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A String of Disjoint Job Block with Processing Time Associated with Probability in Two-Stage Weighted Open Shop Model

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Abstract Open-shop scheduling problem (OSSP) is a well-known topic with wide industrial applications which belongs to one of the vital issues in the field of engineering. This paper deals with a two-stage open shop scheduling problem in which the processing time of jobs is allied with probabilities. The concept of a string of two job blocks which are disjoint in nature is considered so that the first block covers the jobs with a fixed route and the second block covers the jobs with an arbitrary path. Further, the weights of jobs are also introduced due to their applicability and relative importance in the real world. The objective of this study is to propose a heuristic which on execution, provides an optimal or near-optimal schedule to diminish the makespan. Several numerical illustrations are produced in MATLAB 2018a to demonstrate the effectiveness of the proposed approach, and to confirm the performance, the results are compared with the existing methods developed by Johnson and Palmer.

Keywords Scheduling, Open Shop, Job Block, Weights of Jobs, Flow Shop

1. Introduction

Scheduling is a procedure that concerns decision-making and the allotment of the constrained arrangement of assets with the optimizing objectives. Scheduling is an essential tool to keep the end goal to meet customer requirements as quickly as expected and maximize profits. Scheduling is generally categorized as flow shop (FSSP), open shop (OSSP), job shop (JSSP) problems. This paper presents an open shop scheduling model that consists of ‘n’ jobs, each with two operations (tasks). In this system, operation O_{ij} $(i = 1,2;\ j = 1,2, \dots.. n)$ requires processing on machine M_i during an uninterrupted processing time. The task of all the jobs can be scheduled in any route, as long as their execution does not overlap, particularly in inspection, testing, and maintenance. The flow shop scheduling model was originally presented by Johnson with two machines to minimize the makespan (i.e., total elapsed time) [21]. The optimality of Johnson’s model has drawn the attention of many researchers toward this path and extended the work by taking into account numerous parameters [20,19]. The concept of a job-block equal to a single job was initially proposed by Maggu and Das to achieve parity between the cost of delivering service to a need-based client and other clients [17].

From 1954 onwards, Johnson’s works of scheduling models don’t consider job weightage until 1980. In 1980, Miyazaki et al. have studied the concept of weightage on flow shop scheduling in which computational algorithms are used for the optimum or near optimum solution to the problem [16]. These optimum algorithms conform to the objective criterion as the minimization of weighted mean flow-time of jobs. Palmer applied the heuristic technique to minimize the make-span for the problems consisting of n-job m-machine [15]. Anup broadened the research by assigning probabilities with the processing time of jobs as the time to process the jobs is always not precise [11]. Heydari et al. managed the idea of handling the jobs in a string formed with two distinct job-blocks [10]. Gupta et al. proposed an FSSP in which all the jobs are processed in a string of two disjoint job blocks [8].

In the scheduling literature, it is considered that open shop has larger solution space than the other scheduling problems (Flow-shop & Job-shop) and appears to get less consideration in the literature, although it has a significant and universal problem. Some models of the open shop have been developed in the past. An efficient heuristic algorithm was proposed to solve the problem of OSSP to minimize the makespan when it is considered that there are only two machines available in the workstation [18] and another researcher presented O_2/Cmax model with special transportation times and job block criteria [14,12]. Further, an attempt is to extend their study incorporating the significant criteria job block in a probabilistic environment [12,7]. Some of the unique open approaches were made in recent years by considering various parameters [6,4,3,2,1]. This paper aims to find an optimal string that reduces makespan when the model of the open shop with 2-machines and n-jobs is considered under the following constraints:

- Priority is given to the i_1 job over $i_2, \ldots, i_k$ in job block $(i_1, i_2, \ldots \ldots \ldots i_k)$.
- The process that once began on a machine can't be ceased except if the activity is finished.
- The time of the jobs which can be consumed in transportation from one machine to another is negligible.
- Summation of all the probabilities allied with processing time of jobs is equal to one.
- All the Jobs and Machines are accessible at zero time.

1.1. Practical Situation

Open shop scheduling is a popular optimization method for real-world domestic problems such as scheduling of examination, classroom assignments, scheduling and timetabling for games competitions, scheduling system TV programs, conference presentations, etc. The scheduling paradigm also has a broader application in industry, particularly in testing and maintenance, where machines might be in any sequence, but work possessing makes no difference. A large carport with specific shops is an example of an open job shop where a car may require additional maintenance, such as replacing exhaust and the alignment of wheels. These two tasks might be completed in any order on the given machines; however, it is not conceivable to play out these two tasks simultaneously. OSSP is used in various applications, including vehicle quality control centers, semiconductor fabrication, satellite communications, etc.

Further, the concept of a string of two job blocks on the OSSP becomes significant. It may be due to its urgency or demand. For instance, in a steel manufacturing industry where some jobs such as heating and molding must be passed out as a fixed job block and other jobs such as cutting, crushing, chroming, etc., can be managed in a block disjoint the first block optimally. Hence, a string of disjoint job block criteria plays a vital role in real-world applications.

1.2. Preliminaries

Scheduling is generally categorized as flow shop (FSSP), open shop (OSSP) and job shop (JSSP) problems.

1.2.1. Flow shop scheduling:

A flow shop is that machine environment in which the arrangement of each employment (job) is completely specified and all the employments (jobs) visit the work stations in the same route.

1.2.2. Open shop scheduling problem:

Open shop is that environment where all the jobs are usually processed in immaterial orders i.e. all the employments (jobs) visit the work stations in any route. The schedulers are permitted to decide the order for each activity, and distinctive occupations may have distinct orders.

1.2.3. Job shop scheduling:

As a flow-shop, a job-shop does not have the same restriction on work flow. In a job-shop environment, every job has a separate processing sequence that means jobs can be run in any route on given machines.

2. Material and Method for -Formulation of Open Shop Model with Weights

(i). Let ‘n’ jobs $J_1, J_2, \ldots, J_n$ be processed through 2-machines M_j $(j = 1, 2)$ in an arbitrary order with no passing allowed.

(ii). Assume M_{ij} denote the total time to process of job Ji on machine M_j and p_{ij} denote the probabilities allied with M_{ij}.

(iii). Let $S = (\alpha, \beta)$ be a string consisting of two job-blocks which are disjoint in nature.

(iv). Consider $F_j(i)$ be the partial flow time of J_i counted from the starting time of the first job J_i on M_1 to the completion time of J_i on M_2 , in particular, $F_m(i)$ is called as flow-time of J_i.

(v). Each job is assigned a weight w_i according to its importance.

(vi). The performance measure is to minimize the total elapsed time.

Let the above data in table 1 form be described as follows:

Table 1. Mathematical formulation

Jobs J_i	Machine M_1		Machine M_2		Weights
	M_{i1}	p_{i1}	M_{i2}	p_{i2}	w_i
J_1	M_{11}	p_1	M_{12}	p_{12}	w_1
J_2	M_{21}	p_2	M_{22}	p_{22}	w_2
J_3	M_{31}	p_3	M_{32}	p_{23}	w_3
.	.	.	.	.	.
.	.	.	.	.	.
J_n	M_{n1}	p_n	M_{n2}	p_{2n}	w_n

2.1. Model Notations

Various notations used in this paper are given below:

- i : Index for jobs.
- $M_{i1}^{'}$: Probable time to process the i[th] job on machine M_1
- $M_{i2}^{'}$: Probable time to process the i[th] job on machine M_2
- G_i : Time to process the job 'i' on fictitious machine G
- H_i : Time to process the job 'i' on fictitious machine H.
- p_{i1} : Probability allied with M_{i1}.
- p_{i2} : Probability allied with M_{i2}.
- w_i : Weight assigned to job J_i.

3. The Proposed Model

The algorithm for two-stage OSSP problem is when jobs are processed in string 'S,' including weights of jobs, can be decomposed into the following steps:

Step1: Compute the probable processing times $M_{i1}^{'}$ and $M_{i2}^{'}$ $(i = 1, 2, 3, \dots\dots, n)$ for machine route M_1 to M_2 on machines M_1 & M_2 respectively by the formula:

(a) $M_{i1}^{'} = M_{i1} \times p_{i1}$
(b) $M_{i2}^{'} = M_{i2} \times p_{i2}$

Step2: Construct fanciful machines G and H with run times G_i and H_i as defined below:

(a) If min $(M_{i1}^{'}, M_{i2}^{'}) = M_{i1}^{'}$ then calculate $G_i = M_{i1}^{'} + w_i$ and $H_i = M_{i2}^{'}$
(b) If min $(M_{i1}^{'}, M_{i2}^{'}) = M_{i2}^{'}$ then calculate $H_i = M_{i2}^{'} + w_i$ and $G_i = M_{i1}^{'}$

Step3: Calculate weighted flow times G_i' and H_i' for respective *machines G and H by the following equation:

(a) $G_i' = G_i / w_i$
(b) $H_i' = H_i / w_i$

If a block of jobs contains three or more than three jobs, then compute the weighted flow times using the associative property, i.e. $((J_1, J_2), J_3) = (J_1, (J_2, J_3))$.

The flowchart of the proposed algorithm on two-stage OSSP problem is shown in Figure 1.

Step4: Consider a job-block containing two jobs, say J_k and J_m, with a fix order of jobs. Let this job block be equivalent to a single job J_α, i.e., $J_\alpha = (J_k, J_m)$. Now calculate the working time of job J_α on fanciful machines G and H as defined below:

(a) $G_{J\alpha}' = G_{JK}' + G_{Jm}' - \min(G_{Jm}', H_{Jk}')$
(b) $H_{J\alpha}' = H_{JK}' + H_{Jm}' - \min(G_{Jm}', H_{Jk}')$

Step5: Consider another job block β consisting of $(n - \{J_k, J_m\})$ jobs with an arbitrary route. Apply Johnson's method [21] to obtain the optimum route of jobs in block β. Consider new block is equivalent to γ. Now find the processing time of the block J_γ on machines G and H as defined in step4.

Step6: Convert the given problem into new by substituting the jobs $\{J_k, J_m\}$ by equivalent job J_α and $(n - \{J_k, J_m\})$ jobs by equivalent job J_γ. Then the modified problem can be presented as below:

Table 2. Modified Problem

Jobs (i)	Machine G	Machine H
	t_{i1}	t_{i2}
J_α	$G_{J\alpha}'$	$H_{J\alpha}'$
J_γ	$G_{J\gamma}'$	$H_{J\gamma}'$

Step7: Compute some specific weights to each machine(m) for J^{th} job as follows:

$$W_j = -\sum_{i=1}^{m} \{m - (2i - 1)\} \times t_{ij}$$

Step8: The optimal string $\boldsymbol{S}$ is obtained by arranging the jobs in the increasing order of their weights.

Step9: Next, obtain another string $\boldsymbol{S'}$ in the same manner as obtaining string $\boldsymbol{S}$ by repeating the procedure from step 1 to step 8 for machine route $M_2 \rightarrow M_1$.

Step10: Formulate Flow in-out tables for strings $\boldsymbol{S}$ & $\boldsymbol{S'}$ and calculate the total elapsed time (makespan) for both strings.

Step11: To obtain the optimum sequence achieving our objective function, compare the results obtained in both machine routes and choose the sequence with minimum makespan.

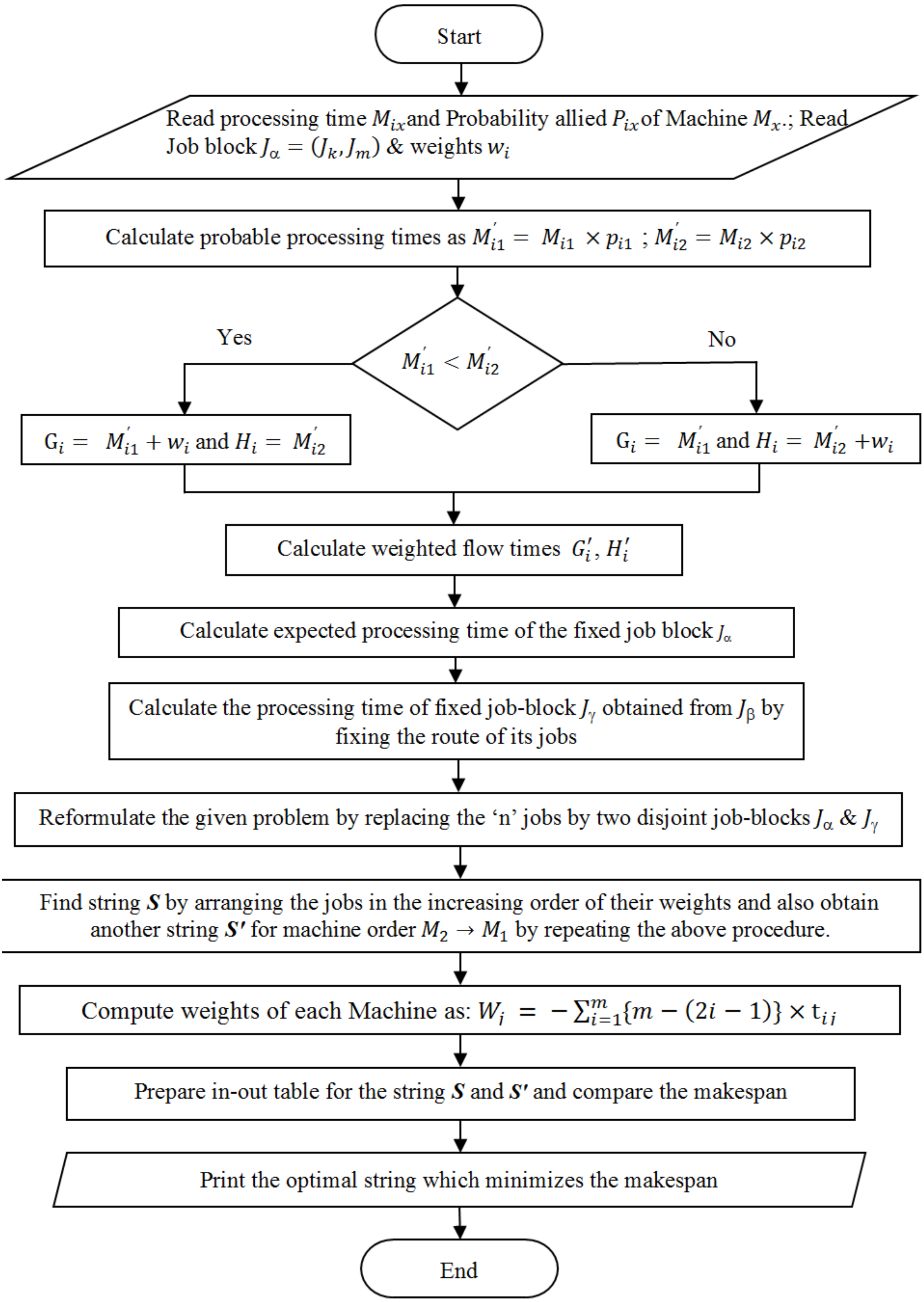

Figure 1. Flow chart of the proposed model on open shop

4. Mathematical Illustration

To assess the execution of the given procedure, a numerical problem is set as below:

Consider the processing of 5 jobs in a string $\boldsymbol{S}$ on two machines M_1 & M_2 in an immaterial order.

Assume the block $\alpha = (J_5, J_3)$ with a fixed order and $\beta = (J_5, J_2, J_4)$ with an arbitrary order running in a string $\boldsymbol{S}$. The probabilities p_{i1} & p_{i2} allied with processing M_{i1} & M_{i2} correspondingly and weightage of the jobs are given in the following Table 3.

Table 3. Data set for the stated problem

Jobs	Machine M_1 M_{i1}	Probabilities p_{i1}	Machine M_2 M_{i2}	Probabilities p_{i2}	Weights w_i
J_1	12	0.2	29	0.2	5
J_2	29	0.2	31	0.1	6
J_3	30	0.3	27	0.2	7
J_4	9	0.2	5	0.3	8
J_5	12	0.1	7	0.2	4

The probable time (G'_i & H'_i) to run all the five jobs J_1 J_2 J_3 J_4 J_5 (say) on fictitious machines G & H is represented in the table below:

Table 4. Probable process times on fictitious machines

Jobs	Machine G	Machine H
	G'_i	H'_i
J_1	1.48	1.16
J_2	0.96	1.52
J_3	1.28	1.77
J_4	0.22	1.19
J_5	1.3	0.35

On the lines of Maggu and Das [17].

Computed values for probable process times for job block $\alpha = (J_5, J_3)$ and $\beta = (J_1, J_2, J_4)$ are shown in Table 5:

Table 5. Probable processing times of equivalent jobs for route G to H

Jobs	Machine G	Machine H
(i)	t'_{i1}	t'_{i2}
J_α	2.23	1.77
J_γ	0.22	1.43

By following the step 7 & 8 of the algorithm, the string $\boldsymbol{S}$ = {α, γ} = {J_5, J_3, J_4, J_2, J_1} is obtained for machine route $M_1 \rightarrow M_2$.

Flow in - Flow out Table for string $\boldsymbol{S}$ is provided in Table 6:

Table 6. Flow in - Flow out table for route $M_1 \rightarrow M_2$

Jobs	Machine M_1			Machine M_2		
	Time In	-	Time out	Time In	-	Time out
J_5	0.0	-	1.2	1.2	-	2.6
J_3	1.2	-	10.2	10.2	-	15.6
J_4	10.2	-	12.0	15.6	-	17.1
J_2	12.0	-	17.8	17.8	-	20.9
J_1	17.8	-	20.2	20.9	-	23.3

Thus, total elapsed time C_{max} = 23.3 units of time corresponds to the string $\boldsymbol{S}$ for the machine route M_1 to M_2. Now, as per step 9, we have obtained another string $\boldsymbol{S'}$ = {γ, α} = {J_1, J_2 J_4, J_5, J_3} for machine route M_2 to M_1, and its flow table is described in Table 7:

Table 7. Flow in-Flow out table for route $M_2 \rightarrow M_1$

Jobs	Machine M_2			Machine M_1		
	Time In	-	Time out	Time In	-	Time out
J_1	0.0	-	5.8	5.8	-	8.2
J_2	5.8	-	8.9	8.9	-	14.7
J_4	8.9	-	10.4	14.7	-	16.5
J_5	10.4	-	11.8	16.5	-	17.7
J_3	11.8	-	17.2	17.7	-	26.7

Thus, the total elapsed time C_{max} = 26.7 units of time for the machine route M_2 to M_1. Hence the above results obtained for different machine routes are described in Table 8.

Hence from the below table, we conclude that the string $\boldsymbol{S}$ = {J_5, J_3, J_4, J_2, J_1} obtained for machine route $M_1 \rightarrow M_2$ provides the minimum total elapsed time.

Therefore, $\boldsymbol{S}$ = {J_5, J_3, J_4, J_2, J_1} is required for the optimal string of jobs possessing optimal elapsed time.

Table 8. Description of results

Machine Route	String/Sequence	Total Elapsed Time/Makespan
$M_1 \rightarrow M_2$	$\boldsymbol{S}$ = {J_5, J_3, J_4, J_2, J_1}	C_{max} = 23.3 units of time.
$M_2 \rightarrow M_1$	$\boldsymbol{S'}$ = {J_1, J_2, J_4, J_5, J_3}	C_{max} = 26.7 units of time.

5. Computational Analysis & Results

To analyze the proposed heuristic method, the number of examples for several groups is taken arbitrarily in which individually group has a different numeral of jobs. Groups are purposed with different job sizes like 4,5,7,10,30,50,60,80 and each group is studied over five different arbitrarily produced tribulations. For each group, the performance measure of the proposed heuristic is compared with already existing heuristics of Johnson and Palmer as revealed in Table 9, and results are plotted in the graph as shown in Figure 2. which reveals that the proposed algorithm curve is lower than the curve of the heuristics presented by Palmer and Johnson [15,21]. It reflects that the results of constructive heuristic are very close to the results of the Palmer and Johnson for small job size $4 \leq n \leq 9$. As the job size n increases, the constructive heuristic provides better results than the heuristics presented by Palmer and Johnson.

Table 9. Computational experiments for total elapsed time (makespan)

Job Size (n)	Palmer [15]	Johnson [21]	Proposed algorithm
4	24.8	24.8	24.1
5	25.6	25.6	23.3
7	20.7	20.5	20.3
10	58.71	56.35	53.73
30	186.13	181.15	170.95
50	843.48	824.65	752.77
60	3748.4	3663.90	3426.5
80	15177.85	14894.48	13882.31

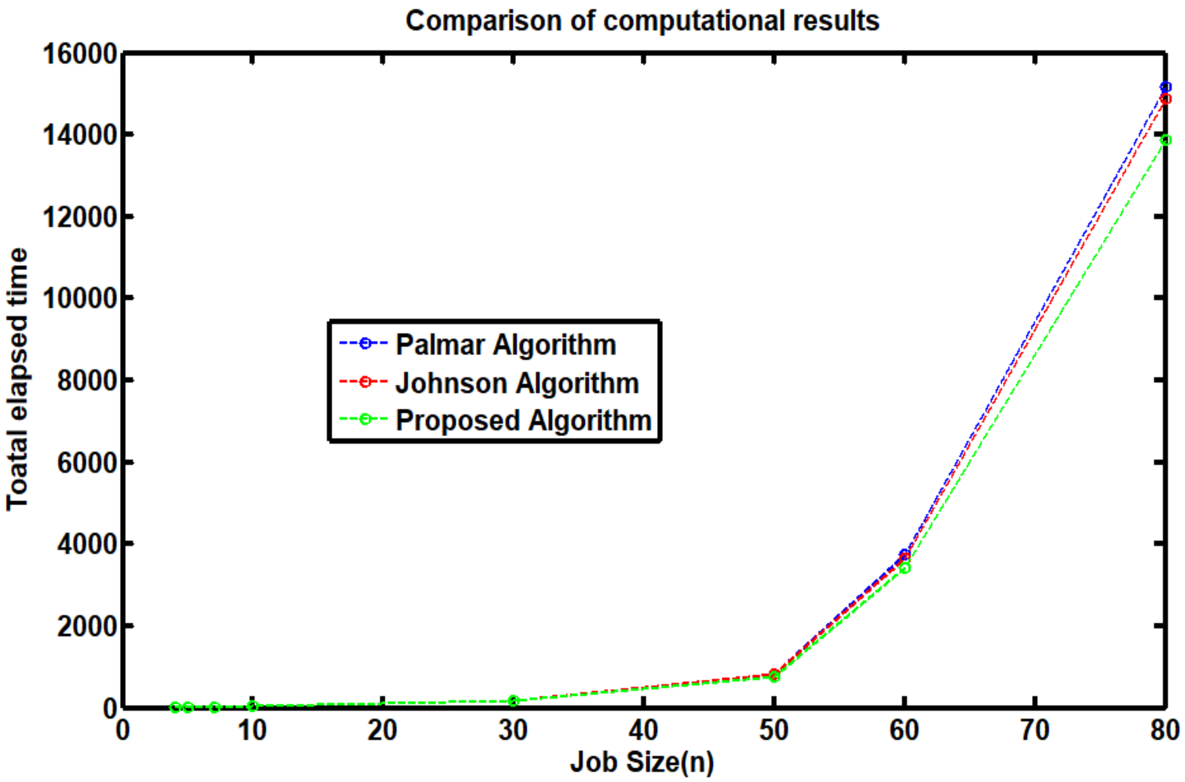

Figure 2. Comparison of computational results

Moreover, to show the effectiveness of computed results, the error rate for each of the problems is calculated by the formula given as:

$$E_{rr} = [(E_{\delta}-E_{\mu})/E_{\mu}]*100$$

where E_{δ} is the total elapsed time of occurred algorithm and E_{μ} is the total elapsed time of the identical job calculated by using the proposed algorithm, tabulated in Table 10 and shown in Figure 3.

Table 10. Percentage error

Job size (n)	Palmer	Johnson
4	2.90	2.90
5	9.87	9.87
7	1.97	0.98
10	9.26	4.88
30	8.88	5.97
50	12.05	9.55
60	9.39	6.93
80	9.33	7.29

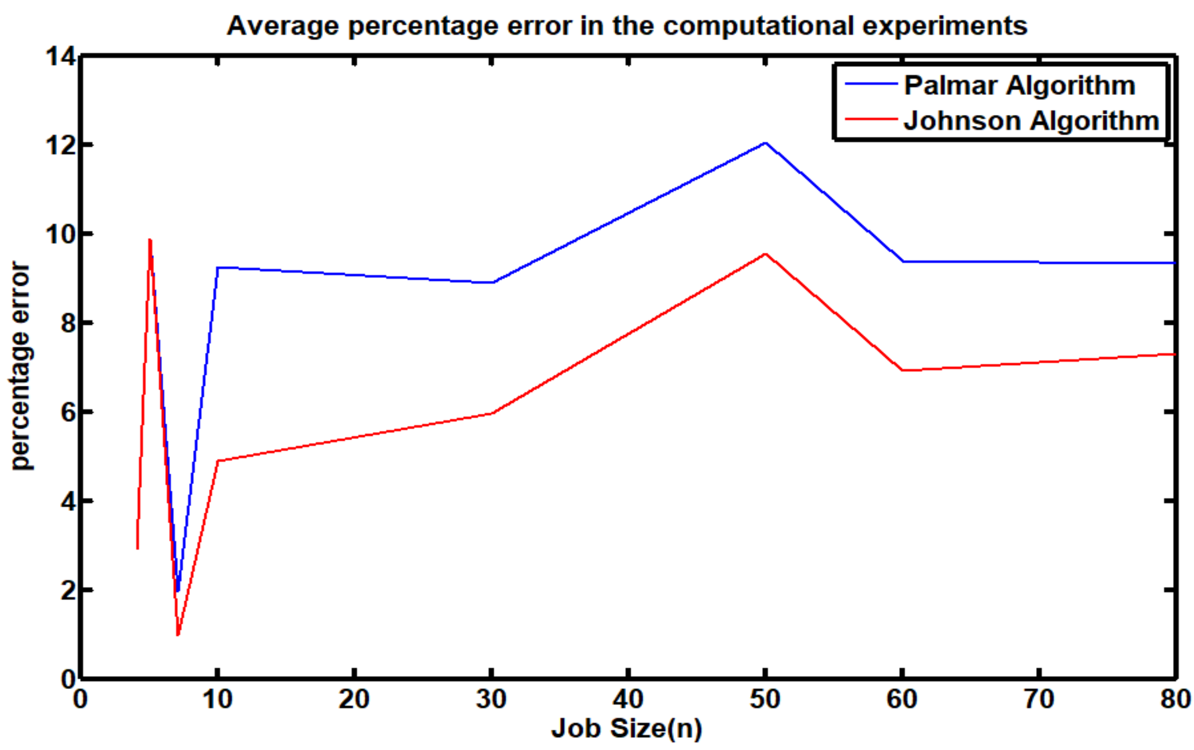

Figure 3. Average percentage error

Table 11. Average of mean percentage error

Algorithm	Average of mean percentage error
Palmer	7.96
Johnson	6.046

6. Discussion and Conclusion

Open shop scheduling problem is a kind of NP problem and has a wider range than other basic scheduling problems. In the past decades, a lot of research has been made in the field of FSSP and OSSP to attain the distinct types of performance measures. But due to the complexity of computation, researchers developed the algorithm for the small job size (1≤n≤8). In our study, we have not constructed a new heuristic to achieve the goal but also attempted to achieve our goal not only for small size (1≤n≤8) but also for medium (9≤n≤30) and large (31≤80) job size.

To check out the efficient heuristic to the decision-maker to find the optimal result for the problems that occur in that environment where jobs run in a string of disjoint job-block. To check out the efficiency of the developed heuristic, the number of illustrations for the generated groups with different job sizes is compiled in MATLAB R2018a. Computational experiments show that the developed heuristic provides better results as compared to the heuristics presented by [15,21,5,9] to achieve the performance measure (minimization of makespan) as the aim of the paper. In Addition, for each job size n, the average percentage error is calculated. The future work could be extended with different parameters like breakdown interval, due dates, the setup time of machines, availability of single transportation etc. The study may additionally be stretched out by introducing the processing time of machines in a fuzzy environment.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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