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Mathematical Analysis of Priority Bi-serial Queue Network Model

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Abstract One of the most comprehensive theories of stochastic models is queueing theory. Through innovative analytical research with broad applicability, advanced theoretical models are being developed. In the present research, we would like to investigate at a queuing network model with low and high priority users and different server transition probabilities. The two service channels used in this study, S 1 and S 2, are connected to the same server, S 3. Customers with low and high priorities are invited by the server $Si(i = 1, 2)$.The objective of the research is to design a model that helps in minimizing congestion in different systems.Poisson distribution is used to characterize both the arrival and service patterns.The functioning of this system takes place in a stochastic domain. The differential difference equations have been established, and the consistency of behaviour of the system has been examined. The generating function approach, the law of calculus, and a statistical formula are used to assess the model's performance. Numerical analyses and graphical presentations are used to show the model's outcomes.The results of the model are displayed graphically and through numerical analyses. This model can be used in a number of real situations, including administration, manufacturing, hospitals, banking systems, etc. In such situations, the present study is quite beneficial for understanding the system and redesigning it.

Keywords Priority, Bi-series Channel, Graphical Analyze, Queue Length, Stochastic Environment

1 Introduction

People are frequently confronted with the injunction of delays. In our crowded and urbanised society, this tendency is becoming progressively effective. Queues are naturally irritating because they waste time, money, and resources. There is a long history of Queuing theory, and researchers have solved the majority of practical problems. Jackson mentions [1] looked on the behaviour of queues with phase-type service and came up with a steady-state solution.O'Brien [2]worked on queuing problems and their solutions. Reich [3] looked on the distribution of waiting times when users go to a second server after being processed on the first. Suzuki [4] investigated the probability law of queue size and waiting time at the second counter in the steady state when two queues are connected in series. The model's transient behaviour in a stochastic domain was described by Parbhu [5]. In queuing theory, Maggu [6]developed the idea of bi-series queuing systems. Maggu [7] described a transient study of serial queues under service parameter restrictions.The parallel bi-series queues have been studied, and the queue network's spontaneous behaviour has been evaluated.Hafiz Noor Mohammad [8]introduced the concept of batch arrival on bi-series queues at each server. Later on,Singh T.P. [9–11]have been studied transient behavoiur of biseries and parallel queues and introduced the idea of feedback on queues.Queue models in fuzzy environment was analysed by Seema [12].Bi-tandem queue network with feedback was studied by Reeta Bhardwaj [13] Mittal Meenu [14, 15],Threshold effect and the priority queue model along the intermediate queue and its applications in a fuzzy environment.The modelling of a queuing network with biserial bulk arrivals linked to a shared server was explored by Mittal Meenu [16]. We present a bi-series queue network model in this research, with varying shifting transition probabilities of low and high priority users from one server towards the next. The bi-series service consulted an endless number of low and high priority consumers. In a stochastic domain, the model's performance is assessed.

2 Assumptions of Model

1.Arrival and Departure both follow Poisson Law.
2.All activities performed in stochastic environment.
3.Customers with a high priority are given special treatment throughout the system.
4.High-priority visitors, such as women,senior citizen, and VIPs.
5.The term "low priority customers" relates to the overall population.
6.High and low priority visitors have different transition probabilities from one server to the other.

3 Practical Application of the Model

The approach is used in banking, retail malls, manufacturing, hospitals, registration processes, and administration, among other places. Customers seeking a single transaction at a banking system's proceed to the cash counter for payment after completing the essential phase completion service at the respective service counter.

4 Description of the Model

In this queue network model there are three service channels $S_1, S_2\&S_3$.The server $S_1\&S_2$ are in Bi-Series relation. Low and high priority customers arrive at server $S_i(i=1,2)$ with arrival ratesλ_{iL} and λ_{iH} and service rateμ_{iL}& μ_{iH} respectively. The service channel S_3 is common server where service rate is same for all customers linked to bi-series service channel $S_i(i=1,2)$.
After acquiring service at $S1$, High Priority Customers may either proceed to server $S2$ for service with probability α_{12} or proceed to common server S_3 with probability α_{13}, where α_{12}+ α_{13}=1.After receiving service from server S_2, High Priority customers may move to server S_1 with probability α_{21} or server S_3 with probability α_{23} for service, with the condition that α_{21}+α_{23}=1.
Low Priority Customers,after acquiring service at S_1,decide whether to go to server S_2 for service with probability α'_{12} or server S_3 for service with probability α'_{13},where α'_{12}+ α'_{13}=1.After getting service from server S_2,low priority customers may either go to server S_1 with probability α'_{21} for service or proceed to server S_3 with probability α'_{23} for service with the condition that $\alpha'_{21}+\alpha'_{23}$=1.Finally,after acquiring quality service,the customer quits the system.

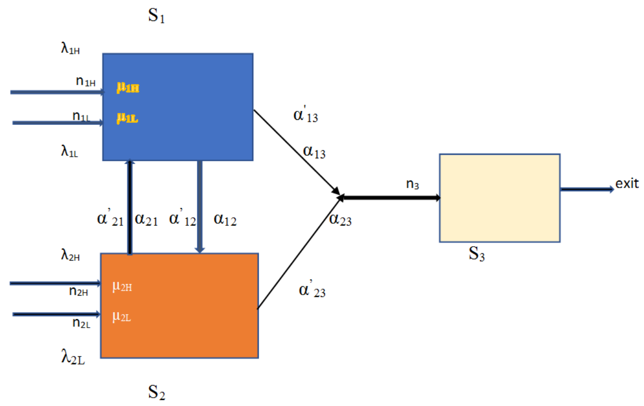

Figure 1. Priority Bi-serial Queue Network Model.

5 Notations

λ_{1L}: Lower priority customers' average arrival rate at S_1
λ_{1H}: Higher priority customers' average arrival rate at S_1
λ_{2L}: Lower priority customers' average arrival rate at S_2
λ_{2H}: Higher priority customers' average arrival rate at S_2
μ_{1L}:The service rate for customers that aren't high on the priority list atS_1
μ_{1H}:High-priority customers' service rate atS_1
μ_{2L}:The service rate for customers that aren't high on the priority list atS_2
μ_{2H}:High-priority customers' service rate atS_2
μ_3: The rate of service at server S_3
α_{12}: moving probability of high priority customer from S_1 to S_2
α_{13}:shifting overall possibility of a high-priority visitor from S_1 to S_3
α_{21}: shifting overall possibility of a high-priority visitor from S_2 to S_1
α_{23}: shifting overall possibility of a high-priority visitor fromS_2 to S_3
α'_{12}: moving probability of low priority customer from S_1 to S_2
α'_{13}: moving probability of low priority customer from S_1 to S_3
α'_{21}: moving probability of low priority customer from S_2 to S_1
α'_{23}: moving probability of low priority customer from S_2 to S_3
n_{1L}: customers waiting in a low priority line at S_1
n_{1H}: customers waiting in a high priority line at S_1
n_{2L}: customers waiting in a low priority line at S_2
n_{2H}: customers waiting in a high priority line at S_2
n_3: customers in queue at S_3

6 Formation of Steady-State Equations

Define $P_{n_{1H},n_{1L},n_{2H},n_{2L},n_3}(t)$ as probability having $(n_{1L},n_{1H}),(n_{2L},n_{2H}),n_3$ calling customers in queues $(Q_1,Q_2),(Q_3,Q_4),Q_5$ in front of servers S_1,S_2,S_3respectively

Differential Difference equation in steady-state form is defined as

$(\lambda_{1H}+\lambda_{1L}+\lambda_{2H}+\lambda_{2L}+\mu_{1H}+\mu_{2H}+\mu_3)P_{n_{1H},n_{1L},n_{2H},n_{2L},n_3}$ $=\lambda_{1H}P_{n_{1H-1},n_{1L},n_{2H},n_{2L},n_3} + \lambda_{1L}P_{n_{1H},n_{1L-1},n_{2H},n_{2L},n_3} + \lambda_{2H}P_{n_{1H},n_{1L},n_{2H-1},n_{2L},n_3} + \lambda_{2L}P_{n_{1H},n_{1L},n_{2H},n_{2L-1},n_3} + \mu_{1H}\alpha_{12}P_{n_{1H+1},n_{1L},n_{2H-1},n_{2L},n_3} + \mu_{1H}\alpha_{13}P_{n_{1H+1},n_{1L},n_{2H},n_{2L},n_{3-1}} + \mu_{2H}\alpha_{21}P_{n_{1H-1},n_{1L},n_{2H+1},n_{2L},n_3} + \mu_{2H}\alpha_{23}P_{n_{1H},n_{1L},n_{2H+1},n_{2L},n_{3-1}} + \mu_3P_{n_{1H},n_{1L},n_{2H},n_{2L},n_{3+1}}$ (A_1)

7 Steady-State Solution

Taking all possible combinations of $n_{1H}, n_{1L}, n_{2H}, n_{2L}, n_3$, 32 steady state equations obtained. To solve the steady state equations (A_1) to (A_{32}), introduce the generating function and its partial generating functions as follows:-

$$G(Y_1,Y_2,Y_3,Y_4,Y_5) = \sum_{\eta_{1H}=0}^{\infty}\sum_{\eta_{1L}=0}^{\infty}\sum_{\eta_{2H}=0}^{\infty}\sum_{\eta_{2L}=0}^{\infty}\sum_{\eta_3=0}^{\infty} Y_1^{\eta_{1H}} Y_2^{\eta_{1L}}Y_3^{\eta_{2H}}Y_4^{\eta_{2L}}Y_5^{\eta_3}P_{\eta_{1H},\eta_{1L},\eta_{2H},\eta_{2L},\eta_3} \tag{1}$$

$$G_{\eta_{1L},\eta_{2H},\eta_{2L},\eta_3}(Y_1) = \Big[\sum_{\eta_{1H}=0}^{\infty} P_{\eta_{1H},\eta_{1L},\eta_{2H},\eta_{2L},\eta_3}Y_1^{\eta_{1H}}\Big] \tag{2}$$

$$G_{\eta_{2H},\eta_{2L},\eta_3}(Y_1,Y_2) = \Big[\sum_{\eta_{1L}=0}^{\infty} G\eta_{1L},\eta_{2H},\eta_{2L},\eta_3(Y_1)Y_2^{\eta_{1L}}\Big] \tag{3}$$

$$G_{\eta_{2L},\eta_3}(Y_1,Y_2,Y_3) = \Big[\sum_{\eta_{2H}=0}^{\infty} G_{\eta_{2H},\eta_{2L},\eta_3}(Y_1,Y_2)Y_3^{\eta_{2H}}\Big] \tag{4}$$

$$G_{\eta_3}(Y_1,Y_2,Y_3,Y_4) = \Big[\sum_{\eta_{2L}=0}^{\infty} G_{\eta_{2L},\eta_3}(Y_1,Y_2,Y_3)Y_4^{\eta_{2L}}\Big] \tag{5}$$

$$G(Y_1,Y_2,Y_3,Y_4,Y_5) = \Big[\sum_{\eta_3=0}^{\infty} G_{\eta_3}(Y_1,Y_2,Y_3,Y_4)Y_5^{\eta_3}\Big] \tag{6}$$

with

$|Y_1| = |Y_2| = |Y_3| = |Y_4| = |Y_5| = 1$

On solving, above equations by using partial generating function technique,the probability distribution function is in the form

$$\begin{aligned} G(Y_1,Y_2,Y_3,Y_4,Y_5) =& \frac{\mu_{1H}(1-\frac{\alpha_{12}}{Y_1}Y_3-\frac{\alpha_{13}}{Y_1}Y_5)G_1}{\lambda_{1H}(1-Y_1)} \\ & \frac{-\mu_{1L}(1-\frac{\alpha'_{12}}{Y_2}Y_4-\frac{\alpha'_{13}}{Y_2}Y_5)G_1}{+\lambda_{1L}(1-Y_2)} \\ & \frac{+\mu_3(1-\frac{1}{Y_5})G_3}{+\lambda_{2H}(1-Y_3)} \\ & \frac{+\mu_{2H}(1-\frac{\alpha_{21}Y_1}{Y_3}-\frac{\alpha_{23}Y_5}{Y_3})G_2}{+\mu_{1H}(1-\frac{\alpha_{12}}{Y_1}Y_3-\frac{\alpha_{13}}{Y_1}Y_5)} \\ & \frac{-\mu_{2L}(1-\frac{\alpha'_{21}Y_2}{Y_4}-\frac{\alpha'_{23}Y_5}{Y_4})G_2}{+\mu_3(1-\frac{1}{Y_5})} \\ & \frac{+\mu_{1L}(1-\frac{\alpha'_{12}}{Y_2}Y_4-\frac{\alpha'_{13}}{Y_2}Y_5)G_4}{+\mu_{2H}(1-\frac{\alpha_{21}Y_1}{Y_3}-\frac{\alpha_{23}Y_5}{Y_3})} \\ & \frac{+\mu_{2L}(1-\frac{\alpha'_{21}Y_2}{Y_4}-\frac{\alpha'_{23}Y_5}{Y_4})G_5}{+\lambda_{2L}(1-Y_4)} \end{aligned} \tag{7}$$

On differentiating(7)according L'Hospital rule,w.r.t to one by one variable and all the variables approaches to 1 as limit of differentiation. the total probability equal to 1
we get the results

$$\lambda_{1H} = \Big[\mu_{1H}(1-G_1) - \mu_{2H}\alpha_{21}(1-G_2)\Big] \tag{8}$$

$$-\lambda_{1L} = \Big[-\mu_{1L}G_1 + \mu_{2L}\alpha'_{21}G_2 + \mu_{1L}G_4 - \mu_{2L}\alpha'_{21}G_5\Big] \tag{9}$$

$$-\lambda_{2H} = \Big[\mu_{1H}\alpha_{12}(1-G_1) - \mu_{2H}(1-G_2)\Big] \tag{10}$$

$$-\lambda_{2L} = \Big[\alpha'_{12}\mu_{1L}G_1 - \mu_{2L}G_2 - \alpha'_{12}\mu_{1L}G_4 + \mu_{2L}G_5\Big] \tag{11}$$

$$\begin{aligned} -\alpha_{13}\mu_{1H} - \alpha_{23}\mu_{2H} + \mu_3 =& \Big[[-\alpha_{13}\mu_{1H} + \alpha'_{13}\mu_{1L}]G_1 \\ & + [-\alpha_{23}\mu_{2H} + \alpha'_{23}\mu_{2L}]G_2 \\ & + \mu_3G_3 - \alpha'_{13}\mu_{1L}G_4 - \alpha'_{23}\mu_{2L}G_5\Big] \end{aligned} \tag{12}$$

On Solving equations (8)-(12) ,we get

$$G_1 = \Big[1 - \frac{\lambda_{1H} + \alpha_{21}\lambda_{2H}}{\mu_{1H} - \alpha_{12}\alpha_{21}\mu_{1H}}\Big] \tag{13}$$

$$G_2 = \Big[1 - \frac{\lambda_{2H} + \alpha_{12}\lambda_{1H}}{\mu_{2H} - \alpha_{12}\alpha_{21}\mu_{2H}}\Big] \tag{14}$$

$$G_3 = 1 - \frac{\alpha_{13}(1-\alpha_{12}^{'}\alpha_{21}^{'})(\lambda_{1H}+\alpha_{21}\lambda_{2H})}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_3} - \frac{\alpha_{13}^{'}(1-\alpha_{12}\alpha_{21})(\lambda_{1L}+\alpha_{21}^{'}\lambda_{2L})}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_3} - \frac{\alpha_{23}(1-\alpha_{12}^{'}\alpha_{21}^{'})(\lambda_{2H}+\alpha_{12}\lambda_{1H})}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_3} - \frac{\alpha_{23}^{'}(1-\alpha_{12}\alpha_{21})(\lambda_{2L}+\alpha_{12}^{'}\lambda_{1L})}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_3} \quad (15)$$

$$G_4 = 1 - \frac{\left[\mu_{1L}(1-\alpha_{12}^{'}\alpha_{21}^{'})(\lambda_{1H}+\alpha_{21}\lambda_{2H}) + \mu_{1H}(1-\alpha_{12}\alpha_{21})(\lambda_{1L}+\alpha_{21}^{'}\lambda_{2L})\right]}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_{1L}\mu_{1H}} \quad (16)$$

$$G_5 = 1 - \frac{\left[\mu_{2L}(1-\alpha_{12}^{'}\alpha_{21}^{'})(\lambda_{2H}+\alpha_{12}\lambda_{1H}) + \mu_{2H}(1-\alpha_{12}\alpha_{21})(\lambda_{2L}+\alpha_{12}^{'}\lambda_{1L})\right]}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_{2H}\mu_{2L}} \quad (17)$$

Also solution is
$P_{n_{1H},n_{1L},n_{2H},n_{2L},n_3} = (1-G_1)^{n_{1H}}(1-G_2)^{n_{1L}}(1-G_3)^{n_{2H}}(1-G_4)^{n_{2L}}(1-G_5)^{n_3}G_1G_2G_3G_4G_5 = w^{n_{1H}}w_2^{n_{1L}}w_3^{n_{2H}}w_4^{n_{2L}}w_5^{n_3}(1-w_1)(1-w_2)(1-w_3)(1-w_4)(1-w_5)$
where

$$w_1 = \left[\frac{\lambda_{1H}+\alpha_{21}\lambda_{2H}}{\mu_{1H}-\alpha_{12}\alpha_{21}\mu_{1H}}\right] \quad (18)$$

$$w_2 = \left[\frac{\lambda_{2H}+\alpha_{12}\lambda_{1H}}{\mu_{2H}-\alpha_{12}\alpha_{21}\mu_{2H}}\right] \quad (19)$$

$$w_3 = \frac{\alpha_{13}(1-\alpha_{12}^{'}\alpha_{21}^{'})(\lambda_{1H}+\alpha_{21}\lambda_{2H})}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_3} - \frac{\alpha_{13}^{'}(1-\alpha_{12}\alpha_{21})(\lambda_{1L}+\alpha_{21}^{'}\lambda_{2L})}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_3} - \frac{\alpha_{23}(1-\alpha_{12}^{'}\alpha_{21}^{'})(\lambda_{2H}+\alpha_{12}\lambda_{1H})}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_3} - \frac{\alpha_{23}^{'}(1-\alpha_{12}\alpha_{21})(\lambda_{2L}+\alpha_{12}^{'}\lambda_{1L})}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_3} \quad (20)$$

$$w_4 = \frac{\left[\mu_{1L}(1-\alpha_{12}^{'}\alpha_{21}^{'})(\lambda_{1H}+\alpha_{21}\lambda_{2H}) + \mu_{1H}(1-\alpha_{12}\alpha_{21})(\lambda_{1L}+\alpha_{21}^{'}\lambda_{2L})\right]}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_{1L}\mu_{1H}} \quad (21)$$

$$w_5 = \frac{\left[\mu_{2L}(1-\alpha_{12}^{'}\alpha_{21}^{'})(\lambda_{2H}+\alpha_{12}\lambda_{1H}) + \mu_{2H}(1-\alpha_{12}\alpha_{21})(\lambda_{2L}+\alpha_{12}^{'}\lambda_{1L})\right]}{(1-\alpha_{12}\alpha_{21})(1-\alpha_{12}^{'}\alpha_{21}^{'})\mu_{2H}\mu_{2L}} \quad (22)$$

and solution exist in steady state condition if $w_1, w_2, w_3, w_4, w_5 < 1$

Marginal Queue Length

$$L_1 = \left[\frac{w_1}{1-w_1}\right] \quad (23)$$

$$L_2 = \left[\frac{w_2}{1-w_2}\right] \quad (24)$$

$$L_3 = \left[\frac{w_3}{1-w_3}\right] \quad (25)$$

$$L_4 = \left[\frac{w_4}{1-w_4}\right] \quad (26)$$

$$L_5 = \left[\frac{w_5}{1-w_5}\right] \quad (27)$$

Average Queue Length

$$L = \frac{w_1}{1-w_1} + \frac{w_2}{1-w_2} + \frac{w_3}{1-w_3} + \frac{w_4}{1-w_4} + \frac{w_5}{1-w_5}$$

8 Analysis of the Model's Behavior

In this phase, while maintaining constant values for the other parameters, we analyse the behaviour of the system's mean and median queue lengths as well as the total amount of time people spent waiting.
(i) How queues behave for various arrival rates at various servers
(ii) How long it takes on average for customers to wait in line depending on various arrival rates (iii) Graphical analysis of average queue length of the system for different values of $\lambda_{1H}, \lambda_{1L}, \lambda_{2H}, \lambda_{2L}$

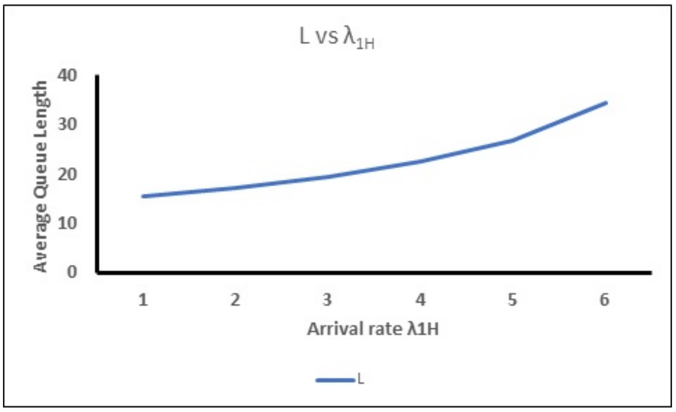

Figure 2. Graphical Representation of queue length of the system w.r.t Table 1

Table 1. The overall system's marginal queue length in proportion to a variable arrival parameter λ_{1H}.

Service Rate	$\mu_{1H} = 17$	$\mu_{1L} = 7$	$\mu_{2H} = 12$	$\mu_{2L} = 8$	$\mu_3 = 14$	Arrival rate	
Probability	$\alpha_{12} = .4$ $\alpha_{13} = .6$	$\alpha'_{12} = .3$ $\alpha'_{13} = .7$	$\alpha_{21} = .8$ $\alpha_{23} = .2$	$\alpha'_{21} = .5$ $\alpha'_{23} = .5$		$\lambda_{1L} = 2, \lambda_{2L} = 1, \lambda_{2H} = 4$	
λ_{1H}	L_1	Ł$_2$	L_3	L_4	L_5	L	E(W)
2	.8173	1.4287	1.8	6.6915	4.6657	15.4032	1.7114
2.2	.8766	1.4879	1.9168	7.8731	5.0018	17.1562	1.8648
2.4	.9397	1.5497	2.0438	9.4822	5.3769	19.3923	2.0630
2.6	1.0070	1.6150	2.1816	11.8028	5.8020	22.4084	2.3342
2.8	1.0792	1.6846	2.3333	15.4473	6.2837	26.8281	2.7375
3	1.1566	1.7569	2.5001	21.9885	6.917	34.3191	3.4317

Table 2. The overall system's marginal queue length in proportion to a variable arrival parameter λ_{2H}.

Service Rate	$\mu_{1H} = 17$	$\mu_{1L} = 7$	$\mu_{2H} = 12$	$\mu_{2L} = 8$	$\mu_3 = 14$	Arrival rate	
Probability	$\alpha_{12} = .4$ $\alpha_{13} = .6$	$\alpha'_{12} = .3$ $\alpha'_{13} = .7$	$\alpha_{21} = .8$ $\alpha_{23} = .2$	$\alpha'_{21} = .5$ $\alpha'_{23} = .5$		$\lambda_{1L} = 2, \lambda_{2L} = 1, \lambda_{1H} = 3$	
λ_{2H}	L_1	Ł$_2$	L_3	L_4	L_5	L	E(W)
3	.8766	1.0608	1.8002	7.8730	3.0014	14.612	1.6235
3.2	.9266	1.1702	1.9164	9.1208	3.4346	16.5686	1.8009
3.4	.9794	1.2920	2.0433	10.7644	3.9772	19.0563	2.0272
3.6	1.0353	1.4288	2.1824	13.0488	4.6659	22.3612	2.3292
3.8	1.0942	1.5821	2.3333	16.4513	5.5820	27.0438	2.7595
4	1.1566	1.7569	2.5001	21.9885	6.917	34.3191	4.8459

Table 3. The overall system's marginal queue length in proportion to a variable arrival parameter λ_{1L}.

Service Rate	$\mu_{1H} = 17$	$\mu_{1L} = 7$	$\mu_{2H} = 12$	$\mu_{2L} = 8$	$\mu_3 = 14$	Arrival rate	
Probability	$\alpha_{12} = .4$ $\alpha_{13} = .6$	$\alpha'_{12} = .3$ $\alpha'_{13} = .7$	$\alpha_{21} = .8$ $\alpha_{23} = .2$	$\alpha'_{21} = .5$ $\alpha'_{23} = .5$		$\lambda_{2H} = 4, \lambda_{2L} = 1, \lambda_{1H} = 3$	
λ_{1L}	L_1	Ł$_2$	L_3	L_4	L_5	L	E(W)
1	1.1566	1.7569	1.8002	3.7260	4.8277	13.2672	1.4741
1.2	1.1566	1.7569	1.9164	4.6208	3.4346	16.5686	1.5865
1.4	1.1566	1.7569	2.0433	5.9297	5.4976	16.3841	1.7429
1.6	1.1566	1.7569	2.1817	8.0332	5.8918	19.0202	1.9812
1.8	1.1566	1.7569	2.3333	11.9699	6.3369	23.5536	2.4034
2	1.1566	1.7569	2.5001	21.9885	6.917	34.3191	3.4317

Table 4. The overall system's marginal queue length in proportion to a variable arrival parameter λ_{2L}.

Service Rate	$\mu_{1H} = 17$	$\mu_{1L} = 7$	$\mu_{2H} = 12$	$\mu_{2L} = 8$	$\mu_3 = 14$	Arrival rate	
Probability	$\alpha_{12} = .4$ $\alpha_{13} = .6$	$\alpha'_{12} = .3$ $\alpha'_{13} = .7$	$\alpha_{21} = .8$ $\alpha_{23} = .2$	$\alpha'_{21} = .5$ $\alpha'_{23} = .5$		$\lambda_{1L} = 2, \lambda_{2H} = 4, \lambda_{1H} = 3$	
λ_{2L}	L_1	Ł$_2$	L_3	L_4	L_5	L	E(W)
.8	1.1566	1.7569	2.3333	15.5836	5.3771	26.2075	2.6742
.9	1.1566	1.7569	2.4151	18.2677	6.0326	29.6289	2.9928
1	1.1566	1.7569	2.5001	21.9885	6.917	34.3191	3.4317
1.1	1.1566	1.7569	2.5895	27.4901	7.8727	40.8658	4.0461
1.2	1.1566	1.7569	2.6844	36.4534	6.8435	48.8948	5.0249
1.3	1.1566	1.7569	2.7836	53.6454	11.0044	70.3469	6.8297

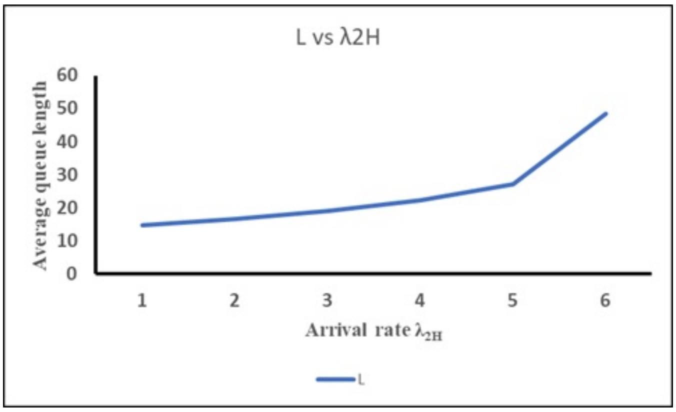

Figure 3. Graphical Representation of Queue length of the system w.r.t Table 2

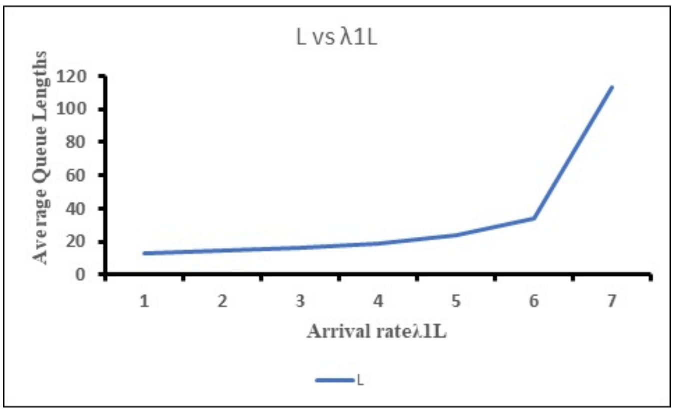

Figure 4. Graphical Representation of Queue length of the system w.r.t Table 3

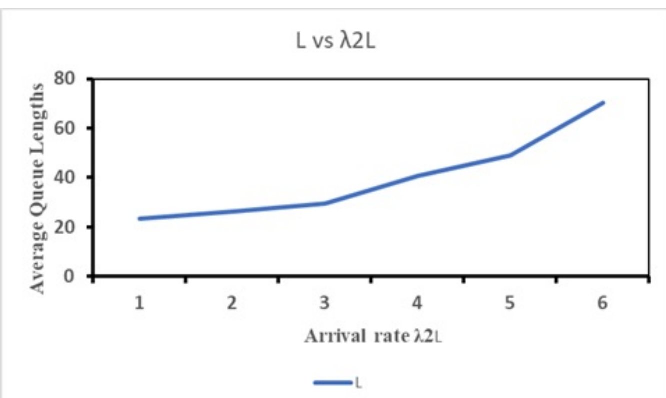

Figure 5. Graphical Representation of Queue length of the system w.r.t Table 4

9 Results and Discussion

• As more high priority customers arrive at server $S1$, it is clear from table 1 that marginal queue lengths, average queue lengths, and Expected Waiting Time rise. At $\lambda_{1H} = 2.6$, $L4$, L increasing more rapidly than before and and other queue lengths increasing with consistent speed. Figure 2 illustrates how quickly the average system queue length increases at $\lambda_{1H} = 3$.
•Table 2 shows a gradual increase in system queue lengths as the number of high priority customers rises at λ_{2H}. And a notable increase in system mean queue length at $\lambda_{2H} = 4$.
•From table 3 and table 4, it is obvious that the arrival of low priority customers has no impact on the wait times or quality of the service for high priority customers. Table 4 shows that the mean queue length and marginal queue lengths will both rise significantly than before when $\lambda_{2L} = 1.3$
• Figures 2, 3, 4, and 5 show all variations in wait lengths and their corresponding rates. The average length of the line and average waiting period of the system can change due to the variations in the arrival rates of high and low priority customers at servers $S1andS2$.

10 Particular Case

If we ignore the concept of priority and the different probabilities of high- and low-priority customers going from one server to another, and rather than adopt a batch arrival approach with always one customer arriving at a time, the findings are consistent with Mittal Meenu. [16]

11 Conclusion

In a stochastic environment, we employ the single arrival and priority approach in our model design for bi-series service channels. This priority bi-series queue network model applicable in various field like banking system,amusement park,manufacturing, hospitals, registration processes, and administration,among other places.The mathematical analysis of the system through numerical calculations and graphs,shows how to redesign the queue system to reduce server congestion and maximize service facility usage.

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