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Variety of Approaches for Generating Volume of Aquara-6 Sunflower Seeds

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Abstract Sunflower is a major oilseed crop in the world. Its seeds are applied to extract oil which gives high nutritional value and can be kept longer than other vegetable oils. In this article, we focus on Aquara-6 sunflower in order to study the way for generating volume of seeds on each head. The distinctive features for this type of sunflower are high yield and large flowers. The aims of this study are to present five ways to calculate the volumes of Aquara-6 sunflower seeds for each head and to compare these methods to volume measurement by replacing them with water of all seeds. This is collected from the physical grain, including width, length, and thickness, of each sunflower head in order to prepare data for calculating volumes of five methods. After that they are estimated via five methods from three regions on sunflower heads and on the whole heads. Moreover, we analyze the comparison between the estimated volumes and the actual volumes of fifteen sunflower heads. The results show that the best way to approximate the volume of seeds on each head is the square pyramid's formular and the second way is the circular cylinder's formular. These methods can be alternative ways to generate the volume of sunflower seeds in the future. In addition, this can be helpful to study the moisture content, especially in the drying process.

Keywords Aquara-6, Sunflower Heads, Sunflower Seeds

1 Introduction

Sunflower is one of the main industrial crops in Thailand. Its scientist name is Helianthus Annus L., which is an annual plant. It can be planted easily and well adapted to tropical conditions. It is resistant to dry and hot conditions. In addition, there are two states of sunflower growth: the first, the apical flower formation and the peduncle expansion and the inflorescences are facing the direction of the sun, that is, facing east in the morning and west in the evening. For the second state is seed formation, where the outermost formation of the sunflower enters the center of the sunflower. In this process, the seeds and base of the sunflower are enlarged [1] and the inflorescences are facing only one direction, that is always east. In addition, sunflower seeds are arranged in spirals. The seed size of sunflowers in the outer circle is large and the center area will be smaller, which can classify oriental seeds into three types, namely, the seeds are used for oil extraction, the seeds are edible and the seeds are used for animal husbandry. Oilseed sunflower is the second most economically important oil crop after soybean and oil palm [2], where sunflower seed oil has high nutritional value and can be stored longer than other vegetable oils because sunflower oil has a high value. Therefore, it is needed by both domestic and international markets for consumption and industrial uses such as varnish, lubricant, and paint. In Thailand, there has been an increase in the cultivation of sunflowers as a supplementary occupation. In order to increase the yield to meet the oil crop industry and consumer demand [3, 4], there are many oil sunflower species, such as Aquara-6, Olison-3, Haison-33, etc.

In this research, we focus on Aquara-6 sunflower giving distinctive features which are high yield and large flowers. The objective of this study was to propose of the ways to calculate volume of sunflower seeds for each head.

There were some researches giving ways to calculate sunflower seeds volume. For example, Mirzabe et al. [5] used the geometric mean diameter to find volume of seeds on the head by dividing a sunflower head into three regions. They randomly collected the physical data of each seed on a head in order to obtain the geometric mean diameter. This gives that the volume of seeds on a head increases when the geometric mean diameter increases. In 2016, Malik and Saini [6] presented volume of the seeds and kernels in mm^3 from equivalent diameter using equation given by Özarslan [7]. In addition, Smaniotto et al. [8] studied the effect of the moisture content on the physical properties of sunflower seeds (cultivar Olisun 3) during drying process.

In fact, the size of sunflower seeds is not equal whole head, we will divide each head into three regions. For each region, the size of seeds is quite similar and equal. The sizes of seeds in the outer region are the largest for each head. So based on idea of Mirzabe et al. [5], we calculate volumes with three methods by using physical parts (width, length and thickness) of each seed and using integration of cross section in the middle part of each head to estimate seed volume.

The structure of this paper is as follows. In section 2, we show all methods using here. Numerical results are described in section 3. Section 4 contains our conclusions.

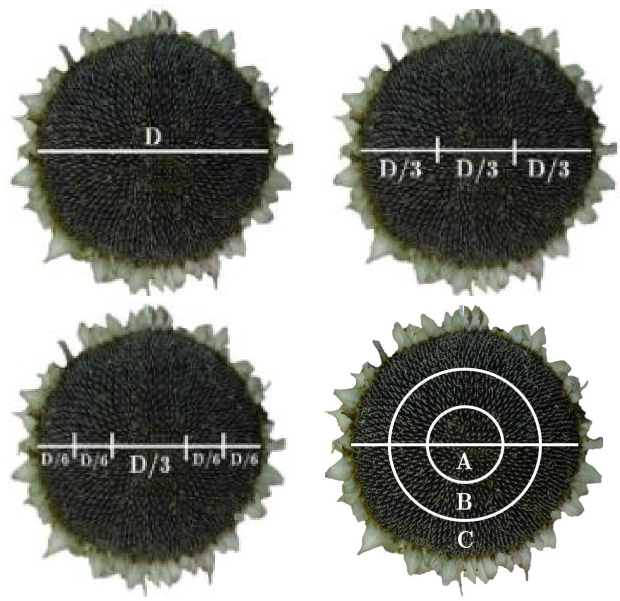

Figure 1. The process to separate each sunflower head into three regions: A, B and C. The first subfigure shows that the sunflower head was measured its diameter, D. The second subfigure presents that each head was divided the length of diameter into three equal parts. Each part had a length with $\frac{D}{3}$. The third subfigure desribes the way that the lengths of two parts, except the middle part, from the second subfigure were split in half. The last subfigure generates three regions: A, B and C using details of the bottom left subfigure.

2 Materials and Methods

In this study, Aquara-6 sunflower seeds were used for all experiments in our research. Here, we will descibe sample preparation and details of three methods for finding seeds volume of each head.

2.1 Sample Preparation

The sunflower heads were provided in February 2016 from the farm of Faculty of Science and Technology, Prince of Songkla University, Pattani Campus, Thailand. The variety of sunflower heads is Aquara-6, which is widely cultivated in Thailand. In addition, the seeds of this variety can produce oil. Thirty different sizes of matured sunflower heads were taken randomly and manually.

According to [5], each head was separated into three regions: A, B and C in Figure 1. To do this, each sunflower head was measured its diameter with four directions, then we generated a mean of diameter, D, for each head. After that, each head was divided the length of diameter into three equal parts. Each part had a length with $\frac{D}{3}$. Next, the lengths of two parts, except the middle part, were split in half as the bottom left subfigure in Figure 1. Finally, each region can be formed on a head as Figure 1. Then, the seeds of each head were extracted from each region with simple random sampling by using Eq.(1):

$$n = \frac{NZ^2s^2}{Ne^2 + Z^2s^2}, \tag{1}$$

where n is the number of sampling sunflowers, N is the number of all sunflowers, e is error measurement of length of diameter of sunflower, s is standard deviation, s^2 is variance, and Z is a significance level. Here, we consider the significance level at 95%, this is $Z = 1.96$ and the error of diameter is 0.5 cm from 40 sunflowers. Then, we put all details in Eq.(1), this gives $n \approx 4$. So, we use the sample size of the sunflower to measure the length of the diameter of each day at least 4 sunflowers.

2.2 Methods for Finding Volume

We propose six ways to find volume (V) of the seeds on each sunflower head. This can be illustrated as follows: a) method of Mirzabe et al. b) square pyramid's formular c) elliptic cylinder's formular d) circular cylinder's formular e) integration of the cross section f) measurement.

2.2.1 Method of Mirzabe et al.

In 2012, Mirzabe et al. [5] studied the physical properties of sunflower seeds and provided the formular for finding the volume of sunflower seeds by using

$$V = \frac{\pi D^3}{6}, \tag{2}$$

where L, W and T are length, width, and thickness of seeds shown in Figure 2, and D is the geometric mean diameter of seed, $D = \sqrt[3]{LWT}$ [6, 7].

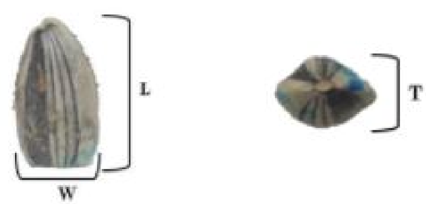

Figure 2. Description of the physical details on each seed with L, W and T, where L is length, W is width and T is thickness.

2.2.2 Square Pyramid's formular

In this method, we find the average length, width, and thinkness of each region in Figure 1 in order to calculate the volume with $V = \frac{1}{6}LW^2$. Each seed could be descibed in Figure 3.

Figure 3. Description for generating the square pyramid part by considering the thickness with square context and the side of the seed with pyramid context.

2.2.3 Elliptic Cylinder's formular

This formular is generated the volume by using $V = \frac{\pi}{4}LWT$ with the average length, width, and thinkness of each region in Figure 1. Each seed could be presented by Figure 4.

Figure 4. Description for generating the elliptic cylinder part by considering the thickness with elliptic context and the side of the seed with cylinder.

2.2.4 Circular Cylinder's formular

For this method, we consider the average length, width, and thinkness of each region in Figure 1 to generate the volume with $V = \frac{\pi}{4}LT^2$. This can be descibed in Figures 2 and 5.

Figure 5. Description for generating the circular cylinder part by considering the thickness with circular context and the side of the seed with cylinder.

2.2.5 Integration of the cross section

In this method, we start with taking a sunflower head to cross section in the middle of the head shown in Figure 6. Then, the first figure in Figure 6 is analyzed along coordinates X and Y with two curves by the program for analyzing points X and Y. Next, all points are constructed by two functions: $f(x)$ and $g(x)$ with a polynomial regression. Then we check accuracy of the functions by a least squares method as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}, \quad (3)$$

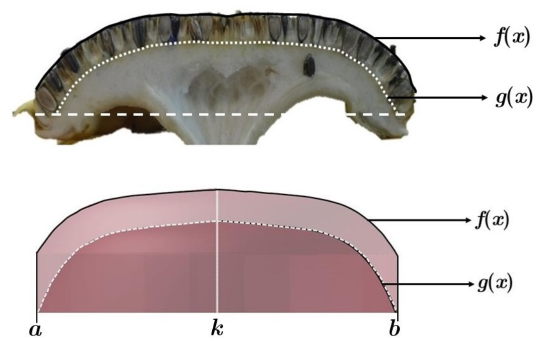

Figure 6. Cross section of a sunflower head. The first subfigure is the actual cross section of the head by constructing all points with two curves and representing with two functions, $f(x)$ and $g(x)$. The second subfigure is a model which represents the first subfigure, where a and b denote a lower bound and a upper bound of both functions, respectively, k is the middle of spindle defined by $\frac{a+b}{2}$.

where R^2 is efficiency of the growing model of sunflowers, y_i is diameter of sunflower head collected in day i, $\hat{y}_i$ is diameter of sunflower head estimated by the model in day i, and $\bar{y}$ is mean of length of diameter of sunflower head collected in day i.
Then, we bring the built-in functions to find volume by rotation of a cylindrical shell. This is defined by

$$V = \int_a^b 2\pi(|k - x|)(f(x) - g(x))dx, \quad (4)$$

where $f(x)$ is the arc on the seed of a sunflower, $g(x)$ is the seed base curve of a sunflower, a is a lower bound of function rotation, b is an upper bound of function rotation, k is a function spindle defined by $\frac{a+b}{2}$, and V is seed volume of a sunflower.
To calculate volume with a function rotation from a to k around the axis $x = k$ and from k to b around the axis $x = k$, the volume generated by the rotation of a cylindrical shell is the mean of the function rotation from a to k and from k to b around the axis $x = k$ shown in Figure 6.

2.2.6 Measurement

Here, the volume of all seeds was replaced by water, then the increased water was taken to find the volume of sunflower seeds with the formula $V = \pi r^2 \Delta h$, where Δh is the height rate of increased water from replacement with sunflower seeds.

3 Numerical Experiments

In this section, we show the numerical results for finding volume of all seeds on each head via six methods mentioned from the previous section. Here, data for length (L), width (W), and thickness (T) of sunflower seeds on each sunflower head by considering in regions A, B and C, respectively, are shown in Table 1.

This is taken 15 sunflowers from 60 sunflowers randomly.

3.1 Method of Mirzabe et al.

For this method, we provide details of length (L), width (W) and thickness (T) for 15 sunflower heads by randomly sampling seeds in Eq.(1). Hence, the number of seeds in each region to measure length, width, and thickness which are 50, 100, and 100 seeds, respectively. Then, we calculate the mean of width, thickness, length of each region in Table 1. Then we calculate the volume of seeds for each region by method of Mirzabe et al. shown in Table 2.

3.2 Square Pyramid's formular

In this method, we calculate the volume of seeds for three regions: A, B, and C with 15 sunflower heads shown in Table 3 based on details from Table 1 by using the formular in section 2.2.2.

3.3 Elliptic Cylinder Method

We use the mean of width, thickness, length of each region in Table 1 to calculate the volume of seeds for each region by this method shown in Table 4.

3.4 Circular Cylinder's formular

Here, the mean of width, thickness, length of each region in Table 1 are provided to generate the volume of seeds for each region by this method as shown in Table 5.

3.5 Integration of the cross section

In this part, we start by creating 2 functions: $f(x)$ and $g(x)$ (see in Figure 6). Then we consider the area between $f(x)$ and $g(x)$. Both functions of each sunflower are presented in Table 6.

3.6 Measurement

Here, we find the volume of all seeds in each sunflower head by replacing them with water. So all details of them can be shown in Table 7. This table shows that volume of seeds on each region and on each head.

In addition, we also give volumes on each region with four methods in Table 8. This shows that the volume of seeds on region A for most sunflower heads can be approximated by Mirzabe et al.'s method. While the volume on region B for most sunflower heads can be approximated by circular cylinder's formular, and the volume on region C for most sunflower heads can be approximated by square pyramid's formular.

Here, we show the average volumes for all seeds on each head with 15 sunflowers by using six different methods in Table 9.

Moreover, we compare volumes from other methods with this methods shown in Table 10.

From Table 10, we can see that there are two methods: square pyramid's formular and circular cylinder's formular, to give the least differences from 15 sunflower heads. This implies that two methods can be the alternative ways to approximate the volume of sunflower seeds of each head. For other three methods, this can adjust and add some conditions to improve for calculating volume of all seeds of each head.

4 Conclusions

In this study, we have proposed five ways to calculate the volume of seeds on each sunflower head by describing the nature of each method and generating volumes of seeds with each region and whole head. The results show that the best way to approximate volume of all seeds on each head is square pyramid's formular and the second way is circular cylinder's formular. In the future, some methods in this research can be reformed and reorganized to aprroximate volume seeds.

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Table 1. Data for presenting mean of width (W), thickness (T), and length (L) of seeds on 15 sunflower heads for each region.

i	W_{A_i}	T_{A_i}	L_{A_i}	W_{B_i}	T_{B_i}	L_{B_i}	W_{C_i}	T_{C_i}	L_{C_i}
1	4	2.157	9.677	5.34	2.984	10.709	5.338	3.271	10.941
2	3.929	2.636	10.111	4.9065	2.8135	10.318	5.4045	3.247	10.7205
3	4.198	2.263	10.416	5.4055	2.892	10.8585	6.3525	4.1469	11.4375
4	4.423	2.461	10.479	5.44	3.123	10.6865	5.7795	3.5275	10.8185
5	4.157	2.496	9.701	4.8745	2.8035	10.0945	5.5195	3.5555	10.695
6	3.937	2.264	10.019	5.3275	2.861	10.729	5.906	3.5275	11.118
7	3.43	2.318	9.219	4.4565	2.5945	9.5885	4.875	3.0605	10.0215
8	4.087	2.828	10.122	5.487	3.1295	10.6405	5.3795	3.265	10.5715
9	4.164	2.773	9.918	4.957	2.9905	9.949	5.225	3.279	9.9705
10	4.215	2.9	9.444	4.3455	2.3745	9.2435	4.593	2.784	9.5655
11	4.034	2.621	9.215	5.062	2.838	9.6725	5.5565	3.5545	9.9339
12	3.95	2.1014	9.4557	4.696	2.3959	9.5959	5.0235	2.73	10.127
13	4.673	2.689	10.827	5.256	2.954	10.5185	5.5205	3.442	10.716
14	5.236	2.827	12.992	6.742	3.607	13.544	7.924	5.172	14.266
15	4.1077	2.214	10.192	5.2893	2.8298	10.625	6.216	4.058	11.192

Table 2. Volume of seeds for each region (A, B and C) by using method of Mirzabe et al.

i	V_{A_i} (mm^3)	V_{B_i} (mm^3)	V_{C_i} (mm^3)
1	11,361	67,245	107,774
2	15,400	47,110	43,615
3	13,257	49,836	110,692
4	13,133	53,208	89,455
5	8,586.3	33,858	69,968
6	13,273	60,420	99,641
7	10,817	44,385	71,989
8	16,531	81,086	100,476
9	14,024	47,004	99,231
10	11,841	29,799	44,551
11	9,535	32,724	66,227
12	9693.9	35,369	61,344
13	15,521	40,170	52,961
14	16,536	62,161	138,067
15	12,850	52,063	87,531

Table 3. Seed volume for each region using the square pyramid's formular.

i	V_{A_i} (mm^3)	V_{B_i} (mm^3)	V_{C_i} (mm^3)
1	3,618	21,416	34,323
2	4,904.3	15,003	13,890
3	4,222	15,871	35,252
4	4,182.3	16,945	28,489
5	2,734.5	10,783	22,283
6	4,227	19,242	31,733
7	3,445	14,135	22,926
8	5,264.6	25,824	31,999
9	4,466.3	14,970	31,602
10	3,771	9,490.1	14,188
11	3,036.6	10,422	21,092
12	3,087.2	11,264	19,536
13	4,943.1	12,793	16,867
14	5,266.1	19,797	43,970
15	4,092.4	16,581	27,876

Table 4. Volume of seeds for each region (A, B and C) using the elliptic cylinder method.

i	V_{A_i} (mm^3)	V_{B_i} (mm^3)	V_{C_i} (mm^3)
1	17,041	100,868	161,661
2	23,099	70,664	65,422
3	19,886	74,754	166,037
4	19,699	79,811	134,182
5	12,880	50,788	104,952
6	19,909	90,631	149,461
7	16,226	66,578	107,983
8	24,796	121,629	150,713
9	21,036	70,507	148,846
10	17,761	44,698	66,827
11	14,302	49,086	99,341
12	14,541	53,053	92,016
13	23,282	60,254	79,442
14	24,804	93,242	207,101
15	19,275	78,095	131,297

Table 5. Volume of seeds for each region (A, B and C) using the circular cylinder's formular.

i	V_{A_i} (mm^3)	V_{B_i} (mm^3)	V_{C_i} (mm^3)
1	9,189.3	56,365	99,062
2	15,498	40,521	39,306
3	10,720	39,994	108,389
4	10,961	45,818	81,898
5	7,733.3	29,210	67,607
6	11,449	48,671	89,269
7	10,966	38,761	67,792
8	17,158	69,371	91,473
9	14,009	42,536	93,410
10	12,220	24,424	40,507
11	9,292.7	27,520	63,549
12	7,735.8	27,068	50,006
13	13,397	33,864	49,531
14	13,371	49,886	135,195
15	10,725	44,833	80,137

Table 6. Functions $f(x)$ and $g(x)$ of 15 sunflower heads shown in Figure 6.

1	$f(x) = -2.7202 \times 10^{-13}x^6 + 5.8406 \times 10^{-10}x^5 - 5.03888 \times 10^{-7}x^4 + 2.2107 \times 10^{-4}x^3 - 5.1693 \times 10^{-2}x^2 + 6.2051x - 383.8853$ $g(x) = -5.5489 \times 10^{-13}x^6 + 1.1604 \times 10^{-9}x^5 - 9.7876 \times 10^{-7}x^4 + 4.237 \times 10^{-4}x^3 - 9.9129 \times 10^{-2}x^2 + 12.0384x - 717.0774$
2	$f(x) = -1.8048 \times 10^{-13}x^6 + 4.0465 \times 10^{-10}x^5 - 3.5507 \times 10^{-7}x^4 + 1.5394 \times 10^{-4}x^3 - 3.5178 \times 10^{-2}x^2 + 4.3537x - 307.0044$ $g(x) = -4.9822 \times 10^{-13}x^6 + 1.10696 \times 10^{-9}x^5 - 9.6972 \times 10^{-7}x^4 + 4.2446 \times 10^{-4}x^3 - 9.80416 \times 10^{-2}x^2 + 11.7064x - 701.2484$
3	$f(x) = -8.7909 \times 10^{-14}x^6 + 1.9691 \times 10^{-10}x^5 - 1.8421 \times 10^{-7}x^4 + 9.0521 \times 10^{-5}x^3 - 2.438 \times 10^{-2}x^2 + 3.4842x - 266.6201$ $g(x) = -2.8352 \times 10^{-13}x^6 + 6.2945 \times 10^{-10}x^5 - 5.6777 \times 10^{-7}x^4 + 2.6354 \times 10^{-4}x^3 - 6.5941 \times 10^{-2}x^2 + 8.5148x - 553.4464$
4	$f(x) = -4.0407 \times 10^{-14}x^6 + 7.5688 \times 10^{-11}x^5 - 6.6464 \times 10^{-8}x^4 + 3.5248 \times 10^{-5}x^3 - 1.1357 \times 10^{-2}x^2 + 2.0101x - 187.9788$ $g(x) = -1.3138 \times 10^{-13}x^6 + 2.5228 \times 10^{-10}x^5 - 2.0223 \times 10^{-7}x^4 + 8.75135 \times 10^{-5}x^3 - 2.2041 \times 10^{-2}x^2 + 3.1757x - 289.2981$
5	$f(x) = -6.1148 \times 10^{-14}x^6 + 1.4702 \times 10^{-10}x^5 - 1.5017 \times 10^{-7}x^4 + 8.0758 \times 10^{-5}x^3 - 2.3843 \times 10^{-2}x^2 + 3.7254x - 303.0589$ $g(x) = -1.2789 \times 10^{-13}x^6 + 2.946 \times 10^{-10}x^5 - 2.8486 \times 10^{-7}x^4 + 1.4532 \times 10^{-4}x^3 - 4.0981 \times 10^{-2}x^2 + 6.1272x - 486.9794$
6	$f(x) = -1.7252 \times 10^{-13}x^6 + 3.9007 \times 10^{-10}x^5 - 3.5263 \times 10^{-7}x^4 + 1.6160 \times 10^{-4}x^3 - 3.9795 \times 10^{-2}x^2 + 5.1436x - 321.8957$ $g(x) = -2.951 \times 10^{-13}x^6 + 6.7181 \times 10^{-10}x^5 - 6.1594 \times 10^{-7}x^4 + 2.8943 \times 10^{-4}x^3 - 7.3859 \times 10^{-2}x^2 + 9.8739x - 627.3513$
7	$f(x) = -4.1557 \times 10^{-14}x^6 + 8.4893 \times 10^{-11}x^5 - 7.7362 \times 10^{-8}x^4 + 4.001 \times 10^{-5}x^3 - 1.2213 \times 10^{-2}x^2 + 2.126x - 216.9726$ $g(x) = -1.8641 \times 10^{-13}x^6 + 3.9740 \times 10^{-10}x^5 - 3.4850 \times 10^{-7}x^4 + 1.6033 \times 10^{-4}x^3 - 4.0821 \times 10^{-2}x^2 + 5.5732x - 425.9794$
8	$f(x) = -1.9603 \times 10^{-13}x^6 + 4.4678 \times 10^{-10}x^5 - 4.0381 \times 10^{-7}x^4 + 1.8277 \times 10^{-4}x^3 - 4.4017 \times 10^{-2}x^2 + 5.6857x - 384.3529$ $g(x) = -3.7901 \times 10^{-13}x^6 + 8.4233 \times 10^{-10}x^5 - 7.4947 \times 10^{-7}x^4 + 3.3889 \times 10^{-4}x^3 - 8.2509 \times 10^{-2}x^2 + 10.6154x - 678.7668$
9	$f(x) = -1.8520 \times 10^{-13}x^6 + 3.9527 \times 10^{-10}x^5 - 3.3979 \times 10^{-7}x^4 + 1.5029 \times 10^{-4}x^3 - 3.6671 \times 10^{-2}x^2 + 4.910x - 339.3038$ $g(x) = -3.6628 \times 10^{-13}x^6 + 7.6895 \times 10^{-10}x^5 - 6.5472 \times 10^{-7}x^4 + 2.8921 \times 10^{-4}x^3 - 7.050 \times 10^{-2}x^2 + 9.2025x - 599.6995$
10	$f(x) = -8.9131 \times 10^{-14}x^6 + 1.9470 \times 10^{-10}x^5 - 1.7847 \times 10^{-7}x^4 + 8.7336 \times 10^{-5}x^3 - 2.4087 \times 10^{-2}x^2 + 3.6049x - 291.6633$ $g(x) = -3.2661 \times 10^{-13}x^6 + 7.0282 \times 10^{-10}x^5 - 6.1513 \times 10^{-7}x^4 + 2.7891 \times 10^{-4}x^3 - 6.9109 \times 10^{-2}x^2 + 8.9959x - 599.3305$
11	$f(x) = -5.1693 \times 10^{-14}x^6 + 1.1622 \times 10^{-10}x^5 - 1.0937 \times 10^{-7}x^4 + 5.4683 \times 10^{-5}x^3 - 1.5794 \times 10^{-2}x^2 + 2.6902x - 262.1396$ $g(x) = -2.3634 \times 10^{-13}x^6 + 5.2865 \times 10^{-10}x^5 - 4.7677 \times 10^{-7}x^4 + 2.2088 \times 10^{-4}x^3 - 5.5978 \times 10^{-2}x^2 + 7.6471x - 555.7966$
12	$f(x) = -2.0292 \times 10^{-14}x^6 + 3.5852 \times 10^{-11}x^5 - 3.3562 \times 10^{-8}x^4 + 2.1664 \times 10^{-5}x^3 - 8.8542 \times 10^{-3}x^2 + 1.9115x - 213.8425$ $g(x) = -1.0793 \times 10^{-13}x^6 + 2.091 \times 10^{-10}x^5 - 1.7113 \times 10^{-7}x^4 + 7.7897 \times 10^{-5}x^3 - 2.1465 \times 10^{-2}x^2 + 3.4316x - 334.3719$
13	$f(x) = -3.9080 \times 10^{-15}x^6 + 2.0097 \times 10^{-11}x^5 - 3.9438 \times 10^{-8}x^4 + 3.2552 \times 10^{-5}x^3 - 1.3027 \times 10^{-2}x^2 + 2.5670x - 245.3833$ $g(x) = -1.3761 \times 10^{-13}x^6 + 3.1063 \times 10^{-10}x^5 - 2.8776 \times 10^{-7}x^4 + 1.3775 \times 10^{-4}x^3 - 3.6127 \times 10^{-2}x^2 + 5.0728x - 396.1284$
14	$f(x) = -1.046 \times 10^{-13}x^6 + 2.1220 \times 10^{-10}x^5 - 1.8375 \times 10^{-7}x^4 + 8.7922 \times 10^{-5}x^3 - 2.4533 \times 10^{-2}x^2 + 3.7599x - 265.4604$ $g(x) = -2.8329 \times 10^{-13}x^6 + 5.8231 \times 10^{-10}x^5 - 4.9121 \times 10^{-7}x^4 + 2.1829 \times 10^{-4}x^3 - 5.4078 \times 10^{-2}x^2 + 7.1213x - 450.4441$
15	$f(x) = -2.5019 \times 10^{-13}x^6 + 5.4125 \times 10^{-10}x^5 - 4.5667 \times 10^{-7}x^4 + 1.8971 \times 10^{-4}x^3 - 4.1302 \times 10^{-2}x^2 + 4.8133x - 315.4945$ $g(x) = -5.9521 \times 10^{-13}x^6 + 1.2499 \times 10^{-9}x^5 - 1.0326 \times 10^{-6}x^4 + 4.2637 \times 10^{-4}x^3 - 9.3291 \times 10^{-2}x^2 + 10.6795x - 636.7752$

Table 7. Diameters (D) on each head, and volumes of all seeds on regions A, B and C (V_A, V_B and V_C), respectively.

i	D_i (cm)	V_{A_i} (mm^3)	V_{B_i} (mm^3)	V_{C_i} (mm^3)	V_i (mm^3)
1	22	22,012.56	52,830.14	79,245.2	154,087.9
2	19.2	13,207.53	22,012.56	39,622.6	74,842.69
3	18.6	13,207.53	35,220.09	52,830.14	101,257.8
4	20.4	8,805.023	39,622.6	57,232.65	105,660.3
5	17.4	13,207.53	17,610.05	39,622.6	70,440.18
6	20.8	13,207.53	35,220.09	48,427.62	96,855.25
7	18	13,207.53	37,421.35	52,830.14	103,459
8	21	17,610.05	46,226.37	68,238.92	132,075.3
9	20.4	15,408.79	37,421.35	74,842.69	127,672.8
10	15	8,805.023	26,415.07	33,018.83	68,238.92
11	15.6	10,125.78	23,333.31	40,503.1	73,962.19
12	16.6	14,528.29	28,616.32	34,779.84	77,924.45
13	16.8	11,886.78	21,132.05	24,654.06	57,672.9
14	23.2	8,615.667	17,610.05	22,012.56	57,672.9
15	19.8	22,012.56	52,830.14	79,245.2	154,087.9

Table 8. The average volume (mm^3) of each seed on each region with four methods: Mirzabe et al.'s method (V_M), square pyramid's formular (V_S), elliptic cylinder's formular (V_E), circular cylinder's formular (V_C).

i	V_{M_A}	V_{M_B}	V_{M_C}	V_{S_A}	V_{S_B}	V_{S_C}	V_{E_A}	V_{E_B}	V_{E_C}	V_{C_A}	V_{C_B}	V_{C_C}
1	43.695	89.303	99.976	25.805	50.896	51.959	65.542	133.96	149.96	35.344	74.854	91.894
2	54.802	74.541	98.454	26.014	41.399	52.189	82.204	111.81	147.68	55.151	64.115	88.726
3	51.785	88.835	157.68	30.594	52.88	76.925	77.678	133.252	236.521	41.874	71.291	154.4
4	59.694	95.013	115.426	34.167	52.709	60.228	89.54	142.52	173.139	49.821	81.818	105.675
5	52.677	72.193	109.84	27.94	39.976	54.304	79.015	108.289	164.76	47.443	62.281	106.133
6	46.735	85.581	121.22	25.882	50.752	64.634	70.103	128.372	181.826	40.313	68.939	108.6
7	38.359	58.02	78.249	18.077	31.739	39.695	57.539	87.03	117.373	38.885	50.667	73.686
8	61.225	95.62	97.172	28.179	53.393	50.988	91.838	143.431	145.758	63.547	81.805	88.465
9	59.933	77.183	89.397	28.661	40.744	45.367	89.899	115.774	134.096	59.868	69.845	84.153
10	60.413	49.915	64.011	27.964	29.091	33.632	90.619	74.872	96.016	62.348	40.912	58.199
11	50.989	72.72	102.678	24.993	41.308	51.118	76.484	109.079	154.017	49.693	61.155	98.525
12	41.076	56.5	72.682	24.589	35.267	42.593	61.614	84.749	109.023	32.779	43.24	59.248
13	71.199	85.467	106.562	39.405	48.43	54.43	106.798	128.2	159.842	61.455	72.052	99.661
14	100.492	172.388	305.987	59.369	102.616	149.277	150.738	258.582	458.98	81.258	138.344	299.622
15	48.516	83.226	147.725	28.662	49.541	72.068	72.773	124.839	221.587	39.23	66.79	144.652

Table 9. The average volumes on whole head (mm^3) from six methods: Mirzabe et al.'s method (V_M), square pyramid's formular (V_S), elliptic cylinder's formular (V_E), circular cylinder's formular (V_C), integration of the cross section (V_I), and method by replacing by water (V_R).

i	V_M	V_S	V_E	V_C	V_I	V_R
1	186,380	101,046	279,570	164,616	201,867	154,088
2	106,124	56,593.5	159,186	95,323.6	168,325	74,842.7
3	173,785	91499.2	260,677	159,103	154,768	101,258
4	155,795	83,710	233,693	138,677	257,360	105,660
5	112,413	57,894.1	168,619	104,550	149,201	70,440.2
6	173,334	96,311	260,001	149,389	264,579	96,855.2
7	127,191	65,896.6	190,787	117,517	158,677	103,459
8	198,092	105,607	297,139	178,002	157,573	132,075
9	160,259	81,877.1	240,389	149,955	186,849	127,673
10	86,191.3	46,256.2	129,287	77,151.2	24,043.1	68,238.9
11	108,486	56,233.1	162,729	100,361	125,675	73,962.2
12	106,406	63828.9	159,610	84,809.6	136,197	77,924.4
13	108,652	58,404	162,978	96,793	131,803	57,672.9
14	216,764	69,033.1	325,146	198,451	283,643	126,300
15	152,445	48,549.2	228,667	135,694	242,074	103,388

Table 10. Comparing volumes (mm^3) from Mirzabe et al.'s method (V_M), square pyramid's formular (V_S), elliptic cylinder's formular (V_E), circular cylinder's formular (V_C), integration of the cross section (V_I) with measurement by replacing water (V_R).

i	$\lvert V_M - V_R \rvert$	$\lvert V_S - V_R \rvert$	$\lvert V_E - V_R \rvert$	$\lvert V_C - V_R \rvert$	$\lvert V_I - V_R \rvert$
1	32,291.9	53,042	125,482	10,528.5	47,778.7
2	31,281.3	18,249.2	84,343.4	20,480.9	93,482.6
3	72,527.2	9,758.56	159,420	57,845.2	53,510.7
4	50,134.7	21,950.3	128,032	33,016.3	151,700
5	41,972.5	12,546	98,178.8	34,109.8	78,760.9
6	76,478.8	544.296	163,146	52,533.9	167,724
7	23,732.4	37,562.4	87,328.2	14,058.4	55,217.7
8	66,017.1	26,468.4	165,063	45,926.3	25,498.2
9	32,586.4	45,795.7	112,716	22,281.8	59,176.5
10	17,952.3	21,982.7	61,048	8,912.24	44,195.8
11	34,524	17,729.1	88,767.1	26,398.9	51,712.9
12	28,482	14,095.5	81,685.2	6,885.13	58,272.9
13	50,979	731.08	105,305	39,120.1	74,130.4
14	90,464	57,266.9	198,846	72,151	157,343
15	49,056.6	54,838.8	125,279	32,306.3	138,686