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# The Locating Chromatic Number for Certain Operation of Origami Graphs

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**Abstract** The locating chromatic number introduced by Chartrand et al. in 2002 is the marriage of the partition dimension and graph coloring. The locating chromatic number depends on the minimum number of colors used in the locating coloring and the different color codes in vertices on the graph. There is no algorithm or theorem to determine the locating chromatic number of any graph carried out for each graph class or the resulting graph operation. This research is the development of scientific theory with a focus of the study on developing new ideas to determine the extent to which the locating chromatic number of a graph increases when applied to other operations. The locating chromatic number of the origami graph was obtained. The next exciting thing to know is locating chromatic number for certain operation of origami graphs. This paper discusses locating chromatic number for specific operation of origami graphs. The method used in this study is to determine the upper and lower bound of the locating chromatic number for certain operation of origami graphs. The result obtained is an increase of one color in the locating chromatic number of origami graphs.

**Keywords** Locating Coloring, Locating Chromatic Number, Origami Graph, Certain Operation of Origami Graphs

## 1 Introduction

The locating chromatic number was introduced by Chartrand et al.[1] in 2002 combination of partition dimension [2] and graph coloring. Partition dimension is developed from metric dimension. Metric dimensions were first introduced by Harary and Melter [3] in 1976. Many applications that can be applied using the concept of metric dimensions include robotic navigation [4], optimization of fire sensor placement [5], and data classification of chemical compounds [6].

Chartrand et al. [1] defined the locating chromatic number of a graph like this. Let $H = (V, E)$ be a finite and connected graph. The distance between two of its vertices $a$ and $b$, denoted by $d(a, b)$, is the length of the shortest path between them. Let $\Pi = \{R_1, R_2, ..., R_f\}$ be a partition of $V(H)$ which is induced by the coloring $r$ with color $\{1, 2, ..., f\}$. The color code of $u$, denoted by $r_\Pi(u)$ is the ordered $f$-tuple $(d(u, R_1), d(u, R_2), ..., d(u, R_k)))$ where $d(u, R_i) = \min\{d(u, w) : w \in R_i\}$ for any $i \in \{1, 2, 3, ..., f\}$. If all vertices of $H$ have different color codes, then $r$ is called a $r$-locating coloring of $H$. The smallest $f$ such that $H$ has a locating $f$-coloring said the locating chromatic number (lcn, in short) denoted by $\chi_L(H)$.

In general, determining the lcn of a graph is a complex problem because there is no algorithm to assess the lcn of any graph [1]. The results obtained by Chartrand et al. [1] among others, that the lcn of paths is 3, lcn of cycle graph $C_n$, where $n \geq 3$ vertices are 3 for odd $n$ and 4 for even $n$. Next, the lcn of double star graph $S_{(a,b)}$ for $1 \leq a \leq b$ and $b \geq 2$ is $b+1$ and the lcn of a complete multipartite graph is $n$. In 2011, Asmiati et al. [7] have succeeded in obtaining the lcn of a uniform amalgamation of stars and Asmiati et al. [8] for general amalgamation of stars. Furthermore, Syofyan et al. [9] are also interested in studying the lcns of tree, especially for Lobster graphs in 2013. Then in 2016, Asmiati [10] obtained the lcns for general caterpillar and firecracker graphs.

Chartrand et al. [11] characterized graphs with lcns $(n-1)$ or $(n-2)$. Furthermore, Asmiati and Baskoro [12] in 2012, succeeded in characterizing some graphs containing cycles with lcn 3. Next, Baskoro and Asmiati [13] determined to characterize tree as having lcn 3. In 2017, Asmiati et al. [14] obtained the classification of Petersen graphs with lcn 4 or 5.

The study of lcn and its variants is still exciting today, as evidenced by the many research results on the lcn of graphs and their variances. Welyyanti et al. [15] determined the lcn of disconnected graphs. Behtoei and Anbarloei [16] determined lcn for joining the graphs. Asmiati et al. [17] have obtained the lcn on a barbell graph containing a complete graph or Generalized Petersen graph, then continued in 2019 by Asmiati et al. [18] for its subdivisions. Ghanem et al. [19] obtained the lcn of power paths and cycles. In 2021, Irawan et al. [20] obtained the lcn on the origami graph and its barbell. Later in the same year, Irawan et al. [21] obtained the lcn of the origami barbell graph. Furthermore, Asmiati et al. [22] in 2021 succeeded in determining the lcn of the path shadowgraph and its barbell.

The following definition of an origami graph is taken from [23]. Let $m \in \mathbb{N}$ with $m \geq 3$. An origami graph $O_m$ is a graph with $V(O_m) = \{u_j, v_j, w_j : j \in \{1, ..., m\}\}$ and $E(O_m) = \{u_jw_j, u_jv_j, v_jw_j : j \in \{1, ..., m\}\} \cup \{u_ju_{j+1}, w_ju_{j+1} : j \in \{1, ..., m-1\}\} \cup \{u_mu_1, w_mu_1\}$. An subdivision of a origami graph $O^*_m$ is a graph with $V(O^*_{mn}) = \{u_j, v_j, x_j, w_j : j \in \{1, ..., m\}\}$ and $E(O^*_m) = \{u_jw_j, u_jv_j, v_jx_j, x_jw_j : j \in \{1, ..., m\}\} \cup \{u_ju_{j+1}, w_ju_{j+1} : j \in \{1, ..., m-1\}\} \cup \{u_mu_1, w_mu_1\}\}$. Irawan et al.[20] discussed lcn for origami graphs and subdivision in the outer edge of origami graphs.

Theorem 1
Let $O_m$ be an origami graph for $m \geq 3$. Then, the lcn of $O_m$ is 4 for $3 \leq m \leq 6$ and 5 for otherwise.

Theorem 2
Lcn for subdivision outer edge of origami graphs for $m \geq 3$ is 4 for $m = 3$ and 5 for otherwise.

Based on two results, we are interested in more research about another operation of origami graphs. We give a new definition about certain operation of origami graphs. A certain operation of origami graphs, denoted by $HO_m$ is obtained from two origami graphs linked a path, $u_j$ to $u_{m+j}$, for each $j \in [1, m]$.

# 2 Results and Discussions

In this section, we discuss the locating chromatic number of a specific operation of origami graphs which we state in one grand theorem.

Theorem 3
Let $HO_m$ be a certain operation of origami graph for $m \geq 3$. Then the locating-chromatic number of $HO_m$, $\chi_L(HO_m)$, is 5 for $m = 3$ and 6 for otherwise.

Proof : To prove this theorem, we divide two cases:
**CASE 1.** $m = 3$
First, we determine the lower bound of $\chi_L(HO_3)$. A certain operation of origami graph $HO_3$ contains two cliques, then by Theorem 1, we have $\chi_L(HO_m) \geq 4$. Suppose $c$ is a 4-locating coloring of $HO_3$. Without loss of generality, we assign $\{c(u_j), c(v_j), c(w_j), c(u_{j+1})\} = \{1, 2, 3, 4\}$. Since $HO_3$ containing two origami graphs $O_3$. Then there is $c(u_j) = c(w_l)$ for $j \neq l$, a contrary. As a result, $\chi_L(HO_m) \geq 5$.

Next, we determine the upper bound of $\chi_L(O_3)$. We assign coloring $r$ using 5 colors like this,

$$r(u_j) = \begin{cases} 4, & \text{for } j = 1; \\ 3, & \text{for } j = 2; \\ 2, & \text{for } j = 3. \end{cases}$$

$$r(v_j) = \begin{cases} 2, & \text{for } j = 2; \\ 3, & \text{for } j = 1, 3. \end{cases}$$

$$r(u_{3+j}) = \begin{cases} 5, & \text{for } j = 1; \\ 1, & \text{for } j = 2; \\ 3, & \text{for } j = 3. \end{cases}$$

$$r(v_{3+j}) = \begin{cases} 2, & \text{for } j = 1, 3; \\ 3, & \text{for } j = 2. \end{cases}$$

$$r(w_{3+j}) = 4, j = 1, 2, 3.$$

The coloring $r$ induces partition $\Pi$ and the color codes of $V(HO_3)$ are: $r_\Pi(u_1) = (1, 1, 1, 0, 1)$; $r_\Pi(u_2) = (1, 1, 0, 1, 2)$; $r_\Pi(u_3) = (1, 0, 1, 1, 2)$; $r_\Pi(u_4) = (1, 1, 1, 1, 0)$; $r_\Pi(u_5) = (0, 2, 1, 1, 1)$; $r_\Pi(u_6) = (1, 1, 0, 1, 1)$; $r_\Pi(v_1) = (1, 2, 0, 1, 2)$; $r_\Pi(v_2) = (1, 0, 1, 2, 3)$; $r_\Pi(v_3) = (1, 1, 0, 2, 3)$; $r_\Pi(v_4) = (2, 0, 2, 1, 1)$; $r_\Pi(v_5) = (1, 3, 0, 1, 2)$; $r_\Pi(v_6) = (2, 0, 1, 1, 2)$; $r_\Pi(w_1) = (0, 2, 1, 1, 2)$; $r_\Pi(w_2) = (0, 1, 1, 2, 3)$; $r_\Pi(w_3) = (0, 1, 1, 1, 2)$; $r_\Pi(w_4) = (1, 1, 2, 0, 1)$; $r_\Pi(w_5) = (1, 2, 1, 0, 2)$; $r_\Pi(w_6) = (2, 1, 1, 0, 1)$. Since the color codes of all vertices $HO_3$ are different, thus $r$ is a locating-chromatic coloring. So $\chi_L(HO_3) \leq 5$.

**CASE 2.** $m \geq 4$

First, we determine the lower bound for the lcn of certain operation origami graph $HO_m$ for $m \geq 4$. The certain operation origami graph $HO_m$ for $m \geq 4$ whose two origami graphs $O_m$, then by Theorem 1, we have $\chi_L(HO_m) \geq 5$ for $m \geq 4$. Assume there is a 5-locating coloring of $HO_m$ for $m \geq 4$. Without loss of generality, we assign $\{c(u_j), c(v_j), c(w_j), c(u_{(j+1)}), c(u_{(m+j+1)})\} = \{1, 2, 3, 4, 5\}$. Observe that $HO_m$ for $m \geq 4$ have $m$ vertices of degree 6, namely $u_j$, $j = 1, 2, \ldots, m$. Then, at least two vertices, $u_l$ and $u_{(m+l)}$, where $l \neq m + l$ have the same color code, a contrary. As a results, $\chi_L(HO_6) \geq 6$ for $m \geq 4$.

To show the upper bound for the lcn of certain operation origami graphs $HO_m$ for $m \geq 4$. Let us differentiate some subcases.

**SUBCASE 2.1** (odd $m$)

First, for odd $\left\lceil\frac{m}{2}\right\rceil, m \geq 5$.
Let $r$ be a coloring of certain operation origami graph $HOm$, odd $\left\lceil\frac{m}{2}\right\rceil, m \geq 5$ we make the partition $\Pi$ of $V(HO_m)$

- $R_1 = \{w_j | 1 \leq j \leq m\}$.
- $R_2 = \{u_j |$ odd $j, 3 \leq j \leq m\} \cup \{v_j|$ even $j, 2 \leq j \leq m-1\} \cup \{u_{m+1}|$ even $j, 2 \leq j \leq m-1\} \cup \{v_{m+j}|$ odd $j, 1 \leq j \leq m\}$.
- $R_3 = \{u_j|$ even $j, 2 \leq j \leq m-1\} \cup \{v_j|$ odd $j, 1 \leq j \leq m\} \cup \{u_{m+1}|$ odd $j, 1 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 2\} \cup \{u_{m+1}|$ even $j, \left\lceil\frac{m}{2}\right\rceil + 2 \leq j \leq m-2\} \cup \{v_{m+j}|$ odd $j, 2 \leq j \leq m-1\}$.
- $R_4 = \{u_j\} \cup \{w_{m+j} | 1 \leq j \leq m\}$.
- $R_5 = \{u_{2m}\}$.
- $R_6 = \{u_{m+\left\lceil\frac{m}{2}\right\rceil}\}$.

The color codes of this partitions are:

- $R_1 = \{w_j | 1 \leq j \leq m\}$.
We have color codes are $r_\Pi(w_1) = (0, 2, 1, 1, 3, \left\lceil\frac{n}{2}\right\rceil)$; $r_\Pi(w_2) = (0, 1, 1, 1, 4, \left\lceil\frac{m}{2}\right\rceil + 1)$; for $3 \leq i \leq \left\lceil\frac{n}{2}\right\rceil - 1, m \geq 9$, $r_\Pi(w_j) = (0, 1, 1, 3, j+2, \left\lceil\frac{n}{2}\right\rceil - j+1)$; for $\left\lceil\frac{m}{2}\right\rceil \leq j \leq m-2, m \geq 5$, $r_\Pi(w_j) = (0, 1, 1, 3, m-j+1, 2, 1-\left\lceil\frac{m}{2}\right\rceil + 2)$; for $m-1 \leq j \leq m, m \geq 5$, $r_\Pi(w_j) = (0, 1, 1, m-j+1, 2, j-\left\lceil\frac{m}{2}\right\rceil + 2)$.

- $R_2 = \{u_j|$ odd $j, 3 \leq j \leq m\} \cup \{v_j|$ even $j, 2 \leq j \leq m-1\} \cup \{u_{m+1}|$ even $j, 2 \leq j \leq m-1\} \cup \{v_{m+j}|$ odd $j, 1 \leq j \leq m\}$.
For $3 \leq j \leq \left\lceil\frac{m}{2}\right\rceil, m \geq 9$, $r_\Pi(u_j) = (1, 0, 1, 2, j+1\left\lceil\frac{m}{2}\right\rceil - j+1)$; for $\left\lceil\frac{m}{2}\right\rceil \leq j \leq m-2, m \geq 5$, $r_\Pi(u_j) = (1, 0, 1, 2, m-j+1, j-\left\lceil\frac{m}{2}\right\rceil + 1)$; for $j = m$, $r_\Pi(u_j) = (1, 0, 1, 1, 1, j-\left\lceil\frac{m}{2}\right\rceil + 1)$; for $j = 2$, $r_\Pi(v_j) = (1, 0, 1, 2, 4, \left\lceil\frac{m}{2}\right\rceil)$; for $4 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 1, m \geq 9$, $r_\Pi(v_j) = (1, 0, 1, 3, j+2, \left\lceil\frac{m}{2}\right\rceil - 1 + 2)$; for $\left\lceil\frac{m}{2}\right\rceil + 1 \leq j \leq m-1, m \geq 5$, $r_\Pi(u_j) = (1, 0, 1, 3, j-1+2, j-\left\lceil\frac{m}{2}\right\rceil + 2)$; for $2 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 1, m \geq 5$, $r_\Pi(u_{m+j}) = (2, 0, 1, 1, j, \left\lceil\frac{m}{2}\right\rceil - 1)$; for $\left\lceil\frac{m}{2}\right\rceil + 1 \leq j \leq m-1, m \geq 5$,$r_\Pi(u_{m+j}) = (2, 0, 1, 1, m-j, j-\left\lceil\frac{m}{2}\right\rceil)$; for $1 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 2, m \geq 5$, $r_\Pi(v_{m+j}) = (3, 0, 1, 1, j+1, \left\lceil\frac{n}{2}\right\rceil - j+1)$; for $\left\lceil\frac{m}{2}\right\rceil \leq j \leq m, m \geq 5$, $r_\Pi(v_{m+j}) = (3, 0, 1, 1, m-j+1, 1-\left\lceil\frac{m}{2}\right\rceil)$.

- $R_3 = \{u_j|$ even $j, 2 \leq j \leq m-1\} \cup \{v_j|$ odd $j, 1 \leq j \leq m\} \cup \{u_{m+1}|$ odd $j, 1 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 2\} \cup \{u_{m+1}|$ even $j, \left\lceil\frac{m}{2}\right\rceil + 2 \leq j \leq m-2\} \cup \{v_{m+j}|$ odd $j, 2 \leq j \leq m-1\}$.
For $j = 2$, $r_\Pi(u_j) = (1, 1, 0, 1, 3, \left\lceil\frac{m}{2}\right\rceil - j+1)$; for $4 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 1, m \geq 9$, $r_\Pi(u_j) = (1, 1, 0, 2, j+1, \left\lceil\frac{m}{2}\right\rceil - j+1)$; for $\left\lceil\frac{m}{2}\right\rceil + 1 \leq j \leq m-1, m \geq 5$, $r_\Pi(u_j) = (1, 1, 0, 2, m-j+1, j-\left\lceil\frac{m}{2}\right\rceil + 1)$ for $j = 1$, $r_\Pi(u_j) = (1, 2, 0, 1, 3, \left\lceil\frac{m}{2}\right\rceil - j+2)$; for $3 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 2, m \geq 9$,$r_\Pi(v_j) = (1, 1, 0, 3, j+2, \left\lceil\frac{m}{2}\right\rceil - j+2)$; for $\left\lceil\frac{m}{2}\right\rceil \leq j \leq m-2, m \geq 5$,$r_\Pi(v_j) = (1, 1, 0, 3, m-j+2, j-\left\lceil\frac{m}{2}\right\rceil + 2)$; for $j = m$, $r_\Pi(v_j) = (1, 1, 0, 2, 2, j-\left\lceil\frac{m}{2}\right\rceil + 2)$; for $1 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 2, m \geq 5$, $r_\Pi(u_{m+j}) = (2, 1, 0, 1, j, \left\lceil\frac{m}{2}\right\rceil - j)$; for $\left\lceil\frac{m}{2}\right\rceil + 2 \leq j \leq m-2, m \geq 9$, $r_\Pi(u_{m+j}) = (2, 1, 0, 1, m-j, j-\left\lceil\frac{m}{2}\right\rceil)$; for $2 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 1, m \geq 5$, $r_\Pi(v_{m+j}) = (3, 1, 0, 1, j+1, \left\lceil\frac{m}{2}\right\rceil - j+1)$; for $\left\lceil\frac{m}{2}\right\rceil + 1 \leq j \leq m-1, m \geq 5$, $r_\Pi(v_{m+j}) = (3, 1, 0, 1, m-j+1, j-\left\lceil\frac{m}{2}\right\rceil + 1)$.

- $R_4 = \{u_j\} \cup \{w_{m+j} | 1 \leq j \leq m\}$.
For $j = 1$, $r_\Pi(u_j) = (1, 1, 1, 0, 2, \left\lceil\frac{m}{2}\right\rceil)$; for $1 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 1, m \geq 5$, $r_\Pi(u_j) = (3, 1, 1, 0, j+1, \left\lceil\frac{m}{2}\right\rceil - 1)$; for $j = \left\lceil\frac{m}{2}\right\rceil$, $r_\Pi(w_{m+j}) = (3, 1, 2, 0, m-j, 1)$; for $\left\lceil\frac{m}{2}\right\rceil + 1 \leq j \leq m-1, m \geq 5$, $r_\Pi(w_{m+j}) = (3, 1, 1, 0, m-j, j-\left\lceil\frac{m}{2}\right\rceil + 1)$; for $j = m$, $r_\Pi(w_{m+j}) = (3, 1, 1, 0, 1, j-\left\lceil\frac{m}{2}\right\rceil + 1)$.

- $R_5 = \{u_{2m}\}$
$r_\Pi(u_{2m}) = (2, 1, 1, 1, 0, 1, m-\left\lceil\frac{m}{2}\right\rceil + 1)$.

- $R_6 = \{u_{m+\left\lceil\frac{m}{2}\right\rceil}\}$
$r_\Pi(u_{2m}) = (2, 1, 1, 1, 0, 1, m-\left\lceil\frac{m}{2}\right\rceil, 0)$.

Next, for even $\left\lceil\frac{m}{2}\right\rceil, m \geq 7$.
Let $r$ be a coloring of certain operation origami graph $HO_m$, even $\left\lceil\frac{m}{2}\right\rceil, m \geq 7$ we make the partition $\Pi$ of $V(HO_m)$:

- $R_1 = \{w_j | 1 \leq j \leq m\}$.
- $R_2 = \{u_j|$ odd $j, 3 \leq j \leq m\} \cup \{v_j|$ even $j, 2 \leq j \leq m-1\} \cup \{u_{m+1}|$ even $j, 2 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 2\} \cup \{u_{m+1}|$ even $j, \left\lceil\frac{m}{2}\right\rceil + 2 \leq j \leq m-1\} \cup \{v_{m+j}|$ odd $j, 1 \leq j \leq m\}$.
- $R_3 = \{u_j|$ even $j, 2 \leq j \leq m-1\} \cup \{v_j|$ odd $j, 1 \leq j \leq m\} \cup \{u_{m+j}|$ odd $j, 1 \leq j \leq m-2\} \cup \{v_{m+j}|$ even $j, 1 \leq j \leq m-1\}$.
- $R_4 = \{u_1\} \cup \{w_{m+j} | 1 \leq j \leq m\}$.
- $R_5 = \{u_{2m}\}$.
- $R_6 = \{u_{m+\left\lceil\frac{m}{2}\right\rceil}\}$.

The color codes of partitions are:

- $R_1 = \{w_j | 1 \leq j \leq m\}$.
We have color codes are $r_\Pi(w_1) = (0, 2, 1, 1, 3, \left\lceil\frac{m}{2}\right\rceil - j+1)$; for $j = 2$, $r_\Pi(w_2) = (0, 1, 1, 1, 4, \left\lceil\frac{m}{2}\right\rceil - j+1)$; for $3 \leq j \leq \left\lceil\frac{m}{2}\right\rceil - 1, m \geq 7$, $r_\Pi(w_j) = (0, 1, 1, 3, j+2, \left\lceil\frac{m}{2}\right\rceil - j+1)$; for $\left\lceil\frac{m}{2}\right\rceil \leq j \leq m-2, m \geq 7$, $r_\Pi(w_j) = (0, 1, 1, 3, m-j+1, j-\left\lceil\frac{m}{2}\right\rceil + 2)$; for $m-j \leq j \leq m, m \geq 7$, $r_\Pi(w_j) =$

$(0,1,1,m-j+1,2,j-\lceil\frac{m}{2}\rceil+2)$.

- $R_2=\{u_j|$ odd $j,3\leq j\leq m\}\cup\{v_j|$ even $j,2\leq j\leq m-1\}\cup\{u_{m+1}|$ even $j,2\leq j\leq\lceil\frac{m}{2}\rceil-2\}$ $\cup\{u_{m+1}|$ even $j,\lceil\frac{m}{2}\rceil+2\leq j\leq m-1\}\cup\{v_{m+j}|$ odd $j,1\leq j\leq m\}$.
For $3\leq j\leq\lceil\frac{m}{2}\rceil-1,m\geq 7$, $r_\Pi(u_j)=(1,0,1,2,j+1,\lceil\frac{m}{2}\rceil-j+1)$; for $\lceil\frac{m}{2}\rceil+1\leq j\leq m-2,m\geq 7$, $r_\Pi(u_j)=(1,0,1,2,m-j+1,j-\lceil\frac{m}{2}\rceil+1)$; for $j=m$, $r_\Pi(u_j)=(1,0,1,1,1,j-\lceil\frac{m}{2}\rceil+1)$;for $j=2$, $r_\Pi(v_j)=(1,0,1,2,4\lceil\frac{m}{2}\rceil)$; for $4\leq j\leq\lceil\frac{m}{2}\rceil-2,m\geq 11$, $r_\Pi(v_j)=(1,0,1,3,j+2,\lceil\frac{m}{2}\rceil-j+2)$; for $\lceil\frac{m}{2}\rceil+1\leq i\leq m-1,m\geq 7$, $r_\Pi(v_j)=(1,0,1,3,m-j+2,j-\lceil\frac{m}{2}\rceil+2)$; for $2\leq j\leq\lceil\frac{m}{2}\rceil-2,m\geq 7$, $r_\Pi(u_{m+j})=(2,0,1,1,j,\lceil\frac{m}{2}\rceil-j)$; for $\lceil\frac{m}{2}\rceil+2\leq j\leq m-1,m\geq 7$, $r_\Pi(u_{m+j})=(2,0,1,1,m-j,j-\lceil\frac{m}{2}\rceil)$; for $1\leq j\leq\lceil\frac{m}{2}\rceil-1,m\geq 7$, $r_\Pi(v_{m+j})=(3,0,1,1,j+1,\lceil\frac{m}{2}\rceil-j+1)$; for $\lceil\frac{m}{2}\rceil+1\leq j\leq m,m\geq 7$, $r_\Pi(v_{m+j})=(3,0,1,1,m-j+1,j-\lceil\frac{m}{2}\rceil+1)$.

- $R_3=\{u_j|$ even $j,2\leq j\leq m-1\}\cup\{v_j|$ odd $j,1\leq j\leq m\}\cup\{u_{m+j}|$ odd $j,1\leq j\leq m-2\}\cup\{v_{m+j}|$ even $j,1\leq j\leq m-1\}$.
For $j=2$, $r_\Pi(u_j)=(1,1,0,1,3,\lceil\frac{m}{2}\rceil-j+1)$; for $4\leq j\leq\lceil\frac{m}{2}\rceil-2,m\geq 11$, $r_\Pi(u_j)=(1,1,0,2,j+1\lceil\frac{m}{2}\rceil-j+1)$; for $\lceil\frac{m}{2}\rceil\leq j\leq m-1,m\geq 7$,$r_\Pi(u_j)=(1,1,0,2,m-j+1,j-\lceil\frac{m}{2}\rceil+1)$; for $j=1$, $r_\Pi(v_j)=(1,2,0,1,3,\lceil\frac{m}{2}\rceil-j+2$; for $3\leq j\leq\lceil\frac{m}{2}\rceil-1,m\geq 7$, $r_\Pi(v_j)=(1,1,0,3,j+2,\lceil\frac{m}{2}\rceil-j+2$; for $\lceil\frac{m}{2}\rceil+1\leq i\leq m-2,m\geq 7$, $r_\Pi(v_j)=(1,1,0,3,m-j+2,j-\lceil\frac{m}{2}\rceil+2$; for $j=m$, $r_\Pi(v_j)=(1,1,0,2,2,j-\lceil\frac{m}{2}\rceil+2$, for $1\leq j\leq\lceil\frac{m}{2}\rceil-1,m\geq 7$, $r_\Pi(u_{m+j})=(2,1,0,1,j,\lceil\frac{m}{2}\rceil-j)$; for $\lceil\frac{m}{2}\rceil+1\leq j\leq m-2,m\geq 7$, $r_\Pi(u_{m+j})=(2,1,0,1,m-j,j-\lceil\frac{m}{2}\rceil)$; for $2\leq j\leq\lceil\frac{m}{2}\rceil-2,m\geq 7$, $r_\Pi(v_{m+j})=(3,1,0,1,j+1,\lceil\frac{m}{2}\rceil-j+1)$; for $\lceil\frac{m}{2}\rceil\leq j\leq m-1,m\geq 7$, $r_\Pi(v_{m+j})=(3,1,0,1,m-j+1,j-\lceil\frac{m}{2}\rceil+1)$.

- $R_4=\{u_1\}\cup\{w_{m+j}|1\leq j\leq m\}$
for $j=1$, $r_\Pi(u_j)=(1,1,1,0,2,\lceil\frac{m}{2}\rceil)$; for $1\leq j\leq\lceil\frac{m}{2}\rceil-1,m\geq 7$, ${}_\Pi(w_{m+j})=(3,1,1,0,j+1,\lceil\frac{m}{2}\rceil-j)$; for $j=\lceil\frac{m}{2}\rceil$, $c_\Pi(w_{m+j})=(3,1,2,0,m-j,1)$; for $\lceil\frac{m}{2}\rceil+1\leq j\leq m-1,m\geq 7$, $r_\Pi(w_{m+j})=(3,1,1,0,m-j,j-\lceil\frac{m}{2}\rceil+1)$; for $j=m$, $r_\Pi(w_{m+j})=(3,1,1,0,1,j-\lceil\frac{m}{2}\rceil+1)$.

- $R_5=\{u_{2m}\}$.
$r_\Pi(u_{2m})=(2,1,1,1,0,n-\lceil\frac{m}{2}\rceil)$.

- $R_6=\{u_{m+\lceil\frac{m}{2}\rceil}\}$.
$r_\Pi(u_{m+\lceil\frac{m}{2}\rceil})=(2,1,2,1,m-\lceil\frac{m}{2}\rceil,0)$.

**SUBCASE 2.2** (even $m$)

For odd $\lceil\frac{m}{2}\rceil$, $m\geq 4$.
Let $r$ be a coloring of certain operation origami graph $HO_m$, odd $\lceil\frac{m}{2}\rceil$, $m\geq 4$ we make the partition $\Pi$ of $V(HO_m)$ and color codes:

- $R_1=\{w_j|2\leq j\leq m\}$.
for $2\leq j\leq\frac{m}{2},m\geq 6$, $r_\Pi(w_j)=(0,1,1,3,j,\frac{m}{2}-1+2)$; for $\frac{m}{2}+1\leq j\leq m,m\geq 6$, $r_\Pi(w_j)=(0,1,1,3,m-j+2,j-\frac{m}{2}+1)$.

- $R_2=\{u_j|$ odd $j$, $1\leq j\leq m-1\}\cup\{v_j|$ even $j,2\leq j\leq m\}\cup\{u_{m+1}|$ even $j,1\leq j\leq\frac{m}{2}-1\}$ $\cup\{u_{m+1}|$ even $j,\frac{m}{2}+3\leq i\leq m\}\cup\{v_{m+j}|$ odd $j,1\leq j\leq m-1\}$.
for $j=1$, $r_\Pi(u_j)=(1,0,1,2,1,\frac{m}{2}+1)$; for $3\leq j\leq\frac{m}{2},m\geq 6,r_\Pi(u_j)=(1,0,1,2,j-1,\frac{m}{2}-i+2)$; for $\frac{m}{2}+2\leq j\leq m-1,m\geq 6$, $r_\Pi(u_j)=(1,0,1,2,m-j+2,j-\frac{n}{2})$; for $2\leq j\leq\frac{m}{2}-1,m\geq 6$, $r_\Pi(v_j)=(1,0,1,3,j,\frac{m}{2}-m+3)$; for $\frac{m}{2}+1\leq j\leq m,m\geq 6$, $r_\Pi(v_j)=(1,0,1,3,m-j+3,j-\frac{m}{2}+1)$; for $2\leq j\leq\frac{m}{2}-1,m\geq 6$, $r_\Pi(u_{m+j})=(2,0,1,1,j,\frac{m}{2}-j+1)$; for $\frac{m}{2}+3\leq j\leq n,n\geq 6$, $c_\Pi(u_{n+i})=(2,0,1,1,n-i+3,i-\frac{m}{2}-j)$; for $j=1$, $c_\Pi(v_{m+j})=(3,0,1,1,3,\frac{m}{2}+1)$; for $3\leq j\leq\frac{m}{2},m\geq 6$, $r_\Pi(v_{m+i})=(3,0,1,1,i+1,\frac{m}{2}-j+2)$; for $\frac{m}{2}+3\leq j\leq m-1,m\geq 6$, $r_\Pi(v_{m+j})=(3,0,1,1,m-j+4,j-\frac{m}{2})$.

- $R_3=\{u_j|$ even $j$, $2\leq j\leq m\}\cup\{v_j|$ odd $j,1\leq j\leq m-1\}\cup\{u_{m+1}|$ odd $j,1\leq j\leq m-1\}\cup\{v_{m+j}|$ even $j,2\leq j\leq m\}$.
For $2\leq j\leq\frac{m}{2}-1,m\geq 6$, $r_\Pi(u_j)=(1,1,0,2,j-1,\frac{m}{2}-j+2)$; for $\frac{m}{2}+1\leq j\leq m,m\geq 6$, $r_\Pi(u_j)=(1,1,0,2,m-j+2,j-\frac{m}{2})$; for $j=1$, $r_\Pi(v_j)=(2,1,0,3,1,\frac{m}{2}+2)$; for $3\leq j\leq m-1,m\geq 6$, $r_\Pi(v_j)=(1,1,0,3,j,\frac{m}{2}-j+3)$; for $\frac{m}{2}+2\leq j\leq m-1,m\geq 6$, $r_\Pi(v_j)=(1,1,0,3,m-j,j-\frac{m}{2}+1)$; for $j=1$, $r_\Pi(u_{m+j})=(2,1,0,1,2,\frac{m}{2})$; for $3\leq j\leq\frac{mn}{2},m\geq 6$, $r_\Pi(u_{m+j})=(2,1,0,1,j,\frac{m}{2}-j+1)$; for $\frac{m}{2}+2\leq j\leq m-1,m\geq 6$, $r_\Pi(u_{m+j})=(2,1,0,1,m-j+3,j-\frac{m}{2}-1)$; for $2\leq j\leq\frac{m}{2}-1,m\geq 6$, $r_\Pi(v_{m+j})=(3,1,0,1,j+1,\frac{m}{2}-j+2)$; for $j=\frac{m}{2}+1$, $r_\Pi(v_{m+j})=(3,3,0,1,\frac{m}{2}+2,1)$; for $\frac{m}{2}+3\leq j\leq m,m\geq 6$, $r_\Pi(v_{m+j})=(3,1,0,1,m-j+4,j-\frac{m}{2})$.

- $R_4=\{w_j|1\leq j\leq m\}$.
For $j=1$, $r_\Pi(w_{m+j})=(3,1,1,0,3,\frac{m}{2})$; for $2\leq j\leq\frac{m}{2},m\geq 6$, $r_\Pi(w_{m+j})=(3,1,1,0,j+1,\frac{m}{2}-j+1)$; for $j=\frac{m}{2}+1$, $r_\Pi(w_{m+j})=(3,3,1,0,\frac{m}{2}+2,1)$; for $\frac{m}{2}+2\leq j\leq m,m\geq 6$, $r_\Pi(w_{m+j})=(3,1,1,0,m-1+3,j-\frac{m}{2})$.

- $R_5 = \{w_1\}$.
$r_\Pi(w_1) = (2, 1, 1, 3, 0, \frac{m}{2} + 1)$.

- $R_6 = \{w_{m+\frac{m}{2}+1}\}$.
$r_\Pi(w_{m+\frac{m}{2}+1}) = (2, 2, 1, 1, \frac{m}{2} + 1, 0)$.

Next, for even $\lceil \frac{m}{2} \rceil$, $m \geq 4$.
Let $r$ be a coloring of certain operation origami graph $HO_m$, even $\lceil \frac{m}{2} \rceil$, $m \geq 4$ we make the partition $\Pi$ of $V(HO_m)$ and color codes:

- $R_1 = \{w_j | 2 \leq j \leq m\}$.
for $2 \leq j \leq \frac{m}{2}, m \geq 4$, $r_\Pi(w_m) = (0, 1, 1, 3, j, \frac{n}{2} - j + 2, )$; for $\frac{m}{2} + 1 \leq j \leq m, m \geq 4$, $r_\Pi(w_j) = (0, 1, 1, 3, m - j, \frac{m}{2} + 1)$.

- $R_2 = \{u_j |$ odd $j, 1 \leq j \leq m - 1\} \cup \{v_j |$ even $j, 2 \leq i \leq m\} \cup \{u_{m+1} |$ even $j, 2 \leq j \leq m\} \cup \{v_{m+j} |$ for odd $j, 1 \leq j \leq m - 1\}$.
For $j = 1$, $r_\Pi(u_j) = (1, 0, 1, 2, 1, \frac{m}{2} + 1)$; for $3 \leq j \leq \frac{m}{2}, m \geq 8, r_\Pi(u_j) = (1, 0, 1, 2, j-1, \frac{m}{2} - j + 1)$; for $j = \frac{m}{2} + 1$, $r_\Pi(u_j) = (1, 0, 1, 2, \frac{m}{2}, 1)$; for $\frac{m}{2} + 3 \leq j \leq m-1, m \geq 8, r_\Pi(u_j) = (1, 0, 1, 2, m-1+2, j - \frac{m}{2})$; for $2 \leq j \leq \frac{m}{2}, m \geq 4, r_\Pi(v_j) = (2, 0, 1, 1, j, \frac{m}{2} - j + 1)$; for $\frac{m}{2} + 2 \leq j \leq m, m \geq 4$, $r_\Pi(u_{m+j}) = (2, 0, 1, 1, m - j + 3, j - \frac{m}{2} - 1)$; for $j = 1$, $r_\Pi(v_{m+j}) = (3, 0, 1, 1, 3, \frac{m}{2} + 1)$; for $3 \leq j \leq \frac{m}{2} - 1, m \geq 8$, $r_\Pi(v_{m+j}) = (3, 0, 1, 1, j + 1, \frac{m}{2} - j + 2)$; for $j = \frac{m}{2} + 1$, $r_\Pi(v_{m+j}) = (3, 0, 1, 1, \frac{m}{2} + 2, 1)$; for $\frac{m}{2} + 3 \leq j \leq m-1, m \geq 8, r_\Pi(v_{m+j}) = (3, 0, 1, 1, m - j + 4, j - \frac{m}{2})$.

- $R_3 = \{u_j |$ even $j, 2 \leq j \leq m\} \cup \{v_j |$ odd $j, 1 \leq j \leq m - 1\} \cup \{u_{m+1} |$ odd $j, 1 \leq j \leq \frac{m}{2} - 1\}$ $\cup \{u_{m+1} |$ odd $j, \frac{m}{2} + 3 \leq j \leq \frac{m}{2} - 1\} \cup \{v_{m+j} |$ even $j, 2 \leq j \leq n\}$.
For $2 \leq j \leq \frac{m}{2}, m \geq 4$, $r_\Pi(u_j) = (1, 1, 0, 2, j - 1, \frac{m}{2} - j + 2)$; for $\frac{m}{2} + 2 \leq j \leq m, m \geq 4$, $r_\Pi(u_j) = (1, 1, 0, 2, m - j + 2, j - \frac{m}{2})$; for $j = 1$, $r_\Pi(v_j) = (2, 1, 0, 3, 1, \frac{m}{2} + 2)$; for $3 \leq j \leq \frac{m}{2} - 1, m \geq 8$, $r_\Pi(v_j) = (1, 1, 0, 3, j, \frac{m}{2} - j + 3)$; for $j = \frac{m}{2} + 1$, $r_\Pi(v_j) = (1, 1, 0, 3, \frac{m}{2} + 1, 2)$; for $\frac{m}{2} + 3 \leq j \leq m-1, m \geq 8, r_\Pi(v_j) = (1, 1, 0, 3, m - j + 3, j - \frac{m}{2} + 1)$; for $j = 1$, $r_\Pi(u_{m+j}) = (2, 1, 0, 1, 2, \frac{m}{2})$; for $3 \leq j \leq \frac{m}{2} - 1, m \geq 8$, $r_\Pi(u_{n+j}) = (2, 1, 0, 1, j, \frac{m}{2} - j + 1)$; for $\frac{m}{2} + 3 \leq j \leq m - 1, m \geq 8$, $r_\Pi(u_{m+j}) = (2, 1, 0, 1, m - j + 3, j - \frac{m}{2} - 1)$; for $2 \leq j \leq \frac{m}{2}, m \geq 4$, $r_\Pi(v_{m+j}) = (3, 1, 0, 1, j + 1, \frac{n}{2} - 1 + 2)$; for $\frac{m}{2} + 2 \leq j \leq m, m \geq 4, r_\Pi(v_{m+j}) = (3, 1, 0, 1, m - j + 4, j - \frac{m}{2})$.

- $R_4 = \{w_{m+j} | 1 \leq j \leq m\}$.
for $j = 1$, $r_\Pi(w_{m+j}) = (3, 1, 1, 0, 3, \frac{m}{2})$; for $2 \leq j \leq \frac{m}{2}, m \geq 4$, $r_\Pi(w_{m+j}) = (3, 1, 1, 0, j + 1, \frac{m}{2} - j + 1)$; for $j = \frac{m}{2} + 1$, $r_\Pi(w_{m+j}) = (3, 1, 2, 0, \frac{m}{2} + 2, 1)$; for $\frac{m}{2} + 2 \leq j \leq m, m \geq 4$, $r_\Pi(w_{m+j}) = (3, 1, 1, 0, m - j + 3, j - \frac{m}{2})$.

- $R_5 = \{w_1\}$.
$r_\Pi(w_1) = (2, 1, 1, 3, 0, \frac{m}{2} + 2)$.

- $R_6 = \{u_{m+\frac{m}{2}+1}\}$.
$r_\Pi(u_{m+\frac{m}{2}+1}) = (2, 1, 2, 1, \frac{m}{2} + 2, 0)$.

Since for $m$ even all vertices have different color codes, $r$ is a locating coloring for certain operation of origami graphs $HO_m$, so that $\chi_L(HO_m) \leq 6$, for $\frac{m}{2}$ even, $m \geq 4$. The proof of this theorem is complete. □

Figure 1 shows an example of locating chromatic number for certain operations of origami graph $H0_3$ using five colors.

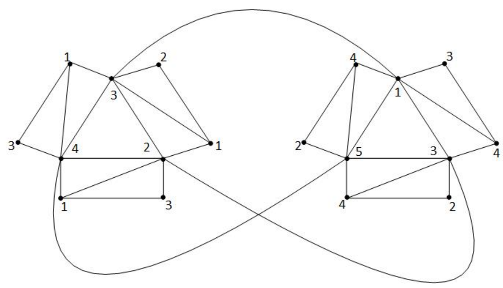


**Figure 1.** A minimum locating coloring of $HO_3$

# 3 Conclusions

The result obtained from this discussion is $\chi_L(HO_m) = \chi_L(O_m) + 1$. The result is such because $HO_m$ contains two origami graphs $O_m$. Therefore, research to determine the effects of other operations of origami graphs is interesting follow-up research.


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