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Area Measurement of Flat Rectangular Surfaces by Students 10-11 Years Old: Traditional vs Inquiry-Based Teaching Intervention

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Abstract The main purpose of this work is to determine the requirements during the design and implementation of inquiry-based teaching interventions (structured and guided inquiry-based teaching), as well as to record their effect on the perception of the area and its measurement by children aged 10-11 in relation with traditional didactic intervention. Initially, the students of three classes completed pre-test with tasks that aimed to detect their knowledge of the concept of area and its measurement. Then three different didactic interventions were applied (traditional, structured and guided inquiry-based), one in each class. Finally, after the completion of the interventions, the students completed a post-test similar to the pre-test in order to compare their answers and to record the results of each teaching intervention. The type of activities and the degree of guidance by the teacher during the interventions were the main criteria for differentiating the design and implementation of the inquiry-based teaching interventions from the traditional one. The results showed that the students of the traditional teaching intervention (TI) had a better general response, both in pre-test and in the post-test, than the other two groups of students. However, a greater improvement in the students' response to the post-test, compared to the pre-test, was recorded in the guided inquiry-based intervention (GIBI), with the structured inquiry-based intervention (SIBI) and the TI following. There was also a greater improvement in the response of low-achieving students, regardless of the teaching method, with students of the GIBI and the TI mid-achieving following. Regarding high-achieving students, greater improvement was observed in students in the SIBI with those in the GIBI following.

Keywords Area, Measurement, Rectangular, Surface, Primary Students, Teaching Intervention, Inquiry-Based, Traditional

1. Introduction

Modern society is characterized by complex, dynamic and powerful information systems, the negotiation of which requires both knowledge and a variety of other skills. For this reason, in recent years there has been a need to redefine the nature of school mathematics, while at the same time there is a shift in the use of more complex problems and inquiry-based teaching [1]. Thus, it is necessary to adopt a didactic approach that favors the simultaneous development of cognitive skills, but also a variety of other skills such as creativity, problem solving, originality, modeling, and argumentation, among others [2]. Skills are necessary for the students as future citizens [3].

An approach based on this philosophy is the inquiry-based approach, which seems to encourage the construction of knowledge by students themselves through inquiry-based methods. Results from the implementation of the inquiry-based approach demonstrate significant benefits both in the cognitive field and in the cultivation of skills and competencies, such as collaboration, communication, and argumentation [4]. However, the complexity and requirements of the approach, as well as the limited research data, make its implementation a particular issue that needs further investigation, since most studies are mainly in Sciences [5] and to secondary school students [6] and not to primary school students [7].

In view of the above considerations and in order to determine the criteria to be taken into account in the design and implementation of inquiry-based teaching interventions, inquiry-based teaching interventions were designed and implemented (Structured Inquiry-Based Intervention(SIBI) and Guided Inquiry-Based Intervention (GIBI)) and compared with Traditional Intervention (TI) applied, in order to comparatively study their effects on the perception of the area and its measurement by children 10-11, as well as on their arguments. Only the first topic will be developed in this paper. The study of students' arguments will be analyzed in another paper. The interest in this research was the implementation of an innovative teaching approach, which was not known to the students of the sample, but also in general in the Greek school reality.

The topic of the area and its measurement was chosen because its importance for everyday life, but also for its research and educational interest since it is a difficult and demanding chapter in the mathematics curriculum and in which there are special difficulties of conceptual and procedural nature and many misunderstandings by students [8]. The research questions posed were the following:

1. How did the type of didactic intervention (SIBI, GIBI, TI) affect the degree of response of the students, of each group, to the tasks?
2. How did the type of didactic intervention (SIBI, GIBI, TI) affect the degree of response of the students of different cognitive levels, of each group, to the tasks?
3. How did the answers of the students, in the tasks, differ in relation to the intervention in which they participated?
4. Which criteria should be considered when designing and implementing inquiry-based teaching interventions (SIBI and GIBI)?

2. Background Literature

2.1. Inquiry-Based Approach

The inquiry-based approach is a complex and unique confection with a vague definition and elements from various theoretical approaches of mathematics education [9]. Its basic philosophy is the work of students with methods like those used for searching scientific knowledge. The inquiry-based approach is a modern teaching practice, which contributes to the cultivation and development of scientific skills, such as the conscious process diagnosis and critical consideration of problems, the distinction of alternative solutions, the research design, the case study, the information search, the construction of models and the formulating of coherent argument, something that is achieved through debates with arguments within an open climate that it is suitable for an exchange of views in democratic conditions [45]. In particular, it contributes to the cultivation and development of scientific skills such as critical problem-solving, distinction of alternative solutions, research design, case investigation, modeling, argumentation etc. [4].

The potential of this approach is due to the different degree of its structure and guidance [11], which results in its categorization into [12]: (a) confirmation inquiry, where students know both the question and the answer, which they confirm with a process that has been suggested in some way; (b) structured inquiry, where the teacher gives students the question and the process that students must follow to reach a conclusion; (c) guided inquiry, where the question is asked, but students must independently plan the process of the research to follow in order to reach a conclusion; and (d) open inquiry, where the students themselves formulate both the question and the research process that they must follow to reach a conclusion. Various studies have been conducted on the effectiveness of each type, the results of which differ. Other research shows that guided inquiry-based intervention is more effective [13], while others demonstrate as more effective the open one [14]. However, researchers, often set conditions to the more open inquiry-based approaches, such as that students have already developed the appropriate skills through a guided and structured approach [15].

According to the existing literature, many researchers model stages for inquiry-based approach in order to use it in teaching practice. Thus, implementation models have been proposed by educators, Science researchers [16] and mathematics researchers [17].

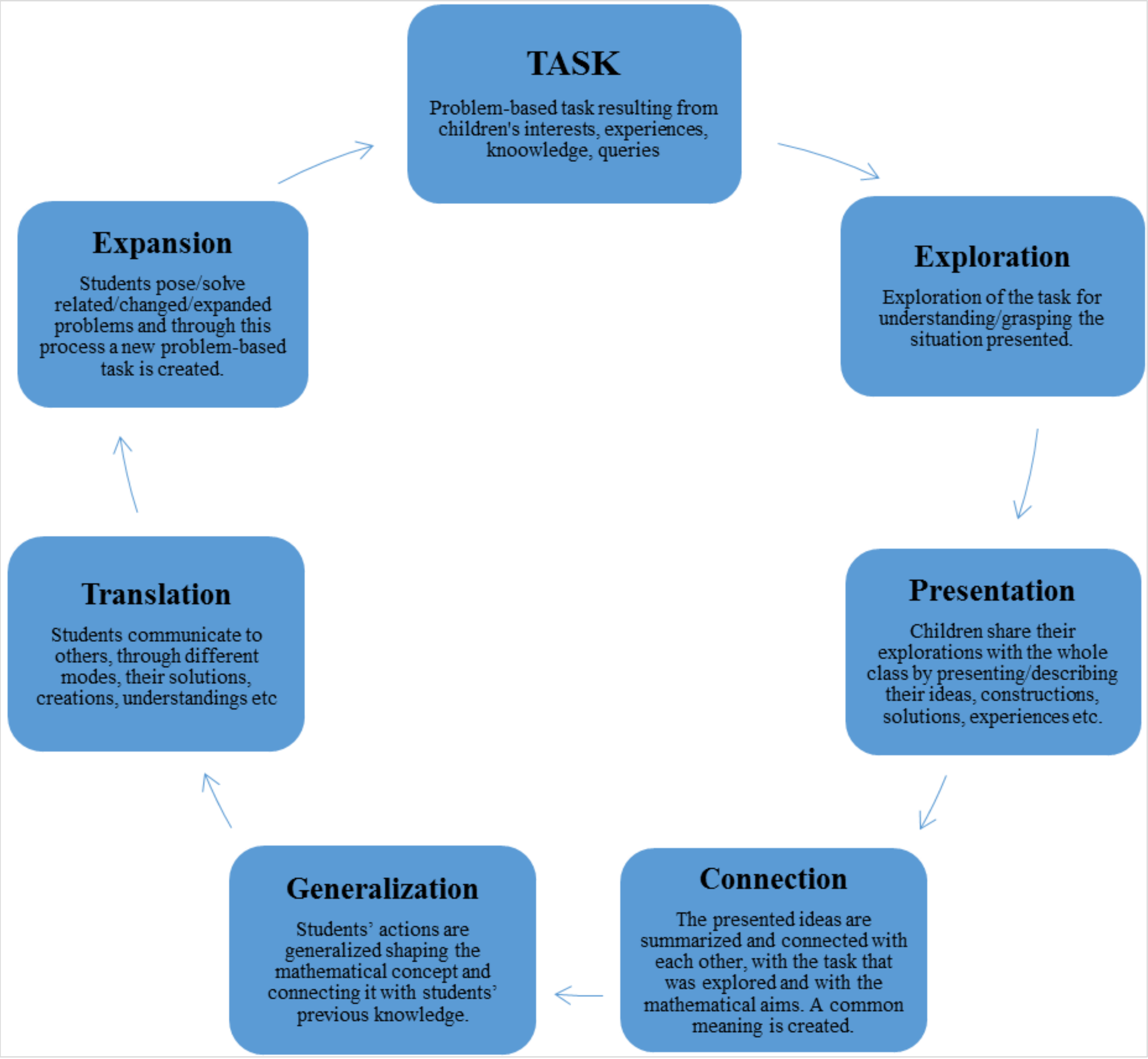

Figure 1. The seven stages of the Framework for Inquiry-Based design and implementation of Activities (FIBA) [10]

A framework that is focused on the design and implementation of mathematics activities is the Framework for Inquiry-Based Activities (FIBA) which consists of seven stages [10][18] (Figure 1):

1. task: A problem-based task is invented and presented by the teacher, through a context, resulting from children's interests, experiences, knowledge, queries. The task, which may have one or more solutions, is designed in such a way that problematizes and incites children, to engage in a problem solving and posing process. The problem to be solved could be a non-standard, unfamiliar, a bit complex and novel situation in order not to be solved just by applying existing knowledge and already-known strategies, but through exploration. 2. Exploration: Children (individually or in teams) use their own (informal) problem-solving strategies to explore the problem introduced by the scenario, to choose/use materials and other auxiliary means, to make conjectures, to pose questions to each other and to the teacher for understanding/grasping the situation and to suggest solutions, to solve the problem. In that stage, students can reflect and think about the problem on their own, before sharing their thoughts with their peers. They are also free to discuss their ideas about the problem with their peers before presenting them to the whole class. 3. Presentation: Children share their explorations with the whole class by presenting/describing their ideas, constructions, solutions, experiences etc. Teacher, in that stage, is an observer and organizer of each team's presentation, orchestrating students' contributions, posing questions to help children describe, explain, and communicate their explorations. He/She also encourages students to pose questions to their peers, from other teams, to ensure that they understand all the presentations. 4. Connection: Teacher, in cooperation with students, summarizes the results, poses questions and encourages students to ask questions that connect the

presented ideas with each other, with the task that was explored and with the mathematical aims, to construct the common meaning that the classroom would share. Teacher's questions must encourage mathematical thinking and reasoning and can be of several types. At that stage, it will become apparent if cooperative strategies are effective in promoting classroom discourse. 5. Generalization: Teacher is generalizing (and mathematizing when and if possible) students' actions, shaping the mathematical concept, connecting it with students' previous knowledge and giving feedback to them. 6. Translation: At that stage, students are asked to communicate to others (students from another class, family etc.), their solutions, creations, understandings etc. through different modes—verbally, gesturally, and schematically. 7. Expansion: Students are asked to pose/solve related/changed/expanded problems. In all the above stages, teacher is also responsible to take students' questions and comments into consideration, turn them into learning opportunities incorporating them to his/her instructional design, creating a new problem-based task.

2.2. Traditional Teaching

In the traditional teaching—the most widespread and used teaching approach until the early '80s and which is widely adopted and applied even today in school reality—the teacher oversees presenting the course as defined by the curriculum and in the school textbook. The teacher functions as a source of knowledge and evaluates what is the most important point of the lesson and where the student should base to learn the content being taught [19].

Students, who usually sit frontally (in pairs on desks facing the board and the teacher) and do not interact much with each other, since the tasks are mainly individual, passively accept the new knowledge from the teacher while practicing the new knowledge in a way as to respond to the intended evaluation format. The individual responsibility of the learning process is particularly reduced, as students do not take initiatives regarding its development [20], since their main purpose is to memorize and apply new knowledge [21].

Traditional teaching consists of three main phases [22]: 1. Orientation phase: the teacher explains the usefulness of the knowledge, detects the already existing knowledge of the students, and outlines the teaching that will follow. 2. Presentation phase: the teacher presents the subject to be taught, after he has structured it according to the basic units which he seeks to present. What prevails in this phase is the large number of examples and applications, accompanied by a variety of comprehension questions to students. 3. Practice phase: The teacher gradually reduces his/her support, while the students practice individually, more and more autonomously in the practical application of knowledge [23].

2.3. Inquiry-Based Teaching VS Traditional Teaching

Comparing the inquiry-based with the traditional teaching approach, it was found that at more demanding levels of critical thinking, inquiry-based teaching has proven to be more effective than traditional forms of teaching [24]. In particular, it has been shown that the retrieval of information from memory as well as the right use of scientific concepts was better in students to whom the guided inquiry-based method was applied compared to those to whom the traditional method was applied. Also, the performance, in a high level of test, of students aged 10-11 who were taught with inquiry was much better than the corresponding performance of students who were taught with the traditional method [25]. In addition, it has been shown that the inquiry-based approach can have a more positive impact on lower performing students, which dispels the belief that this method only applies to the best students [26]. Students aged 10-11 who participated in inquiry-based teaching had a higher success rate, especially in the questions that required explanation, compared to traditional teaching methods [7].

On the other hand, comparison of the inquiry-based with the traditional teaching method around the concept of multiplication in children 8-9 years old showed an improvement in the understanding of the multiplication by the students who were taught with the inquiry-based method but not to a significant degree [27]. From the in-depth study of the above research, it results that the parameters that affect the effectiveness of the inquiry-based approach are the degree of guidance from the teacher and the time of its implementation. In terms of the degree of guidance from the teacher, the guided and structured inquiry-based approach seems to be preferred as the most effective [13], but also the basis for the subsequent use of the open inquiry approach [15]. In terms of implementation time, it has been recorded that the syllabus covered through the inquiry-based approach is less, compared to the traditional one, however its impact on the low achieving students is more positive [26]. Also, many benefits of the inquiry-based method are highlighted and enhanced in the long run, as students become familiar with the method [28].

2.4. Measuring Area

Significant difficulties have been documented in the research for measuring area [8] [29]. These difficulties originate from the curriculum, the school textbooks, the additional educational material used during the teaching procedure, and the way of teaching [30]. Research has shown that in activities that entail the measurement of the area, most of the mistakes originate from lack of understanding of the conceptual procedure [31] and not from mistakes in the measurement itself [32]. This is believed to occur due to the emphasis, given from the early years of education, on the use of and memorization of the area formula [33]. In addition, students often confuse area with perimeter [34], which leads to an inability to understand two-dimensional formation as the product of width and length [32]. Incomplete understanding of the algebraic formula of measurement of the area of rectangular surfaces often leads to their super-generalization, regardless of the type of shape [35].

The development of students' skills in measuring area, using units and/or measuring instruments for the transition to the formula, is developed in seven stages [34] [36]: 1) Children have little or no ability to organize and structure two-dimensional surface (eg cannot cover a rectangle with tiles without gaps and overlaps) 2) Children achieve the encrustation, but cannot count the number of tiles they did use (they do not do a systematic count, for example they measure the tiles of the periphery and then the interior, but in a non-systematic way) 3) They cover the surface and measure the tiles, but without structure (row and column) 4) They do not use rows and columns in a systematic way (eg they measure some but not all rows as units) 5) They structure the rectangle as a set of rows 6) They count with continuous repetition of rows (eg counting each row of 5, "5,10,15,20") 7) They count with continuous repetition of rows in relation to the number of the squares of a column (eg counting by 5) 8) They understand that the dimensions of the rectangle come from the number of squares of the columns and rows and thus calculate the area from these dimensions (area formula).

3. Materials and Methods

3.1. Research Model

The methodology followed in this research is based on design-based research, a methodology used to create new theories and frameworks for conceptualizing learning, teaching, and educational reform [37]. Its main feature is that it aims to improve educational practices, while supporting the implementation and control of the effectiveness of innovations to solve problems in education [38].

This methodology was adopted because it allows changes at all stages of research design. These changes allow the simultaneous development, testing and change in the design and implementation of didactic interventions in specific learning environments, with the aim of describing the conditions under which such an approach can operate, in the specific sample and in the specific cognitive object.

3.2. Population and Sample

The students who participated in the research, attended in three classes of 5th grade (10-11 years old), of a public school in a suburb of Attica and had similar cognitive and social characteristics. The number of students in each class was nine (9), except from one in which there were ten (10). The choice of this grade (5th) was made because it is the grade in which the transition takes place from surface coverage with standard and non-standard units of measurement, to the mathematical formula for measuring the area, a very important transition due to the changes by the cognitive system of students [39].

The students of each of the three classes were divided into three cognitive levels, high, medium, and low, depending on their performance in mathematics during the year at the discretion of their teachers. The last one was based on the performance in mathematics during the previous school year. These levels were confirmed after the analysis of the results in the pre-test. This separation was made on the one hand for the creation of teams with a similar level of knowledge, for the SIBI and the GIBI groups, and on the other hand for the analysis of the effect of each didactic intervention on the degree of the response of each group to the tasks of pre-test and post-test. Thus, groups were created, where the cognitive level of the students who constituted them did not have big differences in order to investigate the role of each teaching intervention in each different group. In particular, Webb (1985,1989) summarizing related research, states that working in groups is productive mainly when they consist of students with a similar cognitive level [47,48].

3.3. Data Collection Tools

Both before and after teaching interventions, students filled a test with the same tasks (the test before the interventions is called *pre-test* and the test after the interventions is called *post-test*). The tasks where the same because the point was the possible different answers before and after each intervention. For the design of the tasks of the pre-test and post-test, we studied the difficulties and misunderstandings identified in the literature for the measurement of the area of flat surfaces, the tasks, the goals, and the proposals of the school pack (school textbook, workbooks, curriculum), which is provided to teachers, as well as related activities from mathematics competitions and research. Issues of area and perimeter of rectangular surfaces were included, as well as division of surfaces, unit repetition, structure formation, unit counting, and numerical value rendering in surface measurement. The tasks were formulated in such a way that the students had to justify their answer. It is worth mentioning that these tasks didn't have to do with the inquiry-based method, so they didn't have to be based on real life. Also, the tasks were accompanied by the following materials: paper square centimeters, strips of 5 and 10 square centimeters and various rectangles divided into square centimeters, which were available to each student for use (see Figure 2). Rulers were also provided to the students. The completion of each test lasted one teaching hour and the students' answers constituted the data of the research.

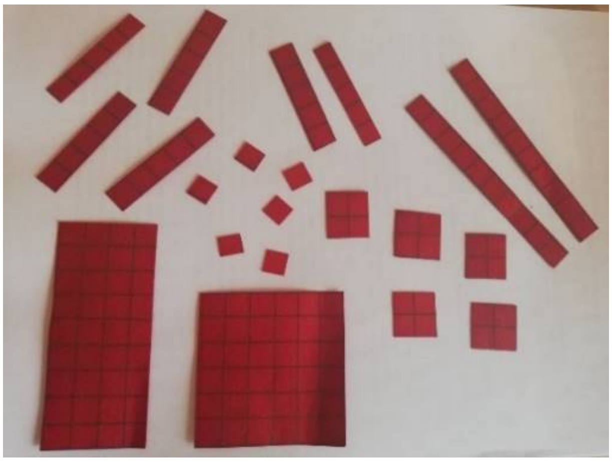

Figure 2. Supporting surface covering material

3.4. Data Analysis

The data analysis was performed both qualitatively and quantitatively. The data came from the answers that children gave in the tasks of the test before and after the interventions (in the pre-test and post-test). The children answered in the tasks individually, like a normal test, after they have been given a box with supporting covering material (Figure 2). For the qualitative analysis of the data and the coding of the students' answers, the 'requirements' for each task of the test were recorded, which are necessary for an answer to be considered correct and documented. These requirements were the indicators of the degree of response of each student to each task. Below are the 'requirements' for each task. Before these the documentation of using of each task will be mentioned according to the mathematical concepts that wanted to be tested every time. Besides that, every concept in the tasks has been taught in each one of the interventions.

1st Task

Look at the following flat shapes A and B. What shape is A? What shape is B? What do you notice about their perimeter and area? Justify your answer.

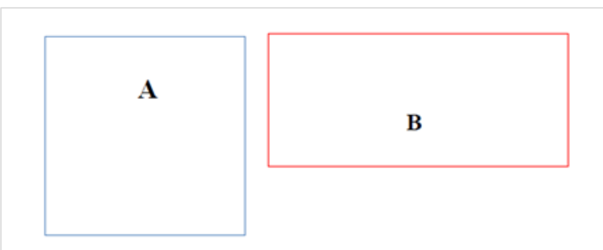

Figure 3. Flat shapes (square and rectangular) in the 1st task of the tests

In this task it is detected the perception of concepts involved in understanding the concept of area and its measurement such as: division, repeating a unit of measurement, maintaining, structuring the grid. Its linear measurement and matching ability obtained, with the number of the area of each surface can also lead to some conceptual conclusions. Specifically, the point was to see if the students will use the formula of the method of the coverage after the intervention and how. Apart this, the specific task can lead to conclusions about the misconceptions that exist around the concept of area, such as: "Area is the same when shape changes forms?" and "Can shapes with different perimeters have the same area?".

The coding of students' answers depended on the following task's requirements.

1st task's requirements:

i). Finding the length of the sides of the shape A and of the shape B.
ii). Finding the perimeter of the shape A and of the shape B.
iii). Finding the area of the shape A and of the shape B.
iv). Comparison of the perimeter and the area of the two shapes

2nd Task

The students of a 5th grade class of a school were given the following shape to find its area. Some children said that since some part of it has been erased, it is not possible to find its area. However, some other children said that there is a way to find its area. What is your opinion? Can you find the area of the figure below? If not, justify your answer. If so, find its area and justify your answer. (The squares on the paper were 1 square centimeter)

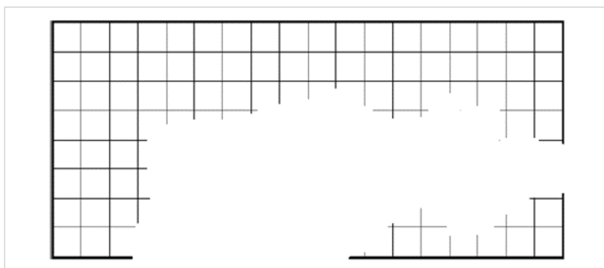

Figure 4. Rectangular shape in the 2nd task of the tests

The aim of the specific task was to detect the strategy that will be used by the students before and after each intervention (counting square centimeters, coverage with the material given to them, using the algorithmic formula). Furthermore, it was aimed to find the understanding of construction of the grid and the way the algebraic formula is derived at rectangular shapes that will be taught in the interventions. This will lead to conclusions that will show if the exploration of the formula made to them more clear how we use it.

The coding of students' answers depended on the following task's requirements.

2nd task's requirements:

i). Finding the length of the sides of the above shape.

ii). Knowledge of the area formula of a rectangular shape.
iii). Application of the area formula, for the above shape, and finding the arithmetic result.
iv). Reference to the units of the above shape's area.

In case the students did not use the area formula, either because they had not yet been taught it, or because they chose to use the surface coverage units, they were required to:

i). Completion of the surface with materials/units given to them or draw the surface grid by filling in the square centimeters on the paper.
ii). Knowledge of what should be counted and specifically the connection of the area with the number of units required to cover the surface.
iii). Right counting of the materials/units/grids and finding an arithmetic result.
iv). Reference to the units of the above shape's area.

3rd Task

A garden consists of 8 same square areas, as in the figure below. The perimeter of the garden is 28m. What is the area of the garden surface? Justify your answer. Note that 1 cm of the figure corresponds to 1 m in reality. (The squares on the paper had a side of 2 cm)

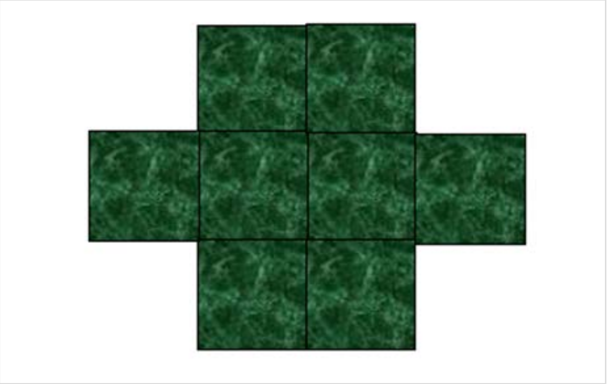

Figure 5. Flat figure in 5th task of the tests

The aim of the specific task was, also, to detect the strategy that will be used by the students before and after each intervention (counting square centimeters, coverage with the material given to them, using the algorithmic formula). Furthermore, it was analyzed the understanding of the concepts that involved in the concept of area and its measurement (Clements & Stephan, 2004; Reynolds & Wheatley, 1996) such as: partitioning, repeating units of measurement and building a pattern. No similar activity was used in the interventions carried out. However, it helps us to study, among other things, whether the students understood the common characteristics of the rectangle and the square in terms of measuring their areas.

The coding of students' answers depended on the following task's requirements.

3rd task's requirements:

i). Finding the length of the sides of the squares.
ii). Knowledge of the area formula of a square shape.
iii). Application of the area formula for the above shape.
iv). Reference to the units of the above shape's area.

In case the students did not use the area formula, either because they had not yet been taught it, or because they chose to use the surface coverage units, they were required to:

i). Completion of the surface with materials/units given to them.
ii). Knowledge of what should be counted and specifically the connection of the area with the number of units required to cover the surface.
iii). Right counting of the materials/units/grids and finding an arithmetic result.
iv). Reference to the units of the above shape's area.

4th Task

The children of a school decided to make a flower bed in the school garden to plant flowers. They can use a part of the garden, which they will fence with 16 meters of wire mesh. What shape should they give to their flower bed to have the largest surface? Justify your answer to convince the other children.

In this task, students are expected to distinguish the concept of area from that of perimeter. It is, also, a task that encourages experimentation while detecting the misconception of students, where they perceive the area with a cumulative view confusing the area with the perimeter (Allerton & Nunes, 1994; Kidman & Nason, 2003) or whether they resorted to mathematical formula even in cases where it was an obstacle (Divisova, 2012). Finally, it is evaluated the ability to document and argue before and after each teaching intervention. This is the only task that has the less common data with the activities in interventions, but it is so important because their answers show both the conceptual and procedural understanding of the students.

The coding of students' answers depended on the following task's requirements.

4th task's requirements:

i). Finding the requested shape (the square) with perimeter 16 meters and the largest area. The square is the requested shape because students in this grade haven't learn yet the way that they measure the area of a circle. However, if somehow they find the circle it is totally accepted.
ii). Finding of two isoperimetric shapes.
iii). Finding of three isoperimetric shapes.
iv). Finding of four isoperimetric shapes.
v). Knowledge of the area formula.
vi). Application of the formula for measuring the area.
vii). Knowledge of the formula of measuring the perimeter.

viii). Application of the formula for measuring the perimeter.

For the quantitative analysis of the results, the averages of the responses to the tasks' requirements of each group of students were calculated. Specifically, in each of the tasks of the tests, each of the requirements was scored with 0.5 points. When the student's answer included everything set by the response indicators, it was scored with the maximum of points. Thus, the first three tasks were scored with 2 points in case of correct and complete answer, while the fourth with 4 points.

3.5. Collection of Data

The didactic interventions took place in three 5th grade classes (students aged 10-11 years old). Each of the three classes was taught with a different didactic intervention: traditional intervention (TI), structured inquiry-based intervention (SIBI) and guided inquiry-based intervention (GIBI), by the researcher who designed them. The students in the TI group worked individually, while in the SIBI and the GIBI group they worked in teams.

The teaching interventions took place in June 2020 and the duration of each one was two (2) teaching hours, as this is the time set by the school curricula for the teaching of the area measurement of rectangular surfaces.

The students of all three classes, both before and after the didactic interventions, completed individual pre-test and post-test, which aimed on the one hand to detect their existing knowledge for the perception of the area and its measurement and on the other hand to investigate the evolution of their knowledge, after the interventions.

From the four types of inquiry-based teaching interventions, which differ in the degree of guidance, it was chosen to use the SIBI and the GIBI. The reason that open inquiry and confirmation inquiry were not chosen was that the former offer too much freedom which was disorienting to the children as they had no familiarity with any of the types of the method and the latter offer a restriction on both its configuration question and in the research process which was considered pointless as it would have many similarities with the TI.

In inquiry-based interventions the teacher supports, encourages, urges, guides and facilitates process of inquiry [45]. Also, he connects students' mathematical views and reasoning with formal mathematical knowledge. Maaß and Doorman (2013) describe the teacher's role in PRIMA: "Teachers are proactive: they support pupils who are struggling and challenge those who are succeeding through the use of carefully chosen strategic questions" (p. 887) [46].

In the GIBI the researcher/ teacher asked questions with minimal guidance, and she gave the basic instructions and then left the children to discover the knowledge on their own, encouraging them throughout the teaching intervention (Table 1). In case where the children needed feedback, by asking question, she guided them through supporting questions. In the SIBI, the researcher/teacher posed guidance questions to help children discover the new knowledge step by step (Table 1). In both the GIBI and the SIBI the role of students was expected to be active through the inquiry they were required to carry out, in collaboration and interaction with their classmates and the researcher/teacher to discover and construct new knowledge through the designed activities. The designing and the implementation of the activities was based on the stages of the FIBA framework [10]. Like the inquiry-based method, so does the framework, which was adopted, is not a series of steps to be followed, but a series of steps that guide the process.

In the TI, the researcher / teacher had complete control over the teaching process by presenting the new knowledge to the students (Table 1). The role of students was expected to be passive, to act as receivers of knowledge with limited self-efficacy and as determined by the activities and tasks of the textbook and workbook, as they participate in their usual teaching process.

Table 1. Indicative differences between didactic interventions

	TI	SIBI	GIBI
Degree of guidance from the teacher	Absolute control of the teaching process. Presentation of new knowledge and listening by students	Questions of building the research process for the discovery of knowledge step by step	Questions to guide the research process to discover knowledge step by step
Role of students	Passive receivers of new knowledge. Limited self-efficacy	Active role of exploration in a structured inquiry environment through collaboration and interaction	Active role of exploration in a guided inquiry environment through collaboration and interaction
Student work	Individual work	Work in teams	Work in teams
Teacher role	Source and transmission of new knowledge	Structuring an inquiry process	Guiding an inquiry process
Teaching material	Activities from the school textbook and workbook	Designed structured inquiry-based activities	Designed guided inquiry-based activities

3.6. Activities in Traditional Intervention

The activities used in the TI were the activities of the textbook and workbook provided to students by the state. The main feature of these activities is the quick transition to the formula and the subsequent solution of activities with exclusive use of the taught formula.

First, they discussed the use of the concept of area in our daily lives and then referred to its standard units of measurement.

This was followed by the teaching of area measurement with the help of the algorithm formula. Then, the students proceeded to individually solve exercises from the school textbook, which had to do with applying the formula to rectangular and square surfaces. At some point during the lesson, the teacher raised the question of applying the new formula to other shapes and thus they discussed the common characteristics of the rectangle and the square and why we use the same mathematical formula.

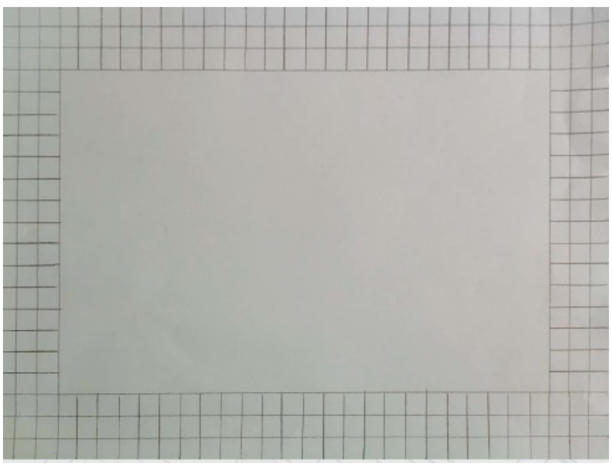

Figure 6. Cardboard, which was given in each student group

3.7. Activities in Inquiry-based Interventions

A particular problem arose during the planning of the IB interventions. As the IB is a multifaceted and complex construct with unclear definition, it had to clarify characteristics that define it as innovative, as well as the differences which distinguish it from other theories. The inquiry-based interventions used a specially designed series of activities that were developed in such a way as to concern students and involve them in a process of gradual discovery of the algebraic formula of measuring the area of rectangular surfaces (surface coverage, grid creation, algebraic formula). To create these activities the school textbook and the content of syllabus were taken into account. These activities were non-formal different from those in textbook, while defining an innovative situation where students to solve them they had to work together and use pre-existing knowledge and inquiry strategies.

The activities in inquiry-based interventions were based on a real-life scenario where the students had to find a solution. This solution was actually the mathematical concept that was to be taught. The procedure of the activities was based on the the order in which concepts related to area are taught in Primary school until the teaching of algebraic formula. Especially, each group was given the following cardboard, which was a floor plan of an available plot of one architect, in which a house will be built (see Figure 6).

Then they had to divide it into rooms as they wished and measure the area of the yard (either with the help of the educational material given to them or with the square centimeters drawn) (see Figures 7,8).

Having already been taught the standard units of measurement, the first activity involved covering an area with them (the yard). The goal here was to match the number of units they used with the surface of the area.

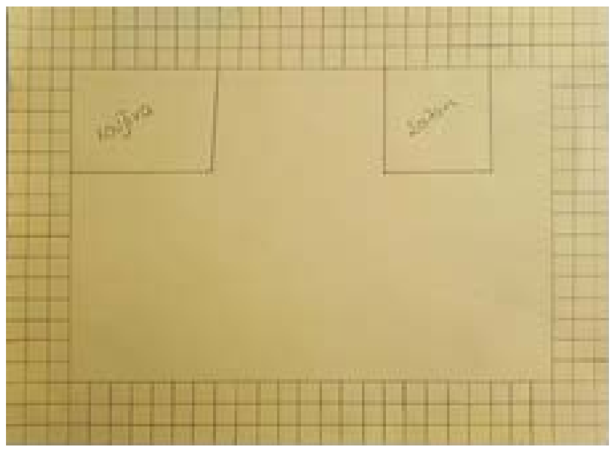

Figure 7. Division of rooms in the floor plan by student groups

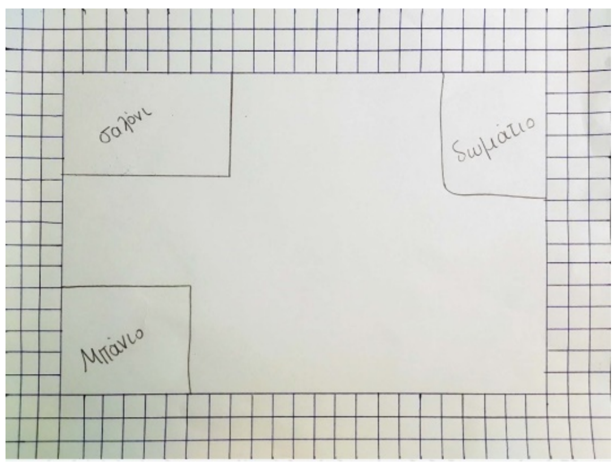

Figure 8. Division of rooms in the floor plan by student groups

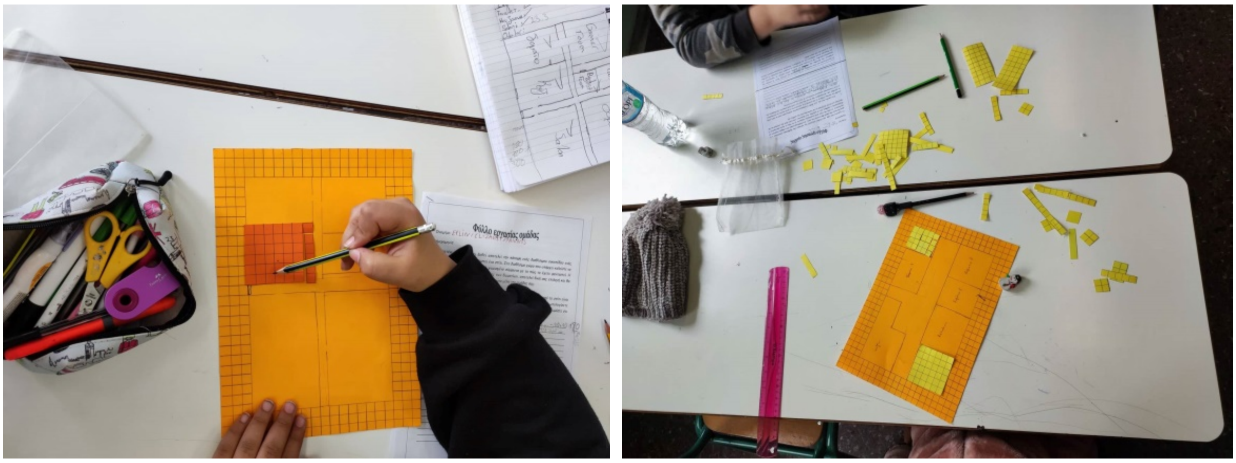

Figure 9. Covering the surface of a room with the help of the material

After the yard, students were asked to measure the area of a specific room with or without the teaching material (square centimeters, strips of square centimeters and bigger squares consisted of smaller squares). The specific activity was again about covering a surface without gaps and overlaps but also structuring a grid in the rectangular (or square) room with the help of the material (see Figure 9). For this activity, one of the key concepts involved in learning to measure area was taken into account: partitioning (the ability to divide a two-dimensional space into distinct parts).

The main difference between the two types of inquiry-based interventions was that in the structured inquiry-based approach there were more questions that guided them in the procedure like: *"What does the number that you found mean?", "What unit of measurement did you use?", "Can the algebraic formula you discovered be applied to other geometric shapes*?". In other words, the students were asked more often to argue for what they did in relation to the students of the SIBI.

In SIBI the teacher focused the students' attention on the dimensions of the kitchen, wanting to discover some repeating pattern of square centimeters on each side. So, the following dialogue ensued.

T: How many such columns do we have with 6 square centimeters?

S14:15 (all agree)

*S16: So 15*6*

T: Why did you multiply?

S17: To find the perimeter

T: The perimeter we are looking for?

S17: Hey, the area, I'm getting all confused

T: But why did you use multiplication?

S17: So that we don't count the squares one by one, since each column has the same ones.

The other group seemed to struggle more, so the teacher tried to guide them with some additional questions.

T: Having covered only one column, one side here you can tell me how many square centimeters have you put in?

S: 6

T: How many such columns of 6 squares do you have?

*S10: 11. So 11*6*

Both groups while doing the correct calculations did not include in the answer them the unit of measurement, with the result that the teacher, after each answer, sets the following questions: *"What does the number you found mean?" "Is this what you were looking for?"*

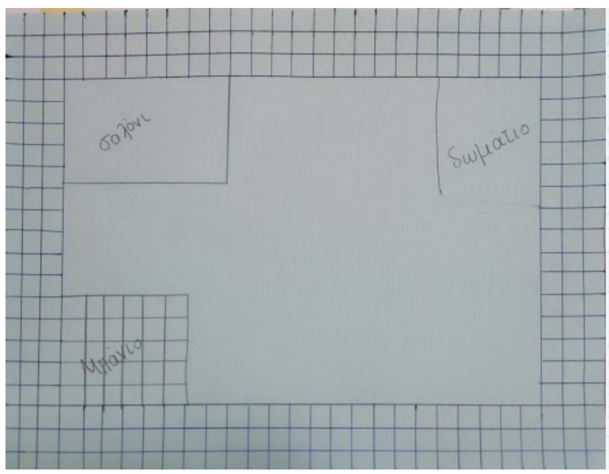

Figure 10. Divided surface into square centimeters

On the other hand, the students of the GIBI did not receive as many coaching questions from the teacher, nor were they asked to justify their strategies in this phase. In fact, one group chose to use the material given to them, while the other divided the surface of the room into square centimeters (see Figure 10).

The last activity was about discovering the algebraic formula for measuring the area of a rectangle, as they are asked to measure the area of the whole house. The goal was to divide the rectangular surface into lines and columns as well as to connect the number of square centimeters that fit on each side with its length. The challenge was that the material was not enough to cover it, so they were led to find that each column and each row consists of the same number of square centimeters.

In the SIBI, the students did not seem to have much difficulty and followed the same strategy as the previous question. They used the material to measure the one square centimeter column and then calculated its diagonal. Specifically, they recorded: "*We will calculate the two sides with the ruler and multiply them together. We did multiplication to find it easily, that's the point of area instead of counting one by one the cans, we measured both sides".*

The students in GIBI uses again the strategy of covering the surface with the material, and when they found that this was not sufficient, they proposed to measure the rooms of the house one by one. Then the teacher suggested that they think about whether there is a faster way without giving any additional instruction. After many tests with the help of the material they proposed to think of the house as a large room and cover only one line with square centimeters and see how many such there are in the house.

The tasks that described above was constituted the phase of *exploration*. The next phase was the *presentation*, where the groups presented their results, trying to substantiate as much as possible their answers. This phase of the procedure was the most difficult as the students seemed hesitant to explain the way they acted through arguments. So, the teacher asked some questions to the representatives such as: "What did you explore today with your group?", "What evidence did you need to come up with them". Thus, the groups discussed with each other and exchanged strategies. The students in SIBI seemed quite reluctant to describe how they worked with them their groups and the reasoning they relied on to derive the specifics conclusions. So, they mentioned the way that they calculated the area of the figure.

S7: We measured one side and then did multiplication.

S9: We also measured the two sides to find the area.

T: Why did you do that?

S14: So that we don't count all the squares one by one.

At a similar level of argumentation, the students of the GIBI moved, who only epigrammatically described what they had discovered.

S20: We did multiplication because multiplication multiplies it as many times as it is

each column.

S19: That's why we did it too, but first we thought of counting the squares.

S20: Well, it was the same.

After the presentation of the results, the groups had to connect all the data that came from and conclude to common meanings (*Connection).* The common meaning in our case was the algebraic formula of measuring the area of a rectangular surface. The last phases of interventions (*Generalization and Expansion)* were both focused on the principle of conservation of area and the relation of the algebraic formula for measuring the area of a rectangle, with other flat shapes such as square and triangle (based on the common shapes' features). The students of both inquiry interventions easily found that the shape that "looks" like the rectangle is the square, while they justified it by saying that "*The square has 4 equal sides, while the rectangle only in 2", "But they also have all right angles*".

In all three didactic interventions the learning goal was the same, the conceptual, as well as the procedural understanding of the concept of area and its measurement. Emphasis was placed on distinguishing the area from the perimeter, as well as on the transition from surface coverage, through grid construction, to the algebraic formula for calculating the area of rectangular surfaces.

The skills that are defined for children before 5th grade in terms of area are the perception of the concept of the rectangular surface and its construction in rows and columns using square units of measurement, since the children already know how to measure lengths. The skills identified for 5th grade focus on introducing the formula for area measurement and distinguishing between perimeter and area. At the same time, students are expected to know the usual units of area and be able to make conversions between them. Finally, by teaching the area in this grade, students are asked to make size arrangements and use the appropriate unit to measure situations that are familiar to them.

4. Results

The results of the present research, emerged from the individual answers of students to the tasks of pre-test and post-test, which were analyzed based on the response indicators mentioned above. Even though the sample of students in the present study was small, the study proceeded to a quantitative analysis of the results, to highlight the general trend and the effect of the type of intervention in each case.

4.1. Quantitative Analysis of Results

For the quantitative analysis of the results, the averages of the students' response to the pre-test and post-test were calculated after scoring their correct answers based on the requirements of the tasks. The recording of the responding average is listed in the table below (see Table 2) and includes a comparison of students' response rates per intervention, and per cognitive level.

Table 2. Averages of students' response to the pre-test and post-test according to their cognitive level and their group

	Averages before the interventions				Averages after the interventions			
Type of intervention	Students of high cognitive level	Students of intermediate cognitive level	Students of low cognitive level	Total average of each intervention	Students of high cognitive level	Students of intermediate cognitive level	Students of low cognitive level	Total average of each intervention
TI	7	3,75	0,25	3,67	6,33	6,25	4,5	5,69
SIBI	3,17	0,83	0,67	1,56	5,83	1,33	3,67	3,61
GIBI	2,75	0,5	0,2	1,15	4,75	3,5	4,7	4,32

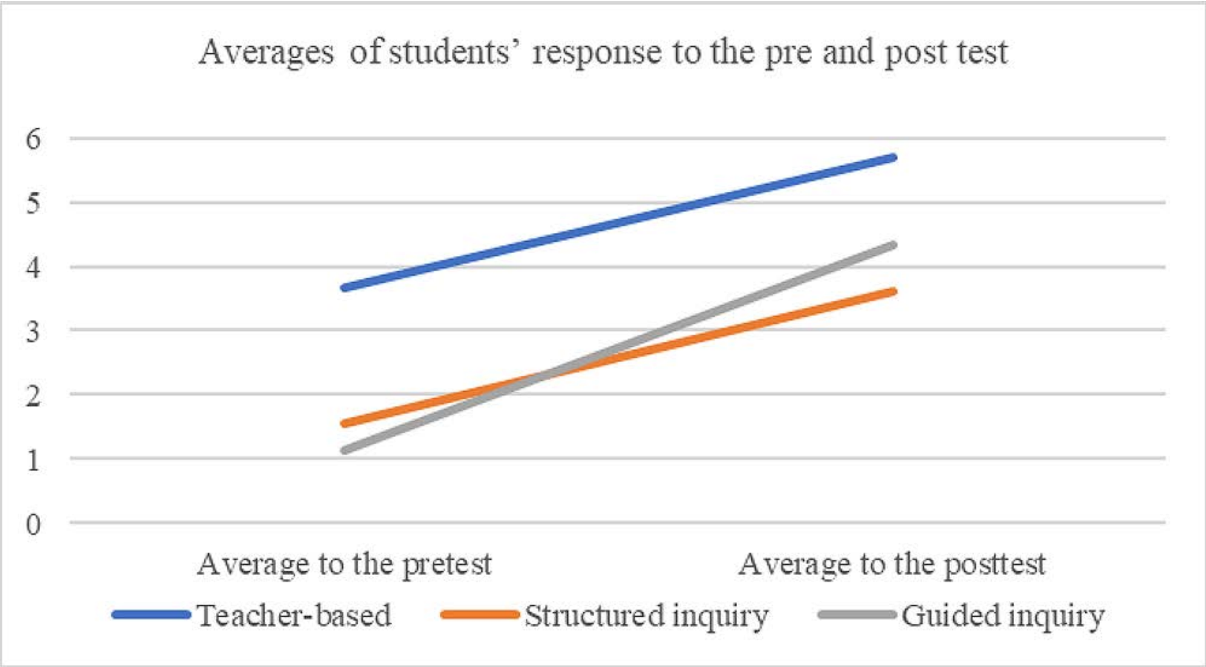

Chart 1. Averages of students' response to the pre-test and post-test

The results of the pre-test showed that most of the sample was not able to indicate the area of the shapes nor to calculate its size. The answers given by the children in the pre-test concerned the measurement of the length of the sides of the shapes and in some cases the measurement of their perimeter.

The results of the post-test showed that the students, regardless of the intervention in which they participated, indicated the area of the shapes, and knew the process of measuring the area of flat surfaces. This measurement was accomplished in the majority with the help of the mathematical formula, with a few exceptions of students who used the material given to them (see Figure 2).

The overall results of both the pre-test and the post-test show that the students of the TI group had a better overall response than the other two groups of students (see Table 2).

According to the comparison of the results of the pre-test and post-test of all children (see Table 2) it seems that the students improved the degree of their response to the area issues regardless of the didactic intervention in which they participated, with the students of the GIBI group showing the greatest improvement and the students of the SIBI group and the TI group follow.

The effect of each didactic intervention on the degree of response of students of different cognitive levels to the tasks of pre-test and post-test seemed to be different, without major differences. As for the high-performing students, those who participated in the SIBI group seemed to respond better to the pre-test and post-test tasks, with those of the GIBI group following, while the response average of students to the TI group, decreased. Intermediate students also improved their response, with students in the GIBI group showing the greatest improvement and students in the TI group following, while students in the SIBI group not showing significant improvement. Greater was the improvement in the response of low-performing students, regardless of the teaching method.

From the above data, it is found that the students of the

TI group may have achieved higher averages in their overall response after the interventions, however equally important was the degree of improvement of the response of the students of the GIBI. This improvement can be due to many reasons, one of which is the peculiarities of this intervention. The involvement of students in an environment of collaboration, discovery, and construction of new knowledge with the help of material (independent of the textbook) which they were accustomed to, but also the degree of guidance of the teacher seemed to be the main factors that improve their response.

This is congruent with recent research [40], which emphasizes the importance of student participation in this type of teaching compared to traditional teaching methods, as well as other which showed that at least in terms of students' conceptual development, discovery learning environments are more effective than using a single type of learning environment [41].

Table 3. Categories of mistakes and examples

	Mistakes about:	Examples of mistakes
1.	**The concept of area**	Confusion of area with the concept of perimeter or side length
2.	**The procedure for measuring the area**	Incorrect construction/coverage Incorrect counting after proper construction/coverage Incorrect application of the area formula
3.	**The solving process**	Measuring a part of the area of the figure Impression of the area formula without its application Strategy description without resolution Wrong use of ruler Wrong multiplication

4.2. Qualitative Analysis of Results

For the qualitative analysis of the results, the students' answers were processed in relation to the response indicators set for each task. From the above processing the mistakes that were highlighted had to do with (see Table 3): 1. the concept of area and usually concerned the confusion of area with perimeter or length, 2. the process of measuring the area and usually concerned: a. incorrect construction/coverage, b. incorrect counting after proper construction/coverage and c. wrong formula of measuring area, 3. the solving process: a. measurement of area of the figure, b. imprint of the formula without its application, c. description of an unresolved strategy, d. misuse of a ruler and e. mistakes in the practice of multiplication as well as 4. presentation of the task either not understanding the task or not using the task data.

Students taught with one of the inquiry-based approaches made mistakes in the concept and process of finding the area, mistakes that were not observed that much in the answers of students taught with the TI (see Tables 4, 5, 6, 7). Specifically, students taught with the TI didn't make mistakes in the procedure for measuring the area, while only two cases of mistakes in the concept of area were observed (see Tables 6,7). As for the mistakes in the solving process the findings showed that they appeared in every didactic case.

Regarding the procedure of measuring the area, the students of the SIBI group seemed to make mistakes in covering or structuring the surface. In the group of the GIBI, these kinds of mistakes were observed in answers where students structured the area in a right way but didn't succeed in counting the units of measurement (see Tables 4, 5, 6, 7).

Mistakes in the process of solving the task were recorded by the students of all three didactic interventions. In particular, there were students of the TI group who did not apply the algebraic formula or applied it to a part of the given shape, even though they had understood its use. Measurement of the area of a part of the figure seemed to be also performed by students of the GIBI group (see Tables 4, 5, 6, 7). Regarding SIBI group …most of this kind of mistakes were related to the use of ruler which was wrong. There were also cases that students wrote down either the strategy description, but they continued with it, or a number as a solution without an explanation (see Tables 4, 5, 6, 7).

Consequently, the students' answers for each task of the test are analyzed and the mistakes they made in case they failed to answer correctly are commented.

Starting from the first task most of the correct answers were recorded by students of the TI (see Table 4).

Table 4. Types of student mistakes in the first task of the post-test

Mistakes as to	TI	SIBI	GIBI
Concept of area	0	2	1
Procedure of measurement	0	2	0
Solving process	1	0	5
Correct answers	8	5	4

Specifically, eight (8) were the students of the TI group who calculated both the perimeter and the area of the two shapes. Five (5) out of those, proceeded to compare the numerical value of the area and of the perimeter, as required by the text of the task. From the SIBI group, only one was the student who managed to answer the task completely, comparing the sizes with each other, while from the GIBI group none. The rest of those who were ranked in the table in the correct answers, correctly calculated the area and the perimeter for each flat shape but did not proceed to compare their arithmetic values (perimeter and area) (see Table 4).

The mistakes observed in the SIBI group had to do with the concept or the process of measuring the area (see Table

4). In terms of the concept, the students found the length of the sides of the shapes but confused the way of calculating the perimeter with that of the area, as a result of which they could not find either of the two sizes (eg. S14 A(a)=9*9*4*4=72 cm). As for the process of measuring the area, the students either used the auxiliary material (see Figure 2) and covered the two shapes with its help but ignoring the rules of coverage (with gaps and overlaps) or drew a grid, but not correctly resulting in incorrect answers (see Figure 11).

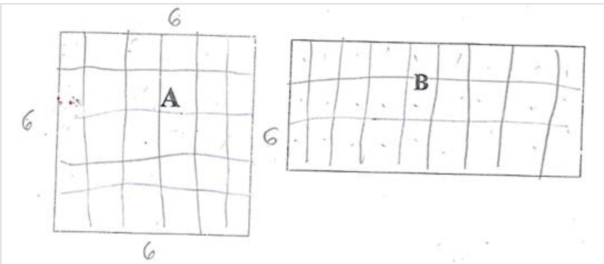

Figure 11. Indicative student answer, with the grid drawn in the two shapes

In the case of the GIBI group, the mistakes observed had to do in one case with the area concept-they confused the area with the perimeter-and in another with the solving process-which began with the identification and naming of the shapes and ended with the measuring and the comparison of the perimeter of the shapes, but not the measuring and the comparison of their area.

This task did not show any difficulty in understanding the text, as it was reminiscent of the texts of textbook tasks. It is worth noting that most students applied the algebraic formula to measure the area, while the students that used the material given to them belonged to the SIBI group. The latter indicates that the greater degree of autonomy of the students during the intervention, may have made them more familiar with the material and its use to find the area, which may indicate a stronger procedural understanding.

In the second task, the students' correct answers were almost the same in the case of the TI and the GIBI group (see Table 5). However, although the task-solving process was correct, the area units were not always mentioned in students' answers. The reference to the area units is an important issue as it determines whether the students know what they measure and what are the units of that measurement. Thus, out of the answers that were characterized as correct, the answers that mentioned the area units of the measurement were five (5/6) for the TI group, none (0/2) in the case of the SIBI group and two (2/5) in the case of the GIBI group.

The students of the TI group that did not answer correctly completed the process of solving the task as they measured the squares along the sides of the shapes, plotted the mathematical formula of the area, but did not proceed with its application (see Table 5). However, the process up to the point where it was recorded was correct.

In the SIBI group, apart from one student who covered the shape correctly but did not count the square centimeters correctly, and one student that confused the area with the perimeter, the rest made mistakes using the ruler, which resulted in incorrect shapes' sides measurements and in incorrect numerical value of area measurement (see Table 5). It is easy to see that the answer may not have been correct, but it shows an understanding of how to measure area.

In the case of GIBI group, students made almost the same, as the above, mistakes. They, in some cases, correctly constructed the surface in the form of a grid but failed to correctly count the square centimeters of which it consisted of. Finally, errors were observed that had to do with the incorrect use of the ruler (see Table 5).

In the third task, only one correct answer was identified in each group of students.

However, in the case of the SIBI group, what is interesting is that all students tried to record an answer, regardless of its correctness (see Table 6). The concept of area was replaced by that of perimeter in one case of students from each group.

Among the answers of the SIBI group, there were those that showed mistakes in the process of finding the area, such as a wrong count of the units which however followed a correct coverage of the surface, as well as a confusion of the area formula. The rest of the students' answers in the SIBI group either contained an incorrect arithmetic value without explanation or an answer in which the data of the task were not used (see Table 6).

In the fourth task most of the mistakes were related to not understanding the task (see Table 7). This may be explained by the fact that this task was very different from the textbook tasks that children were familiar with. There were isolated cases of students, to the SIBI and the GIBI groups, that recorded a partially correct response, but it could not be clarified whether it was consciously recorded or incidental, as it was not accompanied by an explanation. Most students failed to record an answer to this task.

Table 5. Types of student mistakes in the second task of the post-test

Mistakes as to	TI	SIBI	GIBI
Concept of area	0	1	0
Procedure of measurement	0	1	3
Solving process	3	5	2
Correct answers	6	2	5

Table 6. Types of student mistakes in the third task of the post-test

Mistakes as to	TI	SIBI	GIBI
Concept of area	1	1	1
Procedure of measurement	0	4	0
Solving process	1	3	0
No reply	6	0	8
Correct answers	1	1	1

Table 7. Types of student mistakes in the fourth task of the post-test

Mistakes as to	TI	SIBI	GIBI
Concept of area	1	2	1
Solving process	1	1	2
No reply	7	5	6
Correct answers	0	1	1

Concerning the first two tasks, the children's response was better after the TI. This may be due to the fact that the specific tasks were similar to the textbook tasks that students are familiar with. In the other two tasks (third and fourth), where both the text and the strategy of measuring the area are more complex, the response of the children did not seem to differ in relation to the intervention in which they participated. The students in these tasks did not record in their majority any answer and so the types of mistakes are isolated cases that cannot be related to the type of intervention in which they participated and thus do not lead to a research conclusion.

5. Conclusion and Discussion

The main purpose of this work was to determine the requirements during the design and implementation of inquiry-based teaching interventions (structured and guided inquiry-based teaching), as well as to record their effect on the perception of the area and its measurement by children aged 10-11 in relation with traditional didactic intervention.

The absence of research results from inquiry-based implementations in mathematics classrooms made the design of such research a difficult task. The characteristics that describe the inquiry-based teaching and the unfamiliarity of the teacher and students with it, made the design and the implementation of the activities quite difficult. At the same time, it turned it into a dynamic process, which was modified according to the needs that arose each time.

The type of the didactic intervention (SIBI, GIBI, TI) affected the degree of response of the students, of each group, to the tasks. The students showed a greater degree of response to the tasks of the tests after the interventions, regardless of the type of intervention in which they participated. Although the improvement in students' response to the post-test was observed to be greater in the case of inquiry-based interventions and especially the GIBI, the difference was not that big. In order to see bigger difference may require more time to implement the interventions, as is mentioned by other research [27].

Regarding the difference between the two types of inquiry-based approaches, it appeared that compared to the initial response averages, those who participated in the GIBI group had a greater improvement, without a statistically significant difference. Thus, this research agrees with a previous one where it was found that the more open the inquiry-based intervention, the more effective it is [14].

The type of didactic intervention (SIBI, GIBI, TI) also affected the degree of response of the students of different cognitive levels, of each group, to the tasks, in different ways. Specifically, students of high cognitive level showed greater improvement in the case of the SIBI, while students of medium and low performing in the case of the GIBI. The limited duration of the interventions and the small sample do not allow us to generalize the specific results and lead to conclusions, however, the tendency to improve the students' response becomes obvious, regardless of the cognitive level to which they belong. This contradicts the notion that the inquiry-based approach is only suitable for high-achieving students [26].

The students’ perceptions of the area and its measurement were different in relation to the intervention in which they participated. Regarding the qualitative differentiation of students' answers and specifically in the study of mistakes in them, it seemed that: The students of the TI group made mistakes in the solving procedure, which mainly concerned the operations or the use of materials and the ruler. May, the fact that the area formula was taught to the TI group, made it understantable, revealing procedural understanding, without necessary and conceptual understanding of the area. The students of the SIBI group made mistakes that concerned both the concept of the area and its measurement but also in general in the solving process. Furthermore, the students of the GIBI group made more mistakes in the solving procedure, than in the concept of area and its measurement.

The criteria that were considered when designing and implementing inquiry-based teaching interventions (structured and guided) were related with the main characteristics of the inquiry-based approach such as the collaborative learning, the role of the teacher, the students and the specific types of activities that lead to exploration and construction of new knowledge. One of the most important criteria is the familiarity’s degree of the students with cooperative teaching methods. In particular, the more they are familiar, the more they can participate in more open types of inquiry-based interventions. It is important that the design of the inquiry-based intervention begins based on more structured forms and opens up as students become familiar with it. In our case this was one of the most important difficulties we faced, because students didn’t have the experience to cooperate with another child.

Besides that, central role in the selection and designing the inquiry-based intervention is played by the mathematical concept that would be chosen and whether it can be discovered by the specific group of students (depending on their age and cognitive level). As for the mathematical concept on which the interventions were based, the learning objectives that formed the context for the designing of the interventions, that were also suggestions of the curriculum, were the understanding of the concept of area, the use of standard units of measurement to cover a surface, the multiplicative procedural measurement, the use of area formula, as well as the reference to the units of the measurement. Based on Greek reality, every teaching intervention must be based on specific principles of the applicable syllabi. The inflexibility of syllabi and the mostly closed textbook activities, make it difficult to create open problematic situations. Thus, one criterion that must be taken into account is the characteristics of the activities of the school textbook, with the aim of gradually enriching them with inquiry ones and their utilization in inquiry-based interventions.

The development of the inquiry questions was designed in such a way as to follow the steps of [36], on the development of the ability of area measurement. Specifically, from the surface coverage, they were led to the creation of the grid and finally to the algebraic area formula. Finally, the tasks that made up the tests were either modified textbook activities or activities from math competitions and research referred to this age.

Despite the positive response and the increased interest of the students in the implementation of the innovative exploratory teaching interventions, the students were also awkward, probably due to the demanding and new role they were called to take on. Previous research has reached similar conclusions [42]. For this reason, perhaps familiarizing students with collaborative teaching methods prior to applying an inquiry approach would help students better address it. The same conclusion was reached by research [28], who reported that the time students spend in inquiry learning environments is proportional to the effectiveness of teaching. This leads to the conclusion that a criterion for choosing the type of inquiry-based method to be applied is the students' previous experience with similar teaching methods.

Considering all the above, we conclude that the design and implementation of an inquiry-based intervention must be based on various factors, which have to do with the approach itself and its characteristics, with the teacher, as well as with the students. A model of the axes of design and implementation of inquiry-based teaching method is represented at Figure 2. The choice of the type of the inquiry-based approach has to do with the age of the students, their familiarity with inquiry and collaborative activities as well as the learning object itself. In particular, the more familiar students and teachers are with inquiry processes, the easier it is to choose a more open type of inquiry-based teaching method. Before making this determination, the teacher must study all those characteristics that distinguish both the types from each other and from other approaches and choose based on the degree of guidance he/she wants to have, the role of the students and the type of work they do. This option will then determine the type of activities that will be designed.

The challenge is to differentiate the design of the activities and their implementation according to the different types of inquiry-based intervention (SIBI and GIBI) chosen. Activities should be based on the textbook, promote dialogue and cooperation between group members, acquire new mathematical knowledge [43], but at the same time be worded in such a way that they differ in the degree of guidance that will be given to the students. Additionally, the design of the interventions must consider the prior knowledge of the students, which in addition to the content will define the duration of the interventions, which will be longer and more flexible than in a TI.

Regarding the implementation of inquiry interventions, time is required to obtain the best results, as both students and the teacher become familiar and work better in the inquiry process. In addition, what is suggested is support, guidance, and scaffolding, which will slowly decline, according to research [44].

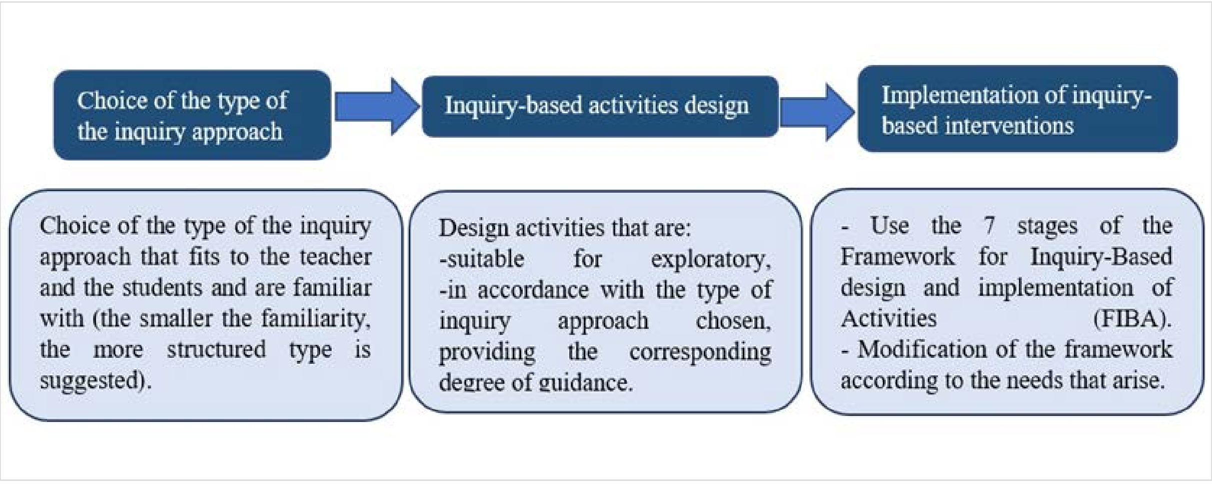

Figure 12. Axes of design and implementation of inquiry-based interventions

6. Limitations

A key limitation of the present study was the number of students sampled. Due to the conditions determined by the pandemic in Greece at that time, the students attended classes at school alternately day by day, so the sections were randomly divided in half. Thus, a repetition of the research to a larger sample, might give conclusions that would allow the creation of generalizations, something that cannot happen in our case. Finally, the longer the duration of the implementation of such a teaching intervention can lead to more reliable conclusions, as the familiarity of students can reduce the rate of negative effects due to awkwardness.

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