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Construction of Rough Graph through Rough Membership Function

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Abstract Rough membership function defines the degree of relationship between conditional and decision attributes of an information system. It is defined by $\mu_X^R : U \to [0,1]$ where X is the subset of U under the relation R where U is the universe of discourse. It can be expressed in different forms like cardinality form, probabilistic form etc. In cardinality form, it is expressed as $\mu_X^R = \frac{|[x]_R \cap X|}{|[x]_R|}$ where as in probabilistic form it can be denoted as $P(x \in X|\, [x]_R) = \frac{P([x]_R \cap X)}{P([x]_R)}$ where $[x]_R$ is the equivalence class of x with respect to R. This membership function is used to measure the value of uncertainty. In this paper we have introduced the concept of graphical representation of rough sets. Rough graph was introduced by He Tong in 2006. In this paper, we propose a novel method for the construction of rough graph through rough membership function $\omega_G^F(f)$. We propose that there is an edge between vertices if $max(\omega_G^F(v_i), \omega_G^F(v_j)) > 0$. The rough graph is being constructed for an information system; here objects are considered as vertices. Rough path, rough cycle, rough ladder graph are introduced in this paper. We develop the operations on rough graph and also extend the properties of rough graph.

Keywords Set Approximations, Rough Membership Function, Rough Graph

1 Introduction

The generic and simple way of making optimal decisions is a graphical representation of the given problem. Information system is the building block for rough set. It contains universe of discourse (Objects) along with conditional and decision attributes. This theory was constructed by Pawlak through the concept of indiscernibility relation. The objects which are having same attribute values are indiscernible, otherwise they are discernible with each other [1]. The indiscernibility relation is represented in the form of matrix and graph[5]. The matrix representation of discernible relation was introduced by Skowron [1]. By using two crisp sets, the approximation space has been considered. The lower approximation consists of all objects which surely belong to the set and the upper approximation consists objects which possibly belong to the set[20]. There are many properties defined on lower and upper approximations [10].

Rough membership function is another approach [6] to define rough set. Through the definition of rough membership function, the set approximations have been defined. Pawlak has defined two types of information system in rough set theory with and without decision attributes respectively [7]. The minimal representation of the information system is named as reduct [1]. The wide range of applications of the rough set are being implemented in the field of decision making approaches, artificial intelligence, machine and deep learning techniques.

The rough graph is a mechanism of embedding graph in the domain of rough. In 2006, He Tong and Kai Shi gave the structure for rough graph and extended as weighted rough graph by giving class weights for edges [17]. He Tong gave COTA algorithm for finding the shortest path through Kruskal and Dijkstra's algorithm for rough optimal tree. The edge rough graph was constructed by the partition of edge set[18]. Bibin Mathew defined vertex rough graph by dividing the vertex set [4]. Shu defined a Rough graph model with a double universe of discourse [8].

Rough Graph was represented by matrix and edge list [14]. The properties of rough graph were defined through similarity degree, precision etc. [4, 15, 16]. Topological properties and optimization techniques of rough set are discussed in [2],[3]. In this paper, we have introduced the novel idea for constructing rough graph through rough membership function. Reduct of an information system can be calculated through rough metric dimension. Section 2, deals about the basic concepts of rough sets with existing graph structure. In section 3, we have proposed the construction process of rough graph through rough membership function followed by the properties of rough walk, rough path and rough cycle in section 4. Section 5,exhibits the operations on rough graph and its properties. In section 6, we discuss about isomorphism on rough graphs followed by concluded remarks and future extension.

2 Preliminaries

Definition 2.1. *Let U be the universe set and an equivalence relation $B \subseteq U \times U$. Let Y be a subset of U. Let $B(x)$ be the equivalence class on U. The lower and upper approximations of Y are given as*

$$\begin{aligned} B_{\nabla}(Y) &= \bigcup_{x \in Y} \{B(x) : B(x) \subseteq Y\} \\ B^{\Delta}(Y) &= \bigcup_{x \in Y} \{B(x) : B(x) \cap Y \neq \phi\} \end{aligned}$$

B- boundary region of Y

$$BN_B(Y) = B^{\Delta}(Y) - B_{\nabla}(Y)$$

The pair $(B_{\nabla}(Y), B^{\Delta}(Y))$ is called a Rough set[1].

Definition 2.2. *Let $U = (W, E)$ be the universe graph with the universe of discourse $U = \{e_1, e_2, \ldots, e_{|U|}\}$ having the attribute set on U which is defined as R= $\{r_1, r_2, \ldots, r_{|R|}\}$, where $W = \{v_1, v_2, \ldots, v_n\}$, $E = \bigcup e_k(v_i, v_j)$. The elements in E are divided into different equivalence classes $[e]_R$ from the attribute set $R \subseteq \mathbb{R}$ on E. By two exact graphs $\underline{R}(T) = (W, \underline{R}(X))$ which is lower approximation of X and $\overline{R}(T) = (W, \overline{R}(X))$ which is upper approximation of X can be used to define it approximately, where*

$$\begin{aligned} \underline{R}(X) &= \{e \in E \mid [e]_R \subseteq X\} \\ \overline{R}(X) &= \{e \in E \mid [e]_R \cap X \neq \phi\} \end{aligned}$$

The pair $\left(\overline{R}(T), \overline{R}(T)\right)$ is called $R-$rough graph[24].

Definition 2.3. *Let $U = (V, E)$ be universe graph, where $V = \{v_1, v_2, \ldots, v_n\}, E = \bigcup e_k(v_i, v_j)$, for every $e \in E$ with the mapping for the edge weight is $\omega : e \rightarrow \omega(e)$. The class weights for the edge equivalence class are given as $\omega[e_{uv}]_R = f(\omega(e))$, where $[e_{uv}]_R-$ edge equivalence class between the vertex u and vertex v with respect to attribute R. The class weight of their edge equivalence class for the rough graph $T = (\underline{R}(T), \overline{R}(T))$ is called weighted rough graph[24].*

Definition 2.4. *Let $\mathcal{A} = (K, R)^e$ be an edge approximation space. Given an edge subset $X \subseteq E(K)$, be the lower and upper approximation of X in $\mathcal{A}$, denoted by $\underline{Q}(X)$ and $\overline{Q}(X)$ respectively and defined as*

$$\begin{aligned} \underline{Q}(X) &= \{x \in E(K) | J_Q(x) \subseteq X\} \\ \overline{Q}(X) &= \{x \in E(K) | J_Q(x) \cap X \neq \phi\} \end{aligned}$$

where, $J_Q(x)$ denotes the edge set[25].

Definition 2.5. *By two exact graphs, $R-$vertex rough graph[8] is defined as $R_{\nabla}(H) = (R_{\nabla}(K), R_{\nabla}(L))$ and $R^{\Delta}(H) = (R^{\Delta}(K), R^{\Delta}(L))$,where*

$$\begin{aligned} R_{\nabla}(K) &= \{v \in V : [v]_R \subseteq K\} \\ R^{\Delta}(K) &= \{v \in V : [v]_R \cap K \neq \phi\} \end{aligned}$$

$$\begin{aligned} R_{\nabla}(L) &= \{(v_i, v_j) \in L : v_i, v_j \in [v]_R \\ &\quad \text{for some } v \in R_{\nabla}(K)\} \\ R^{\Delta}(L) &= \begin{cases} (v_i, v_j) \in E : v_i \in [v_i]_R \text{ and } [v_i]_R \cap L \neq \phi\} \\ v_i : [v_i]_R \text{ and } [v_j]_R \cap L \neq \phi. \end{cases} \end{aligned}$$

Definition 2.6. *Assume $\mathbb{M} = (U, F)$ is an information system and $\phi \neq G \subseteq U$. The Rough membership function[10] for the set G is*

$$\omega_G^F(f) = \frac{|[f]_G \cap G|}{|[f]_F|} \; for\ some\ f \in U$$

Example 2.1. *Let $\mathbb{M} = (U, F)$ be an information system. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ and $G = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Let thirst, hunger, frequent urination, weight loss, tiredness are conditional attributes and diabetic is the decision attribute. The decision system is given as follows* $\boldsymbol{Table\ 1}$.

The rough membership values are,

$$\begin{aligned} \omega_G^F(1) &= \frac{|\{1, 9, 15\} \cap \{1, 2, 3, 4, \ldots, 8\}|}{|\{1, 9, 15\}|} = \frac{1}{3}; \\ \omega_G^F(2) &= 1 \end{aligned}$$

Similarly we can find other membership values.

3 Rough Graph and its Properties

In this section, we have proposed the constructional process of rough graph from an information system through the rough membership function.

Definition 3.1. *Let $\mathfrak{R} = \{V, E, \omega\}$ be a triple consisting of non-empty set $V = \{v_1, v_2, \ldots, v_n\} = U$, where U is a universe, $E = \{e_1, e_2, \ldots, e_m\}$ be an edge set for V and ω be a function $\omega : V \rightarrow [0, 1]$. A rough graph is defined as,*

$$\mathfrak{R}(v_i, v_j) = \begin{cases} \text{If } max(\omega_G^V(v_i), \omega_G^V(v_j)) > 0, & \text{then } v_iv_j \text{ exists} \\ \text{If } max(\omega_G^V(v_i), \omega_G^V(v_j)) = 0, & \text{no edge.} \end{cases}$$

Patients	Thirst	Hunger	Frequent	Weight Loss	Tiredness	Diabetic
P_1	H	H	L	L	H	H
P_2	H	H	L	L	L	H
P_3	H	H	H	L	H	H
P_4	H	H	H	L	L	H
P_5	H	L	H	H	H	H
P_6	H	H	H	H	H	H
P_7	H	L	L	L	L	H
P_8	H	H	H	H	H	H
P_9	H	H	L	L	H	L
P_{10}	H	L	H	L	H	L
P_{11}	H	H	H	L	H	L
P_{12}	H	L	L	L	L	L
P_{13}	L	H	L	H	H	L
P_{14}	L	L	L	H	L	L
P_{15}	H	H	L	L	H	L

Table 1:Decision System

Remark 3.1. *1. Rough graph is always simple and undirected graph.*

Example 3.1. *From example 2.7 the rough graph is constructed as shown below* $\boldsymbol{Figure}$ $\mathbf{1}$.

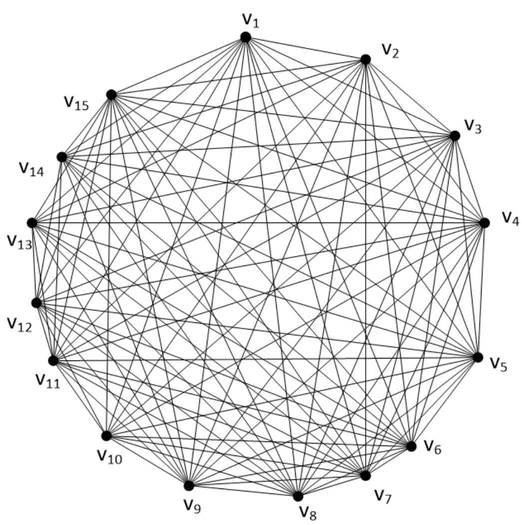

Figure 1. Rough graph

Definition 3.2. *The degree of a vertex v_i of rough graph $\mathfrak{R}$ is defined as the number of edges incident to that vertex. It is denoted by $\Delta_{\mathfrak{R}}(v_i), i \in N$.*

Example 3.2. *From example 3.3, the degree of vertex in rough graph will be*

$$\Delta_{\mathfrak{R}}(v_1) = 14$$

Similarly we can find the degree for other vertices.

Theorem 3.1. *Rough graph is always a connected and pendant free graph.*

Proof. In rough graph $\forall v_i, \Delta_{\mathfrak{R}}(v_i) > 1$, proof follows. □

Remark 3.2. *1. Rough graph satisfies hand-shaking lemma.*

2. In any rough graph, the number of vertices of odd degree is always even.

Definition 3.3.

$$+(v) = \max\{\Delta_{\mathfrak{R}}(v_i)|v_i \in V(G)\}$$
$$-(v) = \min\{\Delta_{\mathfrak{R}}(v_i)|v_i \in V(G)\}$$

If $+(v) = -(v)$, then the graph is said to be regular rough graph.

Definition 3.4. *If every vertex of rough graph is adjacent with all other vertices, then it is said to be complete rough graph. The complete rough graph with n vertices is denoted by $\mathfrak{I}_n$.*

Example 3.3. *By using the following information system* $\boldsymbol{Table}$ $\mathbf{2}$*., the complete rough graph can be constructed.*

Subjects	Assignment	Test	Internal	Result
x_1	Not Submitted	Written	>80	Pass
x_2	Submitted	Not Written	>80	Pass
x_3	Submitted	Written	>90	Pass
x_4	Not Submitted	Written	>50	Fail
x_5	Submitted	Not Written	>50	Fail
x_6	Not Submitted	Written	>90	Fail

Table 2:Information system

The complete rough graph for the table 2 is represented in $\boldsymbol{Figure}$ $\mathbf{2}$.

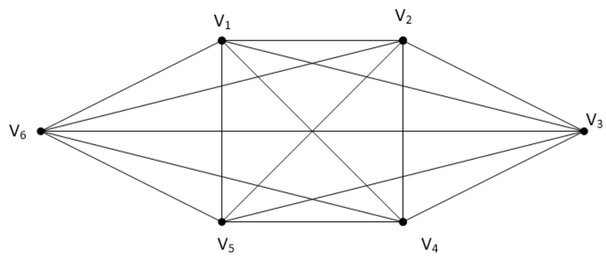

Figure 2. Complete rough graph

Remark 3.3. *1. A complete rough graph with n vertices contains $\frac{n(n-1)}{2}$ edges.*

2. A complete rough graph is always $(n-1)$ regular rough graph.

4 Rough walk and Rough cycle

In this section, we have discussed rough walk, rough cycle and its properties.

Definition 4.1. *An alternating sequence of vertices and edges in a rough graph with $\omega(v_i) \geq 0.5$, is said to be rough walk.*

Example 4.1. *The decision system is given as $\boldsymbol{Table\ 3}$.,*

Section	Inside	Outside	Difference	Churn
1	Middle	Middle	Small	False
2	Large	Large	Small	False
3	Small	Small	Small	False
4	Small	Small	Large	True
5	Middle	Middle	Small	True
6	Middle	Small	Small	True

Table 3:Decision system

The rough walk for the above decision table is $v_1,\ v_2,\ v_4,\ v_5,\ v_2,\ v_6,\ v_3$.
The rough walk for the $\boldsymbol{Table\ 3}$is represented in $\boldsymbol{Figure\ 3}$.

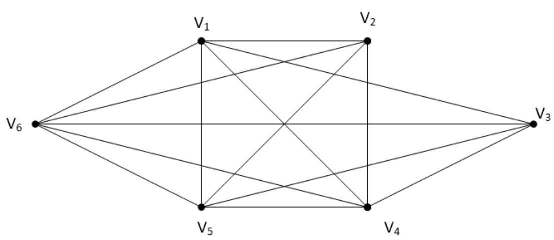

Figure 3. Rough graph

Definition 4.2. *Distinct edges in rough walk, are said to be rough trail and distinct vertices in a rough walk, are said to be rough path. It is denoted by $\mathfrak{P}_n$.*

Example 4.2. *From example 4.2 the rough trail and rough path are given as,*

$$v_1, v_2, v_4, v_5, v_3, v_5 \textit{ is a rough trail.}$$
$$v_1, v_3, v_6, v_4, v_5, v_2 \textit{ is a rough path.}$$

Definition 4.3. *The rough walk formed by distinct vertices $v_1,\ v_2,\ \ldots,\ v_n$ is called rough cycle if $v_1 = v_n$. It is denoted by $\mathfrak{C}_n$.*

Example 4.3. *From example 4.2 rough cycle is constructed as follows in $\boldsymbol{Figure\ 4}$.*

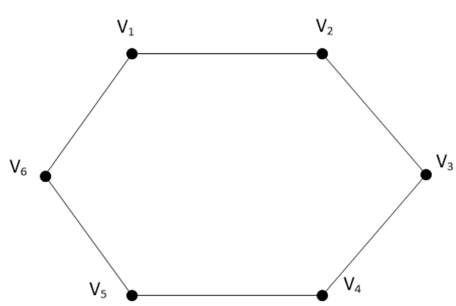

Figure 4. Rough cycle

Theorem 4.1. *In a rough graph, any $v_1 - v_2$ rough walk contains at-least one $v_1 - v_2$ rough path.*

Proof. Since rough walk contains repeated vertices, distinct rough paths can be developed in $v_1 - v_2$
$\therefore$ Any $v_1 - v_2$ rough walk contains $v_1 - v_2$ rough path. □

5 Operations on Rough graph

In this section, we have discussed about the operations on rough graph and some results.

Definition 5.1. *Let $\mathfrak{R}_1\,(V_1, E_1)$ and $\mathfrak{R}_2\,(V_2, E_2)$ be two rough graphs with $V_1 \cap\ V_2 = \phi$. Then rough union of $\mathfrak{R}_1$ and $\mathfrak{R}_2$ is defined as $\mathfrak{R}_1 \cup \mathfrak{R}_2 = (V_1 \cup\ V_2, E_1 \cup\ E_2)$, where*

$$V_1 \cup V_2(x) = \begin{cases} \omega_1(x), & if\ x \in V_1 \\ \omega_2(x), & if\ x \in V_2 \end{cases}$$

Definition 5.2. *Let $\mathfrak{R}_1(V_1, E_1)$ and $\mathfrak{R}_2(V_2, E_2)$ be two rough graphs with $V_1 \cap\ V_2 = \phi$. Then the rough join of $\mathfrak{R}_1$ and $\mathfrak{R}_2$ is defined as joining all vertices of V_1 to the vertices of V_2. (i.e)$\mathfrak{R}_1 \sim \mathfrak{R}_2 = (V_1 \sim V_2, E_1 \sim E_2)$ where,*

$$V_1 \sim V_2(x) = \begin{cases} \omega_1(x), & if\ x \in V_1 \\ \omega_2(x), & if x \in V_2 \end{cases}$$

Example 5.1. *Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be two rough graphs represented in $\boldsymbol{Figure\ 5}$*

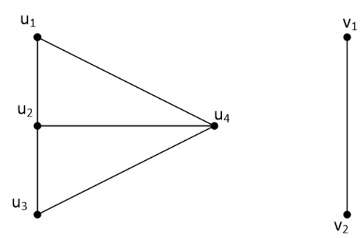

Figure 5. Rough graph $\mathfrak{R}_1$ and Rough graph $\mathfrak{R}_2$

The rough join of $\mathfrak{R}_1$ and $\mathfrak{R}_2$ is represented in $\boldsymbol{Figure\ 6}$.

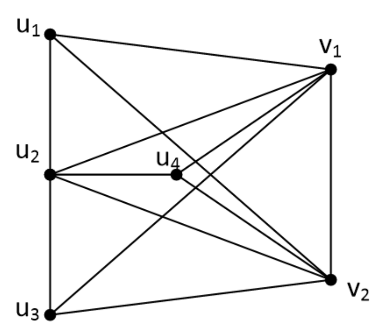

Figure 6. Join of two rough graphs

Definition 5.3. *Let $\mathfrak{R}_1 = (V_1, E_1)$ and $\mathfrak{R}_2 = (V_2, E_2)$ be two rough graphs with the vertex sets as V_1 and V_2 and their* *edge sets as E_1 and E_2 respectively. Then the rough cartesian product of two rough graphs $\mathfrak{R}_1$ and $\mathfrak{R}_2$ is defined as*

$$V_1 \approx V_2 = \{(u, v) \,|u \in V_1 \; and \; v \in V_2\}$$

$$E_1 \approx E_2 = \{(u, v)(x, y)|\omega(u) = \omega(x), vy \in E_2 \; or \; ux \in E_1, \; \omega(v) = \omega(y)\}$$

with $(\omega_1 \approx \omega_2)(u, v) = \omega_1(u) \wedge \omega_2(v)$

Example 5.2. *Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be two rough graphs are defined in $\boldsymbol{Figure}$ **7***

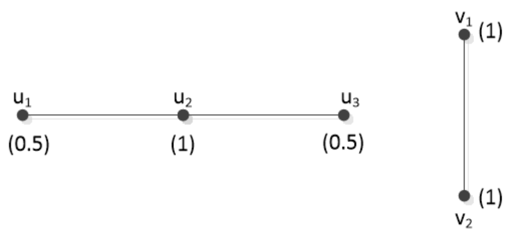

Figure 7. $\mathfrak{R}_1$ and $\mathfrak{R}_2$

$$\begin{aligned} V(H) &\approx V(G) \\ &= \Big\{(u_1, v_1), (u_1, v_2), (u_2, v_2), (u_2, v_1), \\ &\quad (u_3, v_1), (u_3, v_2)\Big\} \end{aligned}$$

*The rough cartesian product of $\mathfrak{R}_1 \approx \mathfrak{R}_2$ is represented in $\boldsymbol{Figure}$ **8***

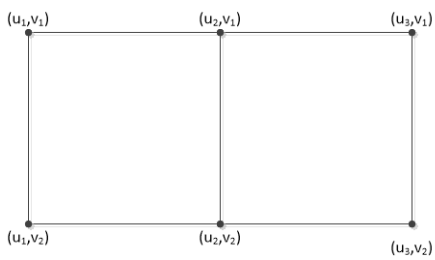

Figure 8. Cartesian product of $\mathfrak{R}_1$ and $\mathfrak{R}_2$

Definition 5.4. *The degree of a vertex in a rough cartesian product for rough graphs is defined as the sum of all edges to a vertex (u_i, v_i) such that*

$$\Delta_{\mathfrak{R}_1 \approx \mathfrak{R}_2} = \omega(u_i) \wedge \omega(v_i) \wedge \omega(v_j), if \; u_i = u_j, v_i = v_j$$

Example 5.3. *By example 5.5, we can find the degree of a vertex for the rough cartesian product is*

$$\Delta(u_1, v_1) = 0.5 + 0.5 = 1$$

Similary we can find for the other vertices.

Definition 5.5. *The grid rough graph is defined as the rough Cartesian product of two rough paths. It is denoted by $\mathfrak{G}_{m\times n}$.*

Theorem 5.1. *The rough Cartesian product of two rough paths is a grid rough graph.*

Proof. The rough Cartesian product on two rough graphs is defined as

$$\begin{aligned} V_1 &\approx V_2 = \{(u, v)|u \in V_1 \; and \; v \in V_2\} \\ E_1 &\approx E_2 = \{(u, v)(x, y)\| \; \omega(u) = \omega(x), vy \in E_2 \\ &\quad \text{or } ux \in E_1, \omega(v) = \omega(y)\} \end{aligned}$$

Since by the rough path, the membership values between two vertices will be ≥ 0.5.
Case $1 : m = n$
Then

$$\begin{aligned} V(\mathfrak{R}_1) &\approx V(\mathfrak{R}_2) \\ &= \Big\{(u_1, v_1), (u_1, v_2), \ldots, (u_1, v_n), (u_2, v_1), \\ &\quad (u_2, v_2), \ldots, (u_2, v_n), \ldots, (u_n, v_1), \\ &\quad (u_n, v_2), \ldots, (u_n, v_n)\Big\} \\ E(\mathfrak{R}_1) &\approx E(\mathfrak{R}_2) \\ &= \{\{(u_j, v_i), (u_j, v_i + 1)\} \cup \{(u_i, v_j)(u_{i+1}, v_j)\}\}, \\ &\quad 1 \leq i \leq n - 1, \; j = 1, 2, 3, \ldots \end{aligned}$$

By using $V(\mathfrak{R}_1) \approx V(\mathfrak{R}_2)$ and $E(\mathfrak{R}_1) \approx E(\mathfrak{R}_2)$, the resulting graph will be $n \times n$-grid rough graph.
Case $2 : m \neq n$
The number of vertices for the rough Cartesian product will be,

$$\begin{aligned} V(\mathfrak{R}_1) &\approx V(\mathfrak{R}_2) \\ &= \{(u_1, v_1), \ldots, (u_1, v_n), \ldots, \\ &\quad (u_n, v_1), \ldots, (u_n, v_n)\} \\ E(\mathfrak{R}_1) &\approx E(\mathfrak{R}_2) \\ &= \{\{(u_m v_i)(u_m v_{i+1})\} \cup \{(u_i v_n)(u_{i+1} v_n)\}\} \end{aligned}$$

By using $V(\mathfrak{R}_1) \approx V(\mathfrak{R}_2)$ and $E(\mathfrak{R}_1) \approx E(\mathfrak{R}_2)$, the resulting graph will be $m \times n$-grid rough graph. □

Example 5.4.

$$\begin{aligned} V(\mathfrak{P}_4) &\approx V(\mathfrak{P}_3) \\ &= \Big\{(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_2, v_1), (u_2, v_2), \\ &\quad (u_2, v_3), (u_3, v_1), (u_3, v_2), (u_3, v_3), (u_4, v_1), \\ &\quad (u_4, v_2), (u_4, v_3), (u_4, v_4)\Big\} \end{aligned}$$

*The Rough path $\mathfrak{P}_4$ and $\mathfrak{P}_3$ is represented in $\boldsymbol{Figure}$ **9***

Figure 9. Rough path $\mathfrak{P}_4$ and $\mathfrak{P}_3$

The Grid rough graph of 4×3 *is represented in* $\boldsymbol{Figure}$ **10**

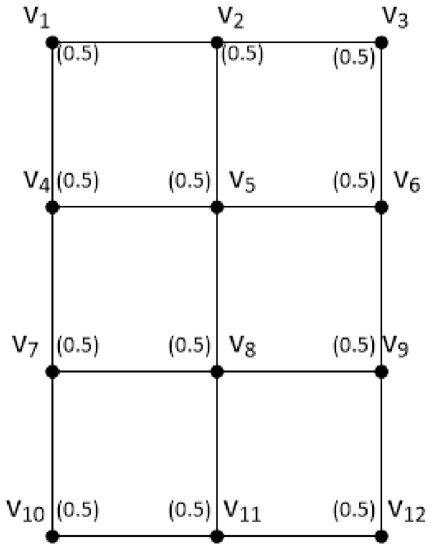

Figure 10. Grid rough graph of 4×3

Definition 5.6. *The ladder rough graph is defined as the rough Cartesian product of rough path and the complete rough graph* $\mathfrak{I}_2$. *It is denoted by* $\mathfrak{L}_m$.

Theorem 5.2. *For the rough Cartesian product of* $\mathfrak{I}_2$ *and* $\mathfrak{P}_n, n \geq 2$, *the resulting graph will be rough ladder graph.*

Proof. By the definition of rough Cartesian product, the vertex set of $\mathfrak{I}_2$ and $\mathfrak{P}_n$ will be

$$\begin{aligned} V(\mathfrak{I}_2) &\approx V(\mathfrak{P}_n) \\ &= \{(u_1, v_1), \dots, (u_1, v_n), (u_2, v_1), \dots, (u_2, v_n)\} \end{aligned}$$

The number of elements in the vertex set $V(G) \approx V(H)$ will be $2 \times n$. The edges between $\mathfrak{I}_2 \approx \mathfrak{P}_n$ will be

$$\begin{aligned} E(\mathfrak{I}_2) &\approx E(\mathfrak{P}_n) \\ &= \{(u_1, v_j)(u_2, v_j) \cup (u_1, v_i)(u_1, v_{(i+1)}) \\ &\quad \cup (u_2, v_i)(u_2, v_{(i+1)})\}, j = 1, 2;\ i = 1, 2, \dots, n \end{aligned}$$

Then the ladder rough graph $(L_{(n-1)})$ is obtained from the vertex set and the edge set. □

Example 5.5. *The* $\mathfrak{I}_2$ *and* $\mathfrak{P}_5$ *are given as* $\boldsymbol{Figure}$ **11**

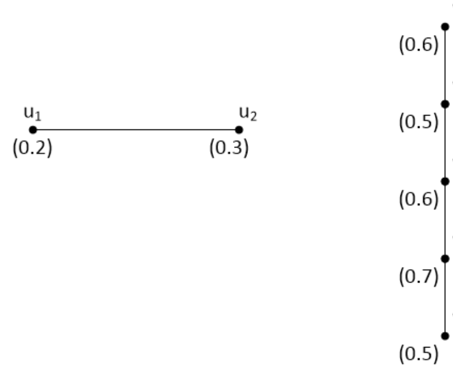

Figure 11. $\mathfrak{I}_2$ and $\mathfrak{P}_5$

$$\begin{aligned} V(\mathfrak{I}_2) &\approx V(\mathfrak{P}_5) \\ &= \Big\{((u_1, v_1), (u_1, v_2), (u_1, v_3), (u_2, v_1), \\ &(u_2, v_1), (u_2, v_2), (u_2, v_3), (u_3, v_1), (u_3, v_2), \\ &(u_3, v_3), (u_4, v_1), (u_4, v_2), (u_4, v_3))\Big\} \end{aligned}$$

The Ladder rough graph is represented in $\boldsymbol{Figure}$ **12**

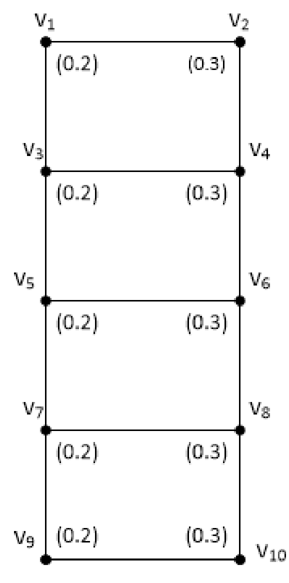

Figure 12. Ladder rough graph

6 Isomorphism on rough graphs

In this section, we have introduced isomorphism on rough graphs and analyzed its properties.In this paper, λ represents vertex membership value and β represents edge membership value.

Definition 6.1. *A homomorphism on rough graph* $g : G \rightarrow G'$

is a map $g : A \to A'$ which satisfies

$$\begin{aligned}\lambda(m) &\leq \lambda'(g(m)) \,\forall\, m \in A \text{ and} \\ \beta(m,n) &\leq \beta'(g(m), g(n)) \,\forall\, m,n \in A\end{aligned}$$

Example 6.1. *Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be two rough graphs represented in $\boldsymbol{Figure}$ 13, $\boldsymbol{Figure}$ 14.*

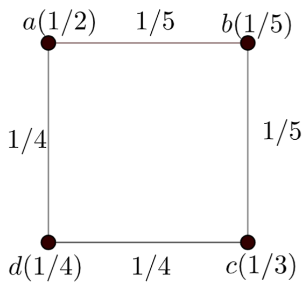

Figure 13. Rough graph $\mathfrak{R}_1$

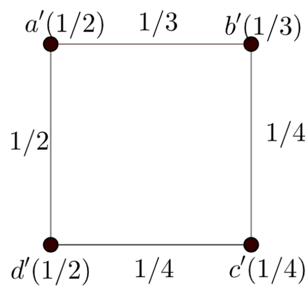

Figure 14. Rough graph $\mathfrak{R}_2$

$$\begin{aligned}\lambda(a) \leq 1/2 \leq \lambda(a') &\qquad \beta(a,b) = 1/5 \leq 1/3 = \beta(a',b') \\ \lambda(b) \leq 1/5 = \lambda(b') &\qquad \beta(b,c) = 1/3 \leq 1/5 = \beta(b',c') \\ \lambda(c) = 1/4 \leq 1/3 = \lambda(c') &\qquad \beta(c,d) = 1/4 \leq \beta(c',d') \\ \lambda(d) = 1/2 \leq 1/4 = \lambda(d') &\qquad \beta(d,a) = 1/2 \leq 1/4 = \beta(d',a')\end{aligned}$$

The mapping will be $g(a) \leq a', g(b) \leq b', g(c) \leq c', g(d) \leq d'$ which satisfies

$$\begin{aligned}\lambda(m) &\leq \lambda'(g(m)) \,\forall\, m \in A \text{ and} \\ \beta(m,n) &\leq \beta'(g(m), g(n)) \,\forall\, m,n \in A\end{aligned}$$

Definition 6.2. *An isomorphism $g : G \to G'$ which is a bijective map that satisfies*

$$\begin{aligned}\lambda(m) &= \lambda'(g(m)) \,\forall\, m \in A \text{ and} \\ \mu(m,n) &= \mu'(g(m), g(n)) \,\forall\, m,n \in A\end{aligned}$$

Example 6.2. *Let G_1 and G_2 be rough graph with vertex and edge membership values($\boldsymbol{Figure}$ 15, $\boldsymbol{Figure}$ 16.)*

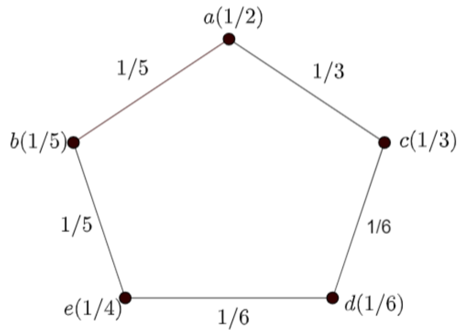

Figure 15. G

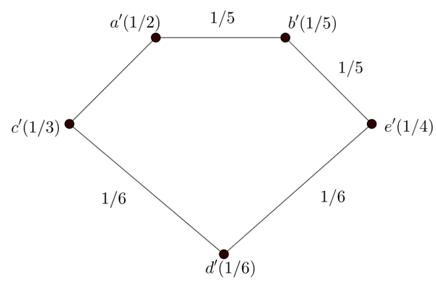

Figure 16. G'

$$\begin{aligned}\lambda(a) = 1/2 = \lambda(a') &\qquad \beta(a,b) = 1/5 = \beta(a',b') \\ \lambda(b) = 1/5 = \lambda(b') &\qquad \beta(a,c) = 1/3 = \beta(a',c') \\ \lambda(c) = 1/3 = \lambda(c') &\qquad \beta(b,e) = 1/5 = \beta(b',e') \\ \lambda(d) = 1/6 = \lambda(d') &\qquad \beta(e,d) = 1/6 = \beta(e',d') \\ \lambda(e) = 1/4 = \lambda(e') &\qquad \beta(c,d) = 1/6 = \beta(c',d')\end{aligned}$$

The mapping will be $g(a) = a', g(b) = b', g(c) = c', g(d) = d', g(e) = e'$ which satisfies

$$\begin{aligned}\lambda(m) &= \lambda'(g(m)) \,\forall\, m \in A \text{ and} \\ \beta(m,n) &= \beta'(g(m), g(n)) \,\forall\, m,n \in A\end{aligned}$$

6.1 Degree of vertex

The degree of a vertex $'v'$ in rough graph will be defined as

$$d(v) = \sum_{\substack{v \neq u \\ v \in A}} \beta(u,v)$$

Remark 6.1. *1. The membership values of edges and vertices are preserved in isomorphism.*

2. A homomorphism of G to itself is an endomorphism of rough graph.

3. An isomorphism of G to itself is an automorphism of a rough graph.

Proposition 6.1. *The order and size are same for any two isomorphic rough graphs.*

Proof. Since two graphs are isomorphic between G and G' with that the sets A and A' which satisfies the condition that

$$\begin{aligned} \lambda(m) &= \lambda'(g(m))\ \forall\ m \in A \\ \beta(m,n) &= \beta'(g(m),g(n))\ \forall\ m,n \in A \\ \text{Order}(G) &= \sum_{m\in A} \lambda(m) \\ &= \sum_{m\in A'} \lambda'(g(m)) = \text{ Order } g(G') \\ \text{Size }(G) &= \sum_{m,n\in A} \beta(m,n) \\ &= \sum_{m',n'\in A'} \beta'(g(m),g(n)) = \text{Size }(G') \end{aligned}$$

□

Proposition 6.2. *Two rough graphs are same in order and size need not be isomorphic to each other.*

Example 6.3. *Consider two graphs G and G' with the sets A and A' and the mapping are defined with $g(x) = x'\ \forall\{x|x \in \{a,b,c,d\}\}(Figure17, Figure18)$*
The order of G and G' is $17/12$. The size of G and G' is $7/6$. But the graphs G and G' are not isomorphic to each other because there is no edge between $'c'$ and $'b'$ in G.
$\therefore$ G and G' are not isomorphic.

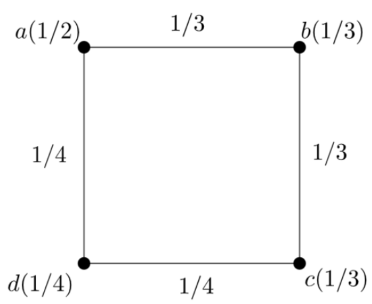

Figure 17. *G*

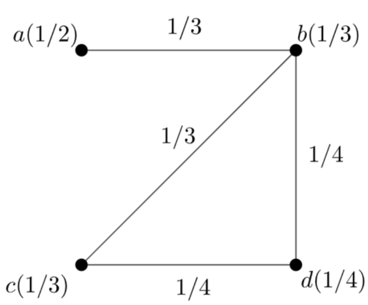

Figure 18. G'

Theorem 6.1. *The degrees of their vertices are preserved, if rough graphs G and G' are isomorphic.*

Proof. By the definition of isomorphism,

$$\beta(m,n) = \beta'(g(m),g(n))\ \forall\ m,n \in A$$

so,

$$d(m) = \sum_{\substack{m\neq n \\ n\in A}} \beta(m,n) = \sum_{\substack{m\neq n \\ n\in A}} \beta'(g(m),g(n)) = d(g(m))$$

□

Proposition 6.3. *If the degrees of their nodes of G and G' are preserved, need not be isomorphic to each other(**Figure 19**,**Figure 20**).*

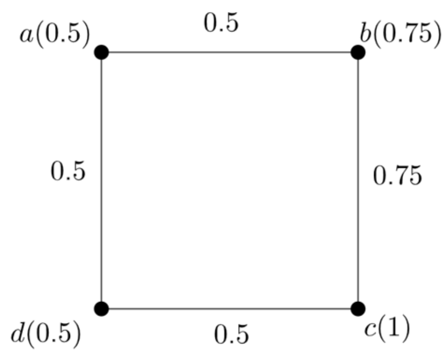

Figure 19. *G*

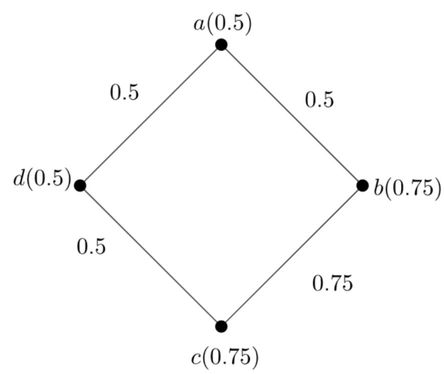

Figure 20. G'

$$\begin{aligned} d(a) &= 1 & d(a') &= 1 \\ d(b) &= 1.25 & d(b') &= 1.25 \\ d(c) &= 1.25 & d(c') &= 1.25 \\ d(d) &= 1 & d(d') &= 1 \end{aligned}$$

Theorem 6.2. *Rough graph isomorphism is an equivalence relation.*

Proof. Let $G = (\lambda, \beta), G' = (\lambda', \beta'), G'' = (\lambda'', \beta'')$ be rough graphs with underlying sets A, A', A'' respectively.
Reflexive: The identity map $g : A \to A \ni: g(m) = m \;\forall\; m \in A$ This g is a bijective map satisfying

$$\begin{aligned}\lambda(m) &= \lambda'(g(m)) \;\forall\; m \in A \\ \beta(m,n) &= \beta'(g(m), g(n)) \;\forall\; m,n \in A\end{aligned}$$

As a result, the rough graph is an isomorphism of g to itself. The reflexive relation is thus satisfied.
Symmetric: Let $g : A \to A'$ be an isomorphism of G onto G' then g is a bijective map

$$\begin{aligned}g(m) &= m', m \in A, \qquad (1)\\ \lambda(m) &= \lambda'(g(m)) \;\forall\; m \in A, \\ \beta(m,n) &= \beta'(g(m), g(n)) \;\forall\; m,n \in A. \qquad (2)\end{aligned}$$

Since g is bijective,by ① $g^{-1}(m^{-1}) = m \forall\; m' \in A'$, using ②

$$\begin{aligned}\omega(g^{-1}(m^{-1})) &= \lambda'(m') \forall\; m' \in A' \\ \beta(g^{-1}(m^{-1}), g^{-1}(m^{-1})) &= \beta(m', n') \forall\; m', n' \in A' (3)\end{aligned}$$

Hence we get a 1-1, onto map $g : A' \to A$, which is an isomorphism from G' to G.

$$\therefore G \cong G' \implies G' \cong G$$

Transitive: Let $g : A \to A'$ and $f : A' \to A''$ be isomorphism of the rough graphs G onto G' and G' onto G'' respectively.Then $g \circ f$ is a 1-1 onto map from A to A'' where

$$(g \circ f)(m) = g(f(m)) \forall\; m \in A$$

As $g : A \to A'$ is an isomorphism $g(m) = m', m, n \in A$

$$\begin{aligned}\lambda(m) &= \lambda'(g(m)) \;\forall\; m \in A \qquad (4)\\ \beta(m,n) &= \beta'(g(m), g(n)) \;\forall\; m,n \in A \qquad (5)\end{aligned}$$

As f is an isomorphic from A' to A'' we have $f(m') = m'', m' \in A'$

$$\begin{aligned}\lambda'(m') &= \lambda''(f(m')) \;\forall\; m' \in A' \qquad (6)\\ \beta'(m', n') &= \beta''(f(m'), f(n')) \;\forall\; m', n' \in A' \qquad (7)\end{aligned}$$

From ④,⑥ and using $g(m) = m', m \in A$

$$\begin{aligned}\lambda(m) = \lambda'(m') &= \lambda''(f(m')) \;\forall\; m' \in A' \\ &= \lambda''(f(g(m))) \;\forall\; m \in A\end{aligned}$$

From ⑤,⑦ we have

$$\begin{aligned}\beta(m,n) &= \beta'(m', n') \;\forall\; m,n \in A \\ &= \beta''(f(m'), f(n')) \;\forall\; m', n' \in A' \qquad (8)\\ &= \beta''(f(g(m)), f(g(m))) \;\forall\; m,n \in A.\end{aligned}$$

$\therefore g \circ f$ is an isomorphism between G and G''. Hence isomorphism between rough graphs is an equivalence relation. □

7 Conclusion

In this paper, we have proposed the constructional technique of rough graph with the help of rough membership function. Also extensive properties of rough graph are briefly discussed in this paper. In our future work, we planned to propose rough metric dimension and its properties. Rough metric dimension may be the alternate technique to calculate reduct and core from an information system. Reduct and core are the concise form of original data without any information loss. The attributes in reduct and core set are most important and removal of these attribute will affect the quality of data. Hence through rough graph we can implement attribute reduction process for any information system which may be imprecise and uncertain.

REFERENCES

[1] Skowron A, Rauszer C, "The discernibility matrices and functions in information systems", Intelligent Decision Support, pp 331–362, 1992.DOI: 10.1007/978-94-015-7975-9_21

[2] Anitha K, "Rough Set Theory on Topological Spaces", Rough Sets and Knowledge Technology, Lecture Notes in Computer Science, vol. 8818, chapter 7, pp 69-74, 2014.DOI:10.1007/978-3-319-11740-9_7

[3] Anitha K, Thangeswari M, "Rough set based optimization techniques",6th International Conference on Advanced Computing and Communication Systems (ICACCS), IEEE, pp 1021-1024, 2020.

[4] Mathew B, John S.J, Garg H, "Vertex rough graphs", Complex Intelligent Systems, Vol 6,no. 2,pp 347–353, 2020. DOI: https://doi.org/10.1007/s40747-020-00133-8

[5] Ognjenovic V, Brtka V, Jovanovic M, Brtka E, Berkovic I, "The representation of indiscernibility relation by graph", IEEE 9th International Symposium on Intelligent Systems and Informatics, pp 91-94, 2011.

[6] Peters J.F, Skowron A, Han L, Ramanna S, "Towards rough neural computing based on rough membership functions: Theory and application", International Conference on Rough Sets and Current Trends in Computing, pp 611–618, 2000.

[7] Zhao Y, Yao Y, Luo F, "Data analysis based on discernibility and indiscernibility", Information Sciences, Vol 177, no. 22, pp 4959–4976, 2007.

[8] Lan S, Xiangzhi H, "Rough set model with double universe of discourse", IEEE International Conference on Information Reuse and Integration, pp 492–495, 2007.

[9] Sarwar M, "Decision-making approaches based on color spectrum and D-TOPSIS method under rough environment", Computational and Applied Mathematics, Vol 39, no 4, pp 1-32, 2020.

[10] Pawlak Z, ”Rough set theory and its applications to data analysis”, Cybernetics and Systems. Vol 29, no 7, pp 661–688, 1998.

[11] Pattaraintakorn P, Cercone N, ”Integrating rough set theory and medical applications”, Applied Mathematics Letters, Vol 21, no 4, pp 400–403, 2008. DOI: https://doi.org/10.1016/j.aml.2007.05.010

[12] Rissino S, Lambert-Torres G, ”Rough set theory- fundamental concepts, principals, data extraction and applications”, Data mining and knowledge discovery in real life applications, 2009. ISBN: 978-3-902613-53-0

[13] Chellathurai S.R, Jesmalar L, ”Core in rough graph and weighted rough graph”, International Journal of Contemporary Mathematical Sciences, Vol 11, no 6, pp 251–265, 2016.

[14] He T, ”Representation form of rough graph”, Applied Mechanics and Materials, Vol 157, pp 874-877, 2012.

[15] He T, ”Rough properties of rough graph”, Applied Mechanics and Materials, VOl 157, pp 517–520, 2012.

[16] He T, Yang R, ”Different Kinds of Rough Graph and Their Representation Forms”, DEStech Transactions on Engineering and Technology Research, no. mdm, DEStech Publications, 2017.

[17] He T, Chen Y, Shi K, ”Weighted rough graph and its application”, IEEE sixth International conference on Intelligent Systems Design and Applications, Vol 1, pp 486–491, 2006.

[18] Liang M, Liang B, Wei L, Xu X, ”Edge rough graph and its application”, Eighth International Conference on Fuzzy Systems and Knowledge Discovery (FSKD), Vol 1, pp 335–338, 2011.

[19] Zhang Q, Xie Q, Wang G, ”A survey on rough set theory and its applications”, CAAI Transactions on Intelligence Technology, Vol 1, no 4, pp 323–333, 2016. DOI: https://doi.org/10.1016/j.trit.2016.11.001

[20] Pawlak Z, ”Rough set theory and its applications”:, Journal of Telecommunications and Information Technology, pp 7-10, 2002.

[21] Anitha, K., Aruna Devi, R., Munir, M., & Nisar, K. S, ”Metric dimension of rough graphs”, International Journal of Nonlinear Analysis and Applications, Vol 12, pp 1793-1806, 2021.