure 9. (1,0,0)-FMML of T(S_5).

Case (ii): (1,1,0) – FMML
Consider a mapping: V∪E $\rightarrow$ {1,2,3…5n - 4} as

$$\phi(u_l) = \frac{6n-2l-2}{2}; 1 \le l \le n$$

$$\phi(v_l) = \frac{2n+2l-2}{2}; 1 \le l \le n-1$$

$$\phi(u_l v_l) = \frac{2l}{2}; 1 \le l \le n-1$$

$$\phi(v_l u_{l+1}) = \frac{6n+2l-4}{2}; 1 \le l \le n-1$$

$$\phi(u_l u_{l+1}) = \frac{10n-2l-6}{2}; 1 \le l \le n-1$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*(f_l) = \left\lfloor \frac{1}{6}[15n-9] \right\rfloor = \frac{5n-3}{2}; 1 \le l \le n-1$$

$$\phi^*(f_0) = \left\lfloor \frac{1}{5n-4}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) + \left(\sum_{l=1}^{n-1} u_l v_l\right) + \left(\sum_{l=1}^{n} u_l u_{l+1}\right) + \left(\sum_{l=1}^{n} v_l u_{l+1}\right) \right] \right\rfloor = \frac{5n-3}{2}$$

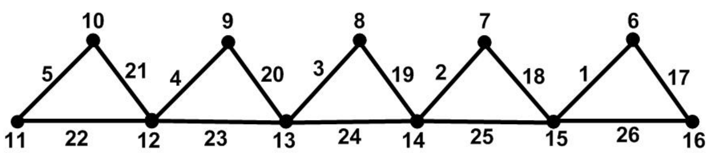

Figure 10. (1,1,0)-FMML of (T(S_6)).

Theorem 2.2.4: Quadrilateral snake graph Q(S_n), n $\geq$ 2 admits (1,0,0) , (1,1,0) – F-Face Magic Mean labeling.

Proof: Let G (V, E, F) be a triangular snake graph (Q_n) for n $\geq$ 2 for mean magic labeling of graph which contains (3n – 2) vertices, (4n – 4) edges and (n – 1) interior faces and one exterior face. Let the vertex set be V where V = {u_l : $1 \le l \le$ n ; v_l,y_l : $1 \le l \le$ n – 1} and edge set be E where E = { $u_l u_{l+1}$, $u_l v_l$, $v_l y_l y_l u_{l+1}$: $1 \le l \le$ n – 1} and face set be F where F = {f_l: $u_l v_l y_l u_{l+1}$;1 $\le l \le$ n – 1} ∪ {f_0 : u_l : $1 \le l \le$ n ; v_l,y_l: $1 \le l \le$ n – 1, $u_l u_{l+1}$, $u_l v_l$, $v_l y_l y_l u_{l+1}$: $1 \le l \le$ n – 1}. where f_l is the interior face of G and f_0 is the exterior face of G. For (1,0,0) – face mean magic labeling the deg (f_l) = 4 : $1 \le l \le$ n – 1; deg (f_0) = 3n – 2 and for (1,1,0) – face mean magic labeling the deg (f_l) = 8: $1 \le l \le$ n – 1; deg (f_0) = 7n – 6.

Type (i): (1, 0, 0) – FMML
Consider a mapping: V (G) $\rightarrow$ {1, 2, 3…3n - 2} as

$$\phi(u_l) = 2n - l; 1 \le l \le n$$

$$\phi(v_l) = l;\ 1 \le l \le n-1$$

$$\phi(y_l) = 2n + l - 1; 1 \le l \le n-1$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*(f_l) = \left\lfloor \frac{1}{4}[6n-2] \right\rfloor = \left\lfloor \frac{3n-1}{2} \right\rfloor; 1 \le l \le \text{n} - 1$$

$$\phi^*(f_0) = \left\lfloor \frac{1}{3n-2}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) + \left(\sum_{l=1}^{n-1} y_l\right) \right] \right\rfloor = \left\lfloor \frac{3n-1}{2} \right\rfloor$$

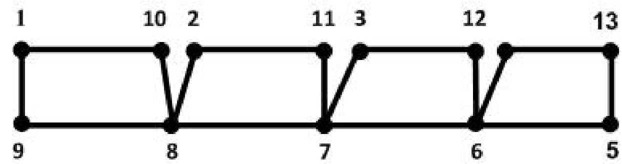

Figure 11. (1, 0, 0)-FMML of (Q(S_5)).

Type (ii) :(1, 1, 0) – FMML
Consider a mapping: V∪E $\rightarrow$ {1, 2, 3…7n - 6} as

$$\phi(u_l) = 3n + l - 3; 1 \le l \le n$$

$$\phi(v_l) = n + l - 1;\ 1 \le l \le n-1$$

$$\phi(y_l) = 7n - l - 5;\ 1 \le l \le n-1$$

$$\phi(u_l v_l) = l; 1 \le l \le n-1$$

$$\phi(v_l y_l) = 6n - l - 4;\ 1 \le l \le n-1$$

$$\phi(y_l u_{l+1}) = 5n - l - 3; 1 \le l \le n-1$$

$$\phi(u_l u_{l+1}) = 3n - l - 2; 1 \le l \le n-1$$

Hence the induced face magic mean labeling ϕ^* is given by

$$\phi^*(f_l) = \left\lfloor \frac{1}{8}[28n-20] \right\rfloor = \left\lfloor \frac{7n-5}{2} \right\rfloor 1 \le l \le n-1$$

$$\phi^*(f_0) = \left\lfloor \frac{1}{7n-6}\left[\left(\sum_{l=1}^{n} u_l\right) + \left(\sum_{l=1}^{n-1} v_l\right) + \left(\sum_{l=1}^{n-1} y_l\right) + \left(\sum_{l=1}^{n-1} u_l v_l\right) + \left(\sum_{l=1}^{n-1} u_l u_{l+1}\right) + \left(\sum_{l=1}^{n-1} y_l u_{l+1}\right) \right.\right.$$

$$+\left(\sum_{l=1}^{n-1} y_l v_l\right)\Bigg]\Bigg] = \left\lfloor\frac{7n-5}{2}\right\rfloor$$

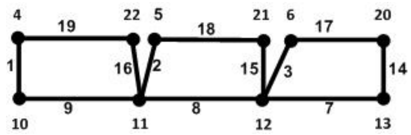

Figure 12. (1, 1, 0)-FMML of Q(S_6).

Note: Assignment of labels to the vertices and edges to be done in such a way that the mean weight of exterior face also remains constant.

3 Conclusions

In this paper we have obtained (1, 0, 0) – F -Face Magic Mean labeling of Ladder graph, Tortoise graph and Middle graph of a path and further investigation is done for (1, 0, 0), (1,1,0) – F - Face Magic Mean labeling of Ortho Chain Square Cactus graph, Para Chain Square Cactus graph, Triangular Snake graph and Quadrilateral Snake graph. For future study, the same graphs can be taken for verification of (1,1,1) – F - Face Magic Mean Labeling. Analogous work can be carried out for antimagic concept. More progression can be shown if the above study is applied in the field of cryptography.

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