







$\mathbf{Z} = (\mathbf{Z}'_1, \dots, \mathbf{Z}'_N)$  is a  $N(T-2) \times m$  matrix and  $\hat{\alpha}$  value defined as follows:

$$\hat{\alpha} = \operatorname{argmin}_{\alpha} (\bar{\mathbf{v}}' \mathbf{Z}) \mathbf{A}_N (\mathbf{Z}' \bar{\mathbf{v}}) = \frac{\bar{\mathbf{y}}'_{it-1} \mathbf{Z} \mathbf{A}_N \mathbf{Z}' \bar{\mathbf{y}}_{it}}{\bar{\mathbf{y}}'_{it-1} \mathbf{Z} \mathbf{A}_N \mathbf{Z}' \bar{\mathbf{y}}_{it-1}} \quad (5)$$

Furthermore Lee and Yu[23] formulated the GMM estimate for the dynamic panel spatial model with a fixed effect where the model developed is the high order model for equations as much as  $p$ , as follows:

$$\begin{aligned} \mathbf{y}_{it} &= \sum_{j=1}^p \lambda_{j0} \mathbf{W}_{ij} \mathbf{y}_{nt} + \gamma_0 \mathbf{y}_{i,t-1} + \sum_{j=1}^p \rho_{j0} \mathbf{W}_{ij} \mathbf{y}_{i,t-1} \\ &\quad + \mathbf{X}_{it} \boldsymbol{\beta}_0 + \mathbf{c}_{i0} + \alpha_{t0} \mathbf{I}_i + \mathbf{v}_{it}, \quad (6) \\ \mathbf{v}_{it} &\sim N(\mathbf{0}, \sigma_v^2), \quad t = 1, 2, \dots, T, \quad i = 1, 2, \dots, n \end{aligned}$$

so that optimal estimation of parameters with GMM is obtained in the following way:

$$\hat{\boldsymbol{\theta}}_{o,nT} = \operatorname{argmin}_{\boldsymbol{\theta} \in \Theta} \mathbf{g}'_{nT}(\boldsymbol{\theta}) \boldsymbol{\Sigma}_{nT}^{-1} \mathbf{g}_{nT}(\boldsymbol{\theta}) \quad (7)$$

that have asymptotic distribution

$$\sqrt{n}(\hat{\boldsymbol{\theta}}_{o,nT} - \boldsymbol{\theta}_0) \xrightarrow{d} N(\mathbf{0}, \operatorname{plim}_{n \rightarrow \infty} \frac{1}{T-1} (\mathbf{D}'_{nT} \boldsymbol{\Sigma}_{nT}^{-1} \mathbf{D}_{nT})^{-1}).$$

Further Yang and Lee [20] developed a model Arrelano, Bond [24] for simultaneous models written in the form of a big vector as in equations (9),

$$\begin{aligned} (\boldsymbol{\Gamma}'_{m0} \ddot{\mathbf{A}} \mathbf{J}_n) \operatorname{vec}(\mathbf{Y}_{nm,t}^*) &= (\boldsymbol{\psi}'_{ms0} \ddot{\mathbf{A}} \mathbf{J}_n) \mathbf{W}_{nm} \operatorname{vec}(\mathbf{Y}_{nm,t}^*) \\ &\quad + [\mathbf{P}'_{ms0} \otimes \mathbf{J}_n + (\boldsymbol{\Phi}'_{ms0} \otimes \mathbf{J}_n) \mathbf{W}_{nm}] \operatorname{vec}(\mathbf{Y}_{nm,t}^{(*,-1)}) \end{aligned} \quad (8)$$

with the single equation approach to the  $l$ -th equation, Yang and Lee[20] write the equation (9) into,

$$\begin{aligned} \mathbf{J}_n \mathbf{y}_{nl,t} &= -\mathbf{J}_n \mathbf{Y}_{l,mm,t} \boldsymbol{\gamma}_{l,m0} + \mathbf{J}_n \ddot{\mathbf{Y}}_{l,mm,t}^* \boldsymbol{\psi}_{l,ms0} + \mathbf{J}_n \mathbf{Y}_{l,mm,t-1}^* \mathbf{P}_{l,ms0} \\ &\quad + \mathbf{J}_n \ddot{\mathbf{Y}}_{l,mm,t-1}^{(*,-1)} \boldsymbol{\phi}_{l,ms0} + \mathbf{J}_n \mathbf{X}_{l,nt}^* \boldsymbol{\pi}_{l,ms0} + \mathbf{J}_n \mathbf{u}_{nl,t}^*. \end{aligned} \quad (9)$$

The IV matrix used is,

$$\begin{aligned} \mathbf{Q}_{nm,t} &= [\mathbf{Y}_{nm,t-1}, \mathbf{W}_{ln} \mathbf{y}_{nl,t-1}, \dots, \mathbf{W}_{mn} \mathbf{y}_{nm,t-1}, \mathfrak{B}_n^2 \mathbf{Y}_{nm,t-1} \\ &\quad , \mathbf{X}_{n,t}^*, \mathbf{W}_{ln} \mathbf{X}_{n,t}^*, \dots, \mathbf{W}_{mn} \mathbf{X}_{n,t}^*, \mathfrak{B}_n^2 \mathbf{X}_{n,t}^*] \end{aligned} \quad (10)$$

with an order as large as  $n \times k_q$ , where

$$\begin{aligned} \mathfrak{B}_n^2 \mathbf{Z} &= [\mathbf{W}_{1n} \mathbf{W}_{1n} \mathbf{Z}, \dots, \mathbf{W}_{1n} \mathbf{W}_{mn} \mathbf{Z}, \mathbf{W}_{2n} \mathbf{W}_{1n} \mathbf{Z}, \dots, \\ &\quad , \mathbf{W}_{mn} \mathbf{W}_{nm} \mathbf{Z}]. \end{aligned} \quad (11)$$

The moment of linear conditions formed by Yang and Lee [20] is

$$E[\mathbf{Q}'_{nm,T-1} (\mathbf{I}_{T-1} \otimes \mathbf{J}_n) \mathbf{u}_{nl,T-1}^*] = 0 \quad (12)$$

where,

$$\begin{aligned} \mathbf{u}_{nl,t}^*(\boldsymbol{\theta}_l) &= \mathbf{y}_{nl,t}^* + \mathbf{Y}_{l,mm,t}^* \boldsymbol{\gamma}_{l,ms} - \ddot{\mathbf{Y}}_{l,mm,t-1}^{(*,-1)} \boldsymbol{\psi}_{j,ms} \\ &\quad - \mathbf{Y}_{l,mm,t-1}^{(*,-1)} \mathbf{P}_{l,ms} - \ddot{\mathbf{Y}}_{l,mm,t-1}^{(*,-1)} \boldsymbol{\phi}_{l,ms} - \\ &\quad + \mathbf{X}_{j,nt}^* \boldsymbol{\pi}_{l,ms0}, \end{aligned} \quad (13)$$

the empirical moment to estimate its parameters is defined as:

$$g_{nl,T}(\boldsymbol{\theta}_l) = \frac{1}{n(T-1)} \mathbf{Q}'_{nm,T-1} (\mathbf{I}_{T-1} \otimes \mathbf{J}_n) \mathbf{u}_{nl,T}^*(\boldsymbol{\theta}_l) \quad (14)$$

and for estimate IV obtained by equation:

$$\hat{\boldsymbol{\theta}}_{nl,T,IV} = \operatorname{argmin}_{\boldsymbol{\theta}} g_{nl,T}(\boldsymbol{\theta}_l)' \mathbf{a}'_{nl,T} \mathbf{a}_{nl,T} g_{nl,T}(\boldsymbol{\theta}_l) \quad (15)$$

whereas to estimate 2SLS,  $\mathbf{a}'_{nl,T} \mathbf{a}_{nl,T} = \boldsymbol{\Sigma}_{nl,T}^{-1}$ .

## 2.2. Methods

### 2.2.1. Model Specifications and Model Assumptions

The spatial dynamic panel simultaneous (SDPS) equation model utilized in this study is an autoregressive spatial model with a stable time space dynamic category. The specification of the simultaneous equation model for  $m$  equation is totally composed as in equation (17);

$$\begin{aligned} \mathbf{y}_{1,it} &= \delta_1 \mathbf{W}_{1,it} \mathbf{y}_{1,it} + \eta_1 \mathbf{W}_{1,it} \mathbf{y}_{1,it-1} + \tau_1 \mathbf{y}_{1,it-1} + \\ &\quad + k_1 \mathbf{y}_{2,it} + \boldsymbol{\beta}_1 \mathbf{X}_{1,it} + \boldsymbol{\alpha}_{1,0t} \mathbf{l}_n + \mathbf{v}_{1,it} \\ &\quad \vdots \end{aligned}$$

$$\begin{aligned} \mathbf{y}_{p,it} &= \delta_p \mathbf{W}_{p,it} \mathbf{y}_{p,it} + \eta_p \mathbf{W}_{p,it} \mathbf{y}_{p,it-1} + \tau_p \mathbf{y}_{p,it-1} \\ &\quad + k_p \mathbf{y}_{p-1,it} + \boldsymbol{\beta}_p \mathbf{X}_{p,it} + \boldsymbol{\alpha}_{p,0t} \mathbf{l}_n + \mathbf{v}_{p,it} \end{aligned} \quad (16)$$

$$\mathbf{v}_{p,it} = \mathbf{c}_{i0} \rho_{p,i} + \boldsymbol{\varepsilon}_{p,it}, \quad |\tau| + |\delta| + |\eta| < 1 \quad (17)$$

with,  $\mathbf{y}_{p,it}$  is a column vector endogen variable from equation  $p$  which is  $nT \times 1$ , where  $p = 1, 2, \dots, m$ , and  $\mathbf{y}_{1,it} = (y_{11t}, \dots, y_{1,nt})'$ ,  $\mathbf{y}_{2,it} = (y_{12t}, \dots, y_{2,nt})'$ .  $\mathbf{y}_{p,it-1}$  is a predetermined lag-1 variable vector of the  $nT \times 1$ , where  $\mathbf{y}_{1,it-1} = (y_{11t-1}, \dots, y_{1,nt-1})'$ ,  $\mathbf{y}_{2,t-1} = (y_{12t}, \dots, y_{2,it-1})'$ .  $\mathbf{v}_{p,it} = (v_{1,1t}, \dots, v_{t})'$  is a  $nT \times 1$  column vector of the error with  $\boldsymbol{\varepsilon}_{p,it} \sim N(\mathbf{0}, \mathbf{I}_{\sigma_p^2})$ .  $\mathbf{W}_{p,it}$  is defined as a spatial weight matrix sized  $nT \times nT$ . Spatial weight matrix according to Anselin [11] can be determined based on contiguity or general area boundaries e.g. side intersections between regions, based on distance boundaries between regions and based on social distance, or more complex economic conditions.  $\mathbf{X}_{p,it}$  is an explanatory variable matrix of  $nT \times k$ ,  $\mathbf{c}_{p,i}$  is an individual effect vector measuring  $n \times 1$ ,  $\boldsymbol{\alpha}_{p,t}$  is a time effect vector measuring  $T \times 1$ ,  $\mathbf{l}_n$  is a vector worth one sized  $NT \times 1$ , and  $\mathbf{W}_{p,it} = \mathbf{I}_T \otimes \mathbf{W}_n$  is the spatial weight matrix of panel data. The innovation of this study is to include elements of individual fixed effects  $\mathbf{c}_{p,it}$  entered into the error component (heteroscedasticity cases occur).

A few of the assumptions utilized in this research are:

**Assumption 1:** The diagonal element of  $W_n$ 's spatial weight is zero. It means that there is no spatial dependence between same region, and it is specifically determined based on the model compiled by the researcher, which has normalized the lines, so that the sum of each row is equal to 1. The spatial weight matrix is a rectilinear matrix, where each component of a matrix of value 1 is given for regions that have spatial proximity, and a value of 0 for regions that have no spatial dependency including dependency to itself (indicated in the diagonal position of the matrix). Spatial dependency can be seen based on the proximity of the location of the region in the map, and it can also be based on proximity in other respects.

**Assumption 2:** Matrix of spatial weight  $W$ , the value of the element and its order are the same of each model in the simultaneous equation, so  $W_{1,it} = W_{2,it} = \dots = W_{p,it} = W_{it}$  with  $p = 1, 2, \dots, m$  number of equations.

**Assumption 3:** Ergodic Stationerity. If  $a_{ij}$  is an unique and constant element of  $(y_{i1}, \dots, y_{im}, Z_{i1}, \dots, Z_{im}, H_{i1}, \dots, H_{im})$  then  $\{a_{ij}\}$  is jointly ergodic and stationery.

**Assumption 4:** Elements of the exogenous variable  $X$ ,  $c_{p,it}$ , and  $\alpha_{p,0t}$  are nonstochastic, constant and uniform limited by  $n$  and  $t$ .  $X$  matrix have full rank  $k_x$ , so  $X'X$  is non-singular.

**Assumption 5:** The error matrix obtained from the model is  $v_{p,it} = c_{i0}p_{p,it} + \epsilon_{p,it}$ , where  $\epsilon_{p,it}$  value are identical, independent and normal distribution  $N(0, \sigma_\epsilon^2)$ .

**Assumption 6:** The variable instrument matrix  $H$  is non stochastic containing at least an independent linear column ( $X_{nt}, W_{nt}X_{nt}$ ). Element  $H$  is uniformly bounded by an absolute value. In addition  $H$  has the following properties:

- a)  $Q_{HH} = \lim_{n \rightarrow \infty} n^{-1}H'H$  is a non singular finite matrix
- b)  $Q_{HZ_{p,it}} = \lim_{n \rightarrow \infty} n^{-1}H'E(Z_{p,it})$  is a finite matrix that

has a full column rank,  $p = 1, \dots, m$ .

**Assumption 7:** Vectors  $y_{n0}$  observable.

2.2.2. Estimated Parameters

The study used a panel data model with a time fixed effect only and incorporated individual effects into the error model (Yu, Jong, and Lee, [25]). Estimation of GMM 2SLS with a single equation approach, for example for the equation that the  $p$  begins with the process of elimination of the time effect by multiplying the variables in the equation simultaneously with eigenvectors  $F_{n,n-1}$ . It comes from the matrix  $J_n = I_n - \frac{1}{n}I_n I_n'$ .  $F_{n,n-1}$  is the eigen vector value of eigen values that is worth one.  $I_n$  is a vector containing its elements worth one as many as  $n$  lines, and because  $F'_{n,n-1}(\alpha_{p,0t} \otimes I_n) = 0$  so equation (17) to be:

$$\begin{aligned} (F'_{n,n-1} \otimes I_t)Y_{p,it}k_p &= (F'_{n,n-1} \otimes I_t)[W_{1,it}y_{1,it}, W_{2,it}y_{2,it}, \dots, \\ &W_{p,it}y_{p,it}] \delta_p + (F'_{n,n-1} \otimes I_t)Y_{p,it-1}\tau_p \\ &+ (F'_{n,n-1} \otimes I_t)[W_{1,it}y_{1,it}, W_{2,it}y_{2,it}, \dots, \\ &W_{p,it}y_{p,it-1}] \eta_p + (F'_{n,n-1} \otimes I_t)X_{p,it}\beta_p \\ &+ (F'_{n,n-1} \otimes I_t)v_{p,it}. \end{aligned} \tag{18}$$

If  $(F'_{n,n-1} \otimes I_t)Y_{p,it} = Y_{p,it}^*$  and this applies to other variables then equations (19) change to be:

$$\begin{aligned} Y_{p,it}^*k_p &= [W_{1,it}y_{1,it}^*, W_{2,it}y_{2,it}^*, \dots, W_{p,it}y_{p,it}^*] \delta_p + Y_{p,it-1}^*\tau_p \\ &+ [W_{1,it}y_{1,it}^*, W_{2,it}y_{2,it}^*, \dots, W_{p,it}y_{p,it-1}^*] \eta_p \\ &+ X_{p,it}^*\beta_p + v_{p,it}^*. \end{aligned} \tag{19}$$

Further instrumental variables (IV) are determined based on Kelejian and Purcha [26] which are a combination of all exogenous variables with spatial weight and are written as follows:

$$\begin{aligned} Q_{p,it} &= [Y_{p,it-1}^*, W_{1,it}y_{1,it-1}^*, \dots, W_{p,it}y_{p,it-1}^*, B^2Y_{p,it-1}^*, \\ &X_{p,it}^*, W_{1,it}X_{1,it}^*, \dots, W_{p,it}X_{p,it}^*, B^2X_{p,it}^*] \end{aligned} \tag{20}$$

with,

$$B^2R = [W_{1,it}W_{1,it}R, \dots, W_{1,it}W_{p,it}R, W_{2,it}W_{1,it}R, \dots, W_{p,it}W_{p,it}R],$$

in this equation  $R = Y_{p,it-1}^*$  or  $X_{p,it}^*$ , and follows assumption 2 that the spatial weight matrix used is the same on each model ( $W_{1,it} = W_{2,it} = \dots = W_{p,it} = W_{it}$ ) in the simultaneous equation then the element  $B^2R = [W_{it}^2R]$  and Instrumental variables  $Q_{p,it}$  can be simplified to,

$$\begin{aligned} Q_{p,it} &= [Y_{p,it-1}^*, W_{it}y_{1,it-1}^*, \dots, W_{it}y_{p,it-1}^*, W_{it}^2Y_{p,it-1}^*, \\ &X_{p,it}^*, W_{it}X_{1,it}^*, \dots, W_{it}X_{p,it}^*, W_{it}^2X_{p,it}^*] \end{aligned} \tag{21}$$

and  $Q_{p,it} = [Q'_{p,i1}, \dots, Q'_{p,iT}]'$ .

$v_{p,it}$  for the single equation equation  $p$  is searched with the taking after equation:

$$\begin{aligned} v_{p,it} &= k_p y_{p-1,it} + y_{p,it} - \delta_p W_{it}y_{p,it} - \eta_p W_{it}y_{p,it-1} \\ &+ \tau_p y_{p-1,it-1} - \beta_p X_{p,it} \end{aligned} \tag{22}$$

Kelejian and Purcha [26] stated the initial value  $v_{p,it}$  can be obtained by estimating parameters using the OLS method, or determining the value of the initial value parameter with constraints according to the limits of stable model parameters, namely  $|\tau| + |\delta| + |\eta| < 1$  so that the moment of condition for the single equation is formulated as follows:

$$g_{p,it}(\theta_p) = \frac{1}{(n-1)T} Q'_{p,it} (I_{n-1} \otimes J_T) v_{p,it} \tag{23}$$

and because  $g_{p,it}(\theta_p) = 0$  cannot be solved directly then it takes a quadratic form of the moment the condition is expressed with:

$$Q_{p,it}(\theta) = g_{p,it}(\theta)' M_p g_{p,it}(\theta). \tag{24}$$

The estimate parameter  $\hat{\theta}_p$  gotten by minimizing equations (25), in a way  $\frac{\partial Q_{p,it}(\theta)}{\partial \theta} = 0$  so that if the equation (25) is described by entering the moment of condition in the equation (24) then,

$$Q_{p,it}(\theta) = \left( \frac{1}{(n-1)T} Q'_{p,it} (\mathbf{I}_{n-1} \otimes \mathbf{J}_T) v_{p,it} \right) \times M_p \left( \frac{1}{(n-1)T} Q'_{p,it} (\mathbf{I}_{n-1} \otimes \mathbf{J}_T) v_{p,it} \right) \tag{25}$$

and if it is regrettable  $H'_p = Q'_{p,it} (\mathbf{I}_{n-1} \otimes \mathbf{J}_T)$ , where  $\mathbf{I}_{n-1}$  is a dotted identity matrix  $n-1$  and  $\mathbf{J}_T = \mathbf{I}_T - \frac{1}{T} \mathbf{1}_T \mathbf{1}'_T$ ,  $\bar{Y}_{p,it} = (\mathbf{I} - \mathbf{W}_{it} \delta_p) y_{p,it}^*$ ,  $v_{p,it} = \bar{Y}_{p,it} - Z_{p,it} \theta_{GMM}$ ,  $\theta_{GMM} = (\delta_p, \varphi'_p)'$  dan  $\varphi_p = (k_p, \eta_p, \tau_p, \beta'_p)'$  and define;

$$Z_{p,it} = [W_{it} y_{p,it-1}^*, y_{p-1,it}^*, X_{p,it}, y_{p,it}^*] \tag{26}$$

so that the estimated value of the GMM parameter for the model is obtained as follows:

$$\hat{\theta}_{GMM} = (Z'_{p,it} H_p M_p H'_p Z_{p,it})^{-1} (Z'_{p,it} H_p M_p H'_p \bar{Y}_{p,it}) \tag{27}$$

Coefficient  $\hat{\theta}_{GMM}$  still contains coefficients  $\delta$ , then endogenous variables  $\bar{Y}_{p,it}$  returned to such an original form until  $\bar{Y}_{p,it} = (\mathbf{I} - \mathbf{W}_{it} \delta_p) y_{p,it}^*$ , then equation (28) will be;

$$\hat{\theta}_{GMM} = (Z'_{p,it} H_p M_p H'_p Z_{p,it})^{-1} \times (Z'_{p,it} H_p M_p H'_p ((\mathbf{I} - \mathbf{W}_{it} \delta_p) y_{p,it}^*)) \tag{28}$$

### 2.2.3. GMM 2SLS Estimation with Single Equation Approach

The assessment of SDPS model parameters can be done in two ways The first is to use a single equation approach such as the 2SLS method, where the estimation of parameters utilizing GMM is carried out on each show separately, while when using a system approach then all variables are combined into one using vectorization of endogenous, and exogenous variables, and the model is assumed to be a recursive model Yang and Lee [20].

The single equation approach forms a model of simultaneous equations according to the number of equations present in the system rather than in the form of high order equations, where the first step is to estimate the parameters in the first equation and the value is obtained by following the way Kelejian and Purcha[26] using the 2SLS method, which is divided into two stages of

estimated calculation. The first is to determine the matrix:

$$P = H_p (H_p' H_p)^{-1} H_p' \quad p = 1, 2 \dots m \tag{29}$$

next determine the matrix  $\tilde{K} = PK$ , with  $K$  is a matrix whose each column contains vectors of all endogenous and exogenous variables,

$$K = [W_{it} y_{p,it}, Z_{p,it}] \tag{30}$$

then the estimated value of the parameter is,

$$\hat{\alpha}_p = (\tilde{K}' Z_{p,it})^{-1} \tilde{K}' y_{p,it}. \tag{31}$$

The estimated value of this parameter next in the second stage is used to get the error value  $\hat{v}_{p,it} = y_{p,it} - Z_{p,it} \alpha_p$ , and assuming identical and independent model errors. This GMM weight is used to get the error value  $v_{p,it}$  used to obtain estimated GMM weight  $M_1$  on estimated parameters  $\theta$  in the second stage on the simultaneous model can be obtained with the following formula:

$$\hat{M}_p = (H'_p L_p Q_p) \tag{32}$$

with  $L_p = \frac{(v'_{p,it} \times v_{p,it})}{nT - k}$  so that the estimated parameters of dynamic GMM 2SLS are as follows:

$$\begin{aligned} \hat{\theta}_{GMM} (M_1) &= (Z'_{p,it} H_p \hat{M}_p H'_p Z_{p,it})^{-1} (\Delta Z'_{p,it} H_p \hat{M}_p H'_p y_{p,it}^* - (Z'_{p,it} H_p \hat{M}_p H'_p Z_{p,it})^{-1} (Z'_{p,it} H_p \hat{M}_p H'_p W_{it} y_{p,it}^*) \\ &\times (A'_p W_{it} y_{p,it}^* H_p \hat{M}_p H'_p A_p W_{it} y_{p,it}^*)^{-1} \\ &\times (A'_p W_{it} y'_{p,it} H_p \hat{M}_p H'_p A_p y_{p,it}^*)) \end{aligned} \tag{33}$$

with,  $A_p = (\mathbf{I} - Z_{p,it} (Z'_{p,it} H_p \hat{M}_p H'_p Z_{p,it})^{-1} Z'_{p,it} H_p \hat{M}_p H'_p)$

### 2.2.4. Estimation Bias Measurement with Monte Carlo Simulation

The Monte Carlo simulation for the SDPS condition employs two conditions within the framework with one exogenous variable  $X$  as takes after:

$$\begin{aligned} y_{1,it} &= \delta_1 W_{it} y_{1,it} + \eta_1 W_{it} y_{1,it-1} + \tau_1 y_{1,it-1} \\ &\quad + k_1 y_{2,it} + \beta_1 x_{1,it} + \alpha_{1,0t} I_n + v_{1,it} \\ y_{2,it} &= \delta_2 W_{it} y_{2,it} + \eta_2 W_{it} y_{2,it-1} + \tau_2 y_{2,it-1} \\ &\quad + k_2 y_{1,it} + \beta_2 x_{2,it} + \alpha_{2,0t} I_n + v_{2,it} \end{aligned} \tag{34}$$

The simulation is carried out using several scenarios of the initial parameter value of  $\theta_0$  which is referred to as the TRUE value, with the aim of obtaining the average value and standard deviation from the estimated bias  $(\theta_0 - \hat{\theta}_{GMM})$ . The TRUE parameter scenario is based on the stable model limitations mentioned at the beginning as  $|\tau| + |\delta| + |\eta| < 1$ .

Providing initial value for parameter is a step taken by Kelejian and Purcha [26] as the first step for estimating 2 SLS with GMM. Model Equation (35) can be applied to

econometric models such as the Foreign Direct investment (FDI) model, where spatial weights are given to countries that have international trade. Discussions on the subject are being conducted by researchers for subsequent research.

Scenario P1 is a scenario when a parameter value  $\delta$  dominating other spatial parameters where value  $\delta$  is the magnitude of the spatial influence of endogenous variables on endogenous variables in the equation. In this scenario the parameters  $k$  and  $\beta$  as the influence of predetermined endogenous variables and exogenous variables of value indicating the existence of the same influence on endogenous variables in simultaneous equations, and the condition is applied to all scenarios aimed at accelerating computational processes by assuming the influence of variables without the same spatial weight. Scenario P2 is a scenario when a parameter value  $\eta$  dominating other spatial parameters, where value  $\eta$  is the magnitude of spatial influence of endogenous variable lag, and scenario P3 is a scenario when parameter values  $\tau$  as dynamic effect parameters dominate other spatial parameters.

Each situation (scenario) is simulated with distinctive  $N$  and  $T$  values, to see the greatness of the average change and standard deviation (SD) of the estimated bias between the assessed value of the parameter and the genuine value in the event that  $N$  and  $T$  changed. Each situation is estimated with GMM-2SLS for a dynamic show called GMM-2SLS Dynamic (GMM-2SLSD) with a single equation approach. Simulation begins by randomly generating data for variables  $x_{pNT \times 1} \sim U(0,1)$ , and  $v_{pNT \times 1} \sim U[0,1]$  each for  $p = 1, 2$ . Spatial weight  $\mathbf{W}$  is also obtained randomly with the provision that the diagonal value of matrix  $\mathbf{W}$  is worth 0, then continued with the normalization of the row, here the  $\mathbf{W}$  weight matrix for model 1 is equal to the  $\mathbf{W}$  weight matrix for model 2. Furthermore, based on the values  $x$  and  $v$  are searched endogenous variables, and endogenous lag, which further transforms the data for all endogenous and exogenous variables.

The simulation is carried out as many as 100 repetitions

( $R$ ) (while taking into account the stable model limits of the resulting parameter values), then the results of calculating the bias value of each parameter are calculated the average value and standard deviation. Discussion of simulation results is explained in the next section.

### 3. Results and Discussion

#### 3.1. Monte Carlo Simulation Results when $T$ Value Remains

This simulation used  $T = 10$  and  $N = 4, 8, 16,$  and  $64$  with the aim to see when the  $T$  is finite and fixed whether the addition of the number of  $N$  individuals affects the estimation bias and consistency of the estimated value and consistency of the estimated GMM-2SLSD parameters of the developed SDPS model. Estimates are assumed to be biased if the average bias value tends to be large. The consistency of the estimate is measured from the standard deviation bias (SD) which tends to be small, and there is no too much change when the number of  $N$  is added.

Based on the results of data simulation using the R program, if you pay attention to models 1 and 2, autoregressive spatial parameters  $\delta$  have an average value of negative bias both for P1 (Table 1), P2 (Table 2) and P3 (Table 3). It means that the estimated magnitude is smaller than the TRUE value of the parameter, but the value is still below one so it can still be said to be unbiased, while the SD value biased both in P1, P2 or P3 have almost the same value, which is between 0.173 to 0.353. Biased SD values that are below one with not much different values indicate consistent estimates. Changes in the value of  $N$  have an influence on the average value of the bias on the parameter  $\delta$  where the average value of the bias tends to increase by about 0.10 as the number of  $N$  increases, while the SD bias tends not to change too much in value ranging from 0.20 to 0.30. It shows that the parameter estimate is consistent.

**Table 1.** Value of Standard Deviation (SD) and Mean Estimated Bias for Scenario P1 with T=10, R = 100

P1	TRUE	Bias	Model 1				Model 2			
			N=4	N = 8	N = 16	N = 64	N=4	N = 8	N = 16	N = 64
$\delta$	0.5	Mean	-0.460	-0.525	-0.494	-0.510	-0.525	-0.507	-0.510	-0.470
		SD	0.238	0.306	0.314	0.234	0.207	0.239	0.324	0.261
$\tau$	0.2	Mean	-0.257	-0.185	-0.175	-0.197	-0.319	-0.189	-0.206	-0.192
		SD	0.265	0.173	0.150	0.092	0.326	0.250	0.166	0.059
$k$	1	Mean	-0.846	-0.933	-0.823	-0.914	-0.813	-1.045	-0.920	-1.238
		SD	0.886	0.827	0.776	0.741	1.543	1.544	1.145	1.068
$\eta$	0.3	Mean	-0.461	-0.369	-0.389	-0.234	-0.381	-0.408	-0.442	-0.395
		SD	0.371	0.392	0.493	0.520	0.409	0.422	0.457	0.521
$\beta$	1	Mean	-0.213	-0.046	-0.096	-0.068	-0.023	0.046	0.029	-0.045
		SD	0.390	0.230	0.170	0.290	0.639	0.469	0.253	0.279

Source: Data processing, with R program

**Table 2.** Value of Standard Deviation (SD) and Mean Estimated Bias for Scenario P2 with T=10, R = 100

P2	TRUE	Bias	Model 1				Model 2			
			N=4	N = 8	N = 16	N = 64	N=4	N = 8	N = 16	N = 64
$\delta$	0.3	Mean	-0.175	-0.204	-0.222	-0.330	-0.219	-0.179	-0.200	-0.304
		SD	0.208	0.353	0.291	0.242	0.184	0.305	0.308	0.228
$\tau$	0.2	Mean	-0.314	-0.323	-0.286	-0.186	-0.365	-0.235	-0.288	-0.201
		SD	0.258	0.206	0.163	0.091	0.293	0.253	0.169	0.066
$k$	1	Mean	-0.905	-0.982	-1.062	-1.086	-0.93	-1.135	-0.979	-0.906
		SD	0.745	0.963	0.907	0.957	1.470	1.599	0.930	0.897
$\eta$	0.5	Mean	-0.565	-0.577	-0.631	-0.486	-0.588	-0.587	-0.505	-0.493
		SD	0.352	0.459	0.471	0.500	0.356	0.437	0.479	0.493
$\beta$	1	Mean	-0.160	-0.109	-0.087	-0.043	0.009	0.062	-0.012	0.001
		SD	0.353	0.267	0.176	0.242	0.496	0.448	0.208	0.249

Source: Data processing, with R program

**Table 3.** Value of Standard Deviation (SD) and Mean Estimated Bias for Scenario P3 with T=10, R = 100

P3	TRUE	Bias	Model 1				Model 2			
			N=4	N = 8	N = 16	N = 64	N= 4	N = 8	N = 16	N = 64
$\delta$	0.2	Mean	-0.234	-0.194	-0.170	-0.227	-0.183	-0.187	-0.201	-0.248
		SD	0.231	0.256	0.292	0.265	0.173	0.273	0.321	0.215
$\tau$	0.5	Mean	-0.546	-0.507	-0.505	-0.501	-0.549	-0.552	-0.472	-0.480
		SD	0.295	0.205	0.199	0.094	0.353	0.235	0.175	0.140
$k$	1	Mean	-0.977	-0.963	-1.084	-0.978	-1.059	-1.012	-0.848	-1.089
		SD	0.798	1.004	1.164	0.698	1.120	0.939	1.091	0.939
$\eta$	0.3	Mean	-0.389	-0.343	-0.375	-0.277	-0.370	-0.399	-0.356	-0.317
		SD	0.341	0.403	0.458	0.546	0.406	0.444	0.431	0.531
$\beta$	1	Mean	-0.202	-0.109	-0.101	-0.040	0.015	0.032	-0.031	0.018
		SD	0.247	0.218	0.240	0.127	0.470	0.297	0.240	0.199

Source: Data processing, with R program

Another spatial parameter is  $\eta$ , that is, a parameter that indicates the presence of spatial dependencies on endogenous variables. Based on Tables 1, 2, and 3 the average values of the parameter biases are also all negative values for both model 1 and model 2 and range from -0.630 to -0.230 and SD bias has values of 0.341 to 0.546, and the greater the value of  $N$ , the SD value tends to be getting bigger, but the change is not too large and still below 1 so it can be assumed that the estimate is consistent, while for model 2 the SD value tends to shrink so that the estimated parameters are more consistent.

Estimation parameters for variables endogen lag  $\tau$  for model 1 and 2 showed the average value of the estimated bias tends to be negative between -0.521 to -0.136, but the value is still below 1, so it tends to be unbiased, where the SD value of the bias tends to decrease as the number of  $N$

increases but the value is still below 1 (between 0.132 to 0.316) so that it can be assumed to be consistent estimates.

The average bias value for estimation parameters endogenous simultaneous variable ( $k$ ) shows a greater value than other parameters although both tend to be negative, where the value is between -1.162 to -0.761. It also happened to SD bias value which tends to be large as well, and the increase in the value of  $N$  also tends to add to the SD value bias parameter, but it is different from the average value of the bias for the estimation of the exogenous variable parameter  $\beta$  that tends to be small and decreases in value when  $N$  increases, whereas the change in the parameter scenario does not significantly affect the change in the average value of the bias and the SD bias significantly. The result of estimate is still unbiased and consistent because the average value is still below 1.



**Table 4.** Value of Standard Deviation (SD) and Mean Estimated Bias for Scenario P1 with N=10, R = 100

P1	TRUE	Bias	Model 1				Model 2			
			T = 4	T = 8	T = 16	T = 64	T = 4	T = 8	T = 16	T = 64
$\delta$	0.5	Mean	-0.481	-0.522	-0.538	-0.493	-0.498	-0.526	-0.495	-0.509
		SD	0.249	0.265	0.283	0.201	0.262	0.221	0.231	0.209
$\tau$	0.2	Mean	-0.204	-0.213	-0.215	-0.216	-0.191	-0.157	-0.191	-0.221
		SD	0.258	0.224	0.227	0.127	0.261	0.188	0.168	0.154
$k$	1	Mean	-1.063	-0.941	-0.872	-0.761	-0.974	-0.928	-1.207	-1.162
		SD	0.735	1.069	1.678	1.829	1.000	1.162	1.545	2.673
$\eta$	0.3	Mean	-0.354	-0.376	-0.381	-0.336	-0.297	-0.360	-0.341	-0.334
		SD	0.470	0.499	0.364	0.296	0.548	0.489	0.379	0.260
$\beta$	1	Mean	-0.098	-0.147	-0.144	0.007	-0.007	0.019	0.019	0.029
		SD	0.336	0.313	0.563	0.357	0.513	0.280	0.275	0.263

Source: Data processing, with R program

**Table 5.** Value of Standard Deviation (SD) and Mean Estimated Bias for Scenario P2 with N=10, R=100

P2	TRUE	Bias	Model 1				Model 2			
			T = 4	T = 8	T = 16	T = 64	T = 4	T = 8	T = 16	T = 64
$\delta$	0.3	Mean	-0.281	-0.322	-0.323	-0.288	-0.297	-0.309	-0.295	-0.298
		SD	0.266	0.283	0.289	0.158	0.297	0.312	0.279	0.173
$\tau$	0.2	Mean	-0.136	-0.222	-0.198	-0.221	-0.199	-0.163	-0.215	-0.179
		SD	0.247	0.165	0.154	0.132	0.285	0.202	0.208	0.133
$k$	1	Mean	-0.946	-1.050	-0.757	-0.888	-1.015	-1.141	-0.695	-0.920
		SD	0.656	0.744	1.052	1.774	1.390	1.195	2.166	2.229
$\eta$	0.5	Mean	-0.508	-0.593	-0.606	-0.572	-0.454	-0.578	-0.602	-0.544
		SD	0.514	0.470	0.331	0.228	0.491	0.495	0.402	0.272
$\beta$	1	Mean	-0.174	-0.107	-0.084	-0.046	0.010	0.018	-0.075	-0.001
		SD	0.306	0.232	0.222	0.133	0.572	0.397	0.549	0.166

Source: Data processing, with R program

**Table 6.** Value of Standard Deviation (SD) and Mean Estimated Bias for P3 Scenarios with N=10, R = 100

P3	TRUE	Bias	Model 1				Model 2			
			T = 4	T = 8	T = 16	T = 64	T = 4	T = 8	T = 16	T = 64
$\delta$	0.2	Mean	-0.200	-0.198	-0.192	-0.224	-0.144	-0.251	-0.229	-0.242
		SD	0.210	0.302	0.236	0.220	0.263	0.283	0.186	0.153
$\tau$	0.5	Mean	-0.506	-0.484	-0.521	-0.497	-0.445	-0.518	-0.521	-0.492
		SD	0.316	0.228	0.202	0.197	0.277	0.220	0.192	0.154
$k$	1	Mean	-1.045	-0.938	-1.094	-0.864	-1.054	-0.829	-0.905	-0.820
		SD	0.764	0.772	1.372	2.515	0.797	0.913	1.349	2.408
$\eta$	0.3	Mean	-0.446	-0.332	-0.423	-0.323	-0.251	-0.326	-0.430	-0.384
		SD	0.492	0.438	0.356	0.230	0.502	0.465	0.357	0.202
$\beta$	1	Mean	-0.182	-0.125	-0.080	-0.030	0.035	0.039	0.001	-0.019
		SD	0.287	0.226	0.169	0.190	0.354	0.258	0.244	0.157

Source: Data processing, with R program

**3.2. Monte Carlo Simulation Results When *N* Value Remains**

The Monte Carlo simulation is done to see the effect of the change in the value of *T* = 4, 8, 16, 64 when the value *N* = 10 remains for each change in the value of *T*. Based on Table 4, 5, and 6, it is seen for  $\delta$  in scenario P1, P2 and P3 the average value of the bias is also all negative (for model 1 and model 2) and the values follow the TRUE's value in every scenario (between -0.538 to -0.200), and the value tends to be below one which indicates the estimate is still unbiased even though the value is below the value of the TRUE parameter, however, when viewed from the bias SD value there is something interesting here because it is different from the discussion in the previous section when the fixed *T* and *N* values change, where the biased SD value tends to decrease consistently when the number of *T* increases, so it can be said that for spatial dynamic models the increase in the number of *T* is quite significant influencing the decrease in the value of the SD bias, which indicates the estimated parameters  $\delta$  are getting more consistent, for parameter  $\eta$  the average bias estimation all the value are negative and same with  $\delta$  parameter, the values also follow of the TRUE value (between -0.606 to -0.251). The biased SD value tends to decrease consistently when the number of *T* increases too. It also occurs in parameters  $\tau$ , *k*, and  $\beta$ .

Based on the results for the SDPS model the consistency of estimated parameters is sensitive to changes in the number of *T* compared to the change in number *N* used in the panel data model.

**3.3. RMSE Value of Model**

The results of the parameter bias based on the P1, P2, and P3 scenarios must also measure the accuracy of the estimate using the RMSE value of the error generated by the model based on its estimation method. Table 7 shows the results of the simulation using the GMM-2SLS estimation method for model 1 (M1) and model 2 (M2), where when the number of *N* increases the average value of RMSE and its standard deviation for models 1 and 2 of the 3 parameter scenarios do not show a change in value that is number of *T* increases the average value of RMSE shows an sufficient to mean that there is no decreased or increased trend when the value of *N* increases, however when the upward trend, but for SD RMSE for model 1 shows an upward trend, but for model 2 it is the opposite.

Based on the results of the Monte Carlo simulation, the scenario of changing the value of the TRUE parameter as the initial parameter does not show the influence on the average result of the estimated bias, while the change in the value of *N* and *T* shows the effect of the increase in SD bias if *T* increases compared to the increase in the value of *N*.

The SDPS model can be applied to economic modeling that has relationships between models simultaneously and there are spatial elements in it, for example the FDI (Foreign Direct Investment) model between countries, which affects each other with GDP (Gross Domestic Product) and with exogenous variables are variables that exert an influence on FDI and GDP. The spatial weight used is a special spatial weight that is the presence or absence of trade relations between countries. Currently researchers are developing the application of the SDPS model to the FDI and GDP models.

**Table 7.** Value of Standard Deviation (SD) and Mean of RMSE for Each Scenario P1, P2 and P3

Parameter	Value	M1	M2	M1	M2	M1	M2	M1	M2
		T = 10, N = 4		T = 10, N = 8		T = 10, N = 16		T = 10, N = 64	
P1	Mean	0.380	0.165	0.376	0.186	0.381	0.153	0.357	0.175
	SD	0.220	0.114	0.181	0.156	0.157	0.087	0.173	0.123
P2	Mean	0.340	0.164	0.388	0.183	0.380	0.156	0.390	0.179
	SD	0.150	0.159	0.236	0.185	0.221	0.081	0.241	0.089
P3	Mean	0.350	0.160	0.380	0.159	0.435	0.157	0.358	0.179
	SD	0.170	0.121	0.271	0.074	0.282	0.096	0.136	0.096
		N = 10, T = 4		N = 10, T = 8		N = 10, T = 16		N = 10, T = 64	
P1	Mean	0.330	0.206	0.407	0.168	0.500	0.152	0.575	0.109
	SD	0.160	0.149	0.263	0.086	0.499	0.122	0.548	0.125
P2	Mean	0.310	0.264	0.361	0.178	0.428	0.181	0.578	0.101
	SD	0.140	0.202	0.147	0.125	0.251	0.196	0.507	0.095
P3	Mean	0.320	0.200	0.353	0.167	0.471	0.140	0.677	0.098
	SD	0.130	0.102	0.140	0.078	0.366	0.079	0.803	0.098

Source: Data processing, with R program

## 4. Conclusions

The conclusion of this paper is that the GMM-2SLS estimation method developed for the SDPS model is a consistent unbiased presumption, because the results of the Monte Carlo simulation can prove that the average bias tends to be negative but still below 1 and the standard deviation value is also small. The consistency of parameter estimates for the GMM-2SLS method is quite good because the standard deviation bias value is small or below one.

The change in the parameter scenario as a TRUE parameter has no effect on the average value and standard deviation of the estimated bias of the parameters obtained. The increase in the number of  $T$  gives a fairly significant change in the decrease in the SD value of all parameters compared to the increase in the number of  $N$ , so it can be concluded that in this SDPS model the consistency of the estimated parameter values can be achieved easily if the number of  $T$  is added. The increase in the number of  $T$  gives a fairly significant change in the decrease in the SD value of all parameters compared to the increase in the number of  $N$ , so it can be concluded that in this SDPS model the consistency of the estimated parameter values can be achieved easily if the number of  $T$  is added. Currently the author is developing the GMM-3SLS estimate for the SDPS model, and it is expected that the simulation results for the GMM-3SLS estimate are much better in terms of un-biasedness and consistency.

The SDPS model can be applied to spatial econometric models in the presence of a heteroscedasticity case and based on the simulation results that have been generated, the researcher suggests using  $N$  and  $T$  which are large enough so that the parameter estimation results are more consistent.

## Acknowledgements

Our thanks go to the Department of Statistics, Faculty of Science and Data Analytics, Institut Teknologi Sepuluh Nopember Surabaya Indonesia who has supported the completion of this paper, as well as the Director General of Higher Education (DIKTI) Indonesia who has provided scholarships for doctoral studies of the main author.

## Conflict of Interests

The authors state that in this paper there isn't conflict of interests.

## REFERENCES

[1] J.P. Elhorst, R. Devillers, F. Group, D. Del, International T,

- Jos é M, Spatial Econometrics From Cross-Sectional Data to Spatial Panels, Springer Briefs In Regional Science, Vol. 16, 436–457, 2014.
- [2] M. Kapoor, H.H. Kelejian, IR. Prucha. Panel data models with spatially correlated error components, *Journal Econom*, Vol. 140, No. 1, 97–130.2007.
- [3] J. LeSage, RK. Pace. Introduction to spatial econometrics, *Introduction to Spatial Econometrics*, 1–341, 2009.
- [4] L. Anselin. Lagrange Multiplier Test Diagnostics for Spatial Dependence and Spatial Heterogeneity, *Geography Analysis*, Vol. 20, No. 1, 1–17, 1988.
- [5] TG. Conley. GMM estimation with cross sectional dependence, *Journal Economic*, Vol. 92, No.1, 1–45, 2009.
- [6] IR. Prucha, HH. Kelejian, A Generalized Moments Estimator for the Autoregressive Parameter in a Spatial Model, *Int Econ Rev (Philadelphia) [Internet]*, Vol. 40, No. 2, 509–33, 1999.
- [7] JP. Lesage. Theory and Practice of Spatial Econometrics, *Spatial Econometric Analysis*, Vol. 10, No.3, 1–400, 2015.
- [8] BH. Baltagi, P. Egger, E. Hall, N. York. Estimating Models of Complex FDI: Are There Third-Country Effects, *Journal Econometrics*, Vol. 140, No. 1, 260-281, 2007.
- [9] J. Yu, R. de Jong, L. Lee fei. Quasi-Maximum Likelihood Estimators for Spatial Dynamic Panel Data with Fixed Effects when Both  $N$  and  $T$  are Large, *Journal Econometrics*, Vol. 146, No. 1, 118–34, 2008.
- [10] M. Pfaffermay. The Spatial Random Effects and the Spatial Fixed Effects Model: [Internet], 229th ed, Kunst. editor, Vienna: Economic Series IHS, 1–60, 2008.
- [11] L. Anselin, H. Badi. *Spatial Econometrics*, Blackwell 310–330, 2003.
- [12] L. Anselin, J. Le Gallo, H. Jayet. *Spatial Panel Econometrics*, *Econometric Panel Data*, Vol. 19, 625–60, 2008.
- [13] J.P. Elhorst. *Spatial Panel Data Models In: Handbook of Applied Spatial Analysis*. Springer, Berlin, Heidelberg, 2010.
- [14] Jan. Mutl. *Dynamic Panel Data Models with Spatially Correlated Disturbances*, University of Maryland, 2006.
- [15] L. Su, Z. Yang. QML estimation of dynamic panel data models with spatial errors, *Journal of Econometrics*, Vol. 185, 230–258, 2013.
- [16] L. Lee fei, J. Yu, Some recent developments in spatial panel data models, *Regional Sciences Urban Econometric [Internet]*, Vol. 40, No. 5, 255–71, 2010.
- [17] G.H. Gebremariam, T.G. Gebremedhin, P.V. Schaeffer, T.T. Phipps, R.W. Jackson. Employment, income, migration and public services: A simultaneous spatial panel data model of regional growth, *Pap Reg Sci*, Vol. 91, No. 2, 275–97, 2012.
- [18] B.H. Baltagi, Y. Deng. EC3SLS Estimator for a Simultaneous System of Spatial Autoregressive Equations with Random Effects, In Paris: International Conference on Panel Data, 2012.

- [19] K. Yang, L. Lee. Multivariate and Simultaneous Equation Dynamic Panel Spatial Autoregressive Models : Stability and Spatial Co-integration, *Jmp*, 2015.
- [20] K. Yang, L. Lee, Identification and estimation of spatial dynamic panel simultaneous equations models. *Regional Sciences Urban Econometrics*, Vol. 76, 32–46, 2019.
- [21] Mustaqim, Setiawan, Suhartono, S.S.U. Brodjol. Labor Absorption and the Growth of Agricultural Output : A Simultaneous Spatial Durbin Panel Data Model Perspective of Fiscal Decentralization's Impact in Indonesia, *Journal Advance Research in Law and Economics*, Vol. 10, 1–13 2019.
- [22] L. Anselin, J. Gallo Le, H. Jayet. Chapter 18, Spatial Panel Econometrics. In: *The Econometrics of Panel Data*, 60-625, 2008.
- [23] L.F. Lee, J Yu, Efficient GMM estimation of spatial dynamic panel data models with fixed effects, *J Econom*, Vol. 180, No. 2, 97-174, 2014.
- [24] M. Arellano, S. Bond. Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations, *Rev Economic Study*, Vol.58, No.2, 277, 1991.
- [25] L. Lee fei, J. Yu. Estimation of spatial autoregressive panel data models with fixed effects, *Journal Econometrics*, Vol. 154, No. 2, 85-165, 2010.
- [26] H.H. Kelejian, I.R. Prucha. Estimation of simultaneous systems of spatially interrelated cross sectional equations, *Journal Econometrics*, Vol. 118, No. 1–2, 27–50, 2004.