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On λ-Ideal Statistically Convergent of Double Sequences in n-Normed Spaces over Non-Archimedean Fields

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Abstract The main aim of this work is to investigate some important properties of statistical convergence sequence in non-Archimedean fields. Statistical convergence has been discussed in various fields of mathematics namely approximation theory, measure theory, probability theory, trigonometric series, number theory, etc. The concept of summability over valued fields is a significant area of mathematics that has many applications in analytic continuation, quantum mechanics, probability theory, Fourier analysis, approximation theory, and fixed point theory. The theory of statistical convergence plays a notable space in the summability theory and functional analysis. The purpose of this work is to provide certain characterizations of λ ideal statistical convergence of sequence and λ ideal statistical Cauchy sequence in n-normed spaces and the establishment of relevant results in non-Archimedean fields. The λ ideal statistical convergence of sequence and λ ideal statistically Cauchy sequence are defined. A few related theorems are proved in field $\mathcal{K}$. The results of this work are extended to establish statistical convergence of double sequences in n-normed space and some new results have been proved. In this work, the main concept is λ ideal statistical convergence of double sequences in n-normed space over a complete, non-trivially valued, non-Archimedean field. Throughout this article, $\mathcal{K}$ is a complete, non-trivially valued, non-Archimedean field.

Keywords λ Statistical convergence; Ideal; Ideal convergent; Double Sequences; n-Normed Space; Non-Archimedean Fields.

1 Introduction

The general idea of statistical convergence for a sequence of real numbers was introduced by Fast[2] in 1951 and later it was reintroduced by Schoenberg[13] in 1959. Further, this concept was examined and related with summability by Šalát[10], Fridy[3], Mursaleen[7], and many others[12],[14]. The statistical convergence has been studied in the theory of ergodic theory, Fourier analysis and number theory. The notion of statistical convergence was expanded to the double sequences by Mursaleen and Edely[8].

In 1964, a 2-normed space was developed by Gahler[4]. Further, the notion of n-normed spaces was investigated by Gunawan and Mashadi[5] in 2001. Since then, numerous authors have had this idea and obtained various results for statistical convergence in normed space Sahiner et al.[9], Hazarika[6], Sevim Yegul and Erdinc Dundar[11], Yamanc and Gurdal[15]. In this article, $\mathcal{N}$ signifies the set of all positive integers and $(X, \|.,\ldots,.\|)$ represents n-normed spaces. Let $\Im$ be an admissible ideal.

"A sequence $x = \{x_i\}$ is said to be statistically convergent to $\mathcal{L}$ for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n}\Big|\{i \leq n; n \in \mathcal{N} : |x_i - \mathcal{L}| \geq \varepsilon\}\Big| = 0.$$

We write

$$stat - \lim_{i\to\infty} \{x_i\} = \mathcal{L}$$

or

$$x_i \xrightarrow{\text{stat}} \mathcal{L}.$$

The sequence $x = \{x_i\}$ is said to be statistically Cauchy sequence if for every $\varepsilon > 0$, there exists a number $n \in \mathcal{N}$ such that

$$\lim_{n\to\infty}\frac{1}{n}\Big|\{i \le n; n \in \mathcal{N} : |x_{i+1}-x_i| \ge \varepsilon\}\Big| = 0.$$

Let $\lambda = \{\lambda_n\}$ be an increasing sequence of positive integer tending to ∞ such that $\lambda_{n+1} \le \lambda_n + 1, \lambda_1 = 1$, where $n \in \mathcal{N}$ and $I_n = \{n - \lambda_n + 1, n\}$.

Let $\mathcal{K}$ be a non-Archimedean field. A valuation on $\mathcal{K}$ is said to be non-Archimedean if satisfies the following axioms:[1]

(i) $|x| \ge 0$ and $|x| = 0$ iff $x = 0$,

(ii) $|xy| = |x||y|$,

(iii) $|x + y| \le \max[|x|, |y|]$ for all $x, y \in \mathcal{K}$ (Ultrametric Inequality)".

2 Preliminaries

In this section, the basic concept of λ ideal statistical convergence of double sequences in n-normed spaces over non-Archimedean fields has been discussed.

Definition 2.1 An ideal $\Im$ in $\mathcal{R}$ is a non-empty subset $\Im$ of a ring $\mathcal{R}$ of $\mathcal{N} \subset X$ if and only if

(i) $A, B \in \Im$ which implies $A \cup B \in \Im$,

(ii) $A \in \Im, B \in \mathcal{R}, B \subseteq A$, which implies $B \in \Im$.

$\Im$ is said to be non-trivial if $\Im \ne \phi$ and $\mathcal{N} \in \Im$. Then the non-trivial ideal $\Im$ in $\mathcal{N}$ is said to be admissible if $\{x\} \in \Im$ for every $x \in \mathcal{N}$, where $\mathcal{N}$ denotes the collection of all positive integers.
If $\{n\} \times \mathcal{N}$ and $\mathcal{N} \times \{n\}$ belong to $\Im$ for every $n \in \mathcal{N}$, a non-trivial ideal $\Im$ in $\mathcal{N} \times \mathcal{N}$ is said to be admissible.

Definition 2.2 Given a vector space X with the dimension 'd' where $n - 1 \le d$ over a non-Archimedean valuation $|.|$ with a valued field $\mathcal{K}$.
A function $\|., ..., .\| : X \times \cdots \times X \to [0, \infty)$ is called a non-Archimedean n-norm if

(i) $\|x_1, x_2, \ldots, x_n\| = 0$ if and only if $x_1, x_2, \ldots, x_n$ are not linearly independent;

(ii) $\|x_1, x_2, \ldots, x_n\| = \|x_{j_1}, x_{j_2}, \ldots, x_{j_n}\|$ for every permutation $(j_1, j_2, \ldots, j_n)$ of $(1, 2, \ldots, n)$;

(iii) $\|\alpha x_1, \alpha x_2, \ldots, \alpha x_n\| = |\alpha|\|x_1, x_2, \ldots, x_n\|$ for all $\alpha \in \mathcal{K}$;

(iv) $\|x + x', \ldots, x_n\| \le max\{\|x, \ldots, x_n\|, \|x', \ldots, x_n\|\}$ for all $x, x', \ldots, x_n \in X$.

Then $(X, \|., \ldots, .\|)$ is defined as non-Archimedean n-normed space.

Example 2.1 Let $X = \mathcal{R}^3$, be defined with a 2-norm given by
$$\|x, y\| = max\left\{\begin{array}{l} \|x_1y_2 - x_2y_1\| + \|x_1y_3 - x_3y_1\|, \\ \|x_1y_2 - x_2y_1\| + \|x_2y_3 - x_3y_2\| \end{array}\right\},$$
where, $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3)$.

Example 2.2 Let $X = \mathcal{R}^n$, where $\mathcal{R}$ is the field of real numbers defined with the n-norm given by

$$\|x_1, x_2, ..., x_n\| = \left|det\begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nn}\end{bmatrix}\right|.$$

That is, $\|x_1, x_2, ..., x_n\| = |det(x_{ij})|$

where, $x_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \mathcal{R}^n$ for every $i = 1, 2 \ldots, n$.

Example 2.3 Let $(X, \|., \ldots, .\|)$ be an n-normed space of dimension $d \ge n \ge 2$ and the set $\{a_1, a_2, \ldots, a_n\}$ is not linearly dependent in X. Then the function $\|., \ldots, .\|_\infty$ on $x^{(n-1)}$ is given by

$$\|x_1, x_2, \ldots, x_{(n-1)}\|_\infty = max\|x_1, x_2, \ldots, x_{(n-1)}, a_i\|$$

for every $i = 1, 2, \ldots, n$. Here, it is $(n-1)$-norm on X with respect to the set $\{a_1, a_2, \ldots, a_n\}$.

Definition 2.3 A double sequence $x = \{x_{ij}\}$ is said to be statistically convergent to $\mathcal{L}$ if for each $\varepsilon > 0$,

$$\lim_{m,n\to\infty}\frac{1}{mn}\Big|\{(i,j) \in \mathcal{N}; i \le m, j \le n : |x_{ij} - \mathcal{L}| \ge \varepsilon\}\Big| = 0.$$

We write
$$stat_2 - \lim_{i,j\to\infty}\{x_{ij}\} = \mathcal{L}.$$

Definition 2.4 Let $\Im \subset 2^{\mathcal{N}}$ be a non-trivial ideal in X. A double sequence $x = \{x_{ij}\}$ is said to be λ ideal statistically convergent to a limit $\mathcal{L}$ in n-normed space $(X, \|., \ldots, .\|)$ over a non-Archimedean field $\mathcal{K}$ if for each $\varepsilon > 0$,

$$\lim_{m,n\to\infty}\frac{1}{\lambda_{mn}}\Big|\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \ge \varepsilon\} \in \Im\Big| = 0$$

for $z_{j2}, z_{j3}, \ldots, z_{jn} \in X$. We write

$$\Im - stat^2_\lambda - \lim_{i,j\to\infty}\|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|.$$

Let $\lambda = \{\lambda_{mn}\}$ be an increasing double sequences of positive integers tending to ∞.
Here $\lambda_{mn+1} \le \lambda_{mn} + 1, \lambda_1 = 1$, for every $m, n \in \mathcal{N}$, where $I_{mn} = \{m - \lambda_m + 1, m\}, \{n - \lambda_n + 1, n\}$.

Definition 2.5 Let $\Im \subset 2^{\mathcal{N}}$ be a non-trivial ideal in X. A double sequence $x = \{x_{ij}\}$ is said to be λ ideal statistically Cauchy sequence in n-normed space $(X, \|., \ldots, .\|)$ over a non-Archimedean field $\mathcal{K}$. Then there exists a number $p, q \in \mathcal{N}$, where $p = p(\varepsilon, z_{j2}, z_{j3}, \ldots, z_{jn})$ and $q =$

$q(\varepsilon, z_{j2}, z_{j3}, \ldots, z_{jn})$ such that

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big| \{(i,j) \in I_{mn} : \|x_{ij} - x_{pq}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im \Big| = 0.$$

Definition 2.6 Let $\Im \subset 2^{\mathcal{N}}$ be a non-trivial ideal in X. A double Sequence $x = \{x_{ij}\}$ is said to be λ ideal Convergent to a limit $\mathcal{L}$ in n-normed space over a non-Archimedean field $\mathcal{K}$, for every $\varepsilon > 0$,

$$\lambda - \Im - \lim_{i,j\to\infty} \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = 0$$

or

$$\lambda - \Im - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

where for every non-zero $z_{j2}, z_{j3}, \ldots, z_{jn} \in X$".

Example 2.4 Let $x = \{x_{ij}\}$, $\Im$ be an admissible ideal and $\lambda = \{\lambda_{mn}\}$ given by

$$x_{ij} = \begin{cases} j, & if \;\; i = 1, for \; all \;\; j, \\ i, & if \;\; j = 1, for \; all \;\; i, \\ 0, & otherwise. \end{cases}$$

Then x is λ ideal statistically convergent to 0. Hence

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big| \{(i,j) \in I_{mn} : |x_{ij} - 0| \geq \varepsilon\} \in \Im \Big|$$

$$\leq \lim_{m,n\to\infty} \frac{\lambda_{mn} + 1}{\lambda_{mn}} = 0.$$

That is,

$$\Im - stat^2_\lambda - \lim_{m,n\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = 0.$$

Example 2.5 Let $x = \{x_{ij}\}$, $\Im$ be an admissible ideal and $\lambda = \{\lambda_{mn}\}$ given by

$$x_{ij} = \begin{cases} 1, & if \;\; i = 2n, for \; all \;\; j, \\ 1, & if \;\; j = 2n, for \; all \;\; i, \\ 0, & if \;\; i, j \neq 2n. \end{cases}$$

Hence

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big| \{(i,j) \in I_{mn} : |x_{ij} - 1| \geq \varepsilon\} \in \Im \Big|$$

$$\leq \lim_{m,n\to\infty} \frac{\lambda_{mn} + 1}{2\lambda_{mn}} = 0$$

and

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big| \{(i,j) \in I_{mn} : |x_{ij} - 0| \geq \varepsilon\} \in \Im \Big|$$

$$\leq \lim_{m,n\to\infty} \frac{\lambda_{mn} + 1}{2\lambda_{mn}} = 0.$$

That is,

$$\Im - stat^2_\lambda - \lim_{m,n\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = 1$$

and

$$\Im - stat^2_\lambda - \lim_{m,n\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = 0.$$

Here x is λ ideal statistically convergent to both 1 and 0 but this is not possible.

Example 2.6 Let a double sequence $x = \{x_{ij}\}$, $\Im$ be an admissible ideal and $\lambda = \{\lambda_{mn}\}$ given by

$$x_{ij} = \begin{cases} i = 1, & if \;\; m - \sqrt{\lambda_m} + 1 \leq i \leq m, \\ j = 1, & if \;\; n - \sqrt{\lambda_n} + 1 \leq j \leq n, \\ 0, & otherwise. \end{cases}$$

Hence

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big| \{(i,j) \in I_{mn} : |x_{ij} - 1| \geq \varepsilon\} \in \Im \Big|$$

$$\leq \lim_{m,n\to\infty} \frac{\sqrt{\lambda_{mn}}}{\lambda_{mn}} = \frac{1}{\lambda_{mn}{}^{1/2}}.$$

Since

$$\frac{1}{\lambda_{mn}{}^{1/2}} \to 0$$

as $m, n \to \infty$ such that x is λ ideal statistically convergent to 0.

Example 2.7 Let $x = \{x_{ij}\}$, $\Im$ be an admissible ideal and $\lambda = \{\lambda_{mn}\}$ given by

$$x_{ij} = \begin{cases} i, & if \; i \; is \; square, \;\; for \; all \;\; \mathcal{K} \\ 2, & otherwise. \end{cases}$$

Then x is neither convergent nor bounded but λ ideal-statistically convergent to 2.

3 λ Ideal Statistical Convergence and λ Ideal Statistically Cauchy Sequences of Double Sequences

In this section, we prove theorems concerning λ ideal statistical convergence, λ ideal convergence, and λ ideal statistically Cauchy sequence of double sequences in an n-normed space over non-Archimedean field $\mathcal{K}$.

Theorem 3.1 Let $\lambda = \{\lambda_{mn}\}$ and $\{x_{ij}\}$ be a double sequence in an n-normed space $(X, \|.,\ldots,.\|)$, $\Im$ be an admissible ideal. If

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

and

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}', z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

for all $z_{ij} \in X$, $i = 2, 3, \ldots, n$. Then $\mathcal{L} = \mathcal{L}'$ where $\mathcal{L}, \mathcal{L}' \in X$.

Proof: Suppose that $\mathcal{L} = \mathcal{L}'$, there exists a non-zero $z_{j2}, z_{j3}, \ldots, z_{jn} \in X$ such that $\mathcal{L} - \mathcal{L}'$ and $z_{j2}, z_{j3}, \ldots, z_{jn}$ are linearly dependent, for every $\varepsilon > 0$.
Let

$$A_1(\varepsilon) = \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| = 0. \quad (1)$$

$$A_2(\varepsilon) = \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}', z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| = 0. \quad (2)$$

Now

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|\mathcal{L} - \mathcal{L}', z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big|$$

$$= \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|(\mathcal{L} - x_{ij}) - (x_{ij} - \mathcal{L}'), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big|$$

$$\leq max \left\{ \begin{array}{l} \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|(x_{ij} - \mathcal{L}), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big|, \\ \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|(x_{ij} - \mathcal{L}'), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| \end{array} \right\}$$

$= 0$ (using [1] and [2]).
Thus

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|\mathcal{L} - \mathcal{L}', z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| = 0.$$

Therefore, $\mathcal{L} = \mathcal{L}'$.

Theorem 3.2 Given the double sequences $\{x_{ij}\}$, $\{y_{ij}\}$ in an n-normed space $(X, \|.,\ldots,.\|)$ and $\lambda = \{\lambda_{mn}\}$. If $\{y_{ij}\}$ is λ ideal convergence sequence such that $\{x_{ij}\} = \{y_{ij}\}$ for all $\varepsilon > 0$, then $\{x_{ij}\}$ is λ ideal statistically convergent.

Proof: If $\{(i,j) \in I_{mn} : |x_{ij} \neq y_{ij}|\} \in \Im = 0$ and

$$\lambda - \Im - \lim_{i,j\to\infty} \|y_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|.$$

Then for each $\varepsilon > 0$ and $z_{j2}, z_{j3}, \ldots, z_{jn} \in X$

$$\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im \subseteq \{(i,j) \in I_{mn} : |x_{ij} \neq y_{ij}|\} \in \Im.$$

Therefore,

$$\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im \leq max \left\{ \begin{array}{l} \{(i,j) \in I_{mn} : \\ \|y_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im, \\ \{(i,j) \in I_{mn} : |x_{ij} \neq y_{ij}|\} \in \Im \end{array} \right\}. \quad (3)$$

Since,

$$\lambda - \Im - \lim_{i,j\to\infty} \|y_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

for every $z_{j2}, z_{j3}, \ldots, z_{jn} \in X$,it follows that the set

$$\{(i,j) \in I_{mn} : \|y_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im$$

contains finite number of integers.
Hence,

$$\{(i,j) \in I_{mn} : \|y_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im = 0.$$

Using inequality [3], we get

$$\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im = 0$$

for every $\varepsilon > 0$ $z_{j2}, z_{j3}, \ldots, z_{jn} \in X$.
Thus

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|.$$

Theorem 3.3 Given the double sequences $\{x_{ij}\}, \{y_{ij}\}$ in an n-normed space $(X, \|.,\ldots,.\|)$ and $\lambda = \{\lambda_{mn}\}$. Let $\Im$ be an admissible ideal, where $\mathcal{L}$, $\mathcal{L}'$, $\alpha \in \mathcal{K}$. If

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

and

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}', z_{j2}, z_{j3}, \ldots, z_{jn}\|.$$

Then

(i) $\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|x_{ij} + y_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L} + \mathcal{L}', z_{j2}, z_{j3}, \ldots, z_{jn}\|$

(ii) $\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|\alpha x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\alpha\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|$

for every non zero $z_{j2}, z_{j3}, \ldots, z_{jn} \in X, i = 2, 3, \ldots, n$ for all $\alpha \in \mathcal{K}$.

Proof:
(i) Let

$$\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

and

$$\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|y_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}', z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

for all $\varepsilon > 0$ the set $A_1, A_2 \in \Im$.

$$A_1(\varepsilon) = \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| = 0. \quad (4)$$

$$A_2(\varepsilon) = \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|y_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| = 0. \quad (5)$$

$$A_0(\varepsilon) = \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|(x_{ij} + y_{ij}) - (\mathcal{L} + \mathcal{L}'), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| = 0.$$

It is sufficient to show that $A_0 \subset A_1 \cup A_2$. Then $A_0(\varepsilon) = 0$ suppose $A_1, A_2 \in A_0$, that is $j_0, k_0 \in A_0$.
Now,

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i_0,j_0) \in I_{mn} : \|(x_{i_0j_0} + y_{i_0j_0}) - (\mathcal{L} + \mathcal{L}'), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big|$$

$$= \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i_0,j_0) \in I_{mn} : \|(x_{i_0j_0} - \mathcal{L}) + (y_{i_0j_0} - \mathcal{L}'), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big|$$

$$\leq max \left\{ \begin{array}{l} \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i_0,j_0) \in I_{mn} : \|(x_{i_0j_0} - \mathcal{L}), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big|, \\ \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i_0,j_0) \in I_{mn} : \|(y_{i_0j_0} - \mathcal{L}'), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| \end{array} \right\}$$

$= 0$ (using [4] and [5]).

Hence, $j_0, k_0 \in A_0$, that is $A_0 \subset A_1 \cup A_2$.
Therefore,

$$\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|x_{ij} + y_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L} + \mathcal{L}', z_{j2}, z_{j3}, \ldots, z_{jn}\|.$$

(ii) Let

$$\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|.$$

Now,

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \frac{\varepsilon}{|\alpha|}\} \in \Im\Big|.$$

To prove that

$$\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|\alpha x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\alpha\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|,$$

where $\alpha \in \mathcal{K}$, $\alpha \neq 0$.

Now,

$$\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|\alpha x_{ij} - \alpha\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = 0,$$

which implies

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|\alpha x_{ij} - \alpha\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| = 0$$

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|\alpha(x_{ij} - \mathcal{L}), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big| = 0$$

$$= \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \||\alpha|(x_{ij} - \mathcal{L}), z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im\Big|$$

$$= \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big|\{(i,j) \in I_{mn} : \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \frac{\varepsilon}{|\alpha|}\} \in \Im\Big|.$$

Therefore,

$$\Im - stat_{\lambda}^{2} - \lim_{i,j\to\infty} \|\alpha x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\alpha\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|.$$

Theorem 3.4 Let $\{x_{ij}\}$ be a double sequence in n-normed spaces $(X, \|.,\ldots,.\|)$ and $\lambda = \{\lambda_{mn}\}$. The double sequence $\{x_{ij}\}$ is ideal statistical convergence if $\{x_{ij}\}$ is ideal statistically Cauchy sequence.

Proof:
Let

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

for every non-zero $z_{j2}, z_{j3}, \ldots, z_{jn} \in X, i = 2, 3, \ldots, n$ and $\varepsilon > 0$

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = 0.$$

If $p = p(\varepsilon, z_{j2}, z_{j3}, \ldots, z_{jn})$ and $q = q(\varepsilon, z_{j2}, z_{j3}, \ldots, z_{jn})$.

Thus

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{pq} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = 0.$$

Now we have,

$$= \lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \Big| \{(i,j) \in I_{mn} : \|x_{ij} - x_{pq}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon\} \in \Im \Big|.$$

Consider,

$$\|x_{ij} - x_{pq}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon.$$

Now

$$\|x_{ij} - x_{pq}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| + \|\mathcal{L} - x_{pq}, z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

$$\leq max \left\{ \begin{array}{l} \|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\|, \\ \|\mathcal{L} - x_{pq}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \end{array} \right\} = 0.$$

Hence,

$$\|x_{ij} - \mathcal{L}, z_{j2}, z_{j3}, \ldots, z_{jn}\| \geq \varepsilon.$$

Therefore, a double sequence $\{x_{ij}\}$ is ideal statistically Cauchy sequence.
As an immediate consequence of Theorem 3.2, we have the following result.

Theorem 3.5 Let $\{x_{ij}\}$ be a double sequence in n-normed space $(X, \|.,\ldots,.\|)$ and $\lambda = \{\lambda_{mn}\}$ such that

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L} - z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

for every non-zero $z_{ij} \in X$. Then $\{x_{ij}\}$ has a subsequence$\{x_{ij_\mathcal{K}}\}$ such that

$$\Im - stat_\lambda^2 - \lim_{i,j\to\infty} \|x_{ij}, z_{j2}, z_{j3}, \ldots, z_{jn}\| = \|\mathcal{L} - z_{j2}, z_{j3}, \ldots, z_{jn}\|$$

for every non-zero $z_{ij} \in X$.

4 Conclusions

Known results in Archimedean fields have been extended to non-Archimedean fields. Some inclusion relations in λ ideal statistical convergence of double sequences in n-normed spaces and λ ideal statistically Cauchy double sequences in n-normed spaces over non-Archimedean fields have been proved in this article.

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