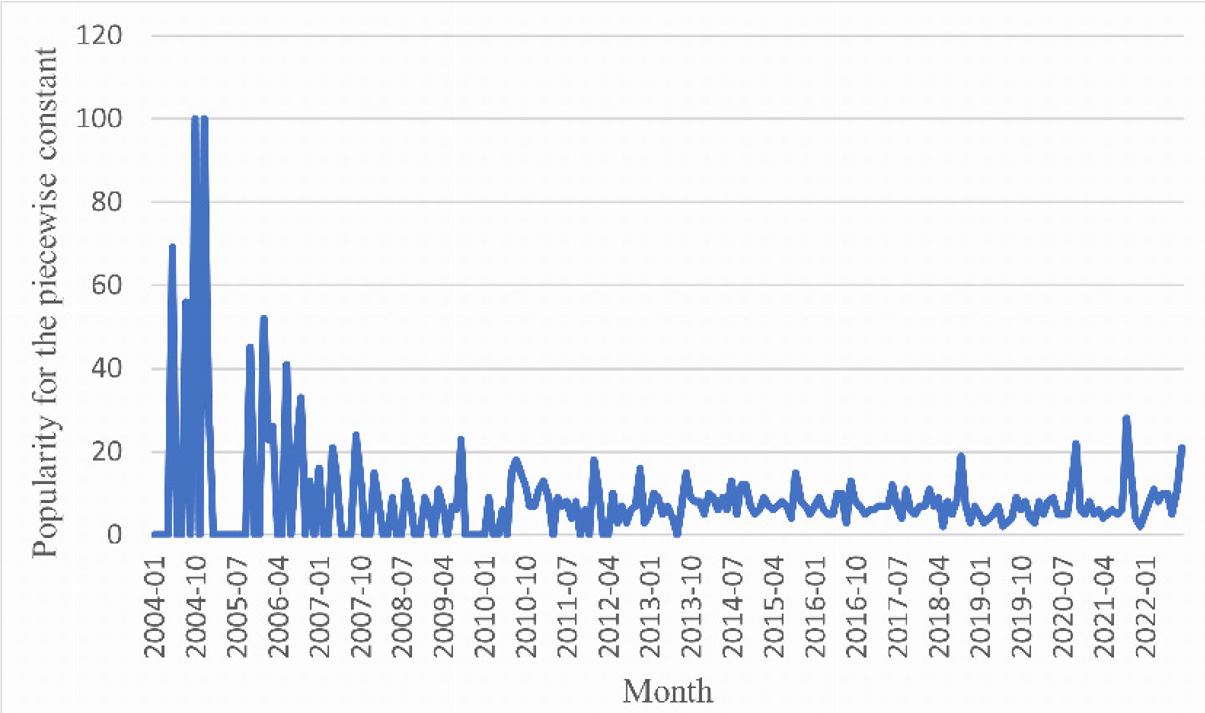

Figure 1. Search results via Google Trends on PC topics (Source: Google trends)

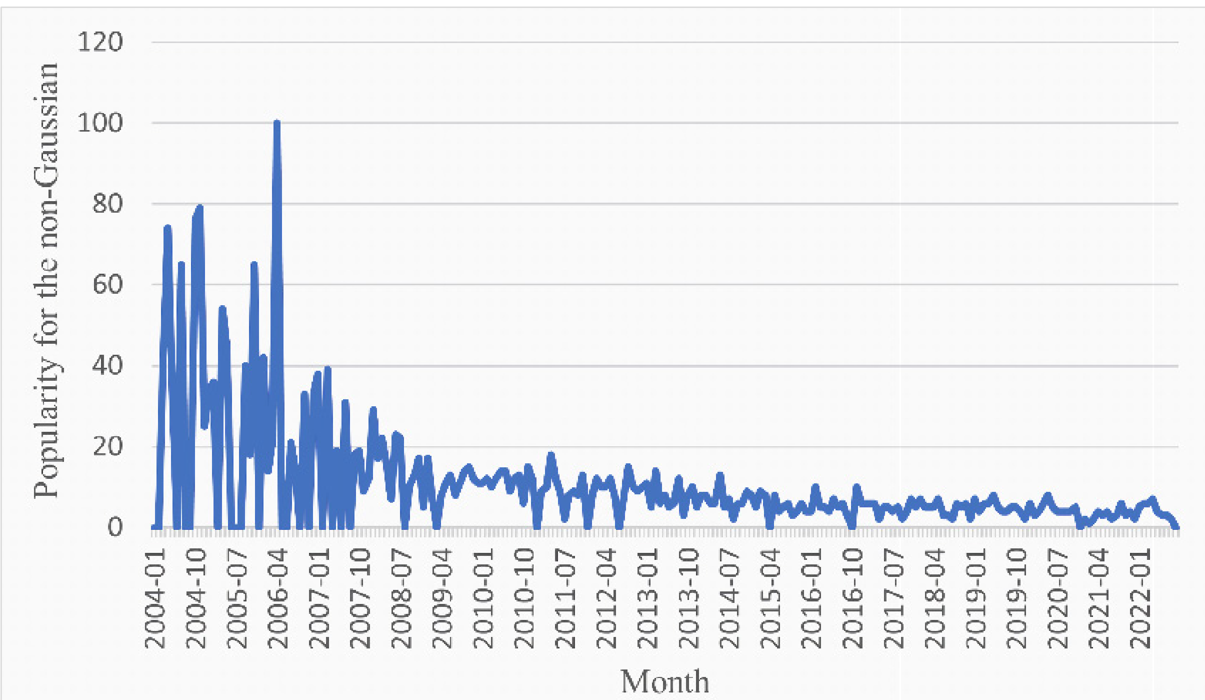

Figure 2. Search results via Google Trends on the topic "non-Gaussian" (Source: Google trends).

Studies related to signal modeling with PC models can be found in various kinds of literature, for example [5-7]. Based on the type of noise, some PC models also use different types of noise, for example, Gaussian [8], Poisson [9], Gamma [10], and Rayleigh [11]. On the other hand, inverse-Gamma (IG) is a distribution studied in some literature, for example [12,13]. In [12], the authors have proposed a mean linear regression model in which the response variable is IG. While in [13], the authors have also used the IG distribution to describe the dynamics of the measurement noise covariance. However, to the author's knowledge, no study has been found on PC models that use IG noise.

1.1. IG Distribution

A random variable w is said to have an IG distribution with parameters $\alpha > 0$ and $\beta > 0$ if the random variable w has a probability function $f(\mathrm{x}|\alpha,\beta)$ as follows [14]:

$$f(\mathrm{w}|\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\mathrm{w}^{-\alpha-1}exp - \frac{\beta}{\mathrm{w}} \tag{1}$$

for w> 0 and is 0 for other w values. In (1), if the random variable w has an IG distribution, then z = 1/w has a Gamma distribution. Inverse Gamma has been used by researchers, for example [15]. In [15], inverse Gamma was used to reduce the effect of shadows on composite statistics.

1.2. PC Model

Let $x_1, \dots, x_n$ represent data where n is the number of observations. This data is said to have a PC model with inverse-gamma multiplicative noise if for $t = 1, \dots, n.$ This data satisfies the following stochastic equation [16]:

$$x_t = r_t w_t \tag{2}$$

In (2), $\boldsymbol{r_t}$ is a step function and is defined as follows:

$$r_t = \begin{cases} a_1, \tau_1 < t \leq \tau_2 \\ a_2, \tau_2 < t \leq \tau_3 \\ \dots \\ a_{k+1}, \tau_{k+1} < t \leq \tau_{k+2} \end{cases} \tag{3}$$

In (3), $\tau_1 = 1$ and $\tau_{k+2} = n$. In this article, w is a multiplicative noise with IG distribution with parameters $\alpha > 0$ and $\beta > 0$.

In this article, the number of change-points is assumed to be unknown. Thus, the PC model can be used to model a wider range of data even though computational techniques in parameter estimation become more difficult. This is because the parameter dimensions are a combination of different dimensional spaces. To overcome this problem, the reversible jump Markov Chain Monte Carlo (RJMCMC) algorithm [17] was adopted to estimate the model parameters. This RJMCMC algorithm has been used in segmentation and change-point, for example [18].

This article aims to: (a) propose a PC model in which the noise has an IG distribution and develop a method for estimating the model parameters. In the stochastic model, there are two types of noise, namely additive and multiplicative. This article focuses on multiplicative noise. The novelty of this article lies in the use of IG multiplicative noise in PC models and the development of estimation methods for PC model parameters.

2. Research Significance

Methods for analyzing data are usually based on a specific model. However, this can result in a large bias when the assumed model is wrong [19]. On the other hand, there are no studies on PC models that use Inverse-Gamma noise. The significant research in this article is the development of a new PC model in which the noise has an inverse gamma distribution and the development of the RJMCMC algorithm to estimate the PC model parameters. So that the findings in this article contribute to reducing errors due to the incompatibility of mathematical models in data modeling. A search through Google Trends on the topic "non-Gaussian" from 2004 to 2022 is shown in Figure 2.

Figure 2 shows that interest in the "non-Gaussian" topic is still stable and has not decreased. One of the non-Gaussian noises is IG noise. So the findings in this article also contribute to providing solutions to research gaps.

3. Methodology

Estimating PC model parameters is challenging to investigate, especially in cases where the number of change-points is unknown [11]. If the number of change-points is not known, the PC model parameters include the number of change-points, change-point locations, and signal amplitudes.

There are two approaches to estimating Bayesian and frequentist. In this article, the PC model parameters are estimated using the Bayesian approach. The Bayesian approach has been used in PC problems, for example [20]. The estimation procedure includes determining the probability function, selecting the prior distribution, and calculating the posterior distribution. The prior distribution for the number of change-points is the Binomial distribution.

Bayesian estimators of model parameters cannot be determined easily because the posterior distribution has a complicated shape. To overcome this, the RJMCMC algorithm is used to calculate the Bayes estimator.

The validity of the estimation procedure was tested using synthetic data. In this simulation study, data synthesis was created using (2) in which the parameters were defined. Then, these parameters are estimated using an estimation procedure using synthetic data. The estimation procedure is said to be valid if the parameter estimator is close to the parameter value of the PC model.

4. Results and Discussion

4.1. Model Formulation and Parameter Estimation

In (1), noise w_t can be written as $w_t = \frac{1}{z_t}$ where z_t has Gamma distribution with parameters α and β . Equation (2) can be written as follows (4):

$$\frac{1}{\mathrm{x_t}} = \frac{1}{\mathrm{r_t}}\mathrm{z_t} \tag{4}$$

Suppose that $y_t = \frac{1}{x_t}$, $h_t = \frac{1}{a_t}$, and $m_t = \frac{1}{r_t}$. Equation (2) can be written as follows (5):

$$y_t = m_t z_t \quad (5)$$

where is the step function given by (6):

$$m_t = \begin{cases} h_1, \tau_1 < t \le \tau_2 \\ h_2, \tau_2 < t \le \tau_3 \\ \dots \\ h_{k+1}, \tau_{k+1} < t \le \tau_{k+2} \end{cases} \quad (6)$$

In (6), let $h = (h_1, \dots, h_{k+1})$, $\tau = (\tau_1, \dots \tau_{k+1})$, $y = (y_1, \dots, y_n)$. Here, the value of α is set. Meanwhile, the parameter values of h, τ, and β were estimated using data y. Parameter estimation using Bayesian approach with the help of reversible jump MCMC. The estimation procedure uses the procedure in [10] but needs to be modified. Although the procedure is similar, the content is very different.

4.2. Algorithm Validation

The estimate parameter procedure is validated using a simulation study. This simulation study uses synthetic data. The parameter values of the PC model are presented in Table 1.

Synthetic data is made based on (2) using the parameter values in Table 1. The resulting synthetic data are presented in Figure 3.

Based on this synthetic data, the reversible jump MCMC algorithm is used to find the estimator of the model parameters. The algorithm was run for 100,000 iterations and a burn-in period of 25,000. The order histogram of the PC model is presented in Figure 4.

The superposition between the data and the parameter estimators is presented in Figure 5. The blue color indicates the signal while the red color indicates the parameter estimation of the signal model. The parameter estimation of the PC model is in the form of a step function.

Table 1. Model parameter values

k	τ	h
6	$\begin{pmatrix} 1 \\ 30 \\ 80 \\ 120 \\ 170 \\ 200 \\ 250 \\ 350 \end{pmatrix}$	$\begin{pmatrix} 0.91 \\ 1.25 \\ 0.67 \\ 1.43 \\ 0.67 \\ 2.50 \\ 1.00 \end{pmatrix}$

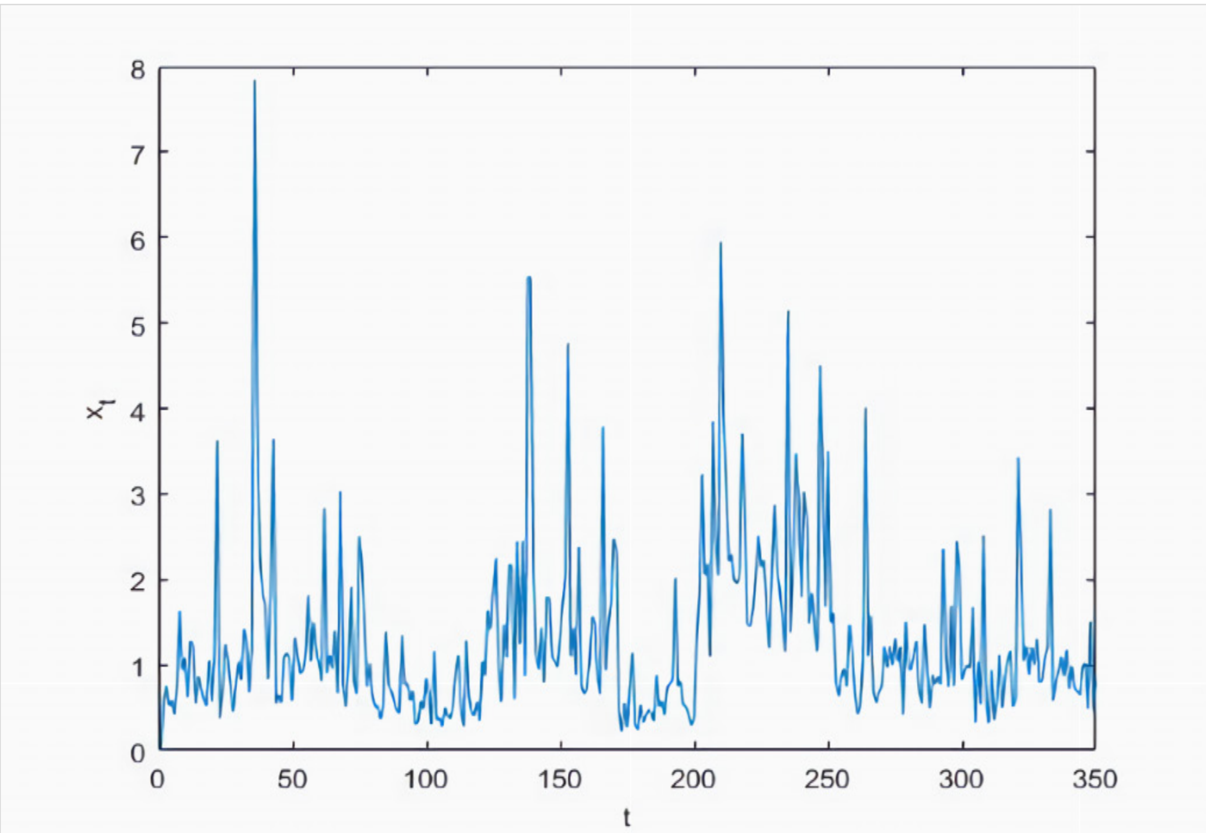

Figure 3. Signal with Inverse-Gamma Noise (k=6).

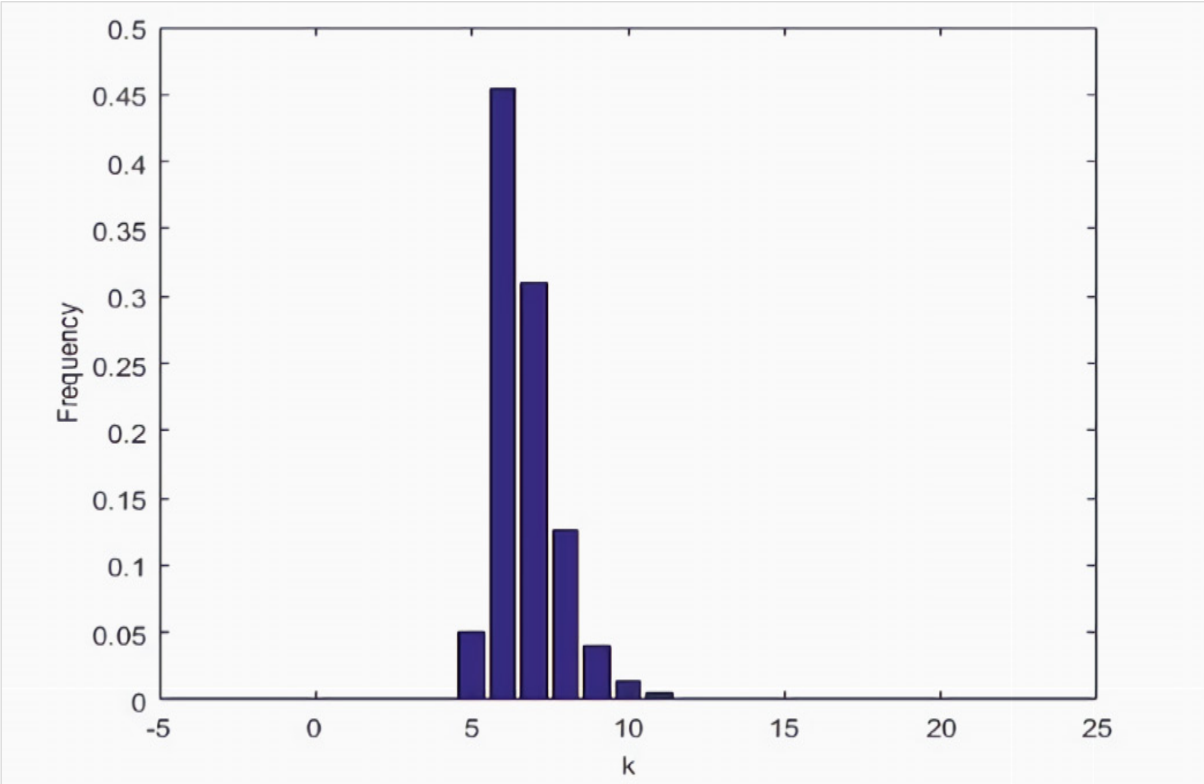

Figure 4. Histogram (k=6)

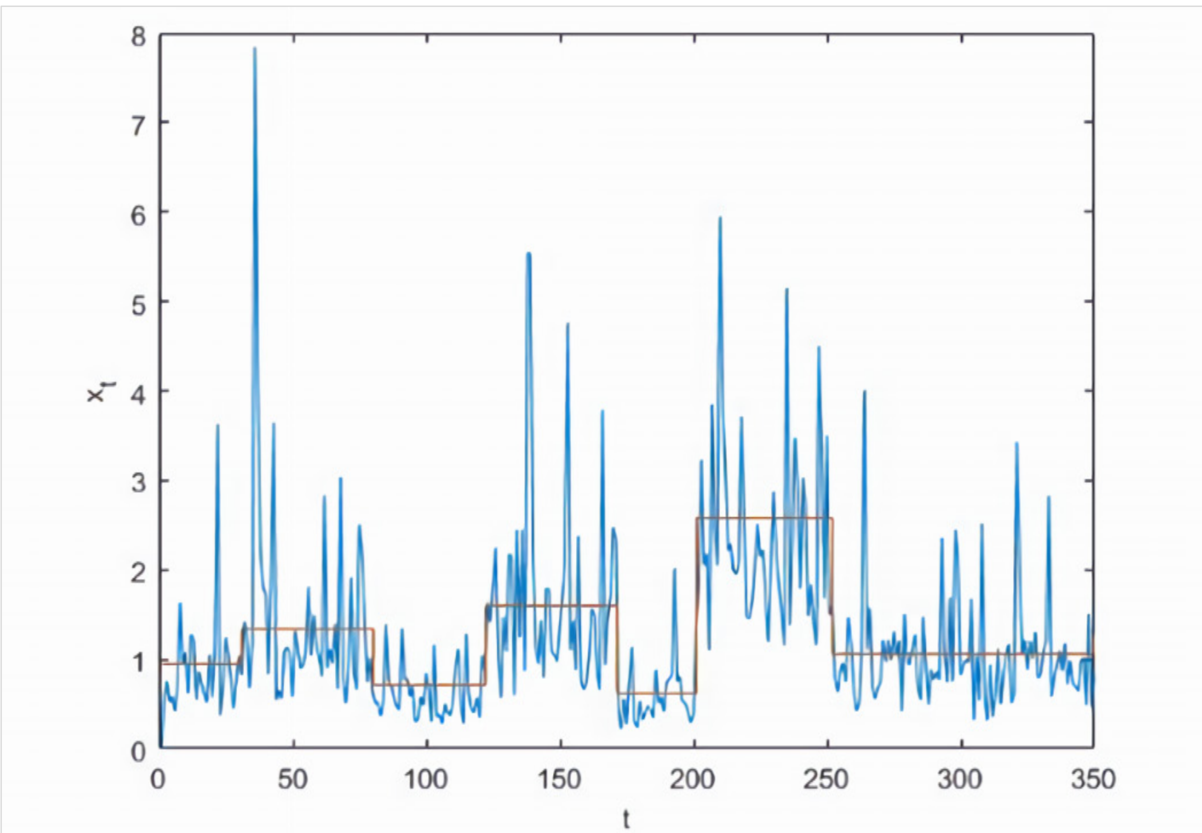

Figure 5. Estimated signal (k=6)

The model parameter estimators are presented in Table 2 below.

Table 2. Model Parameter Estimator

$\hat{k}$	$\hat{\tau}$	$\hat{h}$
6	$\begin{pmatrix}1\\31\\80\\122\\172\\201\\252\\350\end{pmatrix}$	$\begin{pmatrix}0.95\\1.34\\0.71\\1.60\\0.61\\2.58\\1.06\end{pmatrix}$

4.3. Discussion

In Figure 3, without using software, it is not visible how many change-points, where the location of the change points occur, and how high the signal amplitude is between change points. However, using the developed RJMCMC algorithm, model parameters can be determined (Figure 5).

In Figure 4 it can be seen that the maximum change point value occurs at k = 6. This shows that the estimator for the number of change points is 6.

Table 3. Euclidean distance for position

τ	$\hat{\tau}$	$\lvert\tau-\hat{\tau}\rvert$
$\begin{pmatrix}1\\30\\80\\120\\170\\200\\250\\350\end{pmatrix}$	$\begin{pmatrix}1\\31\\80\\122\\172\\201\\252\\350\end{pmatrix}$	3.81

Table 4. Euclidean distance for signal amplitude

h	$\hat{h}$	$\lvert h-\hat{h}\rvert$
$\begin{pmatrix}0.91\\1.25\\0.67\\1.43\\0.67\\2.50\\1.00\end{pmatrix}$	$\begin{pmatrix}0.95\\1.34\\0.71\\1.60\\0.61\\2.58\\1.06\end{pmatrix}$	1.41

The distance between the parameter values and the parameter estimator of the PC model was calculated using the Euclidean distance. The distances are presented in Table 3 and Table 4. This distance calculation is carried out to determine the validity of the RJMCMC algorithm in estimating the parameters of the PC model. The distance between the parameter value and the parameter estimator shows the accuracy of the RJMCMC algorithm. The smaller the distance between the estimator and the parameter value, the better the algorithm's performance. On the other hand, the larger the distance between the estimator and the parameter value, the less good the algorithm's performance will be.

In Table 3 and Table 4, it can be seen that the distance between the parameter estimator and the parameter value is relatively small. This means that the RJMCMC algorithm meets the valid criteria for estimating the parameters of the PC model.

This RJMCMC algorithm has been used in [10]. In [10], the PC model assumes that noise has Gamma distribution, while in [11], the PC model uses Rayleigh distribution as noise. In this article, the noise is assumed to have an Inverse-Gamma distribution. So the findings in this article complement the results of previous research.

This study focuses on the signal amplitude modeled by the constant function. So if the signal amplitude has a model other than a constant function, this algorithm is not suitable to be applied. Therefore, further research can be developed on other signals amplitude models such as linear functions and polynomial functions.

In this study, a PC model with inverse-Gamma noise has been developed and its validity was tested using synthetic data. Further research can be done through the application of real data such as engineering and ecology. Then, the results of applying the algorithm to real data are compared with the existing methods to determine their effectiveness. A method is said to be effective compared to other methods if it gives a smaller error.

There are two types of noise, namely additive noise (for example [21]) and multiplicative noise. In this study, the author has discussed the PC model with multiplicative noise but the PC model with additive noise has not been discussed. The effect of this type of noise is very interesting to study. So that the study of PC models with additive noise can be studied further and then the results of studies on PC models with additive noise are compared with PC models with multiplicative noise in order to know which type of noise is the most suitable for the data.

Synthetic Aperture Radar (SAR) signals have been studied by several researchers until now, for example [22,23]. This new PC model can be applied to model SAR signals. While the estimation procedure using reversible jump MCMC developed in this paper, this procedure can be used to solve problems related to segmentation and point change detection. Thus, the findings in this paper contribute to the academics' improvement.

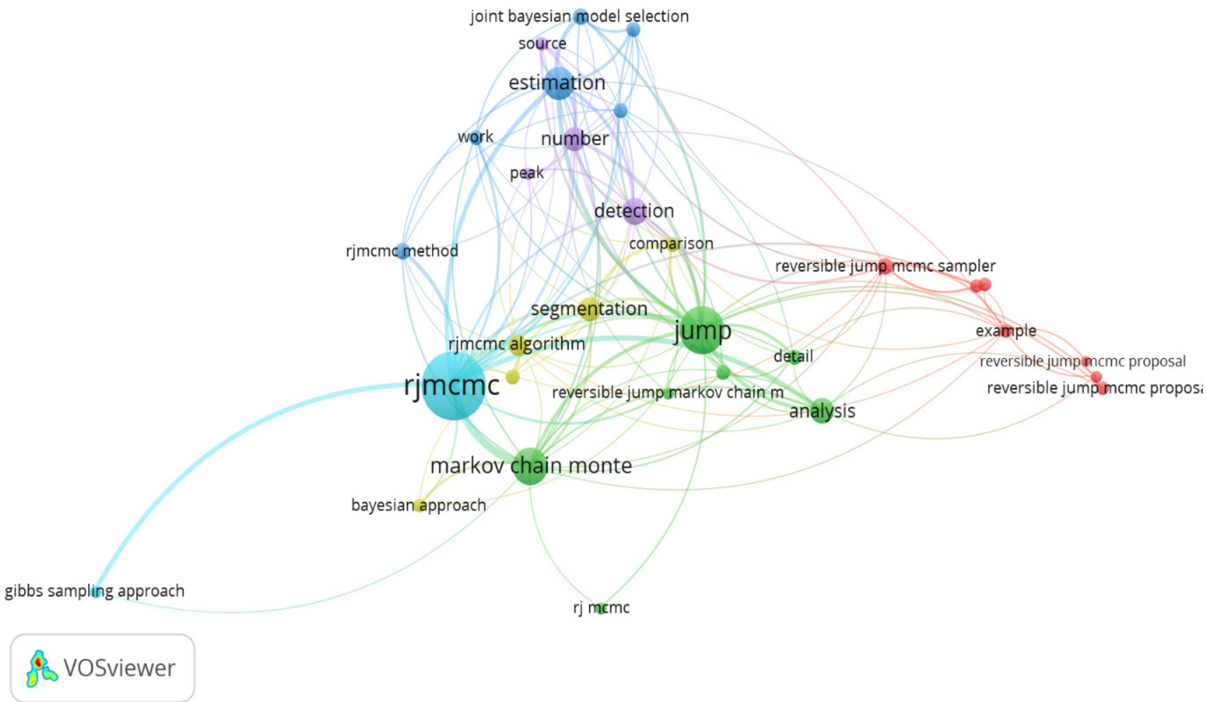

Figure 6. Relationship between RJMCMC and other terms

In this paper, RJMCMC is associated with the terms: signal processing, IG noise, and estimation. To see the relationship between RJMCMC and other terms, this paper also analyzes the relationship between RJMCMC and other terms based on the Google Scholar database from 1994 to 2022. The reference searches that discuss RJMCMC were carried out using Publish or Perish software and resulted in 287 documents. Then these 287 documents were analyzed using Vosviewer software to describe the relationship between RJMCMC and other terms. A map of the relationship between RJMCMC and other terms is presented in Figure 6. The relationship between the two terms is shown by the line connecting the two terms. The two terms that are not directly connected by a line indicate novelty for further investigation.

In the visualization network about the relationship between RJMCMC and other terms, there are 31 items divided into four clusters. The first cluster contains 7 items that discuss convergence and distribution proposal. The second cluster contains 7 items that discuss jump and Markov chain. The third cluster contains 6 items that discuss estimation and Bayesian. The fourth cluster contains 4 items that discuss segmentation. The fifth cluster contains 4 items that discuss detection. While the sixth cluster contains 2 items that discuss the sampling approach.

5. Conclusions

This article proposes a new PC model in which the noise has an inverse gamma distribution and develops the RJMCMC algorithm to estimate the PC model parameters. The developed algorithm has an advantage because this algorithm is able to detect the number of change-points and change-point locations as well as signal values simultaneously. The findings in this article contribute to the development of mathematical models so that errors due to incompatibility of mathematical models are reduced. In this article, the signal value is modeled as a constant function over time between two change-points. Further research can be developed on more complex signal value models, such as linear functions and polynomial functions. In addition, this method is applied to real data and then compared with other existing methods to determine their effectiveness. In terms of noise type, further studies can be compared between PC models with additive noise and multiplicative noise.

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