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Effect of Outstanding Flanges on Stress Concentration in Box Beam

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Abstract Background of the research: A conventional box-type structure is generally used in constructing the core/shear wall, internal frame, and steel box girders. Tube-in-Tube with single and multiple tubes reduces the shear lag effect and lateral deformation in a tall tubular structure. Purpose: However, the present study focuses on the behavior of a modified form of the conventional box beam, i.e., a box beam with outstanding flanges. Thus, this study analyzes a box beam with outstanding flanges and compares the shear lag effect (Stress concentration) and lateral deformation with a conventional box beam. Methodologies: The minimum potential energy principle is applied to analyze the models and validated with the finite element results. Principal results: The transformation is observed to be reducing the shear lag effect at the web-flange junction and lateral deformations of the beam. In addition, the transformation also controls stress reversal. The modification demonstrated in the present study is stiffer than the conventional box-type structure as a core/shear wall or internal frame. Major conclusions and contributions to the field: This modification may be an integral part of tall tubular buildings producing significantly lowered stress concentration and lateral deformation. The present study is also applicable to strengthening steel box girders. It is possible to extend the bottom plate with the outstanding flange to increase the load-bearing capacity or strengthen the existing steel box girder. Therefore, the transformed box beam can be utilized as an economical alternative in constructing tall tubular buildings and retrofitting the steel box girder bridge. It is convenient and straightforward as there is no need to stiffen any part of the core/shear wall or internal frame. Additionally, there is no need to remove elements from the existing girder or alter its dimensions. Add the remaining flange by welding or bolting it onto the bottom flange to extend it. Limitation of the study and future research: A different assumption of stress function can be used in the flange and web when an orthotropic membrane theory is adapted to analyze the frame tube structure.

Keywords Box Beam, Stress Concentration, Outstanding Flange, Shear Lag, Stress Factor

1. Introduction

The assumptions of elementary beam theory (EBT) rendered the beam rigid in shear and resulted in a linear distribution of bending stress across its width. In the case of beams which have wide flanges (I-, T-, U-, and box beam), the shear deformation results in nonlinear bending stress distribution across the flange width, known as shear lag [1-6]. In-plane shear deformation of the flange causes two types of stress variations. First, one is positive shear lag, in which the stress at web-junctions is highest than the other parts of the flange across the width. The second one is negative shear lag which rendered the lowest stress at the web-junctions compared to the other parts [7-16]. As such, the shear lag is a stress concentration problem [17-18].

To analyze shear lag in tubular structures, the framework panels are transformed into equivalent orthotropic membranes, with elastic properties selected to represent the axial and shear behavior of the existing framework [19-21]. Further, the equivalent elastic properties of frame-tube systems were determined considering beam-column-joint deformation with shear deformation of frame members [22-25].

The other methodologies that deal with shear lag in box type structures are the grillage analogy method [26], the folded plate method [27], the finite strip method [28], and the finite element method [2, 29-33]. Apart from these methods, energy method had been proven to be versatile. Reissner [34] introduced the least work solution method [35]. In this method stress compatibility was utilized to derive the governing differential equations. In contrast, with the variational principle, strain compatibility was utilized to derive the differential equations [7-8,36-41]. Laudiero and Savoia [42] developed a unified approach for analyzing shear strain effects in thin-walled beams for nonuniform bending and torsion.

In the box beams, T beams, I beam, and U beams, the flange is stiffened by a web along the length. The middle portion of the flange lags behind that closest to the flange-web corner due to the shear flow between the flange and web under transverse loading. Under transverse loading, the warping displaced longitudinally the portion of the flange closer to the web-flange largely than the other part. This phenomenon produced the nonuniform bending stress distribution [9,14-15,43]. The warping displacement function followed the polynomial variations [44-47]. The warping function selected is related to the shear lag, which indirectly represents the stress distribution [48-49]. Shear lag is evident in box beams and various structural arrangements such as box girders, tall tubular structures and plated structures.

Box beams are used in structures such as box girder bridges, framed-tube structures, and shear/core walls. Even though box girder bridges consist of the box and outstanding flange only at the top [8], framed-tube structures and shear/core walls consist only of the box in their structural forms [19,45,50]. A Tube-in-Tube is also a structural form of a tall building that includes single or multiple internal tubes of a box shape to enhance overall performance in terms of shear lag [36-39]. Using multiple internal tubes improves the overall performance of the whole structure against lateral loads when considering the shear lag effect. However, it makes the construction of a tall tubular building uneconomical. Improving the performance of a single tube may enhance the entire construction economy of a tall tubular structure.

As a result, this study aims to determine the performance and applicability of an axis-symmetrical cantilever box beam with outstanding flange. A transformed box beam's performance is evaluated based on its deflection, stress factor, and the additional stress created by the different shear loadings. This method is based upon a well-established and validated methodology introduced by Reissner [1].

2. Methodology

A box beam cross-section shown in Figure 1(a) is transformed in other axis-symmetrical cross-sections with outstanding flange (Figure 1b). For simplicity, the transformed cantilever box beam is analyzed considering the shear deformation in the flange, and the web is assumed to follow the EBT. The present study closely followed the Reissner's methodology to model the box beam with outstanding flange [1,6,8,16]. The corresponding span-wise flange displacement considering the shear deformation with normal stress is

$$u(x,y) = \pm h\left[\frac{dz}{dx} + \left(1 - \frac{y^2}{w^2}\right)U(x)\right] \qquad (1)$$

The span-wise coordinate is x, and the other coordinate in plain and normal to x direction is y and z where z (x) denotes the deflection of the beam (Figure 1), and $U(x)$ represents the correction due to shear lag. The correction is applied such a way to preserve the continuity of the displacements at the junction of web and flange along the flanges, that is along $y = \pm w$. Assuming the normal strain in the chord wise direction of the flange negligibly small, the strain energy of the flange is

$$\pi_f = \frac{1}{2}\iint 2t[E\varepsilon_x^2 dx + G\gamma^2]\, dxdy \qquad (2)$$

In Equation 2, E denotes the elasticity modulus and G denotes rigidity modulus, respectively. The span-wise normal strain ε_x and shear strain $\boldsymbol{\gamma}$ are expressed as

$$\varepsilon_{xt} = \frac{\partial u}{\partial x},\ \gamma_t = \frac{\partial u}{\partial y},\ \varepsilon_{xb} = \frac{\partial u}{\partial x},\ \gamma_b = \frac{\partial u}{\partial y} \qquad (3)$$

Expressions for the strains in the flange are

$$\varepsilon_{xt} = -h_t\left[z" + (1 - \frac{y^2}{w^2})U'\right],\ \gamma_t = +h_t\frac{2}{w}\frac{y}{w}U,$$

$$\varepsilon_{xb} = +h_b\left[z" + (1 - \frac{y^2}{w^2})U'\right],\ \gamma_b = -h_b\frac{2}{w}\frac{y}{w}U \qquad (4)$$

In the box beam (Figure 1b) a given distribution of load is applied normal to the plane of top flange along the span length l. If a bending moment $M(x)$ is developed by a load distribution, the elastic potential energy inducing in the structure by the load system is.

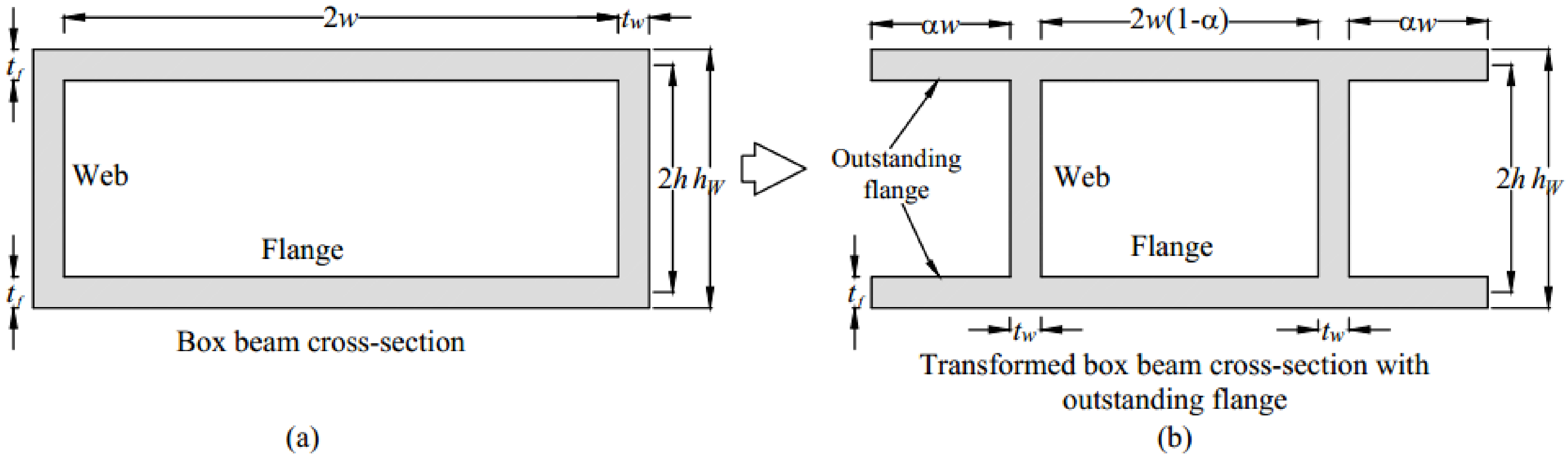


Figure 1. **(a)** Cross-sections of the box beam, **(b)** Cross-section of the box beam with outstanding flange

$$\pi_l = \int_0^l M(x)\frac{d^2z}{dx^2}dx \quad (5)$$

The strain energy of webs is

$$\pi_w = \frac{1}{2}\int_0^l EI_w\frac{d^2z}{dx^2}dx \quad (6)$$

(I_w denotes the principal moment of inertia of webs)

Additionally, the strain energy of the flange can be written as

$$\pi_f = \int_0^l\int_0^{\alpha w}(t_t h_t^2 + t_b h_b^2)\left\{E\left[z'' + (1-\frac{y^2}{w^2})U'\right]^2 + G\left[\frac{2}{w}\frac{y}{w}U\right]^2\right\}dxdy$$
$$+\int_0^l\int_0^{\alpha w}(t_t h_t^2 + t_b h_b^2)\left\{E\left[z'' + \left(1-\frac{y^2}{w^2}\right)U' +\right]^2 + G\left[\frac{2}{w}\frac{y}{w}U\right]^2\right\}dxdy \quad (7)$$

The strain energy of the flange for $t_t = t_b = t_f$ is

$$\pi_f = \frac{1}{2}\int_0^l EI_f\left\{(z'')^2 + \frac{8}{15}(U')^2 + \frac{4}{3}z''U' + \frac{4}{3}\frac{G}{E}\frac{1}{w^2}U^2\right\}dx \quad (8)$$

where $I_f = 4(1+\alpha)wt_f h^2$

The total potential energy is

$$\pi = \int_0^l \frac{1}{2}EI\{(z'')^2 + Mz''\}dx +$$
$$+\int_0^l \frac{1}{2}EI_f\left\{\frac{8}{15}(U')^2 + \frac{4}{3}z''U' + \frac{4}{3}\frac{G}{E}\frac{1}{w^2}U^2\right\}dx \quad (9)$$

where $I = I_f + I_w$

Differential equation and boundary conditions for z and U are obtained by using variation approach. Thus, with x_1 and x_2 denoting the interval of integration,

$$\partial\pi = \int_0^l\left\{\left[EIz'' + M + \frac{2}{3}EI_fU'\right]\partial z'' + EI_f\left[-\frac{8}{15}U'' - \frac{2}{3}z''' + \frac{4}{3}\frac{G}{E}\frac{1}{w^2}U\right]\partial U\right\}dx + \left\{EI_f\left[\frac{8}{15}U' + \frac{2}{3}z''\right]\partial U\right\}_{x_1}^{x_2} = 0 \quad (10)$$

Vanishing the terms multiplying by $\partial z''$ and ∂U in the interval $[x_1, x_2]$ the following differential equations are obtained

$$z'' + \frac{2}{3}\frac{I_f}{I}U' + \frac{M}{EI} = 0 \quad (11)$$

$$EI_f\left[-\frac{8}{15}U'' - \frac{2}{3}z''' + \frac{G}{E}\frac{4}{3w^2}U\right] = 0 \quad (12)$$

$$\left\{EI_f\left[\frac{8}{15}U' + \frac{2}{3}z''\right]\delta U\right\}_{x_1}^{x_2} = 0 \quad (13)$$

From Equation 2 and 3, eliminating the U' and U'' and arranging the term it can be obtained as

$$z'' - \frac{z^{IV}}{k^2} = -\frac{M}{EI} + \frac{n}{k^2}\frac{M''}{EI} \quad (14)$$

where n and k are the two Reissner's parameters

$$n = \frac{1}{1-\frac{5}{6}\frac{I_f}{I}}\ ;\ k = \frac{1}{w}\sqrt{\frac{5}{2}\frac{G}{E}n} \quad (15)$$

From Equation 1 and 2 the bending stress can be derived as

$$\sigma_x = \mp Eh\left[\frac{M}{EI} + \left\{\frac{2}{3}\frac{I_f}{I} + \frac{y^2}{w^2} - 1\right\}U'\right] \quad (16)$$

The solution of Equation 16 for stresses and deflections assuming origin at the free end of the cantilever is given in Equations 17-24 in conjunctions with Reissner [1], Chang and Zheng [8], Singh et al. [16], and Kumar and Singh [6] for shear loading cases as: (a) uniformly distributed load q over the entire span, (b) point load P at the free end, (c) uniformly varied load with intensity q at the support, and (d) uniformly varied load with intensity q at the free end respectively.

$$\sigma_x = \pm\frac{ql^2}{2}\frac{h}{I}\left[\left(\frac{x}{l}\right)^2 + \frac{5}{2}\frac{n}{(kl)^2}\left\{\frac{2}{3}\frac{I_f}{I} + \frac{y^2}{w^2} - 1\right\}\left\{\frac{\cosh(l-x) + kl\sinh kx}{\cosh kl} - 1\right\}\right] \quad (17)$$

$$\sigma_x = \pm Pl\frac{h}{I}\left[\frac{x}{l} + \frac{5}{4}\frac{n}{kl}\left\{\frac{2}{3}\frac{I_f}{I} + \frac{y^2}{w^2} - 1\right\}\left\{\frac{\sinh kx}{\cosh kl}\right\}\right] \quad (18)$$

$$\sigma_x = \pm\frac{ql^2}{6}\frac{h}{I}\left[\left(\frac{x}{l}\right)^3 + \frac{15}{2}\frac{n}{(kl)^2}\left\{\frac{2}{3}\frac{I_f}{I} + \frac{y^2}{w^2} - 1\right\}\left\{-\frac{x}{l} + \left(\frac{kl}{2} + \frac{1}{kl}\right)\frac{\sinh kx}{\cosh kl}\right\}\right] \quad (19)$$

$$\sigma_x = \pm\frac{ql^2}{3}\frac{h}{I}\left[\frac{3}{2}\left(\frac{x}{l}\right)^2 - \frac{1}{2}\left(\frac{x}{l}\right)^3 + \frac{15}{4}\frac{n}{(kl)^2}\left\{\frac{2}{3}\frac{I_f}{I} + \frac{y^2}{w^2} - 1\right\}\left\{\frac{x}{l} - 1 + \cosh kx + \left(\frac{kl}{2} - \sinh kl - \frac{1}{kl}\right)\frac{\sinh kx}{\cosh kl}\right\}\right] \quad (20)$$

$$z(x) = \frac{ql^4}{8EI}\left[\frac{1}{3}\left(\frac{x}{l}\right)^4 - \frac{4}{3}\left(\frac{x}{l}\right) + 1 + \frac{8(n-1)}{(kl)^2}\left\{\frac{1}{2}\left(1 - \left(\frac{x}{l}\right)^2\right) + \frac{\cosh kx - \cosh kl}{(kl)^2} - \frac{\sinh kx - \sinh kl}{(kl)^2\cosh kl}(\sinh kl - kl)\right\}\right] \quad (21)$$

$$z(x) = \frac{Pl^3}{3EI}\left[\frac{1}{2}\left(\frac{x}{l}\right)^3 - \frac{3}{2}\left(\frac{x}{l}\right) + 1 + \frac{3(n-1)}{(kl)^2}\left\{-\frac{x}{l} + \frac{\sinh kx}{kl\cosh kl} - \frac{\tanh kl}{kl} + 1\right\}\right] \quad (22)$$

$$z(x) = \frac{ql^4}{30EI}\left[\frac{1}{4}\left(\frac{x}{l}\right)^5 - \frac{5}{4}\left(\frac{x}{l}\right) + 1 + \frac{15(n-1)}{(kl)^2}\left\{\frac{1}{3}\left(1 - \left(\frac{x}{l}\right)^3\right) + \frac{2}{(kl)^2}\left(1 - \frac{x}{l}\right) + \left(kl + \frac{2}{kl}\right)\frac{\sinh kx - \sinh kl}{(kl)^2\cosh kl}\right\}\right] \quad (23)$$

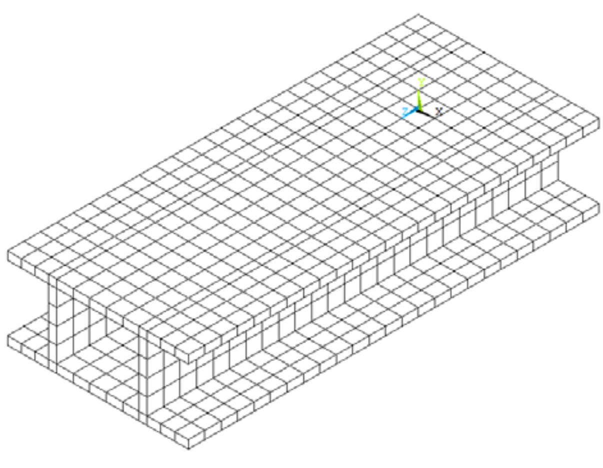
$$z(x) = \frac{11ql^4}{120EI}\left[-\frac{1}{11}\left(\frac{x}{l}\right)^5 + \frac{5}{11}\left(\frac{x}{l}\right)^4 - \frac{15}{11}\frac{x}{l} + 1 + \frac{120}{11}\frac{(n-1)}{(kl)^2}\left\{\begin{matrix}\frac{1}{6}\left(\frac{x}{l}\right)^3 - \frac{1}{2}\left(\frac{x}{l}\right)^2 + \frac{1}{3} - \frac{1}{(kl)^2}\left(1 - \frac{x}{l}\right) + \frac{\cosh kx - \cosh kl}{(kl)^2} + \\ \left(\frac{kl}{2} - \frac{1}{kl} - \sinh kl\right)\frac{\sinh kx - \sinh kl}{(kl)^2\cosh kl}\end{matrix}\right\}\right] \quad (24)$$

3. Result and Discussion

The cantilever box beam with length; l = 250 mm, w = 50 mm, $t_t = t_w$ = 5 mm, h_w = 50 mm, and G/E = 3/8 is considered as an example (Figure 1a). The transformed box beam with outstanding flange is depicted in Fig. 1b. The aspect ratio of the transformed cantilever box beam is defined as $l/(1–\alpha)$ w. For $\alpha = 0$, the aspect ratio becomes l/w, and the transformed cantilever box beam resembles the Reissner [1] box beam (Figure 1a). According to the assumed example, the stiffness ratio (I_f/I) is equal to 0.83 for both types of beams. A Finite-element analysis (FEA) was conducted for the cantilever transformed box beam model for α equals ½ to validate the methodology. The FEA was performed to analyze the model for all loading considered in this study. For the example problem, ANSYS version 15 was used to model the cantilever box beam, considering its dimensional and material properties. The displacement boundary conditions had been placed at the support as ($u(x) = 0$, and $u(z) = 0$), and along the lateral direction ($u(y) = 0$). Therefore, the longitudinal and vertical displacements along the beam were restricted. Moreover, the lateral displacement within the box was constrained.

The model had been discretized using a solid element (SOLID186). A mesh convergence study was performed to eliminate maximum possible errors. A 10 mm element size including 850 elements efficiently analyzed the models. The mesh model of the cantilever box beam with outstanding flanges is depicted in Figure 2a. The point load was applied symmetrically on the top of the webs at the free end of the cantilever. Similarly, the uniformly distributed load and uniformly varying load was applied symmetrically at the top of webs and along the length of the cantilever. The variation in normal stresses under uniformly distributed loading of intensity 10 kN/m is shown in Figure 2b. A variation of deflection in the transformed box beam for α equals ½ is depicted in Figure 3. Regardless of the singularity produced at the support, the end stress could be calculated based on the intersection of the y-axis and tangent at the point where abrupt variations of normal stresses begin in the graph [51]. Additionally, the stress factor variation at the web-flange junction and along the length is demonstrated in the Figure 4. Both the theoretical and FEA results are in reasonable agreement.

A positive shear lag was appeared at the fixed end, and negative shear lag occurred at some subsequent distance from the fixed end for all loading except point loading irrespective of the loading [4,7-8,11,14]. The shear deformation of the flange is responsible to redistribute the bending stress. Therefore, the shear deformation of the flange results in stress concentration at the web-flange junction of a box beam [1,8]. As α increased, a significant shear flow was observed toward the center of the flange of the transformed box beam, compared to the traditional box beam and box girder.



(a) (b)

Figure 2. FEA model of box beam with outstanding flanges: **(a)** Mesh model, and **(b)** Variation of normal stresses under uniformly distributed load of intensity 10 N/m

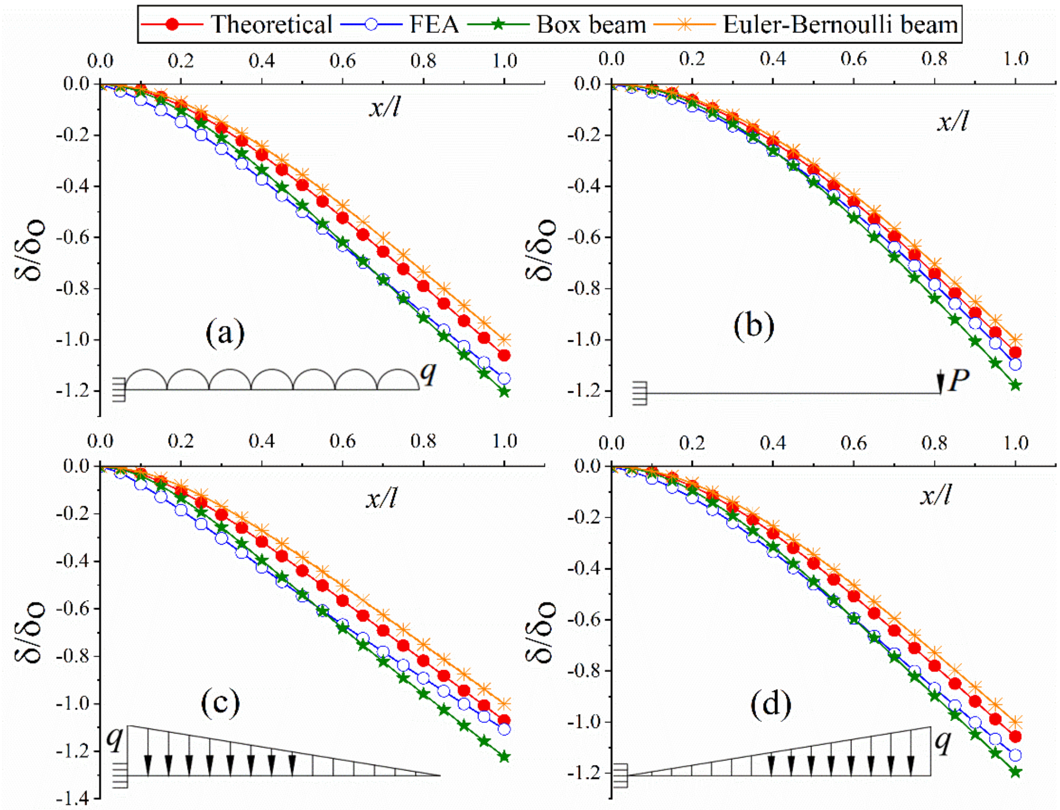


Figure 3. Comparison of the deflection of the box beam with outstanding flange

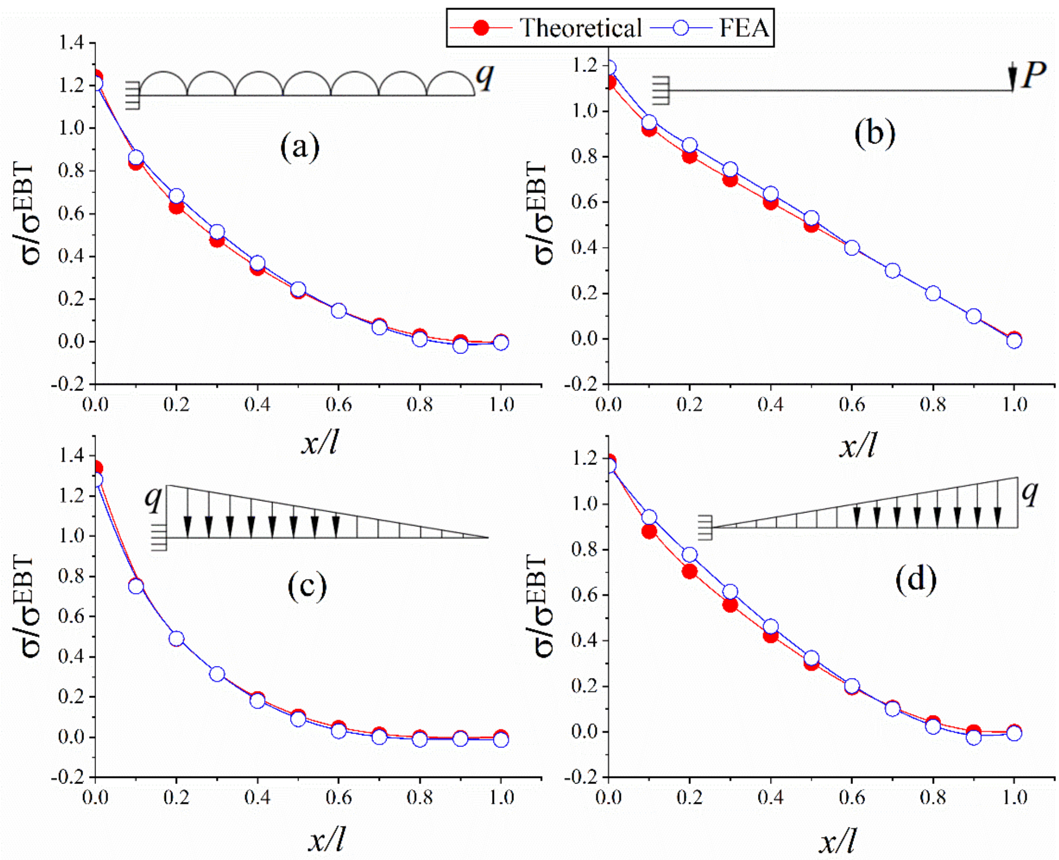


Figure 4. Comparison of the bending stresses along the length at the web-flange junctions

The high shear flow causes less shear deformation in the flange plate, which results in low deflection and low additional stresses. A low additional stress at the fixed end indicates a reduced positive shear lag effect. As a result, the negative shear lag decreased as well.

The stiffness ratio (I_f/I) is constant throughout the study because the transformation does not affect the cross-sectional area. Deflections, stresses, and stress factors are considered for shear lag effects with variation in α from zero to ½. According to Chang and Zheng [8], and Kuzmanovic and Graham [37], the maximum value of α = ½ is used to analyze box girder bridges. The maximum stress factor is denoted as σ^{max}/σ_o, i.e., the maximum stress factor in relation to the stress calculated by EBT. Additionally, the additional stress factor is represented as $\Delta\sigma/\sigma_o$, i.e., the ratio between the stress produced in addition to that obtained by EBT and the stress calculated from EBT. The following sections present a parametric analysis of the variation in deflection and additional deflection, stress factor, and additional stress factor concerning the variation in α.

3.1. Variation in Maximum Deflection

The shear deformation in the flange plate of the box beam produces an additional deflection [8]. This additional deflection attributed to the shear lag effect. A shear lag effect is greatly influenced by such a transformation in the box beam shape. It was found that the transformed box beam was stiffer than the original box beam. For loading cases (a), (b), (c), and (d), the maximum deflections are observed to be reduced by 13.45, 11.68, 14.34, and 13.13 percent, when α varies from zero to ½, respectively (Figure 5). The additional deformation caused by the shear lag effect is the primary concern in the analysis and design. For each loading case, the additional deformation at the cantilever's free end was reduced by 70.34, 69.54, 68.78, and 70.99 percent, respectively.

3.2. Variation in Maximum Stress Factor

In a box beam maximum stress occurs at the web flange junction under transverse loading [1]. For each loading case, the maximum stress is reduced by 14.52, 10.12, 16.68, and 13.26 % in the transformed box beam (Figure 6). Because the transformation does not affect the cross-sectional area of the walls used in the construction of the box beam, the consumption of material is the same as for the primary box beams. Thus, the wall thicknesses in the transformed cantilever box beam can be modified and reduced for the specified loading to get maximum stress equal to obtained in the primary cantilever box beam, thereby enhancing efficiency and economy.

3.3. Additional Stress Factor and Negative Shear Lag

As a result of the flange plate's shear deformation, additional stresses are produced. The variation in additional stresses at the web-flange junction for different loading conditions is depicted in Figure 7. It is evident that the maximum additional stress factors were found to be reduced by 46.81, 50.00, 44.11, and 49.01 %, respectively. It depends on the sign of the additional stress factor whether a negative shear lag will develop at some subsequent distance from the fixed end or not [16]. Except for point loading, the intensity of the negative stress factor increases approximately after one-third of the length from the fixed end for other loading cases.

The additional negative stresses were responsible for the stress reversal in the end quarter length of the box beam [15-16]. When α equals ½, the additional negative stress factor becomes negligible in a transformed cantilever box beam. Approximately negligible negative shear lag effect was observed in this case. Thus, with this type of transformation, the problem of stress reversal is automatically mitigated. Thus, preventing stress reversal further reduced the complexity of the design.

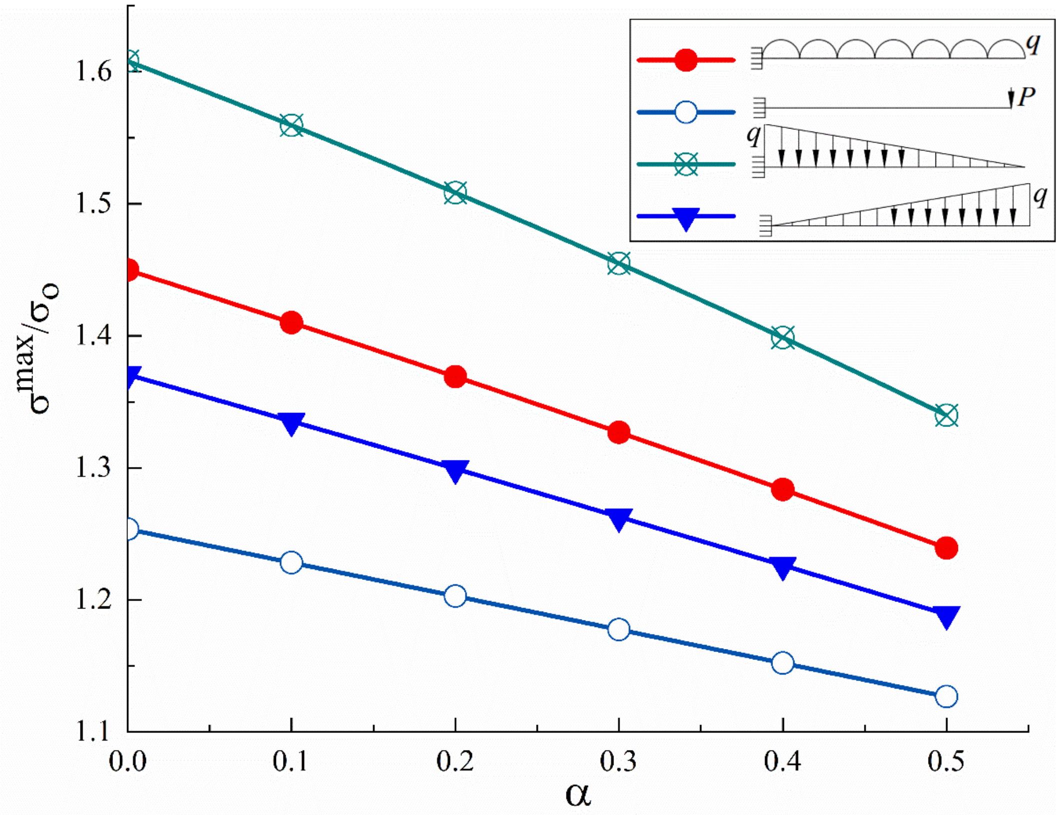


Figure 5. Variation in maximum deflection with respect to ‘α’

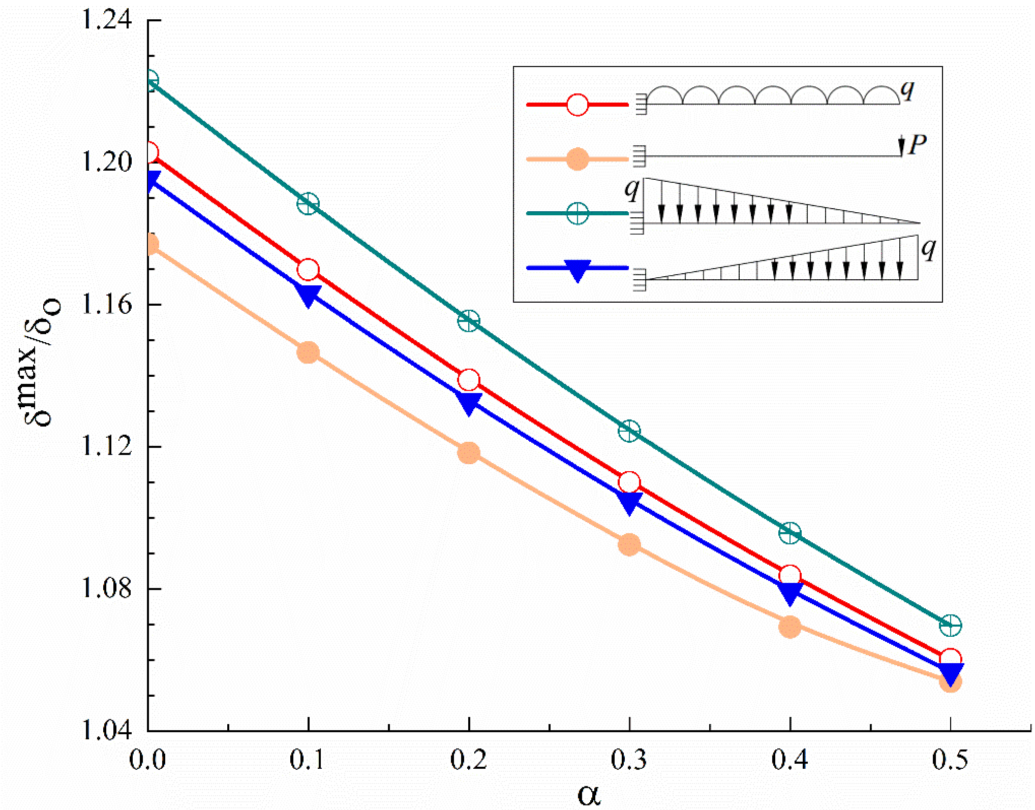


Figure 6. Variation in maximum stress factor with respect to ‘α’

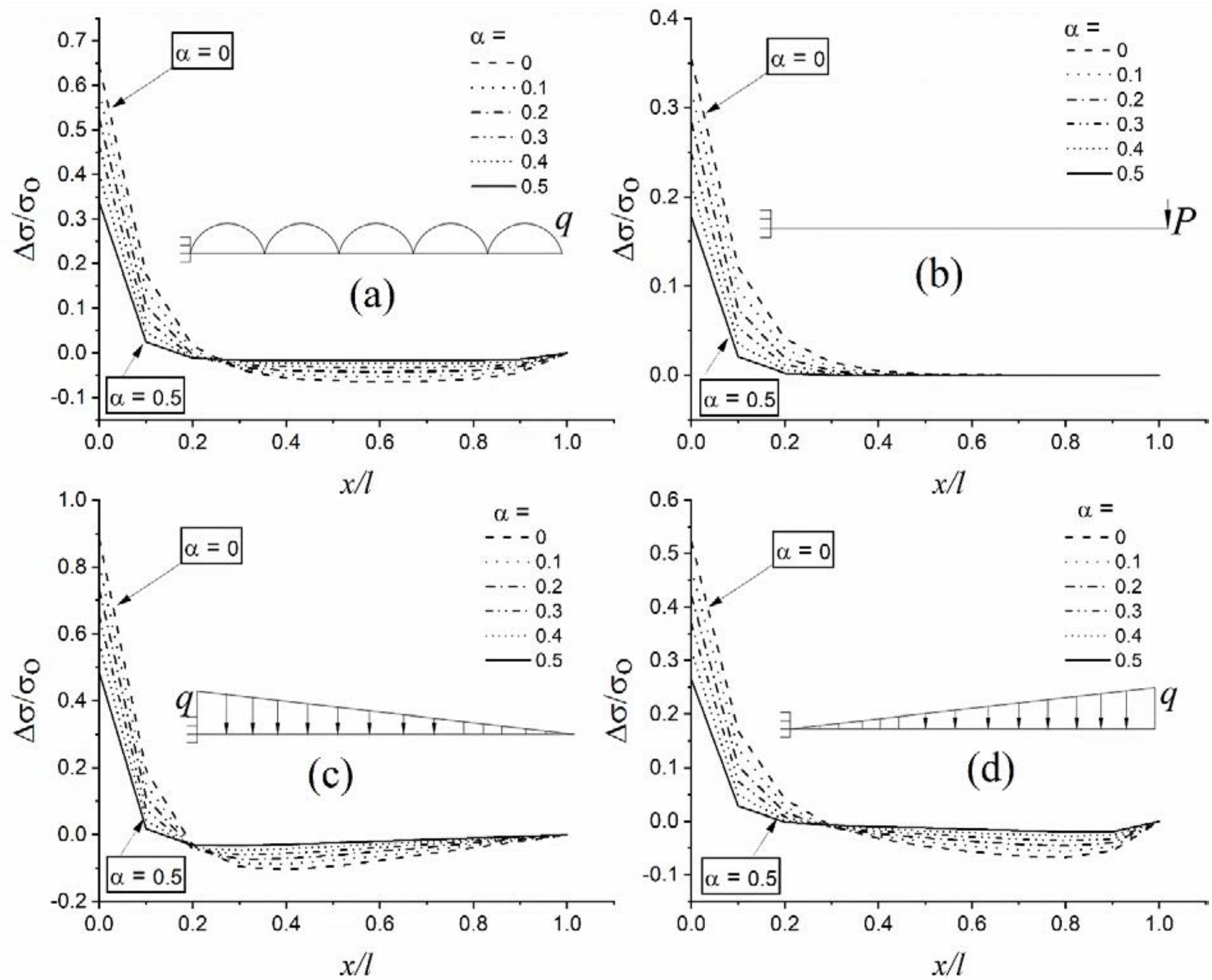


Figure 7. Additional stress factor and negative shear lag regions

4. Applications

This modification can be applied to box beams in the form of box girder bridges or to tubular structures. An extended application of the transformation is to utilize this modification in the frame tube structure. A different assumption of stress function can be used in the flange and web when an orthotropic membrane theory is adapted to the frame tube structure. Tube-in-Tube with single and multiple tubes are generally applied to reduce the lateral deformation in a frame tube structure. The modification demonstrated in the present study is stiffer and more economical than the conventional box-type structure as a core/shear wall or internal frame. This modification may serve as an integral part of tall tubular buildings producing significantly lowered stress concentration in terms of shear lag. The present study is also applicable to strengthening steel box girders. Steel box girders are generally constructed with an outstanding flange at the top. It is possible to extend the bottom plate with the outstanding flange. In order to increase the load-bearing capacity or strengthen the existing steel box girder, the bottom flange can be extended by αw.

It is convenient and straightforward as there is no need to remove parts from the existing girder or alter its dimensions. Add the outstanding flange by welding or bolting it onto the bottom flange to extend it. The maximum stress concentration factor, for example, decreases from 1.19 to 1.12 in the case of a box beam with $\alpha = 0.5$ on the top flange and 0.25 on the bottom flange for uniform loading. This methodology also applies to box girders with different support conditions, such as simply supported, propped cantilever, and continuous box girders.

5. Conclusions

An evaluation of the suitability of transforming a box beam into a box beam with an outstanding flange is presented in this study by considering the shear lag effect. The transformed box beam is found to be stiffer than the primary box beam. This type of transformation reduced the stress concentrations at the web-flange junction and lateral deformations of the beam. Stress reversal can also be prevented by adopting this type of transformation. As a result, the transformed box beam can be used as an economical alternative in tall tubular buildings. Furthermore, this type of transformation is equally applicable to retrofitting steel box girders by increasing the bottom flange width.

Appendix 1: Notations

The following notations are used in this study

E, G	=	Young's modulus, shear modulus
h	=	Distance of a fiber from the neutral axis
h_w	=	Web height
I, I_f, I_w	=	Moment of inertia
k, n	=	Reissner's parameters
l	=	Span length
M(x), M	=	Bending moment
P, q	=	Loading's intensity
t_t, t_b, t_f, and t_w	=	Plate thickness and web width
u (x), u (z) u (x, y)	=	Longitudinal, vertical displacement and spanwise sheet displacement
U(x) or U	=	Shear lag correction
w	=	Half of the flange width
x, y, z	=	Coordinates
x_1, x_2	=	Integration interval
δ, δ^{max}	=	Deflection, and maximum deflection
δ_o	=	Maximum deflection in EBT
ε_x, γ	=	Linear and shear strain
π, π_l, π_w, π_f	=	Potential energy
σ_x, $\Delta\sigma$	=	Bending, and additional bending stress
σ^{max}	=	Maximum bending stress
σ^{EBT}	=	Bending stress in EBT
σ_o	=	Maximum bending stress in EBT

Declarations

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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