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Multiplication and Inverse Operations in Parametric Form of Triangular Fuzzy Number

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Abstract Many authors have given the arithmetic form of triangular fuzzy numbers, especially for addition and subtraction; however, there is not much difference. The differences occur for multiplication, division, and inverse operations. Several authors define the inverse form of triangular fuzzy numbers in parametric form. However, it always does not obtain $\tilde{a}(r)\otimes\frac{1}{\tilde{a}(r)}\neq\tilde{\imath}(r)$, because we cannot uniquely determine the inverse that obtains the unique identity. We will not be able to directly determine the inverse of any matrix in the form of a triangular fuzzy number. Thus, all problems using the matrix $\tilde{A}$ in the form of a triangular fuzzy number cannot be solved directly by determining $\tilde{A}^{-1}$. In addition, there are various authors who, with various methods, try to determine $\tilde{A}^{-1}$ but still do not produce $\tilde{A}\otimes\tilde{A}^{-1}=\tilde{I}$. Consequently, the solution of a fully fuzzy linear system will produce an incompatible solution, which results in different authors obtaining different solutions for the same fully fuzzy linear system. This paper will promote an alternative method to determine the inverse of a fuzzy triangular number in parametric form. It begins with the construction of a midpoint $m(\tilde{a})$ for any triangular fuzzy number $\tilde{a}=(a,\alpha,\beta)$, or in parametric form $\tilde{a}(r)=\left[\underline{a}(r),\overline{a}(r)\right]$. Then the multiplication form will be constructed obtaining a unique inverse which produces $\tilde{a}(r)\otimes\frac{1}{\tilde{a}(r)}=\tilde{\imath}(r)$. The multiplication, division, and inverse forms will be proven to satisfy various algebraic properties. Therefore, if a triangular fuzzy number is used, and also a triangular fuzzy number matrix is used, it can be easily directly applied to produce a unique inverse. At the end of this paper, we will give an example of calculating the inverse of a parametric triangular fuzzy number for various cases. It is expected that the reader can easily develop it in the case of a fuzzy matrix in the form of a triangular fuzzy number.

Keywords Triangular Fuzzy Number, Multiplication, Inverse in Parametric Form, Triangular Fuzzy Liner System

1. Introduction

Fuzzy linear systems are used in various fields of science, especially engineering, finance, and economics [1-4,8]. Some models of fuzzy linear systems include linear systems in the form of triangular fuzzy numbers. The arbitrary triangular fuzzy numbers can be changed in the form of a parametric form triangular fuzzy number, as introduced by some authors, including [5-17,19,23,34].

Some arithmetic forms for triangular fuzzy number operations are introduced by some authors but there is a little difference for addition and subtraction operations. Meanwhile, for multiplication and division/inverse operations, there are some models. For example, [5,7,17-22] use the concept of min max s for multiplication, but do it differently for the division. On the other hand, [4-6,24-28] provide an alternative to multiplication in various cases. The author's focus is "why many authors do not provide alternatives for calculating the inverse of a triangular fuzzy number, such as [1,4-7,18-21,25-27]". Furthermore, why is there no author who completes triangular fuzzy number linear system using the concept of determinate or inverse fuzzy matrix? It is suggested that each author looks for an alternative solution and avoid using the inverse of the triangular fuzzy number, even trying to partition it into a real matrix. For example, [25] uses the ST method, while [8] does this by separating the parts $\underline{a}_{ij}(r)$ with $\overline{a}_{ij}(r)$ into separate equations. Furthermore, [23] uses the functions $\underline{f}(\propto)$ and $\overline{f}(\propto)$ and then calculates the limit, while other method used by various authors [1], [5-6,9-16,19,23,29-30] in solving the linear system of fuzzy numbers was either in the basic form or in the form of parametric.

The basics of the problem of various arithmetic operations and various methods of solving the system of linear are given for the arbitrary triangular fuzzy number $\tilde{a}(r)$. There is no element $\frac{1}{\tilde{a}(r)}$, so that $\tilde{a}(r)\otimes\frac{1}{\tilde{a}(r)} = \tilde{I} = [1,0,0]$.

Based on the description above, the authors define that the form of the multiplication of two fuzzy numbers for $\tilde{a}(r) \neq \tilde{0}$ will be able to determine a single element $\tilde{x}(r)$, i.e., $\tilde{a}(r)\otimes\tilde{x}(r) = \tilde{x}(r)\otimes\tilde{a}(r) = \tilde{I}$; in this case, means $\tilde{x}(r) = \frac{1}{\tilde{a}(r)}$. Furthermore, the concept of multiplication and inverse can be used easily in solving triangular fuzzy number linear systems and other problems that require the concept of determinant and inverse matrix triangular fuzzy numbers.

2. Preliminaries

Some basic concepts of fuzzy number have been defined in [3,9-16,27].

2.1. Definition

A triangular fuzzy number $\tilde{a} = (a, \alpha, \beta)$ is a fuzzy set on R with the membership function given which satisfies:

1. $\tilde{a}(x)$ is upper semi-continuous
2. $\tilde{a}(x) = 0$, outside the interval [0,1]
3. $\tilde{a}(x)$ is a monotonic increasing function on $[a - \alpha, a]$
4. $\tilde{a}(x)$ is a monotonic decreasing function on $[a, a + \beta]$
5. $\tilde{\mu}(x) = 1, for\ x = a$

Notation of the triangular fuzzy number used in this research is $\tilde{a} = (a, \alpha, \beta)$, where a is the center of triangular fuzzy number, α is the distance of left wide, and β is right wide; this notation has been used in [3,9-16,27]. The membership function of triangular fuzzy number $\tilde{a} = (a, \alpha, \beta)$ is:

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 - \frac{a-x}{\propto}, & \text{if } a - \propto \leq x \leq a \\ 1 - \frac{x-a}{\beta}, & \text{if } a \leq x \leq a + \beta \\ 0, & \text{other} \end{cases}$$

A fuzzy number $\tilde{a}(r)$ in parametric form can be notated as $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)]$ with $\underline{a}(r) = a - (1 - r)\alpha$ and $\overline{a}(r) = a + (1 - r)\beta$.

2.2. Definition

A fuzzy number $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)]$ is a function which satisfies:

a. $\underline{a}(r)$ is a bounded left continuous non-decreasing function at (0,1], and right continuous at 0,
b. $\overline{a}(r)$ is a bounded left continuous non-increasing function at (0,1], and right continuous at 0,
c. $\underline{a}(r) \leq \overline{a}(r), r \in [0,1]$

Two fuzzy numbers $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)]$ and $\tilde{b}(r) = [\underline{b}(r), \overline{b}(r)]$are equal if $\underline{a}(r) = \underline{b}(r)$ and $\overline{a}(r) = \overline{b}(r)$. The forms $\tilde{a} = (a, \alpha, \beta)$ and $\tilde{b} = (b, \gamma, d)$ are equal if $a = b, \alpha = \gamma$ and $\beta = d$. Definition of similarity between two fuzzy numbers is agreed by some authors. However, there are not many authors who state explicitly about positivity of triangular fuzzy number as [1,4,7,25]. They denote that triangular fuzzy number $\tilde{a} = (a, \alpha, \beta)$ is non-negative if $a \geq 0$, while [26], denote $\tilde{a}$ is positive and $\tilde{a}$ is negative if $a + \beta < 0$. Furthermore, algebra of interval in parametric form as given by [1,3,5,7,17,23,34] is as follows.

2.3. Definition

Two fuzzy numbers in parametric form $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)]$ and $\tilde{b}(r) = [\underline{b}(r), \overline{b}(r)]$, and scalar $k \in R$ is defined as follows:

a) $\tilde{a}(r) + \tilde{b}(r) = [\underline{a}(r) + \underline{b}(r), \overline{a}(r) + \overline{b}(r)]$
b) $\tilde{a}(r) - \tilde{b}(r) = [\underline{a}(r) - \overline{b}(r), \overline{a}(r) - \underline{b}(r)]$
c) $\tilde{a}(r)\otimes\tilde{b}(r) = [\min S, \max S]$
    a) With
    $S = \{\underline{a}(r)\underline{b}(r), \underline{a}(r)\overline{b}(r), \overline{a}(r)\underline{b}(r), \overline{a}(r)\overline{b}(r)\}$
d) $k\tilde{a}(r) = \begin{cases} [k\,\overline{a}(r), k\underline{a}(r)], & if\ k < 0 \\ [k\underline{a}(r), k\overline{a}(r)], & if\ k \geq 0 \end{cases}$
e) $m(\tilde{a}) = \frac{\underline{a}(r) + \overline{a}(r)}{2}$

If we apply them to triangular fuzzy number in the form $\tilde{a} = (a, \alpha, \beta)$ and $\tilde{b} = (b, \gamma, d)$, the algebra is given by [1,5,7,17-20,23-25]. If it is changed to parametric form, then the multiplication operation will be the same as Definition 2.3. However, the multiplication operation is different from what is given by [4,7,26-28,32-33], while the concept of positivity of triangular and trapezoidal fuzzy number uses a wide area concept as given in [9-16].

3. Material and Method

As noted above, the addition, subtraction, and scalar multiplication use Definition 2.3 (a), (b), and (d). Meanwhile, the multiplication of fuzzy number will be formulated using another concept. Before formulating the concept of multiplication, we will define the positivity of triangular fuzzy number. In this study, triangular fuzzy numbers are chosen, because the use of triangular fuzzy numbers provides easy and simple calculations. This is because the triangular fuzzy number has the characteristic of its membership function which is linear, although the use of fuzzy numbers with non-triangular membership functions may also be studied.

3.1. Definition

A triangular fuzzy number $\tilde{a} = (a, \alpha, \beta)$ is positive if $a > 0$, and $\tilde{a}$ is negative if $a < 0$.

In parametric form, $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)] = [a - (1-r)\alpha, a + (1-r)\beta]$

Define $(\tilde{a}) = \frac{\underline{a}(1)+\overline{a}(1)}{2} = a$, so the positivity concept based on $m(\tilde{a})$ is as follows:

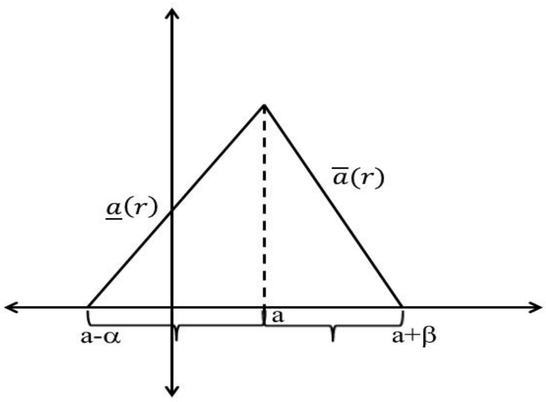

Figure 1. Triangular Fuzzy Number

3.2. Research Model

Parametric triangular fuzzy number $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)]$ is positive if $m(\tilde{a}) > 0$, and it is non-negative if $m(\tilde{a}) \geq 0$. Furthermore, it is negative if $m(\tilde{a}) < 0$, and it is non-positive if $m(\tilde{a}) \leq 0$, and it is zero if $m(\tilde{a}) = 0$, which is notated with $\tilde{a}(r) \approx \tilde{0}(r)$. If $\tilde{a}(r) = [0,0]$ so $\tilde{a}(r)$ is pure zero with the notation $\tilde{a}(r) \approx \tilde{0}_p(r)$.

Both definitions clearly mean that the definition of positivity fuzzy number is equivalent to Definition 3.1 and 3.2.

Furthermore, we define the multiplication formula of arbitrary two parametric triangular fuzzy numbers $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)]$ and $\tilde{b}(r) = [\underline{b}(r), \overline{b}(r)]$ are

$$\tilde{a}(r) \otimes \tilde{b}(r) = \big(\underline{a}(r)m(\tilde{b}) + \underline{b}(r)m(\tilde{a}) - m(\tilde{a})m(\tilde{b}), \overline{a}(r)m(\tilde{b}) + \overline{b}(r)m(\tilde{a}) - m(\tilde{a}).m(\tilde{b})\big) \quad (3.1)$$

Simplifying equation (3.1) to be

$$\tilde{a}(r) \otimes \tilde{b}(r) = \big(\underline{a}(r)b + \underline{b}(r)a - ab,\ \overline{a}(r)b + \overline{b}(r)a - ab\big) \quad (3.2)$$

Remark

By definition 3.2 and equation (3.2), we have

(i) If $\tilde{a}(r)$ and $\tilde{b}(r)$ are positive, then $\tilde{a}(r) \otimes \tilde{b}(r)$ is positive.
(ii) If $\tilde{a}(r)$ and $\tilde{b}(r)$ are negative, then $\tilde{a}(r) \otimes \tilde{b}(r)$ is positive.
(iii) If $\tilde{a}(r)$ is negative and $\tilde{b}(r)$ is positive, then $\tilde{a}(r) \otimes \tilde{b}(r)$ is negative.
(iv) If $\tilde{a}(r)$ is positive and $\tilde{b}(r)$ is negative, then $\tilde{a}(r) \otimes \tilde{b}(r)$ is negative.

4. Results and Discussion

Based on Definition 3.2 and equation (3.2), we can construct $\frac{1}{\tilde{a}(r)}$ for arbitrary parametric triangular fuzzy number $\tilde{a}(r)$, such as

$$\tilde{a}(r) \otimes \tilde{x}^*(r) = \tilde{I}(r) = [1,1] \quad (4.1)$$

4.1. Theorem

Arbitrary fuzzy number $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)]$ where $m(\tilde{a}) \neq 0$, there are

$$\tilde{x}^*(r) = \frac{1}{\tilde{a}(r)} = \left[\frac{2m(\tilde{a}) - \underline{a}(r)}{\big(m(\tilde{a})\big)^2}, \frac{2m(\tilde{a}) - \overline{a}(r)}{\big(m(\tilde{a})\big)^2}\right]$$

then equation (3.1) applies with the multiplication as on equation (3.1) or (3.2).

Proof

First, we determine the value of $m(\tilde{x}^*)$, that is

$$m(\tilde{x}^*) = \underline{x}(1) = \frac{2m(\tilde{a}) - \underline{a}(1)}{\big(m(\tilde{a})\big)^2} = \frac{2m(\tilde{a}) - \overline{a}(1)}{\big(m(\tilde{a})\big)^2}$$

Because the value of $m(\tilde{a}) = \underline{a}(1) = \overline{a}(1) = a$, we get $m(\tilde{x}^*) = \frac{2a-a}{a^2} = \frac{1}{a}$, so that

$$\tilde{a}(r) \otimes \tilde{x}^*(r) = \tilde{I} = \big[\underline{a}(r)m(\tilde{x}^*) + \underline{x}(r)m(\tilde{a}) - m(\tilde{a})m(\tilde{x}^*), \overline{a}(r)m(\tilde{x}^*) + \overline{x}(r)m(\tilde{a}) - m(\tilde{a}).m(\tilde{x}^*)\big]$$

$$= \left[\underline{a}(r)\frac{1}{a} + \frac{2m(\tilde{a}) - \underline{a}(r)}{\big(m(\tilde{a})\big)^2}.m(\tilde{a}) - a.\frac{1}{a}, \overline{a}(r)\frac{2}{a} + \frac{2m(\tilde{a}) - \overline{a}(r)}{\big(m(\tilde{a})\big)^2}.m(\tilde{a}) - a.\frac{1}{a}\right]$$

$$= \left[\underline{a}(r)\frac{1}{a} + \frac{2a - \underline{a}(r)}{a} - 1, \overline{a}(r)\frac{1}{a} + \frac{2a - \overline{a}(r)}{a} - 1\right] = [1,1]$$

4.2. Example

An example is given in Table 4.1.

Table 4.1. An example of arbitrary parametric triangular fuzzy number $\tilde{a}(r)$

$\tilde{a}(r)$	$\tilde{x}^*(r) = \frac{1}{\tilde{a}(r)}$	$\tilde{a}(r)\otimes\frac{1}{\tilde{a}(r)}$
$(4+r, 6-r)$	$\left[\frac{2.5-(4+r)}{25}, \frac{2.5-(6-r)}{25}\right]=\left[\frac{6-r}{25}, \frac{4+r}{25}\right]$	$\left[\left((4+r).\frac{1}{5}+\frac{6-r}{25}.5\right)-1, (6-r).\frac{1}{5}+\frac{4+r}{25}5-1\right]$ = [1,1]
(-2+r, 1-2r)	$\frac{2.(-1)-(-2+r)}{1}, \frac{2.(-1)-(1-2r)}{1}$[-r, -3+2r]	[(-2+r)(-1)+(-r)(-1)-1, (1-2r)(-1)+(-3+2r)(-1)-1 = [1,1]
(-2 + 3r, 5-4r)	$[2.1-(-2+3r), 2.1-(5-4r)]$=[4-3r, -3+4r]	[-2+3r).1+(4-3r).1 - 1, (5-4r).1+(-3+4r).1-1]=[1,1]

If $\tilde{b}(r) = [\underline{b}(r), \overline{b}(r)]$ then $m\left(\frac{1}{\tilde{b}(r)}\right) = \left[\frac{2m(\tilde{b})-\underline{b}(r)}{\left(m(\tilde{b})\right)^2}, \frac{2m(\tilde{b})-\overline{b}(r)}{\left(m(\tilde{b})\right)^2}\right] = \frac{1}{m(b)}$

4.3. Corollary

For arbitrary fuzzy numbers $\tilde{a}(r) = [\underline{a}(r), \overline{a}(r)]$ and $\tilde{b}(r) = [\underline{b}(r), \overline{b}(r)]$, we have

$$\frac{\tilde{a}(r)}{\tilde{b}(r)} = \tilde{a}(r)\otimes\frac{1}{\tilde{b}(r)} = [\underline{a}(r), \overline{a}(r)]\otimes\left[\frac{2m(\tilde{b})-\underline{b}(r)}{\left(m(\tilde{b})\right)^2}, \frac{2m(\tilde{b})-\overline{b}(r)}{\left(m(\tilde{b})\right)^2}\right]$$

$$= \left[\underline{a}(r)\frac{1}{m(\tilde{b})} + \frac{2m(\tilde{b})-\underline{b}(r)}{\left(m(\tilde{b})\right)^2}m(\tilde{a}) - \frac{m(\tilde{a})}{m(\tilde{b})}, \quad \overline{a}(r)\frac{1}{m(\tilde{b})} + \frac{2m(\tilde{b})-\overline{b}(r)}{\left(m(\tilde{b})\right)^2}m(\tilde{a})\right]$$

$$= \left[\frac{\underline{a}(r)m(\tilde{b}) + 2m(\tilde{b})m(\tilde{a}) - \underline{b}(r)m(\tilde{a}) - m(\tilde{a})m(\tilde{b})}{\left(m(\tilde{b})\right)^2}, \frac{\overline{a}(r)m(\tilde{b}) + 2m(\tilde{b})m(\tilde{a}) - \overline{b}(r)m(\tilde{a}) - m(\tilde{a})m(\tilde{b})}{\left(m(\tilde{b})\right)^2}\right]$$

$$= \left[\frac{\underline{a}(r)m(\tilde{b}) - \underline{b}(r)m(\tilde{a}) + m(\tilde{a})m(\tilde{b})}{\left(m(\tilde{b})\right)^2}, \frac{\overline{a}(r)m(\tilde{b}) - \overline{b}(r)m(\tilde{a}) + m(\tilde{a})m(\tilde{b})}{\left(m(\tilde{b})\right)^2}\right]$$

In the same, way as proofs Theorem 4.1 and Corollary 4.3, the following theorem can be proven for arbitrary triangular fuzzy numbers [34-35].

4.4. Theorem

Let $\tilde{a}(r), \tilde{b}(r)$ and $\tilde{c}(r)$ be parametric triangular fuzzy, respectively, we have

a. $\tilde{a}(r)\otimes\tilde{0}(r) = \tilde{0}(r)$
b. $\tilde{a}(r)\otimes\tilde{I}(r) = \tilde{I}(r)$
c. $\tilde{a}(r)\otimes\tilde{b}(r) = \tilde{b}(r)\otimes\tilde{a}(r)$
d. $\left(\tilde{a}(r)\otimes\tilde{b}(r)\right)\otimes\tilde{c}(r) = \tilde{a}(r)\otimes\left(\tilde{b}(r)\otimes\tilde{c}(r)\right)$
e. $\left(\tilde{a}(r)\oplus\tilde{b}(r)\right)\otimes\tilde{c}(r) = \tilde{a}(r)\otimes\tilde{c}(r)\oplus\tilde{b}(r)\otimes\tilde{c}(r)$
f. If $\tilde{a}(r)\otimes\tilde{x}(r) = \tilde{b}(r)$ where $\tilde{a}(r) \not\approx \tilde{0}(r)$, then$\tilde{x}(r) = \frac{\tilde{b}(r)}{\tilde{a}(r)}$
g. If $\tilde{a}(r)\otimes\tilde{b}(r) = \tilde{0}(r)$, then $\tilde{a}(r) \approx \tilde{0}(r)$ or $\tilde{b}(r) \approx \tilde{0}(r)$
h. If $\tilde{a}(r)\otimes\tilde{b}(r) = \tilde{a}(r)\otimes\tilde{c}(r)$ where $\tilde{a}(r) \not\approx \tilde{0}(r)$, then $\tilde{b}(r) = \tilde{c}(r)$
i. If $\tilde{a}(r) \not\approx \tilde{0}(r)$, then $\frac{1}{\tilde{a}(r)} = \tilde{0}(r)$ and $\frac{1}{\frac{1}{\tilde{a}(r)}} = \tilde{a}(r)$
j. If $\tilde{a}(r) \not\approx \tilde{0}(r)$ and $\tilde{b}(r) \neq \tilde{0}(r)$, then $\frac{1}{\tilde{a}(r)\otimes\tilde{b}(r)} = \frac{1}{\tilde{a}(r)}\otimes\frac{1}{\tilde{b}(r)}$

Proof: Clearly

4.5. Remark

For arbitrary parametric triangular fuzzy number $\tilde{a}(r)$, we have:

$$\frac{\tilde{a}(r)}{\tilde{a}(r)} = \left[\frac{\underline{a}(r)m(\tilde{a}) - \underline{a}(r)m(\tilde{a}) + m(\tilde{a})m(\tilde{a})}{\left(m(\tilde{a})\right)^2}, \frac{\overline{a}(r)m(\tilde{a}) - \overline{a}(r)m(\tilde{a}) + m(\tilde{a})m(\tilde{a})}{\left(m(\tilde{a})\right)^2}\right] = [1,1]$$

5. Conclusion

For arbitrary triangular fuzzy numbers in parametric form $\tilde{a}(r) = \left[\underline{a}(r), \overline{a}(r)\right]$ and $\tilde{b}(r) = \left[\underline{b}(r), \overline{b}(r)\right]$, the addition, subtraction, and scalar multiplication operations are the same as those found by most authors such as the rules (a), (b) and (d) on definition in sub 2.3. Meanwhile, multiplication is as in equation (3.1). For inverse, we used the rule of Theorem on sub 4.1. The rules of algebra operation for this triangular fuzzy number in parametric form can be said to be better than the existing form of operation because its multiplication and division operations are more complete and include wider case.

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