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Some Results on Theory of Numbers, Partial Differential Equations and Numerical Analysis

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Abstract In this article, given a number $P \in \mathbb{N}$ that ends in one and assuming that there are integer solutions $(A; B) \in \mathbb{N} \times \mathbb{N}$ for the equations $P = (10x + 9)(10y + 9)$ or $P = (10x + 7)(10y + 3)$ or $P = (10x + 1)(10y + 1)$, the straight line was used passing through the center of gravity of the triangle bounded by the vertices $(A; A)$, $(B; A)$, $(A; B)$. Considering $A \geq 25$, we manage to divide the domain of the curve $P = (10x + 9)(10y + 9)$ into two disjoint subsets, and using Theorem (2.2) of this article, we find the subset where the integer solution of the equation $P = (10x + 9)(10y + 9)$ is found. Similar process is done when $P = (10x + 1)(10y + 1)$, in case P is of the form $P = (10x + 7)(10y + 3)$ or $P = (10x + 3)(10y + 7)$. These curves are different and to obtain a process similar to the one carried out previously, we proceeded according to Observation 2.2. Our results allow minimizing the number of operations to perform when our problem requires to be implemented computationally.

Furthermore, we obtain some conditions to find the solution of the equations:

$$-\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial}{\partial x_i}u\right) + cu = f \text{ en } D.$$
$$u = 0 \text{ en } \partial D.$$

where u is of class $\mathbf{C}^2$, $n = 2$ and D is a bounded open domain of $\mathbb{R}^2$ with piecewise smooth boundary ∂D. All the operations carried out to find the solution have been carried out assuming that these exist, and we have found the conditions that f must satisfy for the coefficients a_{ij}.

We finish by finding an optimal domain for the real solution of a given polynomial of degree five. This process carried out on said given polynomial can also be carried out to reduce the degree of a given polynomial and thus obtain information about its roots.

Keywords Prime Numbers, Diophantine Equations, Differential Equations, Number Theory, Functional Analysis

1 Introduction

In the articles [1] and [10] some results are obtained on integer solutions of quadratic polynomials in two variables. In Theorem (2.1) of [1] it is obtained that the face of the integer solution of the equation $P = (10x + 9)(10y + 9)$ is given by

$AB = \frac{(P-81)}{200}\left(1+\frac{N}{M}\right)$ and $A+B = \frac{(P-81)}{180}\left(1-\frac{N}{M}\right)$ where N and M are relatively prime and P is a natural number ending in one. Similar results are obtained for quadratic polynomials in two variables when they are of the form

$$P = (10x+1)(10y+1) \text{ or } P = (10x+7)(10y+3)$$

Combining the results obtained in [1] and [10], via the Theorems (2.1), (2.2) using a line that passes through the center of gravity of the triangle bounded by the vertices (A, A), (B, A) and (A, B) we manage to divide the domain of the equation $P = (10x+9)(10y+9)$ into two disjoint subsets and via Theorem (2.2), we determine in which of the subsets the integer solution lies. We show this process with an example.

In section 3, for an illustration of the Lax-Milgram Theorem, see Theorem 3.4. 1 of [4], consider a second order elliptic equation defined on an open set D in $\mathbb{R}^2$ with piecewise smooth boundary ∂D. For the equation

$$-\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial}{\partial x_i}u\right) + cu = f \text{ en } D.$$
$$u = 0 \text{ en } \partial D.$$

generalized solutions are obtained.

In Section 4, we obtain an optimal interval where the solution of a given polynomial of degree 5 lies. This interval can be used as a starting point for numerical analysis algorithms to find the approximate root of this and similar polynomials. The results presented have originality in their proof both in number theory, EDPs and polynomials.

2 On Results in Number Theory

Theorem 2.1. *Let P be a natural number ending in one. If there are $(A, B) \in \mathbb{N}\times\mathbb{N}$ with $A < B$, that is $P = (10A+9)(10B+9)$, then there exists $a \in \langle 0; 0.5\rangle$, $A \geq 25$ such that for $1 < P^a < 2$ we have:*

$$\frac{P^{1/2}}{20} - \frac{9}{10} < A < \frac{p^{1/2}-q}{10} \quad \textit{and} \quad \frac{P^{1/2}-9}{10} < B < \frac{2P^{1/2}-9}{10}$$

and for $2 \leqslant P^a \leqslant \dfrac{P^{1/2}}{259}$ we have:

$$25 \leq A \leq \frac{P^{1/2}}{20} - \frac{9}{10} \quad \textit{and} \quad \frac{2P^{1/2}-9}{10} \leq B \leq \frac{P}{2590} - \frac{9}{10}$$

Proof. Let $(A; B) \in \mathbb{N}\times\mathbb{N}$ be the integer solution of the equation

$$P = (10x+9)(10y+9) \tag{1}$$

Then $P = (10A+9)(10B+9)$.

Let C be the center of gravity of the triangle with vertices $(A; A)$, $(B; A)$ and $(A; B)$ then $C = \frac{1}{3}(2A+B; 2A+B)$.

We affirm that the intersection of (1) and the line

$$\mathcal{L}: \quad x + y = \frac{4A+2B}{3} \tag{2}$$

is other than empty.

To prove this information, we use the Theorem (2.2) of [10], where

$$A = \frac{P^{\frac{1}{2}-a}-9}{10}, \quad B = \frac{P^{\frac{1}{2}+a}-9}{10} \tag{3}$$

and $a \in \langle 0; 0.5\rangle$.

From the relations (1) and (2) we have

$$P = (10x+9)\left(10\left(\frac{4A+2B}{3} - x\right) + 9\right) \tag{4}$$

From the relation (4) we obtain:

$$100x^2 + x(90 - 10\tau) + P - 9\tau = 0 \tag{5}$$

where $\tau = \frac{10}{3}(4A+2B)+9$.

From (5) we get:

$$x = \frac{\tau - 9}{20} \pm \frac{\sqrt{(\tau+9)^2 - 4P}}{20} \tag{6}$$

For an intersection to exist, it is clear that we must prove that:

$$(\tau + 9 - 2\sqrt{P})(\tau + 9 + 2\sqrt{P}) \geq 0$$

that is, we must prove that:

$$\tau \geq 2\sqrt{P} - 9 \tag{7}$$

Using the relation (3) we obtain that

$$\tau + 9 = P^{\frac{1}{2}} \left(\frac{4}{3}P^{-a} + \frac{2}{3}P^{a} \right) \tag{8}$$

Therefore, from the relations (7) and (8) we obtain:

$$P^{\frac{1}{2}} \left(\frac{4}{3}P^{-a} + \frac{2}{3}P^{a} \right) \geq 2P^{1/2} \tag{9}$$

Then there is a solution, if from the relation (9)

$$2 \leq P^{a} \tag{10}$$

Using the relation (3), the relation (10) and the data $P^a \leq \frac{P^{1/2}}{259}$ we get one of the desired results. For $1 < P^a < 2$, we see that the line $\mathcal{L}$ does not intersect the equation (1) and using the relation (3) we obtain the mentioned results. □

Theorem 2.2. *Let P be a natural number that ends with one. If there exist $(A, B) \in \mathbb{N} \times \mathbb{N}$ such that $P = (10A+9)(10B+9)$, $A \geq 25$ then it exists $a \in \langle 0; \frac{1}{2} \rangle$ such that:*

$$a = \frac{1}{2}\frac{ln2}{lnP} \quad or \quad a = \frac{ln(\sqrt{2}+1)}{lnP} \quad or \quad a = \frac{lnM}{lnP}, \quad where$$

$$M = \left[\frac{1}{2}\left(2 - \frac{259n}{P^{1/2}}\right) + \sqrt{\left(2 - \frac{259n}{P^{1/2}}\right)^2 + 4n} \right], \quad n = \frac{\left(\frac{P}{259} - 2P^{1/2}\right)}{\left(\frac{P^{1/2}}{2} - 259\right)}$$

Proof. Using the Theorem (2.1) and the relation (3) of the mentioned Theorem, we have:

$$\text{For} \quad 2 \leq P^a \leq \frac{P^{1/2}}{259}; \quad B = nA + \gamma \tag{11}$$

from this relation and the Theorem (2.1) we have:

$$25 \leq A \leq \frac{P^{1/2}}{20} - \frac{9}{10} \quad \text{and} \quad \frac{2P^{1/2} - 9}{10} \leq nA + \gamma \leq \frac{P}{2590} - \frac{9}{10} \tag{12}$$

From the relation (12), one of the possibilities obtained is:

$$n = \frac{\left(\frac{P}{259} - 2P^{1/2}\right)}{\left(\frac{P^{1/2}}{2} - 259\right)} \quad \text{and} \quad \gamma = \frac{2P^{1/2} - 9}{10} - 25n. \tag{13}$$

From the relation (11) and (13) we have:

$$a = \frac{1}{2}\frac{lnM}{lnP}, \quad \text{where} \quad M = \frac{1}{2}\left[\left(2 - \frac{259n}{P^{1/2}}\right) + \sqrt{\left(2 - \frac{259n}{P^{1/2}}\right)^2 + 4n} \right] \tag{14}$$

For $1 < P^a < 2$;

$$B = 2A + \frac{9}{10} \tag{15}$$

and from this relation we get $a = \frac{1}{2}\frac{ln2}{lnP}$.

If in (15) we have $B = A + \gamma$, using Theorem ((2.1))

$$\frac{P^{1/2}}{20} - \frac{9}{10} < A < \frac{P^{1/2} - 9}{10} \quad \text{and} \quad \frac{P^{1/2} - 9}{10} < A + \gamma < \frac{2P^{1/2} - 9}{10} \tag{16}$$

from (16) we get $\gamma = \dfrac{P^{1/2}}{20}$ and $a = \dfrac{ln\left(\frac{1+\sqrt{17}}{4}\right)}{lnP}$. □

Corollary 2.1. *Let P be a natural number that ends with one. If there exist $(A, B) \in \mathbb{N} \times \mathbb{N}$ with $A < B$, that is $P = (10A + 1)(10B + 1)$, then there exists $a \in \langle 0; 1/2\rangle$, such that for $1 < P^a < 2$ and $A \geq 25$ we have:*

$$\frac{P^{1/2}}{20} - \frac{1}{10} < A < \frac{P^{1/2} - 1}{10} \quad \textit{and} \quad \frac{P^{1/2} - 1}{10} < B < \frac{2P^{1/2} - 1}{10}$$

and for $2 \leq P^a \leq \frac{P^{1/2}}{251}$ we have

$$25 \leq A \leq \frac{P^{1/2}}{20} - \frac{1}{10} \quad \textit{and} \quad \frac{2P^{1/2} - 1}{10} \leq B \leq \frac{P}{2510} - \frac{1}{10}.$$

Proof. It is to follow the process given in Theorem (2.1). □

Corollary 2.2. *Let P be a natural number ending in one. If there $(A, B) \in \mathbb{N} \times \mathbb{N}$, $A < B$ such that $P = (10A + 1)(10B + 1)$, $A \geq 25$; then there exists $a \in \langle 0; 1/2\rangle$, such that*

$$a = \frac{1}{2}\frac{ln2}{lnP} \quad or \quad a = \frac{ln\left(\frac{1+\sqrt{17}}{4}\right)}{lnP} \quad or \quad a = \frac{lnM}{lnP}$$

where

$$M = \frac{1}{2}\left(2 - \frac{251n}{P^{1/2}} + \sqrt{\left(2 - \frac{251n}{P^{1/2}}\right)^2 + 4np}\right)$$

with $n = \dfrac{\left(\dfrac{P}{251} - 2P^{1/2}\right)}{\dfrac{P^{1/2}}{2} - 251}$.

Proof. It is to follow the process given in Theorem (2.2). □

Remark 2.1. *If we want that $(A, B) \in \mathbb{N} \times \mathbb{N}$ when replacing $a \in \langle 0; \frac{1}{2}\rangle$ in the equations $A = \frac{P^{\frac{1}{2}-a}-9}{10}$ o $A = \frac{P^{\frac{1}{2}-a}-1}{10}$ o $B = \frac{P^{\frac{1}{2}+a}-9}{10}$ o $B = \frac{P^{\frac{1}{2}+a}-1}{10}$ these values should be rounded, also n and γ are rounded.*

Remark 2.2. *Let P be a natural number ending in one. The equation $P = (10x + 7)(10y + 3)$ is not symmetric about the line $y = x$. If there are integer solutions $(A; B) \in \mathbb{N} \times \mathbb{N}$, $A < B$ such that $P = (10A + 7)(10B + 3)$, $A \geq 25$ we could not apply the reasoning or processes carried out in Theorem 2.1 and Theorem 2.2.*

To carry out the same process in Theorems 2.1 and 2.2 and obtain similar results, we observe that

$$P = (10x + 7)(10y + 3) = (10x + 3)(10y + 7) \tag{17}$$

From the relation (17) we obtain the functions

$$y = f(x) = \begin{cases} \dfrac{P}{100x + 70} - \dfrac{3}{10}, & if\ 0 \leq x \leq \frac{\sqrt{4+P}-5}{10} \\ \dfrac{P}{100x + 30} - \dfrac{7}{10}, & if\ \frac{\sqrt{4+P}-5}{10} \leq x \leq \frac{P-21}{70} \end{cases} \tag{18}$$

or

$$y = f(x) = \begin{cases} \dfrac{P}{100x+30} - \dfrac{7}{10}, & \text{if } 0 \le x \le \frac{\sqrt{4+P}-5}{10} \\ \dfrac{P}{100x+70} - \dfrac{3}{10}, & \text{if } \frac{\sqrt{4+P}-5}{10} \le x \le \frac{P-21}{30} \end{cases} \tag{19}$$

which are symmetric in relation to the line $y = x$. Therefore, the relations (18) and (19) are applied the same process carried out in Theorems 2.1 and 2.2 and similar results are obtained when $P = (10x+7)(10y+3)$ or $P = (10x+3)(10y+7)$.

Example 2.1. *Given $P = 1,291,301$ for $A \ge 25$. In the case that $P = (10A+9)(10B+9)$, where $(A,B) \in \mathbb{N} \times \mathbb{N}$ we obtain via the Theorem (2.1) of [1], the following:*

i) $0.9149073718 \le \frac{N}{M} \le 0.9685686767, A \ge 25$ *where*

$$AB = \left(\frac{P-81}{200}\right)\left(1+\frac{N}{M}\right) \text{ and } A+B = \left(\frac{P-81}{180}\right)\left(1-\frac{N}{M}\right).$$

Then $12363 \le AB \le 12709$ *and* $225 \le A+B \le 655$.

From Theorem (2.2) we get

ii) $a = \frac{1}{2}\frac{ln2}{ln1291301} = 0.024630064$, *then $A = 79$, $B = 159$ (rounded value). Or*

iii) $a = \frac{ln\left(\frac{\sqrt{17}+1}{4}\right)}{ln1291301}$, *then $A = 88$, $B = 145$ (rounded value). Or*

iv) $a = \frac{ln(2.9622519063)}{ln1291301} = 0.0771755631$, *then $A = 37$, $B = 334$ (rounded value).*

So we have from (ii), (iii) and (iv) that $AB = 12,561$ or $AB = 12760$ or $AB = 12358$. Then, if there is a solution of (i) and (ii) we have that $56 < A < 112$.

Theorem 2.3. *Let P be a natural number that ends with one. If there exist $(A,B) \in \mathbb{N} \times \mathbb{N}$ such that $P = (10A+9)(10B+9)$, then exists $\lambda, k, \gamma \in \mathbb{N}$ such that:*

$$\frac{\lambda A + \gamma B}{2k} < \frac{\sqrt{P}-9}{10}$$

Proof. Let be the line $L : x + y = \frac{\lambda A+\gamma B}{k}$, then we affirm that the line L does not intersect the curve of equation: $P = (10x+9)(10y+9)$. Guided by Theorem (1), we see that for our case

$$\tau = \frac{10(\lambda A + \gamma B)}{k} \tag{20}$$

From (20) and the relation (7) we have:

$$\frac{10\lambda A}{k} + \frac{10\gamma B}{k} < 2\sqrt{P} - 9 \tag{21}$$

Of relationships (21) and (3) we have:

$$P^{1/2}\left(\frac{\lambda}{k}P^{-a} + \frac{\gamma}{P}P^{a}\right) - \frac{9(\lambda+\gamma)}{2k} + 9 < 2P^{1/2} \tag{22}$$

In (22) we place the condition

$$\lambda + \gamma = 2k \tag{23}$$

From the relation (22) and (23) we obtain:

$$\frac{\lambda}{k}P^{-a} + \frac{\gamma}{k}P^{a} < 2 \tag{24}$$

From (24) it follows that

$$1 < P^{a} < \frac{2k}{\gamma} - 1 \tag{25}$$

That is $L : x + y = \frac{\lambda A+\gamma B}{k}$ does not intersect the curve if $P^a < \frac{2k}{\gamma} - 1$. □

3 On Results in Partial Differential Equations

In this example, we find a solution u of class C^2 defined on an open and bounded domain $D \subset \mathbb{R}^2$ for the problem

$$\begin{aligned} -\sum_{i,j=1}^{2} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial}{\partial x_j}u\right) + cu &= f \text{ in } D \\ u &= 0 \text{ in } D. \end{aligned}$$

For this f must satisfy the relation (46), and this is obtained under the assumption that these operations are feasible.
Indeed,

$$-\left[\frac{\partial}{\partial x_1}\left(a_{ij}\frac{\partial}{\partial x_j}u\right) + \frac{\partial}{\partial x_2}\left(a_{2j}\frac{\partial}{\partial x_j}u\right)\right] + cu = f$$

$$-\left[\frac{\partial}{\partial x_1}\left(a_{11}\frac{\partial}{\partial x_1}u\right) + \frac{\partial}{\partial x_1}\left(a_{12}\frac{\partial}{\partial x_2}u\right) + \frac{\partial}{\partial x_2}\left(a_{22}\frac{\partial}{\partial x_2}u\right) + \frac{\partial}{\partial x_2}\left(a_{21}\frac{\partial}{\partial x_1}u\right)\right] + cu = f$$

$$-\left[\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial}{\partial x_1}\left(a_{12}\frac{\partial}{\partial x_2}u\right) + \frac{\partial^2 u}{\partial x_2^2} + \frac{\partial}{\partial x_2}\left(a_{21}\frac{\partial}{\partial x_1}u\right)\right] + cu = f \tag{26}$$

From the relation (26) we obtain:

$$a_{11}\frac{\partial^2 \mu}{\partial x_1^2} + \frac{\partial a_{11}}{\partial x_1}\frac{\partial \mu}{\partial x_1} + \frac{\partial a_{12}}{\partial x_1}\frac{\partial \mu}{\partial x_2} + a_{12}\frac{\partial^2 \mu}{\partial x_1 \partial x_2} + \frac{\partial a_{21}}{\partial x_2}\frac{\partial \mu}{\partial x_1} + a_{21}\frac{\partial^2 \mu}{\partial x_2 \partial x_1} + a_{22}\frac{\partial^2 \mu}{\partial x_2^2} + \frac{\partial a_{22}}{\partial x_2^2}\frac{\partial \mu}{\partial x_2} - c\mu = -f \tag{27}$$

$$\text{Sean } A_1 = a_{11}\frac{\partial^2 \mu}{\partial x_1^2},\ A_2 = \frac{\partial a_{11}}{\partial x_1}\frac{\partial \mu}{\partial x_1},\ A_3 = \frac{\partial a_{12}}{\partial x_1}\frac{\partial \mu}{\partial x_2},\ A_4 = a_{12}\frac{\partial^2 \mu}{\partial x_1 \partial x_2},\ A_5 = \frac{\partial a_{21}}{\partial x_2}\frac{\partial \mu}{\partial x_1}$$

$$A_6 = a_{21}\frac{\partial^2 \mu}{\partial x_2 \partial x_1},\ A_7 = a_{22}\frac{\partial^2 \mu}{\partial x_2^2},\ A_8 = \frac{\partial a_{22}}{\partial x_2^2}\frac{\partial \mu}{\partial x_2},\ A_9 = -c\mu \tag{28}$$

Let $F : \mathbb{R}^9 \longrightarrow \mathbb{R},$ be defined by $F(y_1, ..., y_n) = \sum_{i=1}^{9} A_i y_i$, therefore, following the same scheme already described in previous results, we obtain the following:

$$A_1 + ... + A_9 = \lambda[A_1^2 + ... + A_9^2] \tag{29}$$

From the relation (29) we obtain if $\lambda \neq 0$, $\lambda = \lambda(x_1, x_2)$

$$\begin{aligned} A_1 - \frac{1}{2\lambda} &= \frac{3}{2|\lambda|}b_1 \\ &\vdots \\ A_9 - \frac{1}{2\lambda} &= \frac{3}{2|\lambda|}b_9 \end{aligned} \tag{30}$$

Adding the first 8 relations in (31) and subtracting the 9th relation we get

$$-f - \frac{8}{2\lambda} + \frac{1}{2\lambda} = \frac{3}{2|\lambda|}\left[b_1 + ... + b_8 - b_9\right] \tag{31}$$

It is clear that in (30) we have $b_1^2 + ... + b_9^2 = 1$
From the relation (31) for $\lambda > 0$ we obtain

$$\lambda = \frac{-3}{2f}\left[b_1 + b_2 + ... + b_8 - b_9 + \frac{7}{3}\right] \tag{32}$$

From the relation (30) we obtain

$$\frac{\partial^2 \mu}{\partial x_1^2} = \frac{(3b_1 + 1)}{2\lambda a_{11}} \tag{33}$$

$$\frac{\partial^2 \mu}{\partial x_2^2} = \frac{(3b_7 + 1)}{2\lambda a_{22}} \tag{34}$$

Supposing that

$$\mu(x_1, x_2) = T(x_1)G(x_2) \tag{35}$$

The solution of the equation (36)

$$\frac{\partial^2 \mu}{\partial x_1^2} + \frac{\partial^2 \mu}{\partial x_2^2} = \frac{3b_1 + 1}{2\lambda a_{11}} + \frac{3b_7 + 1}{2\lambda a_{22}} \tag{36}$$

is obtained using the results of Example 3.2 of [10], so we have

$$T^{''}(x_1) = k_1 T(x_1)\,,\; G^{''}(x_2) = k_2 G(x_2) \tag{37}$$

An analysis of the relationships (33) , (34), (33)÷ (34) , (36) concludes that:
$3b_1 + 1 = \alpha_1 a_{11}$; $3b_7 + 1 = \alpha_2 a_{22}$; α_1, α_2 constants and also $\lambda(x_1, x_2) = \dfrac{\theta}{\mu}$, θ is a constant and satisfies

$$\theta = \frac{\alpha_1}{2k_1} = \frac{\alpha_2}{2k_2} \tag{38}$$

From the relation (30)

$$A_9 = \frac{3b_9 + 1}{2\lambda} = -c\mu\,,\; \theta = -\frac{\alpha_9}{2}\,,\; 3b_9 + 1 = \alpha_9 c \tag{39}$$

From the relation $A_2 = \dfrac{3b_2 + 1}{2\lambda} = \dfrac{\partial a_{11}}{\partial x_1}\dfrac{\partial \mu}{\partial x_1}$, we obtain

$$3b_2 + 1 = 2\theta\left(\frac{\partial a_{11}}{\partial x_1}\right)\left(\frac{T^{'}(x_1)}{T(x_1)}\right) \tag{40}$$

We proceed similarly for A_3, A_4, A_5, A_6, A_7 y A_8 with which we determine, $b_3, b_4, b_4, b_5, b_6, b_7$ y b_8 , namely

$$3b_3 + 1 = 2\theta\left(\frac{\partial a_{12}}{\partial x_1}\right)\left(\frac{G^{'}(x_2)}{G(x_2)}\right) \tag{41}$$

$$3b_4 + 1 = 2\theta a_{12}\left(\frac{T^{'}(x_1)}{T(x_1)}\right)\left(\frac{G^{'}(x_2)}{G(x_2)}\right) \tag{42}$$

$$3b_5 + 1 = 2\theta\left(\frac{\partial a_{12}}{\partial x_2}\right)\frac{T^{'}(x_1)}{T(x_1)} \tag{43}$$

$$3b_6 + 1 = 2\theta a_{21}\left(\frac{T^{'}(x_1)}{T(x_1)}\right)\left(\frac{G^{'}(x_2)}{G(x_2)}\right) \tag{44}$$

$$3b_8 + 1 = 2\theta\left(\frac{\partial a_{22}}{\partial x_2}\right)\frac{G^{'}(x_1)}{G(x_1)} \tag{45}$$

The relationships (38), (39),(40),(41),(42),(43),(44) and (45) in (32), we obtain

$$f = -T(x_1)G(x_2)\Bigg[k_1 a_{11} + k_2 a_{22} + \left(\frac{\partial a_{11}}{\partial x_1}\right)\left(\frac{T^{'}(x_1)}{T(x_1)}\right) + \left(\frac{\partial a_{12}}{\partial x_1}\right)\left(\frac{G^{'}(x_2)}{G(x_2)}\right) + a_{12}\left(\frac{T^{'}(x_1)}{T(x_1)}\right)\left(\frac{G^{'}(x_2)}{G(x_2)}\right)$$

$$+\left(\frac{\partial a_{21}}{\partial x_2}\right)\left(\frac{T^{'}(x_1)}{T(x_1)}\right) + a_{21}\left(\frac{T^{'}(x_1)}{T(x_1)}\right)\left(\frac{G^{'}(x_2)}{G(x_2)}\right) + \left(\frac{\partial a_{22}}{\partial x_2}\right)\left(\frac{G^{'}(x_1)}{G(x_1)}\right) + c\Bigg] \tag{46}$$

Since $u(x_1, x_2) = T(x_1)G(x_2)$, then from the relation (46) we obtain

$$u(x_1, x_2) = \frac{f}{K(x_1, x_2)}, \tag{47}$$

where $K(x_1, x_2)$ is the third factor from the right hand side given in (46).

4 On Results in Numerical Analysis

On page 142 of [3] we are invited to solve the equation

$$x^5 + x + 1 = 0 \tag{48}$$

using the fixed point theorem. This application of the mentioned theorem is not immediate. In this article, we obtain the optimal domain where the solution is found, without using calculation tools such as derivatives and other methods. For this, the techniques described in this article and in previous articles are used. Let $F(x, y) = (x_0^5)x + (x_0)y$, where x_0 is a solution of the equation given in (48)

$$x_0^5 + x_0 + 1 = 0 \tag{49}$$

Therefore, using the same arguments already made, we obtain:

$$x_0^5 + x_0 = \lambda[x_0^{10} + x_0^2] \tag{50}$$

From the relation (50) we obtain

$$x_0^4 + 1 = \lambda[x_0^9 + x_0] \tag{51}$$

It is clear that $x_0 \neq 0$ and $\lambda = \lambda(x_0)$. From (51) and (50), we obtain

$$\lambda < 0 \text{ y } x_0 < 0. \tag{52}$$

Making use of (49) and (50) we obtain the following equation

$$x_0^2 + x_0 + \frac{\lambda + 1}{2\lambda} = 0 \tag{53}$$

Completing squares the equation (53) we get

$$-2 \leq \lambda < 0 \tag{54}$$

From the relation (49) together with (52), the following is obtained:

$$-1 \leq x_0 < 0 \tag{55}$$

From the relations (53) and (55) we obtain that

$$\lambda \leq -1 \tag{56}$$

From the relations (56) and (54) we obtain

$$-2 \leq \lambda \leq -1 \tag{57}$$

Solving the equation in (53) gives

$$x_0 = \frac{-1}{2} + \frac{1}{2}\sqrt{\frac{-\lambda - 2}{\lambda}} \quad \text{o} \quad x_0 = \frac{-1}{2} - \frac{1}{2}\sqrt{\frac{-\lambda - 2}{\lambda}} \tag{58}$$

As from the relation (57) we obtain

$$0 \leq \sqrt{\frac{-\lambda - 2}{\lambda}} \leq 1 \tag{59}$$

from the relations (49), (58) and (59) we obtain that

$$-1 \leq x_0 \leq -\frac{1}{2} \tag{60}$$

This same technique can be used to determine the domain where the roots of the polynomial equation $x^n + x + m = 0$ lie, where n is an odd number greater than 5 and m is any real number. It is also useful for lowering the degree of a polynomial equation as shown in the relation (53). Another example that can be treated via this technique is: $e^x + x + 1 = 0$, following the described process we have the interval $\langle \frac{-1-\sqrt{3}}{2}, -1 \rangle$ which contains the solution of that equation. We conjecture that this technique can be used to solve equations of the form $F(x) = 0$, where F is a continuous subset function of $\mathbb{R}$.

5 Conclusions

Theorems 2.1, 2.2, allow us to choose the subset where the positive integer solution for a is found in the equation $P = (10a + 9)(10b + 9)$ where P is a given number that ends in one. In addition, said subset has half the elements that solve the given equation, and due to the symmetry of the equation in relation to the line $x = y$, it is enough to consider the mentioned case that involves the integer a.

We can conclude that, when programming in the search for the integer solutions of the aforementioned equations, the number of computational operations will be reduced to half of the total number of possible operations.

In relation to the PDE that is solved, classical solutions were obtained in a very particular way and the operations were carried out assuming that they exist.

In Section 4, we find the domain where the real root of the given equation lies without using the Intermediate Value Theorem and without using differential calculus tools.

The problem gets complicated when we try to use the traditional tools for the polynomial equation $x^n + x + m = 0$ where n is an odd number greater than five and m is any real number. Instead with the proposed technique, finding the domain where the real root is found is quite simple.

The technique that we are using to solve various problems deserves special attention because it works for various problems that apparently have nothing in common.

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REFERENCES

[1] Maguina, BM Cerna, H. Blas, and VH López Solís. ”Some results on natural numbers represented by quadratic polynomials in two variables.” In Journal of Physics: Conference Series, vol. 1558, no. 1, p. 012011. IOP Publishing, 2020. DOI:10.1088/1742-6596/1558/1/012011.

[2] Samuel, Pierre. Algebraic Theory of Numbers: Translated from the French by Allan J. Silberger. Courier Corporation, 2013, págs. 1-120.

[3] V.A Trenoguin, B.M Pisarievski, T.S Sovoleva . Problemas y Ejercicios de Análisis Funcional. Moscú : Editorial Mir, 1987, págs. 1-268.

[4] Groetsch, Charles W. Elements of applicable functional analysis, 2nd Printing edition Vol. 55. Marcel Dekker, 1980, págs. 1-300.

[5] B. M. Cerna Maguiña , Janet Mamani Ramos. “Some Results on Integer Solutions of Quadratic Polynomials in Two Variables. ” Mathematics and Statistics, Vol. 9, No. 6, pp. 931 - 938, 2021. DOI: 10.13189/ms.2021.090609.

[6] Iwaniec, Henryk. “Primes represented by quadratic polynomials in two variables.” Acta Arithmetica 5, no. 24 (1974): 435-459.

[7] Goldoni, Luca. “Prime Numbers And Polynomials.” PhD diss., University of Trento, 2010.

[8] Maguina, BM Cerna. “Some results on prime numbers.” International Journal of Pure and Applied Mathematics 118, no. 3, págs. 845-851, 2018. DOI: 10.12732/ijpam.v118i3.29

[9] Maguina, BM Cerna, H. Blas, and VH López Solís. “Some results on natural numbers represented by quadratic polynomials in two variables.” In Journal of Physics: Conference Series, vol. 1558, no. 1, p. 012011. IOP Publishing, 2020. DOI: 10.1088/1742-6596/1558/1/012011

[10] B. M. Cerna Maguiña , Dik D. Lujerio Garcia , Héctor F. Maguiña , “Some Results on Number Theory and Differential Equations,” Mathematics and Statistics, Vol. 9, No. 6, pp. 984 - 993, 2021. DOI: 10.13189/ms.2021.090614.