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A New Methodology on Rough Lattice Using Granular Concepts

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Abstract Rough set theory has a vital role in the mathematical field of knowledge representation problems. Hence, a Rough algebraic structure is defined by Pawlak. Mathematics and Computer Science have many applications in the field of Lattice. The principle of the ordered set has been analyzed in logic programming for crypto-protocols. Iwinski extended an approach towards the lattice set with the rough set theory whereas an algebraic structure based on a rough lattice depends on indiscernibility relation which was established by Chakraborty. Granular means piecewise knowledge, grouping with similar elements. The universe set was partitioned by an indiscernibility relation to form a Granular. This structure was framed to describe the Rough set theory and to study its corresponding Rough approximation space. Analysis of the reduction of granular from the information table is based on object-oriented. An ordered pair of distributive lattices emphasize the congruence class to define its projection. This projection of distributive lattice is analyzed by a lemma defining that the largest and the smallest elements are trivial ordered sets of an index. A Rough approximation space was examined to incorporate with the upper approximation and analysis with various possibilities. The Cartesian product of the distributive lattice was investigated. A Lattice homomorphism was examined with an equivalence relation and its conditions. Hence the approximation space exists in its union and intersection in the upper approximation. The lower approximation in different subsets of the distributive lattice was studied. The generalized lower and upper approximations were established to verify some of the results and their properties.

Keywords Indiscernibility Relation, Granular Lattice, Congruence Class, Distributive Lattice, Lattice Homomorphism Figure

1 Introduction

Rough Set Theory was introduced by Pawlak [1] for a study of vague and uncertain data with complete and incomplete knowledge. An approximation space was farmed to be defined by a universal set and equivalence relation. The methodology was studied as a pair of subsets namely lower and upper approximation. An abstract approximation space was introduced by Cattaneo [2] and the relation between the orthocomplement operates was studied. A wide investigation was conducted to the model and its approaches toward the Rough Set Theory. Yao [3] studied the binary relation between two universal sets as objects and verifies its properties. Emphasis was put on the features of the concept Lattice with Rough Set Theory. Susanta Bera [4] discussed the properties of Rough Lattice and Rough Modular Lattice. An approximation space emphasizes the indiscernibility relation of an object by Pawlak notations.
Richard [5] defined an algebraic structure in a congruence class under the closed operation as meet. The properties of congruence classes were applied in translation by Lattice. A topological space for mapping a homomorphic condition was introduced and its uniqueness theorem was established. Complete Boolean algebra in a new methodology was constructed and its characteristics space in an approximation sense was discussed. Jarvinen [6] examined the different types of lattice and their complement conditions. The operators were discussed with various approximation spaces to define a Rough Set Theory. Estaji [7], an algebraic structure was defined to describe an interconnection between the concept of Rough set and Lattices, and the properties of Rough Ideal and Rough Filter were discussed. A homomorphism function was described for a Prime ideal and a Prime filter for a set of fixed points. Fei Li [8]

investigated Rough groups, and Rough Quotient groups and examined their results with some of their properties. Shao [9] an ordered structure was emphasized with the binary relation on Rough Set Theory, which implies the Lattice Theory. The rough lattice with a specified notation for lower and upper approximation was studied.
Yamaguchi [10] introduced a Grey Rough Set using Lattice operation. An information system based on numerical interval data was investigated. An equivalence relation was defined for a grey rough set based on the Lattice operation. A Methodology was introduced for a non-deterministic Information system. Rana [11] approached significant results on the model based on a Rough interval using Lattice to define operators. A distributive lattice has been examined by a family of Rough intervals. An effective algebraic structure has been investigated in the data for some types of Rough Sets [12]. This illustrates a rough approximation space with the covering of the lattice. Rana [13] formulated the two important concepts of Rough Partial ordered relation with Rough Lattice. Rough Boolean Lattice was constructed to verify its properties with Rough relation and Rough Lattice. Yao [14] constructed an approximation operator using the concept of Lattice. The data were examined by defining a binary relation using the concept of Lattice. An universe set was partitioned by the nested granular to define an equivalence relation [15]. This illustrates the Rough set approximation for a different level of bounded lattice to examine its results and properties.
From the above literature, a distributive lattice is framed to define a Lattice homomorphism function and verify its equivalency condition. The Cartesian product of distributive lattice was partitioned by congruence classes and its properties with granular Lattice were discussed. Hence the results and properties were examined.

2 Preliminaries

By partitioning the distributive Lattice, an algebraic structure was defined for a lower and upper approximation. Throughout this paper, " $\leq$ " represents the order of a given Lattice.

2.1 Rough Set

An information system (IS) was introduced by Pawlak [1] which consists of a non-empty finite set of an object (O), knowledge obtained as an attribute (A) where A is a combination of condition attributes (C) and decision attributes (D) such as $A = C \cup D$, V is a cartesian product of object and attributes $(O \times A)$ and f is a function defined as $f : A \to V$. Hence, the information system is denoted as $IS = \langle O, A, V, f \rangle$.
Let S be a subset of A, then an indiscernibility relation was described as an approximation space is defined as a pair of $(O, [p]_S)$ where $[p]_S$ is an equivalence relation which partition the universal set O. Consider X be a subset of object (O), then the lower and upper approximation is defined as [7, 9]

$$\overline{apr(X)} = \{p \in O | [p]_S \subseteq X\}$$

$$\underline{apr(X)} = \{p \in O | [p]_S \cap X \neq \oslash\}$$

The Boundary, Positive and Negative regions are
1. $BR(X) = \overline{apr(X)} - \underline{apr(X)}$
2. $PR(X) = \underline{apr(X)}$
3. $NR(X) = U - \overline{apr(X)}$
The set is said to be a definable set if $\overline{apr(X)} = \underline{apr(X)}$, else the set is Rough Set or undefinable set.

2.2 Lattice

A partially ordered set $(L, \leq)$ is said to be a Lattice if it satisfies the condition $p \wedge q = p$ and $p \vee q = q$ for all $p, q \in L$. Here $\wedge$ and $\vee$ represent the binary operators, where $p \leq q$.[5]

2.2.1 Distributive lattice

A distributive Lattice is a Lattice with the operator $\wedge$ and $\vee$ if it satisfies $\forall p, q, r \in L$. [5]

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

2.3 Rough Lattice

Consider the approximation space (L, P) where P is an equivalence relation. Let $X \subseteq L$ and then the Rough Set was defined as pair of $P(X) = (\underline{P(X)}, \overline{P(X)})$. [6, 10]

Consider a Rough lattice $\left\langle \overline{P(X)}, \wedge, \vee \right\rangle$ where $\overline{P(X)}$ are sublattice of L in which it satisfies the following condition for $p, q, r \in X$
1. $p \wedge p = p$, $p \vee p = p$ (Idempotency)
2. $p \wedge q = q \wedge p$, $p \vee q = q \vee p$ (Commutativity)
3. $p \wedge (q \wedge r) = (p \wedge q) \wedge r$, $p \vee (q \vee r) = (p \vee q) \vee r$ (Associativity)
4. $p \wedge (p \vee q) = p$, $p \vee (p \wedge q) = p$ (Absorption)
5. $p \leq q$ if $p \wedge q = p$ and $p \vee q = q$ (Consistency).

3 Main Results

This section analyzes the information table into granules and describes the Cartesian product of two distributive lattices. An equivalence class was determined by the lattice homomorphism. A Granular Distributive Lattice with some of the results and properties are introduced.

Definition 3.1. *Consider an approximation space (A, O, Ind) where A is an attribute, O is an object, and $Ind \subseteq A \times O$ is an indiscernibility relation between A and O.*

Definition 3.2. *Let $X \subseteq O$ and $M \subseteq A$ be the dual operator such that* [9, 13]
$M^* = \{x | x \in O, a \in M, (a, x) \in Ind\}$
$X^* = \{a | a \in M, \forall x \in O, (a, x) \in Ind\}$
Here M^* is the family of all objects that are paired with all the attributes M and X^* is the family of all attributes that are fulfilled with all the objects in X.

Definition 3.3. *A pair of (M, X) is said to be a granular where $M \subseteq A$ and $X \subseteq O$. If $X^* = M$ and $X = M^*$, then $(M_1, X_1) \leq (M_2, X_2)$ which implies $M_1 \subseteq M_2$.*

Definition 3.4. *Let (M_1, X_1) and (M_2, X_2) be the two granular if $X_1 = X_2$ and $M_1 \leq M_2$ then $L(M_2, X_2, G)$ be the Lattice (L) and induced by the granular (G).*

Definition 3.5. *Consider $L(M_1, X_1, G_1)$ and $L(M_2, X_2, G_2)$ be the granular. Hence the least upper bound and greatest lower bound of the lattice are defined as* [8, 10]
$(M_1, X_1) \wedge (M_2, X_2) = (M_1 \cap M_2, (X_1 \cup X_2)^{**})$
$(M_1, X_1) \vee (M_2, X_2) = ((M_1 \cup M_2)^{**}, X_1 \cap X_2)$

Definition 3.6. *Let $L(M_1, X_1, G_1)$ and $L(M_2, X_2, G_2)$ be the two granular in the Lattice then $(M, X) \in L(M_1, X_1, G_1)$ then there exist $(M', X') \in L(M_1, X_1, G_1)$ such that $M' = M$ then $L(M_1, X_1, G_1) \leq L(M_2, X_2, G_2)$* [6] .

Definition 3.7. *Consider a distributive Lattice from a Granular as $\{D_\alpha | \alpha \in I\}$ where I be an index set of the partially ordered if*
(i) $\cup D_\alpha = L$
(ii) If $D_\alpha \leq D_\beta$ then $x \in D_\alpha$ and $y \in D_\beta$ there exist a mapping $\pi : D_\alpha \times D_\beta \to L$ such that $[x, y]_\pi \Rightarrow x \equiv y(L)$ if $(x \wedge y') \vee (x' \wedge y) \in L$
Hence for a canonical structure of the distributive lattice, a congruence relation is defined as $[x, y]_\pi$.

Definition 3.8. *Consider M be a sublattice of L then an approximation space is defined as (L, M, π). Hence the lower and upper approximation as*
$\underline{\pi(M)} = \cup\{a \in [x, y]_\pi | \pi(a) \subseteq M\}$
$\overline{\pi(M)} = \cup\{a \in [x, y]_\pi | \pi(a) \cap M \neq \oslash\}$
$BR_\pi(M) = \{\overline{\pi(M)} - \underline{\pi(M)}\}$
A pair of $(\underline{\pi(M)}, \overline{\pi(M)})$ be the Rough Set on M.

Theorem 3.1. *Let π be an equivalence relation on an approximation space (L, M, π) if and only if it satisfies the Lattice homomorphism.*

Proof. *Consider π be an Lattice homomorphism as $\pi : D_\alpha \times D_\beta \to L$ then*
there exist $[x, y]_\pi \Rightarrow x \equiv y(L)$ if $(x \wedge y') \vee (x' \wedge y) \in L$ such that $\pi(x) = a \wedge x$ and $\pi(y) = a \wedge y$ for all $x \in D_\alpha, y \in D_\beta$ and a is a distributive element in $D_\alpha or D_\beta$.
Here, π is an binary relation on L. Then $_x\pi_y \Rightarrow \pi(x) = \pi(y)$
Therefore π is an equivalence relation on approximation space (L, M, π).
Similarly, converse is also true.

Theorem 3.2. *Let (L, M, π) be an approximation space then there exist any two elements in L such that its union exists in upper approximation.*

Proof. *Consider $D_\alpha \leq D_\beta$ then there exist $x \in D_\alpha$ and $y \in D_\beta$*
By Lattice homomorphism, $\pi(x), \pi(y) \in \overline{\pi(M)}$
Therefore, by distributive lattice if $x \in D_\alpha$ and $y \in D_beta$ then
$\pi(x \vee y) = \pi(x) \vee \pi(y) \in \overline{\pi(M)}$ for all $x \vee y \in L$ Hence the result.

Corollary 3.3. *Let (L, M, π) be an approximation space then there exist any two elements in L such that its intersection exists in upper approximation.*

Proof. *Hence the proof is similar to Theorem 3.2*

Properties 3.1. *Let (L, M, π) be an approximation space then the following properties holds:*
(i) $\underline{\pi(M)} \subseteq M \subseteq \overline{\pi(M)}$
(ii) $\underline{\pi(M \cup N)} \supseteq \underline{\pi(M)} \cup \underline{\pi(N)}$
(iii) $\overline{\pi(M \cup N)} \supseteq \overline{\pi(M)} \cup \overline{\pi(N)}$
(iv) $\underline{\pi(M \cap N)} \subseteq \underline{\pi(M)} \cup \underline{\pi(N)}$
(v) $\overline{\pi(M \cap N)} \subseteq \overline{\pi(M)} \cup \overline{\pi(N)}$
(vi) If $M \subseteq N$ then $\overline{\pi(M)} \subseteq \overline{\pi(N)}$ and $\underline{\pi(M)} \subseteq \underline{\pi(N)}$

Example 3.1. *Consider an information table with the object of 5 with the attribute $price = P$, $Room = R$, $Furniture = F$, also with decision attributes $Prestigiousflat = PF$ consisting of "Yes" and "No".*

$Object/Attributes$	$Price$	$Room$	$Furniture$	$PrestigiousFlat$
1	$High$	1	No	Yes
2	$High$	3	Yes	Yes
3	$High$	1	No	No
4	$High$	2	No	Yes
5	Low	2	Yes	No

Table 1. Infomation Table

By partitioning the set as (M, X, G) with the concepts of Granule we can define 23 Granule sets. Hence, reduction of the Granule concept can reduce 9 Granule and form the Hasse Diagram shown below

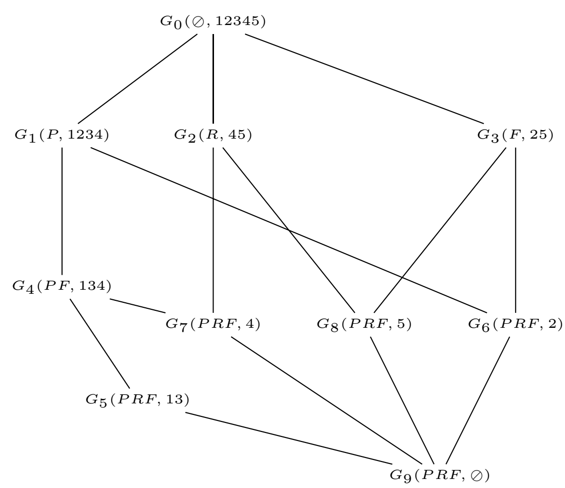

Figure 1. Hasse Diagram

Let us consider $D_\alpha = \{G_0, G_1, G_4, G_5, G_9\}$ and $D_\beta = \{G_0, G_1, G_4, G_5, G_6, G_7, G_9\}$
Then the Cartesian product of $D_\alpha \times D_\beta$
= $\{(G_0, G_0), (G_0, G_1), (G_0, G_4), (G_0, G_5), (G_0, G_6), (G_0, G_7), (G_0, G_9), (G_1, G_0), (G_1, G_1), (G_1, G_4), (G_1, G_5), (G_1, G_6), (G_1, G_7), (G_1, G_9), (G_4, G_0), (G_4, G_1), (G_4, G_4), (G_4, G_5), (G_4, G_6), (G_4, G_7), (G_4, G_9), (G_5, G_0), (G_5, G_1), (G_5, G_4), (G_5, G_5), (G_5, G_6), (G_5, G_7), (G_5, G_9), (G_9, G_0),$

$(G_9, G_1), (G_9, G_4), (G_9, G_5), (G_9, G_6), (G_9, G_7), (G_9, G_9)\}$
Therefore, $[x, y]_\pi = \{\{G_0, G_1, G_4, G_9\}, \{G_0, G_1, G_4, G_9\},$
$\{G_0, G_1, G_4, G_5, G_6, G_7, G_9\}, \{G_0, G_1, G_4, G_5, G_7, G_9\}\}$
Case (i) $M = \{G_0, G_2, G_8, G_9\}$
Then, $\underline{\pi(M)} = \oslash$
and $\overline{\pi(M)} = \{G_0, G_1, G_4, G_5, G_6, G_7, G_9\}$
Case (ii) $M = \{G_0, G_1, G_6, G_9\}$
Then, $\underline{\pi(M)} = \{G_0, G_1, G_6, G_9\}$
and $\overline{\pi(M)} = \{G_0, G_1, G_4, G_5, G_6, G_7, G_9\}$

Lemma 3.4. *Let a distributive lattice* $\{D_\alpha | \alpha \in I\}$ *have the largest and smallest element then* I *be trivially ordered if the same holds with no hypothesis on* D_α.

Proof. *Let us consider* 0 *or* $\oslash$ *and* 1 *or O(Object) be the smallest and largest element in* D_α *where* $\alpha \in I$ *respectively, then*
By the definition of homomorphism function $\pi : D_\alpha \times D_\beta \to L$, *then* π *be a binary relation. Hence* $\pi(x) = \pi(y)$
$\pi(x) = 1$ *if* $\alpha > \beta$
$\pi(x) = x$ *if* $\alpha = \beta$
$\pi(x) = 0$ *if* $\alpha \ngeq \beta$
In case of I *is trivially ordered, there exist* $a \in D_\alpha$ *such that* $\pi(x) = a$ *if* $\alpha \neq \beta$ *and* $\pi(x) = x$ *if* $\alpha = \beta$.
Hence the result

Theorem 3.5. *The Cartesian product of a family of* $\{D_\alpha | \alpha \in I\}$ *be a distributive lattice if and only if* D_α *is a projection for each* $\alpha \in I$.

Proof. *Let* D_α *and* D_β *be the distributive Lattice, where* $\alpha, \beta \in I$
By the definition of homomorphism function $\pi : D_\alpha \times D_\beta \to L$, *then* π *be a binary relation. Hence* $\pi(x) = \pi(y)$
From the above Lemma,
D_α *is projection, there exist an* $\pi(x) = \pi(y)$ *for all* $x \in D_\alpha$, $y \in D_\beta$.
Hence the result.

Theorem 3.6. *Let* (L, M, π) *be an approximation space then Case (i) If* M *is not a subset of* D_β *then the lower approximation is a empty set. Case (ii) If* M *is a subset of* D_β *then the lower approximation is a non-empty set. Case (iii) Always the upper approximation be the* D_β

Proof. *Let* (L, M, π) *be the approximation Space then*
Case (i): Let M *is not a subset of* D_β. *i.e.,* $M \nsubseteq D_\beta$
Hence, $y \in D_\beta$ *which implies* $y \notin M$
Since $y \in \pi(a) \subseteq M$ *then* $y \notin \pi(a)$, *lower approximation is an empty set.*
Case (ii): Let M *is a subset of* D_β. *i.e.,* $M \subseteq D_\beta$
Hence, $y \in D_\beta$ *which implies* $y \in M$
Since $y \in \pi(a) \subseteq M$ *then* $y \in M$ *and* $y \in \pi$ *then lower approximation is always non-empty set.*
Case (iii): For any $x \in D_\alpha$ *and* $y \in D_\beta$ *then,*
By definition of upper approximation, $D_\beta \subseteq \pi(a) \cap M$
which implies $D_\beta \subseteq \overline{\pi(M)}$ *——(1)*
$[x, y]_\pi \in D_\alpha \times D_\beta$ *then there exist* $a \in [x, y]_\pi$ *such that* $[x, y]_\pi \in \pi(a) \cap M$
Since its satisfies the distributive condition, $\pi(a) \cap M \subseteq D_\beta$
which implies $\overline{\pi(M)} \subseteq D_\beta$ *——(2)*
From (1) and (2), $\overline{\pi(M)} = D_\beta$
Hence the result.

It is observed that the Cartesian product of a distributive lattice has its projection. The upper approximation satisfies the Rough lattice and lattice homomorphism condition. Various approximation spaces formed by a different subset of M were examined to determine the knowledge about the information table.

4 Optimistic Analysation

This section investigates the structural study of a granular distributive lattice and determines its equivalence class forms the approximation set. The data is about the 6 patients after the surgery were kept under observation in the ICU ward.

	Temp	*Oxygen*	*BP*	*Consumption*
1	*Low*	*Good*	*Medium*	3
2	*High*	*Good*	*High*	2
3	*Low*	*Good*	*High*	2
4	*Low*	*Better*	*Medium*	2
5	*High*	*Good*	*High*	2
6	*High*	*Better*	*Low*	2

Table 2. After surgury Patient Information

From Table 2, an ICU dataset was analyzed as $\{1, 2, 3, 4, 5, 6\}$ are set of the patients who were treated as an object and $\{Temp, Oxygen, BP, Consumption\}$are the set of attributes of Temperature, Oxygen Supply, Blood Pressure, Consumption Measures of the patient. By partitioning the set as (M, X, G) with the concepts of Granule we can define 55 Granule sets. Hence, the reduction of the Granule concept into 15 Granule and form of the Hasse Diagram shown in Fig 2 consist of $L = \{G_0, G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9, G_{10}, G_{11},$ $G_{12}, G_{13}, G_{14}, G_{15}\}$

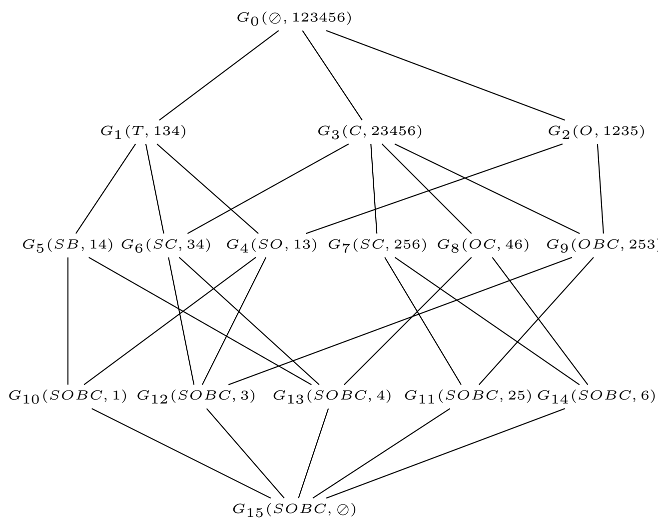

Figure 2. Granular Distributive Lattice

Consider $D_\alpha\{G_0, G_1, G_5, G_{10}, G_{15}\}$ and $D_\beta\{G_0, G_1, G_5, G_6, G_{10}, G_{13}, G_{15}\}$ Hence the ordered pair of distributive lattice as $D_\alpha \times D_\beta$
$= \{(G_0, G_0), (G_0, G_1), (G_0, G_5), (G_0, G_6), (G_0, G_{10}), (G_0, G_{13}), (G_0, G_{15}), (G_1, G_0), (G_1, G_1), (G_1, G_5), (G_1, G_6), (G_1, G_{10}), (G_1, G_{13}), (G_1, G_{15}), (G_5, G_0), (G_5, G_1), (G_5, G_5), (G_5, G_6), (G_5, G_{10}), (G_5, G_{13}), (G_5, G_{15}), (G_{10}, G_0), (G_{10}, G_1), (G_{10}, G_5), (G_{10}, G_6), (G_{10}, G_{10}), (G_{10}, G_{13}), (G_{10}, G_{15}), (G_{15}, G_0), (G_{15}, G_1), (G_{15}, G_5), (G_{15}, G_6), (G_{15}, G_{10}), (G_{15}, G_{13}), (G_{15}, G_{15})\}$

From Definition 3.7, the equivalence class is defined as $[x, y]_\pi \Rightarrow x \equiv y(L)$ if $(x \wedge y') \vee (x' \wedge y) \in L$ and From theorem 3.1, it can be verified with Lattice homomorphism condition below.
$[G_0, G_0]_\pi$ = $(123456 \wedge \oslash) \vee (\oslash \wedge 123456) = \oslash = G_{15} \in L$
$[G_0, G_1]_\pi$ = $(123456 \wedge 256) \vee (\oslash \wedge 134) = 256 = G_7 \in L$
$[G_0, G_5]_\pi$ = $(123456 \wedge 2356) \vee (\oslash \wedge 14) = 2356 \notin L$
Similarly, find for all the ordered pair of the distributive lattice. Hence the equivalence class is defined as $\{\{G_0, G_1, G_{10}, G_{13}, G_{15}\}, \{G_0, G_1, G_5, G_6, G_{10}, G_{13}, G_{15}\}, \{G_1, G_5, G_6, G_{10}, G_{13}, G_{15}\}\}$ and also, consider $M = \{G_0, G_1, G_5, G_{10}, G_{13}, G_{15}\}$ From this lower and the upper approximation as
$\underline{\pi(M)} = \{G_0, G_1, G_10, G_13, G_15\}$
$\overline{\overline{\pi(M)}} = \{G_0, G_1, G_5, G_6, G_{10}, G_{13}, G_{15}\}$

From the upper approximation G_{10} and G_{13} contain the object which satisfies all the attributes and also it contains in G_0, G_1, G_5, G_6. Hence, we concluded that the object 1 and 4 can shift to the general ward from the ICU ward.

It is observed that the 6 patients data were analyzed in the ICU ward and an ordered pair of granular exists to form a distributive lattice using a Hasse diagram. The Cartesian product of two distributive lattices was given and the equivalence class was verified by the lattice homomorphism condition. Consider $M \subseteq L$, then based on M it is easy to diagonalize the patient shift to the general ward or needs more observation regarding the patient.

5 Conclusions

In this paper,

- A lattice homomorphism was discussed to define an equivalence relation.
- The union and intersection of the upper approximation were discussed based on the structure of the distributive lattice.
- The projection of the distributive lattice was verified with the lemma.
- Always the upper approximation is the sub lattice and the lower approximation differed from the case.
- A Real time experimental analysis uses Granular distributive lattice.

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