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A Study on Intuitionistic Fuzzy Critical Path Problems Through Centroid Based Ranking Method

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Abstract In this study the intuitionistic fuzzy version of the critical path method has been proposed to solve networking problems with uncertain activity durations. Intuitionistic fuzzy set [1] is an extension of fuzzy set theory [2] unlike fuzzy set, it focuses on degree of belonging, the degree of non-belonging or non-membership function and the degree of hesitancy which helps the decision maker to adopt the best among the worst cases. Trapezoidal and the triangular intuitionistic fuzzy numbers are utilized to describe the uncertain activity or task durations of the project network. Here trapezoidal and triangular intuitionistic fuzzy numbers are converted into their corresponding parametric form and applying the proposed intuitionistic fuzzy arithmetic operations and a new method of ranking based on the parametric form of intuitionistic fuzzy numbers, the intuitionistic fuzzy critical path with vagueness reduced intuitionistic fuzzy completion duration of the project has been obtained. The authentication of the proposed method can be checked by comparing the obtained results with the results available in pieces of literature.

Keywords Trapezoidal Intuitionistic Fuzzy Number, Triangular Intuitionistic Fuzzy Number, Left Fuzziness Index, Right Fuzziness Index, Parametric Form, Intuitionistic Fuzzy Project Duration

1 Introduction

In any modern manufacturing system, scheduling plays the most vital role in the production. It helps to plan or design the activities to be implemented in a project and control their progress through a production process. The fundamental principle for all scheduling systems is to create a network of activity and event connections. The main objective of the decision-makers is to optimize the total completion duration of the project and to minimize the project cost. The critical path method is an activity-based method designed for the decision-makers for planning, scheduling and controlling of complicated projects. This method elevates the performance of the project by recognizing critical activities and guides the decision-makers to utilize the available resources properly on these critical activities in the project network and ensures the project quality by minimizing the project cost and time. A project is said to be a successful project only when it meets and satisfies the end-users objectives. However, the critical path method deals with complicated projects more effectively. In a real-world situation, to complete any large and complex project within a minimum time period and getting crisp parameters is impossible due to various real-time causes such as activity delays due to bad weather conditions, materials may not be delivered as planned, etc. Hence, we cannot use the standard method for solving the intuitionistic fuzzy project networking problems. This leads to the development of an intuitionistic fuzzy critical path method as it handles ill known quantities more precisely and considers both degrees of belonging and non-belonging.

In 1961, Kelley [3] addressed the importance of critical path method and developed a mathematical proof that includes the essential elements such as time and cost of each project operation. Angelov [4] introduced a new concept to optimization problems with the application of intuitionistic fuzzy sets. Since then, researchers [5][6] were studied and developed many concepts based on intuitionistic fuzzy optimization problems.

Chanas and Zielinski [7] shared their idea of applying the extension principle of Zadeh on the critical path method. Many authors extended their idea of applying the intuitionistic fuzzy concept with a critical path method which ultimately leads to the development of intuitionistic fuzzy arithmetic operations and ranking functions. Ravi Shankar et al [8] introduced an analytical approach to calculate criticality in a fuzzy network and suggested a centroid-based ranking method for trapezoidal fuzzy numbers to obtain the cumulative float time of each task or activity in the fuzzy network. Shakeela Sathish and Ganesan [9] proposed a new approach for solving fuzzy critical path analysis by converting fuzzy numbers into their corresponding crisp numbers.

In 2014, Jayagowri and Geetharamani [10][11] suggested a novel approach to define the critical path in a project network whose task parameters are interpreted by trapezoidal intuitionistic fuzzy numbers. Graded mean integration formula has been defined to reduce trapezoidal intuitionistic fuzzy number to equivalent crisp number. Again in 2015, Jayagowri and Geetharamani analyzed the criticality in the project network by computing total slack time for each path under the intuitionistic fuzzy environment using the metric distance ranking method. In a fuzzy network, Elizabeth and Sujatha [12] developed two distinct algorithms to obtain the critical path, where the duration of each activity is represented as triangular fuzzy numbers and triangular intuitionistic fuzzy numbers. Sophia Porchelvi and Sudha [13][14] proposed an algorithm to perform intuitionistic fuzzy critical path analysis, the length of which is the triangular intuitionistic fuzzy number. Mehlawat and Grover [15] proposed total performance score using strength and weakness index scores to identify the longest path of the project under intuitionistic fuzzy multi-criteria group decision making problem. Kiruthiga and Hemalatha [16] developed decision maker's risk attitude index and decision maker's risk index ranking value for critical path problem under intuitionistic fuzzy nature. Rameshan and Stephen Dinagar [17][18] proposed a new approach for intuitionistic fuzzy critical path problem by interpreting activity durations as octagonal intuitionistic fuzzy numbers and symmetric octagonal intuitionistic fuzzy numbers. Praveena et al [19] defined a new intuitionistic hexadecagonal fuzzy number and its nature by applying this in critical path problem. Mitlif and Sadiq [20] worked on fuzzy critical path problem by proposing new development ranking function to convert the given problem into its equivalent crisp problem and compared their results with other existing results. Priyadharshini and Deepa [21] investigate the critical path problem by defining maximum edge distance method under intuitionistic fuzzy nature where the activity durations are interpreted as triangular intuitionistic fuzzy numbers.

From the above literatures we observe that many authors worked on critical path method under fuzzy and intuitionistic fuzzy nature by using various ranking methods. In this paper we proposed a new centroid based ranking grade for comparing the parametric form of intuitionistic fuzzy numbers. The main objective of this proposed method is it gives vagueness reduced intuitionistic fuzzy project completion duration without converting the given problem into its equivalent crisp problem.

2 Notation

The basic definition of trapezoidal intuitionistic fuzzy number and a specific case of triangular intuitionistic fuzzy number and their arithmetic operations and ranking are discussed in this section, which is useful for our further investigation.

Definition 2.1: The membership function and non-membership function of a **Trapezoidal Intuitionistic Fuzzy Number (TRIFN)** $\tilde{a}^I$ is defined as follows

$$\mu_{\tilde{a}^I}(x) = \begin{cases} 0 & for\, x < a \\ \frac{x-a}{b-a} & for\, a \le x \le b \\ 1 & for\, b \le x \le c \\ \frac{d-x}{d-c} & for\, c \le x \le d \\ 0 & for\, x > d \end{cases}$$

$$\gamma_{\tilde{a}^I}(x) = \begin{cases} 1 & for\, x < a' \\ \frac{b'-x}{b-a'} & for\, a' \le x \le b' \\ 0 & for\, b' \le x \le c' \\ \frac{x-c'}{d'-c} & for\, c' \le x \le d' \\ 0 & for\, x > d' \end{cases}$$

and is denoted by $\tilde{a}^I = ((a,b,c,d),\ (a',b',c',d'))$.

Definition 2.2: The membership function and non-membership function of a **Triangular Intuitionistic Fuzzy Number (TIFN)** $\tilde{a}^I$ with the parameters $a' \ge b$ and $b' \ge c$

$b' \le a$ and $c' \le b$ is defined as follows

$$\mu_{\tilde{a}^I}(x) = \begin{cases} 0 & for\, x < a \\ \frac{x-a}{b-a} & for\, b \le x \le c \\ 1 & for\ x = c \\ \frac{c-x}{c-b} & for\, b \le x \le c \\ 0 & for\, x > c \end{cases}$$

$$\gamma_{\tilde{a}^I}(x) = \begin{cases} 1 & for\, x < a' \\ \frac{b'-x}{b'-a'} & for\, a' \le x \le b' \\ 0 & for\, x = b' \\ \frac{x-b'}{c'-b'} & for\, b' \le x \le c' \\ 1 & for\, x > c' \end{cases}$$

and is denoted by $\tilde{a}^I = ((a,b,c),\ (a',b',c'))$.

2.1 Parametric Representation of Generalized Intuitionistic Fuzzy Numbers

The modal value (location index) of membership and non-membership function of any generalized intuitionistic fuzzy number $\tilde{a}^I$ are represented as $a_0 = \left(\frac{\underline{a}(1)+\bar{a}(1)}{2}\right)$, $a'_0 = \left(\frac{\underline{a}'(0)+\bar{a}'(0)}{2}\right)$ respectively. The non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$ and $a'^* = (\bar{a}' - a'_0)$ represent the left fuzziness index function and right fuzziness index function of membership and non-membership function respectively. In the same way the non-increasing left continuous function $a^* = (\bar{a} - a_0)$ and $a'_* = (a'_0 - \underline{a}^*)$ represent the right fuzziness index function and left fuzziness index functions of membership and non-membership function respectively. Hence every generalized intuitionistic fuzzy number $\tilde{a}^I$ can also be represented by $\left(\tilde{a}^I = \langle a_0, a_*, a^* \rangle, \langle a'_0, a'_*, a'^* \rangle ; \omega_{\tilde{a}}, \nu_{\tilde{a}}\right)$.

2.2 Arithmetic Operations

For any two arbitrary intuitionistic fuzzy numbers $\tilde{a}^I = (\langle a_0, a_*, a^* \rangle, \langle a'_0, a'_*, a'^* \rangle)$ and $\tilde{b}^I = \langle b_0, b_*, b^* \rangle, \langle b'_0, b'_*, b'^* \rangle)$ the arithmetic operations are defined as follows

Addition:

$$\begin{aligned}\tilde{a}^I + \tilde{b}^I &= \left(\langle a_0, a_*, a^* \rangle,\ \left\langle a'_0, a'_*, a'^* \right\rangle\right) \\ &+ \left(\langle b_0, b_*, b^* \rangle,\ \left\langle b'_0, b'_*, b'^* \right\rangle\right) \\ &= (\langle a_0 + b_0, \max(a_*, b_*), \max(a^*, b^*) \rangle \\ &\left\langle a'_0 + b'_0, \max(a'_*, b'_*), \max(a'^*, b'^*) \right\rangle\Big)\end{aligned}$$

Subtraction:

$$\begin{aligned}\tilde{a}^I - \tilde{b}^I &= \left(\langle a_0, a_*, a^* \rangle,\ \left\langle a'_0, a'_*, a'^* \right\rangle\right) \\ &+ \left(\langle b_0, b_*, b^* \rangle,\ \left\langle b'_0, b'_*, b'^* \right\rangle\right) \\ &= (\langle a_0 + b_0, \max(a_*, b_*), \max(a^*, b^*) \rangle \\ &\left\langle a'_0 + b'_0, \max(a'_*, b'_*), \max(a'^*, b'^*) \right\rangle\Big)\end{aligned}$$

2.3 Ranking Function

For any intuitionistic fuzzy number $\tilde{a}^I = \left(\langle a_0, a_*, a^* \rangle,\ \left\langle a'_0, a'_*, a'^* \right\rangle\right)$ we define [20] the centroid point of $\tilde{a}^I$ as $(x(\tilde{a}^I_\mu), y(\tilde{a}^I_\mu);\ x(\tilde{a}^I_\gamma),\ y(\tilde{a}^I_\gamma))$. Where

$$x(\tilde{a}^I_\mu) = \frac{1}{3}\left[3a_0 + \frac{(a^* - a_*)}{1-r}\right]$$

$$y(\tilde{a}^I_\mu) = \frac{1}{3}$$

$$x(\tilde{a}^I_\gamma) = \frac{1}{3}\left[\frac{2(a'^* - a'_*)}{r^*} + 3a'_0\right] \ and$$

$$y(\tilde{a}^I_\gamma)) = \frac{1}{3}$$

The ranking function of intuitionistic fuzzy number $\tilde{a}^I$ is defined by

$R(\tilde{a}^I) = \sqrt{\frac{1}{2}([x(\tilde{a}^I_\mu) - y(\tilde{a}^I_\mu)]^2 + [x(\tilde{a}^I_\gamma) - y(\tilde{a}^I_\gamma)]^2)}$which is the Euclidean distance. For any two intuitionistic fuzzy numbers $\tilde{a}^I = \left(\langle a_0, a_*, a^* \rangle,\ \left\langle a'_0, a'_*, a'^* \right\rangle\right)$ and $\tilde{b}^I = \left(\langle b_0, b_*, b^* \rangle,\ \left\langle b'_0, b'_*, b'^* \right\rangle\right)$ in G(R), we define the ranking of $\tilde{a}^I$ and $\tilde{b}^I$ by comparing the and on R as follows:

(i) If $R(\tilde{a}^I) < R(\tilde{b}^I)$ then $\tilde{a}^I \prec \tilde{b}^I$.

(ii) If $R(\tilde{a}^I) > R(\tilde{b}^I)$ then $\tilde{a}^I \succ \tilde{b}^I$.

(iii) If $R(\tilde{a}^I) = R(\tilde{b}^I)$ then $\tilde{a}^I \approx \tilde{b}^I$.

3 Intuitionistic Fuzzy Critical Path Analysis

The criticality of each activity in a project network is analyzed through the total float values of each activity that are calculated from the difference of the latest finish and earliest finish of the activity or difference of the latest start and earliest start of the activity. The earliest start, earliest finish, latest start and latest finish of each activity are estimated from forward pass and backward pass calculations. The amount of total float values allows the delay in starting that activity without increasing the overall project completion time. The critical path of the project network is the longest path connecting the initial node to the terminal node and consists of only critical activities with the least total float value.

The model $S = \left\langle \tilde{E}, \tilde{A}^I, \tilde{T}^I \right\rangle$ represents intuitionistic fuzzy project network which is an acyclic diagraph containing set of nodes (event) $\tilde{E} = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, ..., \tilde{e}_n\}$ and set of directed edges (activities) $\tilde{A}^I \subset (\tilde{E} \times \tilde{E})$ to be executed in a project network. Each activity $\tilde{a}^I = \{(\tilde{e}_i, \tilde{e}_j)/\tilde{e}_i, \tilde{e}_j \in \tilde{E}\}$ with tail event $\tilde{e}_i$ and head $\tilde{e}_j$ presented as intuitionistic fuzzy number $\tilde{T}^I_{ij} \in \tilde{T}^I$ is the intuitionistic fuzzy time required for the completion of $\tilde{a}^I_{ij}$.

Property 1. Activity $\tilde{a}^I_{ij}$ is said to be predecessor activity of activity $\tilde{a}^I_{pk}$ if and only if there exists a chain between the event j and event p.

Property 2. Activity $\tilde{a}^I_{ij}$ is said to be immediate predecessor of activity $\tilde{a}^I_{pk}$ if and only if either $j = p$ or there exists a chain between the event j and event p in the project network consisting of dummy activities only.

Property 3. For the initial node $\tilde{e}_i$ of the intuitionistic fuzzy project network, set $\tilde{E}S^I_1 = \tilde{L}F^I_1 = ((0,0,0),\ (0,0,0))$. Then the earliest occurrence of the node $\tilde{e}_j$ is

$$\tilde{E}S^I_j = \max(\tilde{E}S^I_i + \tilde{T}^I_{ij}),\ \text{for } j \neq 1,\ j = 2, ..., n$$

Where i is the number of preceding nodes. This is called Forward pass calculation.

Property 4. For the terminal node $\tilde{e}_n$ of the intuitionistic fuzzy project network, set $\tilde{L}F^I_n = \tilde{E}S^I_n$. Then the latest occurrence of the node $\tilde{e}_n$ is

$$\tilde{L}F^I_i = \min(\tilde{L}F^I_j - \tilde{T}^I_{ij});\quad i \neq n.$$

Where j is the number of succeeding nodes. This is called Backward pass calculation.

Property 5. The duration of any path p_k from the first event to terminal event in a network is calculated as

$IFD(p_k) = \sum_{1 \leq i < j \leq n} \tilde{T}^I_{ij}$ such that $\tilde{a}^I_{ij} \in p_k$ and $p_k \in P$.

Definition 3.1: From all possible paths, assume that there exists a path p_c in a project network such that $IFCPM(p_c) = \max\{IFPD(p_i);\ p_i \in P\}$ where $i = 1$ to n then the path p_c is an intuitionistic fuzzy critical path.

Theorem 3.1: Assume that the intuitionistic fuzzy activity duration of all activities in a project network are intuitionistic fuzzy numbers. Then there exists a intuitionistic fuzzy critical path in the project network.

Proof : Let $\tilde{E}S^I_1 = (\langle 0,0,0\rangle, \langle 0,0,0\rangle)$, for $i = 1$ be the starting time of the project network. The earliest intuitionistic fuzzy starting time of the event $\tilde{e}_j$ is the sum of $tildeES^I_i$ and $\tilde{T}^I_{ij}$, then there must uniquely exist a maximal $\tilde{E}S^I_j$ such that

$$\tilde{E}S^I_j = \max(\tilde{E}S^I_i + \tilde{T}^I_{ij}),\ \text{for } j \neq 1,\ j = 2, ..., n.$$

Furthermore let $\tilde{L}F^I_n = \tilde{E}S^I_n$ indicate the latest and earliest start of the last event of the project are same. Then $\tilde{L}F^I_i$ of the ith event can be calculated by subtracting $\tilde{T}^I_{ij}$ from $\tilde{L}F^I_j$ where $i \neq n$. Similarly there uniquely exists a minimal $\tilde{L}F^I_i$ such that $\tilde{L}F^I_i = \min(\tilde{L}F^I_j - \tilde{T}^I_{ij});\ \ i \neq n$ and $\tilde{T}^I_{ij}$ also true for any path $p_k \in P$ which is unique and atleast there exists a path P_c such that $IFCPM(p_c) = \max\{IFPD(p_i);\ p_i \in P\}$ where $i = 1$ to n.

4 Proposed Algorithm

An intuitionistic fuzzy version of the critical path method has been introduced to identify the critical activities of a project network whose activity time durations are interpreted as intuitionistic fuzzy numbers. The proposed arithmetic operation and ranking function are used to handle these intuitionistic fuzzy numbers.

Step 1: Construct fuzzy project network having intuitionistic fuzzy numbers as a activity duration and numbering the events using Ford and Fulkerson's rule.

Step 2: Represent each given intuitionstic fuzzy activity duration into parametric form.

Step 3: Assume that intuitionistic fuzzy earliest starting time of initial event is zero, i.e. $\tilde{E}S^I_i = (\langle 0,0,0\rangle, \langle 0,0,0\rangle),\ \text{for } i = 1$.

Step 4: Compute earliest start for each event using

$$\tilde{E}S^I_j = \max(\tilde{E}S^I_i + \tilde{T}^I_{ij}),\ \text{for } j \neq 1,\ j = 2, ..., n \quad (1)$$

Step 5: Intuitionistic fuzzy earliest finish time for each activity $\tilde{a}_{ij}$ is

$$\tilde{E}F^I_{ij} = (\tilde{E}S^I_i + \tilde{T}^I_{ij}) \quad (2)$$

Step 6: Assume $\tilde{L}F^I_n = \tilde{E}S^I_n$, since intuitionistic fuzzy latest finish of all the end activities are taken as the earliest completion duration of the project network.

$$\tilde{L}F^I_i = \min(\tilde{L}F^I_j - \tilde{T}^I_{ij});\quad i \neq n. \quad (3)$$

Step 7: Intuitionistic fuzzy latest start of each activity $\tilde{a}_{ij}$ is

$$\tilde{L}S^I_{ij} = (\tilde{L}F^I_j - \tilde{T}^I_{ij}). \quad (4)$$

Step 8: Calculate total float

$$\tilde{L}F^I_i = \min(\tilde{L}F^I_j - (\tilde{E}S^I_i + \tilde{T}^I_{ij})). \quad (5)$$

Step 9: The intuitionistic fuzzy activity $\tilde{a}_{ij}$ is said to be a critical activity if and only if its total float $\tilde{T}F^I_{ij} = 0$. Critical path is the longest path from initial event to terminal event in project network having maximum duration. All activities in a critical path are called critical activities.

5 Numerical Examples

In this section, to demonstrate the efficacy of the proposed intuitionistic fuzzy critical path algorithm, two examples discussed by Jayagowri and Geetha Ramani [11] and Sudha and Sophia porchelvi [14] are considered, the length of which is interpreted as trapezoidal intuitionistic fuzzy numbers and a specific case of triangular intuitionistic fuzzy numbers.

Example 5.1: Let us consider a problem discussed by Jayagowri and Geetharamani [11]. Here Table 1 represents the activities involving inspection and replacement of the repaired parts of the boiler. These activity durations are in months and interpreted as trapezoidal intuitionistic fuzzy numbers. The network representation of the intuitionistic fuzzy project network is shown in Fig. 1.

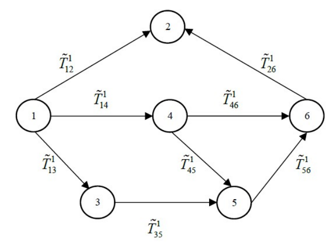

Figure 1. Intuitionistic Fuzzy Project Network

The tabel 2 represents the parametric form of the trapezoidal intuitionistic fuzzy numbers and it can be defined as

$$\tilde{a}^I = ((a, b, c, d),\ (a', b, c', d')) = \left(\langle a_0, a_*, a^*\rangle, \left\langle a'_0, a'_*, a'^*\right\rangle\right)$$

Table 1. Intuitionistic Fuzzy Project Network.

Activity	Description	Activity Duration
$1-2$	Inspection of boiler by boiler engineer and preparation of list of parts to be replaced	$((12,14,16,18),$ $(10,15,16,18))$
$1-3$	Placing the order and purchasing	$((4,10,14,18),$ $(6,10,12,14))$
$1-2$	Dismantling of the defective parts from the boiler	$((8,16,20,24),$ $(9,13,15,17))$
$2-6$	Preparation of Necessary instruction for repairs	$((5,10,15,20),$ $(12,14,15,17))$
$3-5$	Repairing parts in the workshop the boiler	$((12,13,15,18),$ $(16,18,20,22))$
$4-5$	Installation of the repaired parts	$((6,10,16,24),$ $(1,2,5,7))$
$4-6$	Inspection	$((13,15,17,20),$ $(11,15,18,19))$
$5-6$	Trail run	$((5,7,9,15),$ $(6,10,12,14))$

Table 2. Trapezoidal Intuitionistic Fuzzy Arc Length of Each Activity is in the Form $\left(\langle a_0, a_*, a^*\rangle\,,\ \left\langle a'_0, a'_*, a'^*\right\rangle\right)$

Activity	Activity Duration $\tilde{T}^I_{ij}$
1-2	$(\langle 15, 3-2r, 3-2r\rangle\,, \langle 15.5, 0.5-5r^*, 0.5-2r^*\rangle)$
1-3	$(\langle 12, 8-6r, 6-4r\rangle\,, \langle 11, 1+4r^*, 1+2r^*\rangle)$
1-4	$(\langle 18, 10-8r, 6-4r\rangle\,, \langle 14, 1+4r^*, 1+2r^*\rangle)$
2-6	$(\langle 12.5, 7.5-5r, 7.5-5r\rangle\,, \langle 14.5, 0.5+2r^*, 0.5+2r^*\rangle)$
3-5	$(\langle 14, 2-r, 4-3r\rangle\,, \langle 19, 1+2r^*,\ 1+2r^*\rangle)$
4-5	$(\langle 13, 7-4r, 11-8r\rangle\,, \langle 3.5, 1.5{+}r^*,\ 1.5+2r^*\rangle)$
4-6	$(\langle 16, 3-2r, 4-3r\rangle\,, \langle 16.5, 1.5+4r^*,\ 1.5{+}r^*\rangle)$
5-6	$(\langle 8, 3-2r, 7-6r\rangle\,, \langle 10, 4r^*,\ 2+2r^*\rangle)$

From table 3, $1 \to 4, 4 \to 5, 5 \to 6$ are critical activities that imply $1 \to 4 \to 5 \to 6$ is an intuitionistic fuzzy critical path that has been obtained. The length of the intuitionistic fuzzy critical path is the minimum completion period of the intuitionistic fuzzy network of the project. The intuitionistic fuzzy critical path length is, therefore

$$\begin{aligned} &1 \to 4 \to 5 \to 6 \\ &= (\langle 18, 10-8r, 6-4r\rangle\,, \\ &\qquad \langle 14, 1+4r^*, 1+2r^*\rangle) \\ &\quad +(\langle 13, 7-4r, 11-8r\rangle\,, \\ &\qquad \langle 3.5, 1.5+r^*, 1.5+2r^*\rangle) \\ &\quad +(\langle 8, 3-2r, 7-6r\rangle\,, \\ &\qquad \langle 10, 4r^*, 2+2r*\rangle) \\ &= (\langle 39, 10-8r, 11-8r\rangle\,, \\ &\qquad \langle 27.5, 1+4r^*, 2+2r^*\rangle). \end{aligned}$$

Hence the minimum completion duration of the intuitionistic fuzzy project duration is ((29,37,42,50),(22.5,26.5,29.5,31.5)) intuitionistic fuzzy months and ((19,33,45,63);(16,25,32,38)) is the Jayagowri and Geetha Ramani's minimum completion duration of the project network for this same problem.

Example 5.2: We have considered the intuitionistic fuzzy project network as shown in fig 2 having set of events $\tilde{E} = (1,2,3,4,5,6,7,8)$ discussed by Sophia Porchelvi and Sudha [14]. Here a particular case of triangular intuitionistic fuzzy numbers are used to denote activity durations and are given in table 4 also table 5 represents the parametric form of the triangular intuitionistic fuzzy numbers.

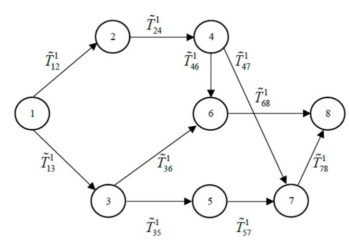

Figure 2. Intuitionistic Fuzzy Project Network

From table 6, $1 \to 2, 2 \to 4, 4 \to 7, 7 \to 8$ are critical activities that imply $1 \to 2 \to 4 \to 7 \to 8$ is an intuitionistic fuzzy critical path that has been obtained. The length of the intuitionistic fuzzy critical path is the minimum completion period of the intuitionistic fuzzy network of the project. The intuitionistic fuzzy critical

Table 3. Computation of Total Float for Each Activity in Intuitionistic Fuzzy Project Network

Activity	Activity Duration	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Total Float
$1-2$	$(\langle 15, 3-2r, 3-2r\rangle, \langle 15.5, 0.5+5r*, 0.5+2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$	$(\langle 15, 3-2r, 3-2r\rangle, \langle 15.5, 0.5+5r*, 0.5+2r*\rangle)$	$(\langle 26.5, 10-8r, 11-8r\rangle, \langle 13, 1+4r*, 2+2r*\rangle)$	$(\langle 11.5, 10-8r, 11-8r\rangle, \langle -2.5, 0.5+5r*, 2+2r*\rangle)$	$(\langle 11.5, 10-8r, 11-8r\rangle, \langle -2.5, 0.5+5r*, 2+2r*\rangle)$
$1-3$	$(\langle 12, 8-6r, 6-4r\rangle, \langle 11, 1+4r*, 1+2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$	$(\langle 12, 8-6r, 6-4r\rangle, \langle 11, 1+4r*, 1+2r*\rangle)$	$(\langle 17, 10-8r, 11-8r\rangle, \langle -1.5, 1+4r*, 2+2r*\rangle)$	$(\langle 5, 10-8r, 11-8r\rangle, \langle -12.5, 1+4r*, 2+2r*\rangle)$	$(\langle 5, 10-8r, 11-8r\rangle, \langle -12.5, 1+4r*, 2+2r*\rangle)$
$1-4$	$(\langle 18, 10-8r, 6-4r\rangle, \langle 14, 1+4r*, 1+2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$	$(\langle 18, 10-8r, 6-4r\rangle, \langle 14, 1+4r*, 1+2r*\rangle)$	$(\langle 18, 10-8r, 11-8r\rangle, \langle 14, 1+4r*, 2+2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$
$2-6$	$(\langle 12.5, 7.5-5r, 7.5-5r\rangle, \langle 14.5, 0.5+2r*, 0.5+2r*\rangle)$	$(\langle 15, 3-2r, 3-2r\rangle, \langle 15.5, 0.5+5r*, 0.5+2r*\rangle)$	$(\langle 27.5, 7.5-5r, 7.5-5r\rangle, \langle 30, 0.5+5r*, 0.5+2r*\rangle)$	$(\langle 39, 10-8r, 11-8r\rangle, \langle 27.5, 1+4r*, 2+2r*\rangle)$	$(\langle 26.5, 10-8r, 11-8r\rangle, \langle 13, 1+4r*, 2+2r*\rangle)$	$(\langle 11.5, 10-8r, 11-8r\rangle, \langle -2.5, 0.5+5r*, 2+2r*\rangle)$
$3-5$	$(\langle 14, 2-r, 4-3r\rangle, \langle 19.5, 1+2r*, 1+2r*\rangle)$	$(\langle 12, 8-6r, 6-4r\rangle, \langle 11, 1+4r*, 1+2r*\rangle)$	$(\langle 26, 8-6r, 6-4r\rangle, \langle 30, 1+4r*, 1+2r*\rangle)$	$(\langle 31, 10-8r, 11-8r\rangle, \langle -17.5, 1+4r*, 2+2r*\rangle)$	$(\langle 17, 10-8r, 11-8r\rangle, \langle -1.5, 1+4r*, 2+2r*\rangle)$	$(\langle 5, 10-8r, 11-8r\rangle, \langle -12.5, 1+4r*, 2+2r*\rangle)$
$4-5$	$(\langle 13, 7-4r, 11-8r\rangle, \langle 3.5, 1.5+r*, 1.5+2r*\rangle)$	$(\langle 18, 10-8r, 6-4r\rangle, \langle 14, 1+4r*, 1+2r*\rangle)$	$(\langle 31, 10-8r, 11-8r\rangle, \langle -17.5, 1+4r*, 2+2r*\rangle)$	$(\langle 31, 10-8r, 11-8r\rangle, \langle -17.5, 1+4r*, 2+2r*\rangle)$	$(\langle 18, 10-8r, 11-8r\rangle, \langle 14, 1+4r*, 2+2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$
$4-6$	$(\langle 16, 3-2r, 4-2r\rangle, \langle 16.5, 1.5+4r*, 1.5+r*\rangle)$	$(\langle 18, 10-8r, 6-4r\rangle, \langle 14, 1+4r*, 1+2r*\rangle)$	$(\langle 34, 10-8r, 6-4r\rangle, \langle 30.5, 1.5+4r*, 1+2r*\rangle)$	$(\langle 39, 10-8r, 11-8r\rangle, \langle 27.5, 1+4r*, 2+2r*\rangle)$	$(\langle 23, 10-8r, 11-8r\rangle, \langle 11, 1.5+4r*, 2+2r*\rangle)$	$(\langle 5, 10-8r, 11-8r\rangle, \langle -3, 1.5+4r*, 2+2r*\rangle)$
$5-6$	$(\langle 8, 3-2r, 7-6r\rangle, \langle 10, 4r*, 2+2r*\rangle)$	$(\langle 31, 10-8r, 11-8r\rangle, \langle -17.5, 1+4r*, 2+2r*\rangle)$	$(\langle 39, 10-8r, 11-8r\rangle, \langle 27.5, 1+4r*, 2+2r*\rangle)$	$(\langle 39, 10-8r, 11-8r\rangle, \langle 27.5, 1+4r*, 2+2r*\rangle)$	$(\langle 31, 10-8r, 11-8r\rangle, \langle -17.5, 1+4r*, 2+2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$

Table 4. Triangular Intuitionistic Fuzzy Activity Duration of Each Activity

Activity	Activity Duration
1-2	$((2,3,4),(5,6,7))$
1-3	$((1,3,4),(4,5,7))$
2-4	$((1,3,5),(4,6,7))$
3-5	$((1,2,3),(3,4,5))$
3-6	$((2,5,7),(6,8,9))$
4-6	$((3,4,6),(5,7,8))$
4-7	$((3,4,5),(6,7,8))$
5-7	$((1,4,5),(5,6,7))$
6-8	$((2,5,6),(7,8,9))$
7-8	$((3,6,8),(7,9,11))$

Table 5. Triangular Intuitionistic Fuzzy Arc Length of Each Activity is in the Form $\left(\langle a_0, a_*, a^*\rangle, \left\langle a_0', a_*', a'^*\right\rangle\right)$

Activity	Activity Duration
1-2	$(\langle 3, 1-r, 1-r\rangle, \langle 6, r^*, r^*\rangle)$
1-3	$(\langle 3, 2-2r, 1-r\rangle, \langle 5, r^*, r^*\rangle)$
2-4	$(\langle 3, 2-2r, 2-2r\rangle, \langle 6, 2r^*, r^*\rangle)$
3-5	$(\langle 2, 1-r, 1-r\rangle, \langle 4, r^*, r^*\rangle)$
3-6	$(\langle 5, 3-3r, 2-2r\rangle, \langle 8, 2r^*, r^*\rangle)$
4-6	$(\langle 4, 1-r, 2-2r\rangle, \langle 7, 2r^*, r^*\rangle)$
4-7	$(\langle 4, 1-r, 1-r\rangle, \langle 7, r^*, r^*\rangle)$
5-7	$(\langle 4, 3-3r, 1-r\rangle, \langle 6, r^*, r^*\rangle)$
6-8	$(\langle 5, 3-3r, 1-r\rangle, \langle 8, r^*, r^*\rangle)$
7-8	$(\langle 6, 3-3r, 2-2r\rangle, \langle 9, 2r^*, 2r^*\rangle)$

path length is, therefore

$$\begin{aligned} &= 1 \to 2 \to 4 \to 7 \to 8 \\ &= (\langle 3, 1-r, 1-r\rangle, \langle 6, r*, r*\rangle) \\ &\quad +(\langle 3, 2-2r, 2-2r\rangle, \langle 6, 2r*, r*\rangle) \\ &\quad +(\langle 4, 1-r, 1-r\rangle, \langle 7, r*, r*\rangle) \\ &\quad +(\langle 6, 3-3r, 2-2r\rangle, \langle 9, 2r*, 2r*\rangle) \\ &= (\langle 16, 3-3r, 2-2r\rangle, \langle 28, 2r*, 2r*\rangle). \end{aligned}$$

Hence the minimum completion duration of the intuitionistic fuzzy project duration is $((13,16,18),(26,28,30))$ and ((9,16,22),(22,28,33)) is the Porchelvi and Sudha's minimum completion duration of the project network for this same problem.

6 Discussion

In real-world situations, uncertainty is unavoidable in the scheduling process. This uncertainty may occur from various causes such as unavailability of resources, activity delays due to bad weather conditions, materials may not be delivered as planned, some activities may not be able to finish on expected time as it may take more or less time than planned, new activities might be included or planned activities might be dropped based on scope of the project, scheduled dates might be changed due to variation of management scenario, etc. These factors make it more difficult for decision-makers to

Table 6. Computation of Total Float for Each Activity in Intuitionistic Fuzzy Project Network

Activity	Activity Duration	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Total Float
$1-2$	$(\langle 3,1-r,1-r\rangle, \langle 6,r*,r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$	$(\langle 3,1-r,1-r\rangle, \langle 6,r*,r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$	$(\langle 3,3-r,3-r\rangle, \langle 6,2r*,2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$
$1-3$	$(\langle 3,2-2r,1-r\rangle, \langle 5,r*,r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$	$(\langle 3,2-2r,1-r\rangle, \langle 5,r*,r*\rangle)$	$(\langle 1,3-2r,2-2r\rangle, \langle 4,2r*,2r*\rangle)$	$(\langle 4,3-3r,2-2r\rangle, \langle 9,2r*,2r*\rangle)$	$(\langle 1,3-3r,2-2r\rangle, \langle 4,2r*,2r*\rangle)$
$2-4$	$(\langle 3,2-2r,2-2r\rangle, \langle 6,2r*,r*\rangle)$	$(\langle 3,1-1r,1-1r\rangle, \langle 6,2r*,r*\rangle)$	$(\langle 6,2-2r,2-2r\rangle, \langle 12,2r*,r*\rangle)$	$(\langle 3,3-3r,2-2r\rangle, \langle 6,2r*,2r*\rangle)$	$(\langle 6,3-3r,2-2r\rangle, \langle 12,2r*,2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$
$3-5$	$(\langle 2,1-r,2-2r\rangle, \langle 6,2r*,r*\rangle)$	$(\langle 3,2-2r,1-r\rangle, \langle 5,r*,r*\rangle)$	$(\langle 5,2-2r,1-r\rangle, \langle 9,r*,r*\rangle)$	$(\langle 4,3-3r,2-2r\rangle, \langle 9,2r*,2r*\rangle)$	$(\langle 6,3-3r,2-2r\rangle, \langle 13,2r*,2r*\rangle)$	$(\langle 1,3-3r,2-2r\rangle, \langle 4,2r*,2r*\rangle)$
$3-6$	$(\langle 5,3-3r,2-2r\rangle, \langle 8,2r*,r*\rangle)$	$(\langle 3,2-2r,1-r\rangle, \langle 5,r*,r*\rangle)$	$(\langle 8,3-3r,2-2r\rangle, \langle 13,2r*,r*\rangle)$	$(\langle 6,3-3r,2-2r\rangle, \langle 12,2r*,2r*\rangle)$	$(\langle 11,3-3r,2-2r\rangle, \langle 20,2r*,2r*\rangle)$	$(\langle 3,3-3r,2-2r\rangle, \langle 7,2r*,2r*\rangle)$
$4-6$	$(\langle 4,1-r,2-2r\rangle, \langle 7,2r*,r*\rangle)$	$(\langle 6,2-2r,2-2r\rangle, \langle 12,2r*,r*\rangle)$	$(\langle 10,3-3r,2-2r\rangle, \langle 19,2r*,2r*\rangle)$	$(\langle 7,3-3r,2-2r\rangle, \langle 13,2r*,2r*\rangle)$	$(\langle 11,3-3r,2-2r\rangle, \langle 20,2r*,2r*\rangle)$	$(\langle 1,3-3r,2-2r\rangle, \langle 1,2r*,2r*\rangle)$
$4-7$	$(\langle 4,1-r,2-2r\rangle, \langle 7,2r*,r*\rangle)$	$(\langle 6,2-2r,2-2r\rangle, \langle 12,2r*,r*\rangle)$	$(\langle 10,3-3r,2-2r\rangle, \langle 19,2r*,2r*\rangle)$	$(\langle 6,3-3r,2-2r\rangle, \langle 12,2r*,2r*\rangle)$	$(\langle 10,3-3r,2-2r\rangle, \langle 19,2r*,2r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$
$5-7$	$(\langle 4,3-3r,1-r\rangle, \langle 6,r*,r*\rangle)$	$(\langle 5,2-2r,1-r\rangle, \langle 9,r*,r*\rangle)$	$(\langle 9,3-3r,1-r\rangle, \langle 15,r*,r*\rangle)$	$(\langle 6,3-3r,2-2r\rangle, \langle 13,2r*,2r*\rangle)$	$(\langle 10,3-3r,2-2r\rangle, \langle 19,2r*,2r*\rangle)$	$(\langle 1,3-3r,2-2r\rangle, \langle 4,2r*,2r*\rangle)$
$6-8$	$(\langle 5,3-3r,1-r\rangle, \langle 8,r*,r*\rangle)$	$(\langle 10,2-2r,2-2r\rangle, \langle 19,2r*,r*\rangle)$	$(\langle 15,3-3r,2-2r\rangle, \langle 27,2r*,r*\rangle)$	$(\langle 11,3-3r,2-2r\rangle, \langle 4,2r*,2r*\rangle)$	$(\langle 16,3-3r,2-2r\rangle, \langle 28,2r*,8r*\rangle)$	$(\langle 1,3-3r,2-2r\rangle, \langle 4,2r*,2r*\rangle)$
$7-8$	$(\langle 6,3-3r,2-2r\rangle, \langle 9,2r*,2r*\rangle)$	$(\langle 10,2-2r,2-2r\rangle, \langle 19,2r*,r*\rangle)$	$(\langle 16,3-3r,2-2r\rangle, \langle 28,2r*,8r*\rangle)$	$(\langle 10,3-3r,2-2r\rangle, \langle 3,2r*,2r*\rangle)$	$(\langle 16,3-3r,2-2r\rangle, \langle 28,2r*,8r*\rangle)$	$(\langle 0,0,0\rangle, \langle 0,0,0\rangle)$

work as they could not obtain accurate estimates of the time for the project activities. Especially when the project has to be done for the first time, it pushes us to seek expert's opinions which ultimately makes them use the term "approximately" or "more or less" or "about".

A classical set or crisp set which is deterministic in nature and always defines the membership function as unity is not suitable to model the terms exhibiting uncertainty or vagueness of the variables that naturally lead to a range of possible values. As an extension of the crisp set, in 1965 the fuzzy set theory has been introduced by Zadeh [1] to model uncertainty or imprecise parameters. However, it tends to be less effective to model imprecise parameters as it concentrates only on the degree of belongingness. This leads to the development of an intuitionistic fuzzy set [2] that considers both degrees of belonging and non-belonging. All the time the degree of belonging or membership function alone is not enough for the decision-makers to adopt the best alternatives whereas the intuitionistic fuzzy set which also concentrates on the degree of non-belonging or non-membership function helps to adopt the best among the worst cases too. This makes intuitionistic fuzzy set as a more powerful tool and helps the decision makers to meet their objectives.

7 Conclusions

Even though the critical path method is simple and effective in the scheduling process when it comes to dealing with ill known quantities which is unavoidable in real-life situations, the proposed intuitionistic fuzzy version of the critical path method helps in handling these ill known quantities more properly. The novelty of the proposed method is checked by comparing the obtained results with the results available in pieces of literature. It is found that the result obtained by the proposed method is more effectual in defining the critical path in an intuitionistic fuzzy sense and also it gives vagueness reduced intuitionistic fuzzy project completion durations. The goal of the proposed method is achieved as it produces vagueness reduced completion duration for the intuitionistic fuzzy project networking problems. The proposed methodology can be extended to picture fuzzy numbers, the picture fuzzy set is a generalization of the fuzzy set and intuitionistic fuzzy set.

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