Mathematics and Statistics 10(4): 816-824, 2022 http://www.hrpub.org
DOI: 10.13189/ms.2022.100412

# A New Ranking Approach for Solving Fuzzy Transportation Problem with Pentagonal Fuzzy Number

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*Received March 18, 2022; Revised July 1, 2022; Accepted July 22, 2022*

***Cite This Paper in the Following Citation Styles***

***(a):*** *[1] V. Vidhya, K. Ganesan , "A New Ranking Approach for Solving Fuzzy Transportation Problem with Pentagonal Fuzzy Number," Mathematics and Statistics, Vol. 10, No. 4, pp. 816 - 824, 2022. DOI: 10.13189/ms.2022.100412.*

***(b):*** *V. Vidhya, K. Ganesan (2022). A New Ranking Approach for Solving Fuzzy Transportation Problem with Pentagonal Fuzzy Number. Mathematics and Statistics, 10(4), 816 - 824. DOI: 10.13189/ms.2022.100412.*



**Abstract** In any decision-making process, imprecision is a significant issue. To deal with the ambiguous environment of collective decision-making, various tools and approaches have been created. Fuzzy set theory is one of the most recent approaches for coping with imprecision. The Fuzzy Transportation Problem (FTP) is a well-known network planned linear programming problem which exists in a variety of situations and has received a lot of attention recently. Many authors defined and solved the fuzzy transportation problem with frequently utilized fuzzy numbers such as triangular fuzzy numbers or trapezoidal fuzzy numbers. On the other hand, real-world problems usually involve more than four variables. To tackle these concerns, the pentagonal fuzzy number is applied to the problems. This article proposes an approach to solving transportation problems whose parameters are pentagonal fuzzy numbers without requiring an initial feasible solution. An algorithm based on the core and spread method and an extended MODI method is developed to determine the optimal solution to the problem. The proposed process is based on the approximation method and gives a more efficient result. An illustrated example is used to validate the model. As a result, the proposed methodology is both simpler and more computationally efficient than the existing approaches.

**Keywords** Pentagonal Fuzzy Number, Optimization, Fuzzy Transportation Problem, Ranking Function, Arithmetic Operation

## 1. Introduction

Fuzzy sets are a generalization of traditional sets theory that allows for intermediate conditions between the whole and nothing. To account for the uncertainty in decision-making, Zadeh [1] proposed the notion of a fuzzy set, which has a membership function. To demonstrate the degree of belongingness to the set under discussion, each member of the universe of discourse is assigned a value from the unit interval [0, 1]. In a fuzzy set, a membership function is used to describe the degree to which an element belongs to a class. The membership value varies from 0 to 1, with 0 indicating that the element is not a member of a class, 1 indicating that it is, and extra values indicating the degree of class membership.

Rather than being precise, as is the case with ordinal numbers, a fuzzy number has values that are unclear. A triangular fuzzy number is a commonly used fuzzy number to represent various decision-makers' opinions. So far fuzzy numbers like trapezoidal fuzzy numbers, triangular fuzzy numbers, pentagonal fuzzy numbers, heptagonal fuzzy numbers, diamond fuzzy numbers, and pyramid fuzzy numbers have also been discussed with their membership functions. Risk analysis, Non-linear equations, and reliability are just a few of the applications for these numbers. Many processes were carried out using the use of ambiguous numbers.

Some of the information we gathered during our observations could not be verified exactly or accurately due to measurement technique errors, equipment faultiness, and other factors. Let's pretend we're measuring the temperature and humidity of the weather at the same time. The temperature is around 25℃ with standard humidity, which means that the temperature is neither exact nor below 25 ℃, affecting standard humidity in the atmosphere. As a result, temperature changes have an impact on the percentage of humidity. This is an all-too-common occurrence. A pentagonal fuzzy number is a novel form of fuzzy number based on this concept of variation.

In today's competitive market, customer happiness has risen to the top of the priority list for businesses, and the pressure on businesses to adopt best practices for producing and providing advantages to customers is increasing. However, it has become increasingly difficult to deliver manufactured items in sufficient quantities to consumers who want them to be less expensive.

The transportation problem (TP) is a least-cost planning problem for transferring a product from manufacturers to warehouses while taking into account the cost of shipping from one location to another. The TP's objective is to obtain the best transportation schedule at the least amount of cost. Hitchcock [2] proposed the basic transportation model in 1941 when the transportation restrictions were dependent on crisp values. Transportation problems can be decided to apply to a wide range of circumstances, including production, scheduling, investment, plant placement, inventory control, and employee scheduling. Many authors have developed a mathematical model for transportation problems in a variety of circumstances.

However, in today's environment, transportation parameters such as requirement, availability, and transportation cost may be unpredictable based on a variety of uncontrollable factors. Zimmermann [3] introduced a Fuzzy Linear Programming (FLP) problem and demonstrated that the method was always highly efficient.

Ladji Kané [4] proposed a two-step technique for explaining a fuzzy transportation problem containing triangular fuzzy numbers. Aurora Nur Aini [5] discussed the technique to explain the problem of transportation without finding an initial feasible solution with the help of the Zero Suffix method and the ASM method. Nirbhay Mathur [6] proposed the minimum demand-supply technique to explain the fuzzy transportation problem containing trapezoidal fuzzy numbers. Many authors [7-10] defined and solved the fuzzy transportation problem with frequently applied fuzzy numbers namely triangular fuzzy numbers or trapezoidal fuzzy numbers. On the other hand, real-world problems usually involve more than four variables. To resolve those problems, the pentagonal fuzzy number is used.

Charles Rabinson [11] proposed the allocation table process (ATM) to determine an initial basic feasible solution for fuzzy transportation problems with pentagonal fuzzy numbers. Apurba Panda [12] discussed the special type of pentagonal fuzzy matrices and their algebraic natures. Also, they discussed the properties such as comparable, nilpotent, and constant pentagonal fuzzy matrices. Pathinathan [13-16] proposed four kinds of similarity measures for pentagonal fuzzy numbers and analyzed the similarity measure between two of them. Sankar Prasad Mondal [17] defined different forms of pentagonal fuzzy numbers and also addressed their arithmetic operations.

The characteristics of pentagonal fuzzy numbers were extended to interval-valued fuzzy numbers and pentagonal fuzzy numbers in the literature. Suhailywati Ramli [18] discussed a portfolio selection model that builds on Markowitz's traditional mean-variance model by representing returns as pentagonal fuzzy numbers. Intan Arfina [19] introduced the alternative arithmetic of pentagonal fuzzy numbers on the concept of positive and negative fuzzy numbers. Sapan Kumar Das [20] introduced a new Neutrosophic pentagonal number score function and established a novel method for obtaining initial fundamental workable answers.

### 1.1. Motivation for the Study

Uncertainty can lead to information loss if it is not properly managed. The majority of existing methods for dealing with fuzzy transportation problems are based on the idea of transforming them into one or more equivalent crisp transportation models. The most important task when converting a fuzzy model to a crisp transportation model is figuring out how to do so. Because the solution obtained by converting the fuzzy model into an equivalent crisp model may not be the original problem, its validity must be verified. The crisp model does not accurately represent the optimization model's fuzzy nature.

As a result, solving a fuzzy transportation problem is dependent on many factors, including the nature of the problems, the decision-makers' choice, and the ranking of the goal as well as its assessment. Motivated by these factors, this article focuses on obtaining an effective fuzzy optimal/efficient solution for a given fuzzy transportation problem without translating it into an equivalent crisp problem, thereby preserving the problem's fuzzy nature.

### 1.2. Novelties of the Work

In an uncertain environment, fuzzy set theory is an effective tool for explaining decision-making problems and other engineering disciplines. The foremost objective of this research paper is to solve the fuzzy transportation problem using pentagonal fuzzy numbers by

1. Using pentagonal fuzzy numbers to express input parameters.
2. Constructing arithmetic operations for pentagonal fuzzy numbers based on core and spread concepts consisting of the operations of addition, subtraction, scalar multiplication, as well as multiplication of two fuzzy numbers.
3. Finding rank based on core and spread concepts.

The following is the outline for the paper. In section 2 we explain the origin of various types of fuzzy numbers and their background work with some basic definitions. The proposed theoretical research method is presented in section 3. The case study of the method is examined in section 4.

## 2. Preliminaries

**Definition 2.1:**

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set of the real line R such that $\{\mu_{\tilde{A}}(x): R \to [0,1]\}, \forall x \in R$ where $\mu_{\tilde{A}}(x)$ is referred to as the membership function of the fuzzy set and is piece-wise continuous.

**Definition 2.2:**

A pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ (Figure 1) with $a_1, a_2, a_3, a_4$ and $a_5$ $(a_1 \le a_2 \le a_3 \le a_4 \le a_5)$ are real numbers and its membership function is as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_3-a_2} & a_2 \le x \le a_3 \\ 1 & x = a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 \le x \le a_4 \\ \dfrac{a_5-x}{a_5-a_4} & a_3 \le x \le a_4 \\ 0 & a_5 \le x \end{cases}$$

### 2.1. Alternate Representation of Fuzzy Numbers

A fuzzy number can also be represented in terms of core (m), left spread (r), and right spread (s). The core of a triangular fuzzy number is a point and the core of a trapezoidal fuzzy number is an interval.

In this article, we represent the pentagonal fuzzy number in the form of $\tilde{A} = (m, r_1, r_2, s_1, s_2)$ as given in Figure 2.

Here m is the centroid point, $r_1$ is the distance of left spread from center m to the point $(m - r_1)$, and $r_2$ is the distance of left spread from the point $(m - r_1)$ to the point $(m - r_1 - r_2)$.

Furthermore, $s_1$ is the distance of right spread from center m to point $(m + s_1)$ and $s_2$ is the distance of right spread from center $(m + s_1)$ to the point $(m + s_1 + s_2)$.

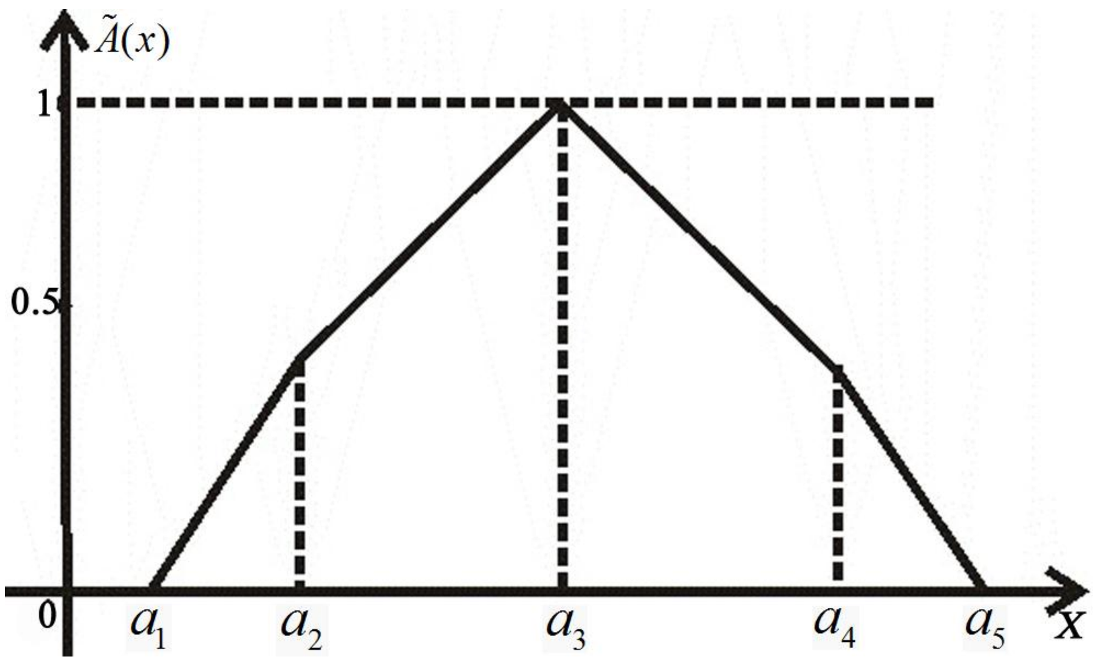


**Figure 1.** Pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ with symmetry

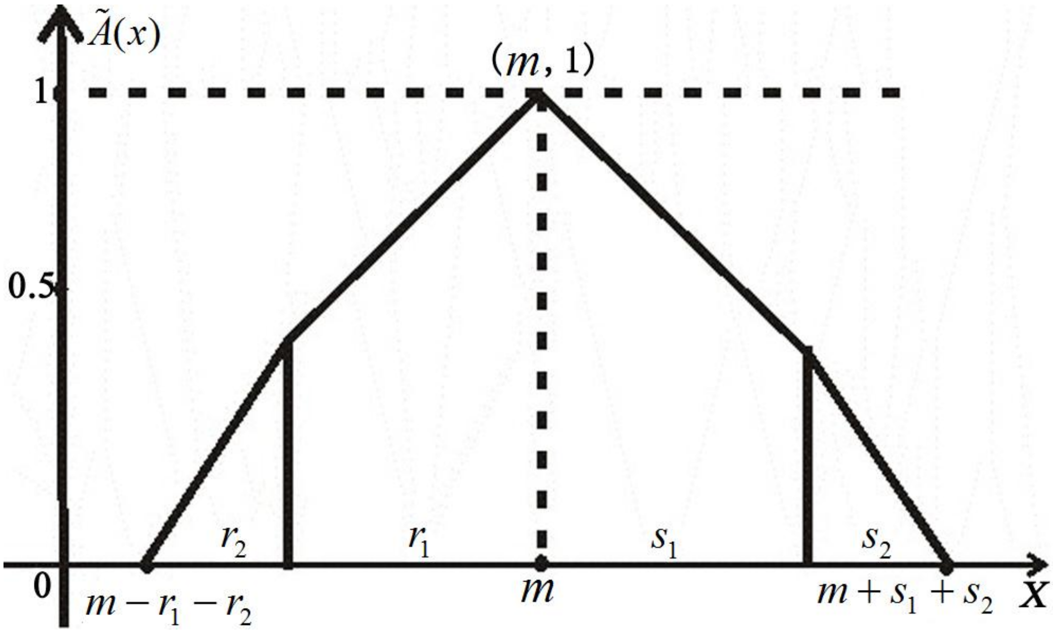


**Figure 2.** Pentagonal fuzzy number $\tilde{A}=(m,r_1,r_2,s_1,s_2)$

**Definition 2.3:**

A fuzzy number $\tilde{A}:R\to[0,1]$ where $\tilde{A}=(m,r_1,r_2,s_1,s_2)$ satisfies the following properties:

(i). $\tilde{A}(x)$ is an upper semi-continuous and $\tilde{A}(x)=0$ outside the interval $[m-r_1-r_2,m+s_1+s_2]$

(ii). There exists a real number x in interval $[m-r_1-r_2,m+s_1+s_2]$ such that

a. $\tilde{A}(x)$ is a monotonic increasing function in interval $[m-r_1-r_2,m-r_1]$ and $[m-r_1,m]$

b. $\tilde{A}(x)$ is a monotonic decreasing function in interval $[m,m+s_1]$ and $[m+s_1,m+s_1+s_2]$

(iii). $\tilde{A}(x)=1$ for $x=m$.

Membership function of the pentagonal fuzzy number $\tilde{A}=(m,r_1,r_2,s_1,s_2)$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{1}{2}+\frac{1}{2}\left(\frac{x-m+r_1}{r_2}\right), & m-r_1-r_2\le x\le m-r_1\\ 1+\frac{1}{2}\left(\frac{x-m}{r_1}\right), & m-r_1\le x\le m\\ 1, & x=m\\ 1-\frac{1}{2}\left(\frac{x-m}{s_1}\right), & m\le x\le m+s_1\\ \frac{1}{2}-\frac{1}{2}\left(\frac{x-m+s_1}{s_2}\right), & m+s_1\le x\le m+s_1+s_2\\ 0, & \text{otherwise}\end{cases}$$

## 2.2. Arithmetic Operations on Fuzzy Numbers

Fuzzy numbers are first represented in their alternative form. Similar to the arithmetic operations defined by Ming Ma [21], new arithmetic operations are used for fuzzy numbers in their alternate form. The arithmetic operations on the pentagonal fuzzy numbers are based upon the left spread and right spread and the core. The core obeys the usual arithmetic whereas the right and left spread obey the lattice rule.

That is for

$$r,s\in L,\ r\vee s=\max\{r,s\}\text{ and }r\wedge s=\min\{r,s\}.$$

For any pentagonal fuzzy numbers, $\tilde{A}=(m_1,r_1,r_2,s_1,s_2)$ and $\tilde{B}=(m_1',r_1',r_2',s_1',s_2')$.

The arithmetic operations are defined as

Addition:

$$\begin{aligned}\tilde{A}+\tilde{B}&=(m_1,r_1,r_2,s_1,s_2)+(m_1',r_1',r_2',s_1',s_2')\\&=(m_1+m_1',\max\{r_1,r_1'\},\max\{r_2,r_2'\},\\&\quad\max\{s_1,s_1'\},\max\{s_2,s_2'\})\end{aligned}$$

Subtraction:

$$\begin{aligned}\tilde{A}-\tilde{B}&=(m_1,r_1,r_2,s_1,s_2)-(m_1',r_1',r_2',s_1',s_2')\\&=(m_1-m_1',\max\{r_1,r_1'\},\max\{r_2,r_2'\},\\&\quad\max\{s_1,s_1'\},\max\{s_2,s_2'\})\end{aligned}$$

Multiplication:

$$\tilde{A}\times\tilde{B}=(m_1,r_1,r_2,s_1,s_2)\times(m_1',r_1',r_2',s_1',s_2')$$
$$=(m_1\times m_1',\max\{r_1,r_1'\},\max\{r_2,r_2'\},$$
$$\max\{s_1,s_1'\},\max\{s_2,s_2'\})$$

Division:

$$\tilde{A}\div\tilde{B}=(m_1,r_1,r_2,s_1,s_2)\div(m_1',r_1',r_2',s_1',s_2')$$
$$=(m_1\div m_1',\max\{r_1,r_1'\},\max\{r_2,r_2'\},$$
$$\max\{s_1,s_1'\},\max\{s_2,s_2'\})$$

It is simple to verify that the following mathematical properties hold:

**Theorem 2.1:**

For all symmetric pentagonal fuzzy numbers $\tilde{A},\tilde{B}$ and $\tilde{C}$, we have

1. $\tilde{A}+\tilde{B}=\tilde{B}+\tilde{A}$
2. $(\tilde{A}+\tilde{B})+\tilde{C}=\tilde{A}+(\tilde{B}+\tilde{C})$
3. $\tilde{A}\tilde{B}=\tilde{B}\tilde{A}$
4. $(\tilde{A}\tilde{B})\tilde{C}=\tilde{A}(\tilde{B}\tilde{C})$
5. $\tilde{A}(\tilde{B}+\tilde{C})=\tilde{A}\tilde{B}+\tilde{A}\tilde{C}$

Proof: It is trivial by definition **2.2.**

### 2.3. Ranking Function

In a fuzzy background, ranking fuzzy numbers is essential in the decision-making process. Ranking of fuzzy numbers is required to find the largest and smallest fuzzy numbers. Various researchers [22-23] have proposed various ranking methods.

In this article, a modified ranking function based on their graded means is used for comparing the fuzzy numbers. For every pentagonal fuzzy number $\tilde{A}=(m,r_1,r_2,s_1,s_2)\in F(\mathbb{R})$, the ranking function $\mathbb{R}:F(R)\to R$ is defined by its graded mean as

$$R(\tilde{A})=\frac{S+4m-R}{4}\quad\text{where}\quad R=r_1+r_2,\ S=s_1+s_2.$$

For any two fuzzy numbers

$$\tilde{\alpha}=(m_1,r_1,r_2,s_1,s_2),\ \tilde{\beta}=(m_1',r_1',r_2',s_1',s_2')\in F(\mathbb{R}),$$

their comparison is done as follows:

a. $\tilde{\alpha}\succeq\tilde{\beta}$ if $R(\tilde{\alpha})\ge R(\tilde{\beta})$

b. $\tilde{\alpha}\preceq\tilde{\beta}$ if $R(\tilde{\alpha})\le R(\tilde{\beta})$

c. $\tilde{\alpha}\approx\tilde{\beta}$ if $R(\tilde{\alpha})=R(\tilde{\beta})$

Any fuzzy number $\tilde{\alpha}$ in $F(R)$ is said to be positive, if $R(\tilde{\alpha})>0$ and is denoted by $\tilde{\alpha}\succ\tilde{0}$.

If $R(\tilde{\alpha})=0$ then $\tilde{\alpha}$ is said to be a zero fuzzy number and is denoted by $\tilde{\alpha}\approx\tilde{0}$.

**Proposition 2.1:**

Let E denotes the set of all symmetric pentagonal fuzzy numbers. Let $\tilde{A},\tilde{B}$, $\tilde{A}+\tilde{C}$ and $\tilde{B}+\tilde{C}$ be elements of E. If $\tilde{A}\succeq\tilde{B}$ then $\tilde{A}+\tilde{C}\succeq\tilde{B}+\tilde{C}$.

Proof:

Consider the following pentagonal fuzzy numbers:

$$\tilde{A}=(m,r_1,r_2,s_1,s_2)$$
$$\tilde{B}=(m_1',r_1',r_2',s_1',s_2')$$
$$\tilde{C}=(m_1'',r_1'',r_2'',s_1'',s_2'')$$

Let $\tilde{A}\succeq\tilde{B}$ then from the relation (a), we have $R(\tilde{A})\ge R(\tilde{B})$.

By adding $R(\tilde{C})$, $R(\tilde{A})+R(\tilde{C})\ge R(\tilde{B})+R(\tilde{C})$

For two arbitrary pentagonal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, we have $R(\tilde{A}+\tilde{B})\ge R(\tilde{A})+R(\tilde{B})$.

Therefore $R(\tilde{A}+\tilde{C})\ge R(\tilde{B}+\tilde{C})$.

Hence $\tilde{A}+\tilde{C}\succeq\tilde{B}+\tilde{C}$.

Suppose $\tilde{A}\succ\tilde{B}$ then $\tilde{A}+\tilde{C}\succ\tilde{B}+\tilde{C}$ where $\tilde{C}\neq 0$.

**Proposition 2.2:**

For all symmetric pentagonal fuzzy number $\tilde{A}=(m,r_1,r_2,s_1,s_2)$, $R(\tilde{A}-\tilde{A})=0$.

Proof:

Let $\tilde{A}=(m,r_1,r_2,s_1,s_2)$ be a symmetric pentagonal fuzzy number.

$$\tilde{A}-\tilde{A}=(m-m,\max(r_1,r_1),\max(r_2,r_2),\max(s_1,s_1),\max(s_2,s_2))$$
$$=(0,r_1,r_2,s_1,s_2)$$

$R(\tilde{A}-\tilde{A})=0$ [since mode $(\tilde{A}-\tilde{A})=0$]

### 2.4. Comparative Analysis

Table 1 shows a comparison of the proposed ranking technique with existing methods based on examples from [24].

Consider

$\tilde{a}=(1,2,3,4,5)$, $\tilde{b}=(-2,1,0,2,4)$, $\tilde{c}=(-2,-1,0,1,2)$ By the proposed method $R(\tilde{a})=3$, $R(\tilde{b})=0.5$, $R(\tilde{c})=0$

Hence $\tilde{a}\succ\tilde{b}\succ\tilde{c}$

**Table 1.** Comparison of the existing method of ranking three fuzzy numbers with the proposed method

| **Ranking Method** | $\tilde{a}$ | $\tilde{b}$ | $\tilde{c}$ | **Ranking order** |
|---|---|---|---|---|
| Graded mean method | 3 | 0.916 | 0 | $\tilde{a} \succ \tilde{b} \succ \tilde{c}$ |
| Signed distance method | 3 | 1 | 0 | $\tilde{a} \succ \tilde{b} \succ \tilde{c}$ |
| Centroid method | 3 | 1.0563 | 0 | $\tilde{a} \succ \tilde{b} \succ \tilde{c}$ |
| Alpha cut method | 3 | 1.05 | 0 | $\tilde{a} \succ \tilde{b} \succ \tilde{c}$ |
| Removal area method | 2.6 | 0.48 | 0.112 | $\tilde{a} \succ \tilde{b} \succ \tilde{c}$ |
| Proposed method | 3 | 0.5 | 0 | $\tilde{a} \succ \tilde{b} \succ \tilde{c}$ |

## 3. Algorithm for the Proposed Method

1. Write each pentagonal fuzzy number in an alternate form. Using the proposed ranking function to obtain the minimum and maximum of a pentagonal fuzzy number.
2. Choose the least cost of each row and deduct it from the remaining cost coefficient.
3. Do it column-wise also. Each row and column should include at least one zero.
4. Find the difference between the least and next least cost in each row and also do it for the column. Then find the greatest difference.
5. Then do the allocation in the zero cost of corresponding row (or) column of greatest difference.
6. Repeat the above steps till the initial basic feasible solution is obtained
7. By utilizing the fuzzy version of the Modified distribution method, check the optimal condition.

## 4. Numerical Example

A balanced fuzzy transportation problem which is given in Table 2 is discussed by Rasha Jalal Miltif [25] in which fuzzy demand, fuzzy availability, and fuzzy cost are pentagonal fuzzy numbers.

Before applying the proposed method, express the problem in an alternate form $\tilde{A} = (m, r_1, r_2, s_1, s_2)$ as follows in Table 3.

By using the proposed ranking and algorithm, the optimal allocation is given in Table 4, and verification of optimality is shown in Table 5. It shows the value of $u_i$ and $v_j$ which contains the costs associated with the allocated cells.

**Table 2.** Fuzzy Transportation Problem

| | $D_1$ | $D_2$ | $D_3$ | Fuzzy Availability |
|---|---|---|---|---|
| $O_1$ | (1,3,4,5,7) | (0,2,3,4,6) | (2,4,5,6,8) | (4,6,7,8,10) |
| $O_2$ | (3,5,6,7,9) | (5,7,8,9,11) | (4,6,7,8,10) | (6,8,9,10,12) |
| $O_3$ | (1,3,4,5,7) | (2,4,5,6,8) | (3,5,6,7,9) | (10,12,13,14,16) |
| Fuzzy Demand | (3,5,6,7,9) | (5,7,8,9,11) | (12,14,15,16,18) | |

**Table 3.** Fuzzy Transportation Problem in the form $(m, r_1, r_2, s_1, s_2)$

| | $D_1$ | $D_2$ | $D_3$ | Fuzzy Availability |
|---|---|---|---|---|
| $O_1$ | (4,1,2,1,2) | (3,1,2,1,2) | (5,1,2,1,2) | (7,1,2,1,2) |
| $O_2$ | (6,1,2,1,2) | (8,1,2,1,2) | (7,1,2,1,2) | (9,1,2,1,2) |
| $O_3$ | (4,1,2,1,2) | (5,1,2,1,2) | (6,1,2,1,2) | (13,1,2,1,2) |
| Fuzzy Demand | (6,1,2,1,2) | (8,1,2,1,2) | (15,1,2,1,2) | |

**Table 4.** Optimal allocation for the Fuzzy Transportation Problem

| | | |
|---|---|---|
| (4,1,2,1,2) | (3,1,2,1,2)<br>(7,1,2,1,2) | (5,1,2,1,2) |
| (6,1,2,1,2) | (8,1,2,1,2) | (7,1,2,1,2)<br>(9,1,2,1,2) |
| (4,1,2,1,2)<br>(6,1,2,1,2) | (5,1,2,1,2)<br>(1,1,2,1,2) | (6,1,2,1,2)<br>(6,1,2,1,2) |

**Table 5.** Fuzzy Version of MODI Method

| | | | | $u_i$ |
|---|---|---|---|---|
| | (2,1,2,1,2)<br>(4,1,2,1,2) | (0,1,2,1,2)<br>(3,1,2,1,2) | (1,1,2,1,2)<br>(5,1,2,1,2) | (-2,1,2,1,2) |
| | (1,1,2,1,2)<br>(6,1,2,1,2) | (2,1,2,1,2)<br>(8,1,2,1,2) | (0,1,2,1,2)<br>(7,1,2,1,2) | (1,1,2,1,2) |
| | (0,1,2,1,2)<br>(4,1,2,1,2) | (0,1,2,1,2)<br>(5,1,2,1,2) | (0,1,2,1,2)<br>(6,1,2,1,2) | (0,1,2,1,2) |
| $v_j$ | (4,1,2,1,2) | (5,1,2,1,2) | (6,1,2,1,2) | |

The solution is non-degenerate, and the optimality is verified by the fuzzy version of the Modified distribution method. Minimum fuzzy transportation cost

$$\begin{aligned}&\approx (7,1,2,1,2)(3,1,2,1,2)+(9,1,2,1,2)(7,1,2,1,2)\\&\quad+(6,1,2,1,2)(4,1,2,1,2)+(1,1,2,1,2)(5,1,2,1,2)\\&\quad+(6,1,2,1,2)(6,1,2,1,2)\\&\approx (149,1,2,1,2)\end{aligned}$$

Hence the fuzzy optimal transportation cost in terms of $\tilde{A}=(a_1,a_2,a_3,a_4,a_5)$ is given by

$$Min\,\tilde{z} \approx (146,148,149,150,152)$$

For the same problem, Rasha Jalal Mitlif [25] obtained only the crisp transportation cost of 149. Also, compared to existing techniques, we determine the optimal solution without affecting the fuzzy nature, and also it has lower spreads as shown in Figure 3 which benefits the decision-maker more.

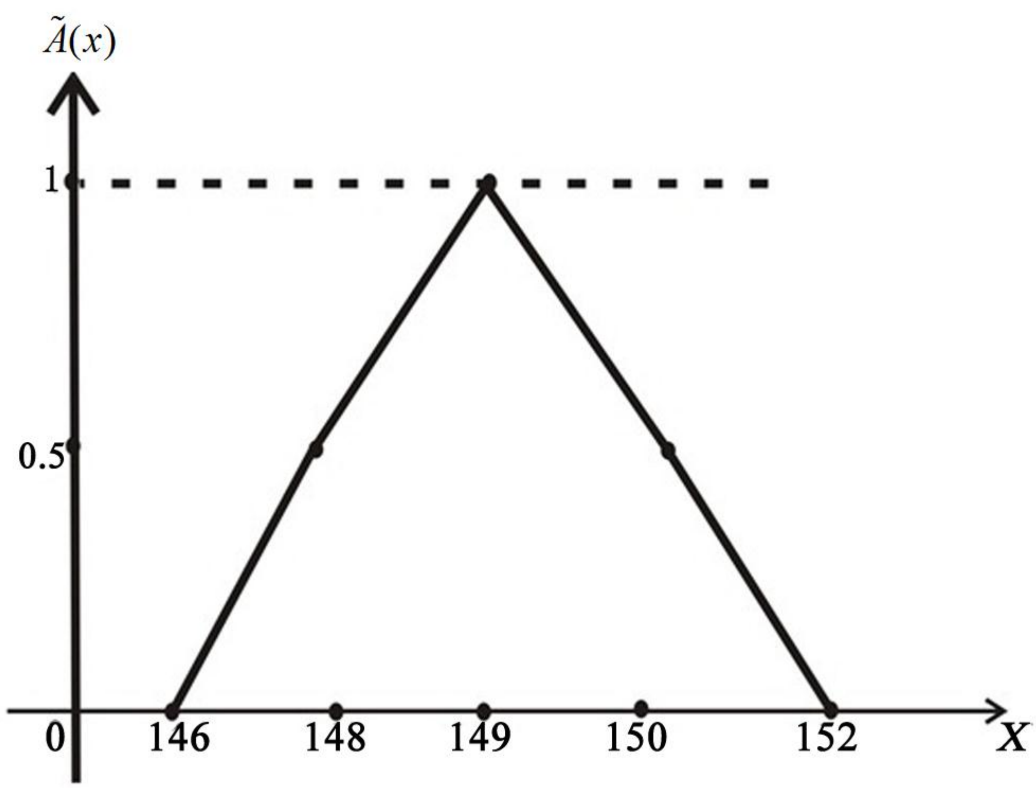


**Figure 3.** Graph of Solution

# 5. Conclusions

Even though numerous transportation problems have been examined with various forms of input data, this study focuses on the solutions to a transportation problem with pentagonal fuzzy numbers in an unpredictable environment. The arithmetic operations on pentagonal fuzzy numbers based on core and spread are employed to find the solutions. The procedure is better organized and takes less time. Decision-makers in the disciplines of logistics and supply chain management might benefit from this strategy because it is faster in terms of calculating procedures.

# Acknowledgements

We are very grateful to experts for their appropriate and constructive suggestions to improve this template.

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