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On Subclasses of Uniformly Convex Spriallike Function Associated with Poisson distribution Series

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Abstract Geometric Function Theory is one of the major areas of mathematics which suggests the significance of geometric ideas and problems in complex analysis. Recently, the univalent functions are given particular attention and they are used to construct linear operators that preserve the class of univalent functions and some of its subclasses. Also, similar attention has been given to distribution series. Many authors have studied about certain subclasses of univalent and bi-univalent functions connected with distribution series like Pascal distribution, Binomial distribution, Poisson distribution, Mittag-Leffler-type P oisson d istribution, G eometric distribution, Exponential distribution, Borel distribution, Generalized distribution and Generalized discrete probability distribution to name few. Some of the important results on Uniformly convex spirallike functions(UCF) and Uniformly spirallike functions(USF) related with such a distribution series are also of interest. The main aim of the present investigation is to obtain the necessary and sufficient c onditions f or Poisson distribution series to belong to the classes $\mathcal{SP}_p(\eta,\zeta)$ and $\mathcal{UCV}_p(\eta,\zeta)$. The inclusion properties associated with Poisson distribution series are taken up for study in this article. Proof of some inequalities on integral function connected to Poisson distribution series has also been discussed. Further, some corollaries and results that follow consequently from the theorems are also analysed.

Keywords Uniformly Spirallike, Poisson Distribution, Convex Spirallike, Convolution

1 Introduction

The class of analytic functions denoted by $\mathcal{A}$ is defined as

$$f(z) = z + \sum_{s=2}^{\infty} a_s z^s, \tag{1}$$

which are analytic in $\mathbb{U}$.

Where $\mathbb{U} = \{z : z \in \mathbb{C} \text{ such that } 0 < |z| < 1\}$. Let $\mathcal{M}$ be the subclass of $\mathcal{A}$ that contains functions of the form,

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k, \ \ z \in \mathbb{U}. \tag{2}$$

If the function $f \in A$ with the following condition

$$\mathcal{R}\left(e^{-i\eta}\frac{zf'(z)}{f(z)}\right) > 0,$$

then it is called spirallike function. For any $\eta \in R$, $|\eta| < \frac{\pi}{2}$ and $z \in \mathbb{U}$.

The subclasses of uniformly spiralike and convex spirallike functions have been introduced by Selvaraj and Geetha [17] as follows:

Definition 1.1. Let f be the function defined in (1). Then $f \in \mathcal{SP}_p(\eta,\zeta)$ iff

$$\mathcal{R}\left\{\left(e^{-i\eta}\frac{zf'(z)}{f(z)}\right)\right\} > \left|\frac{zf'(z)}{f(z)} - 1\right| + \zeta.$$

Where $|\eta| < \frac{\pi}{2}; 0 \leq \zeta < 1$ and $f \in \mathcal{UCV}_p(\eta,\zeta)$ iff $zf'(z) \in \mathcal{SP}_p(\eta,\zeta)$.

Then we have

$$\mathcal{MSP}_p(\eta,\zeta) = \mathcal{SP}_p(\eta,\zeta) \cap \mathcal{M},$$

and

$$\mathcal{UCM}_p(\eta,\zeta) = \mathcal{UCV}_p(\eta,\zeta) \cap \mathcal{M}.$$

In the above equations, when ζ is replaced by 0, we obtain the USF and UCF functions as $\mathcal{SP}_p(\eta,0) = \mathcal{SP}_p(\eta)$ and $\mathcal{UCV}_p(\eta,0) = \mathcal{UCV}_p(\eta)$ respectively. The classes of those functions were proposed by Ravichandran et al.[13] and the same has been discussed by many authors. When $\eta = 0$, Ronning [14] introduced and discussed the classes $\mathcal{UCV}$ and $\mathcal{SP}$.

Definition 1.2.[2] Let $f \in \mathcal{A}$. Then $f \in \mathcal{R}^\tau(\mathcal{X},\mathcal{Y})$ if it holds the following condition

$$\left|\frac{f'(z)-1}{(\mathcal{X}-\mathcal{Y})\tau - \mathcal{Y}[f'(z)-1]}\right| < 1.$$

where $z \in \mathbb{U}, \tau \in \mathbb{C} - \{0\}, -1 \leq \mathcal{Y} < \mathcal{X} \leq 1$.

A variable X follows poisson distribution if

$$P(X = r) = \frac{m^r e^{-m}}{r!},$$

where m is a parameter and r=0,1,2,...

Porwal [10] recently introduced and investigated a power series where the coefficients are probabilities of poisson distributions.

$$\mathcal{K}_m(z) = z + \sum_{k=2}^{\infty} \frac{m^{k-1}}{(k-1)!} e^{-m} z^k, \ \ m > 0; z \in \mathbb{U}.$$

(By ratio test, the radius of convergence of the above power series is obtained as ∞). Porwal and Kumar [12] defined the series as

$$\begin{aligned} \mathcal{F}_m(z) &= 2z - \mathcal{K}_m(z) \\ &= z - \sum_{k=2}^{\infty} \tfrac{m^{k-1}}{(k-1)!} e^{-m} z^k. \end{aligned}$$

And proposed a new linear operator by using the convolution product as $\mathcal{I}_m(z) : \mathcal{A} \to \mathcal{A}$ defined by

$$\begin{aligned} \mathcal{I}_m f(z) &= \mathcal{K}_m(z) * f(z) \\ &= z + \sum_{k=2}^{\infty} \tfrac{m^{k-1}}{(k-1)!} e^{-m} a_k z^k \end{aligned}$$

Very recently, many authors studied about certain subclasses of univalent and bi-univalent functions associated with distribution series (See [3], [6], [1], [15], [16], [11], [18], [9]). And also many authors like B.A.Frasin[4], G.Murugusundramoorthy [7], S.Porwal[10] (see also [8],[19],[5]) investigated on the different subclasses of univalent functions related with poisson distribution series. Here we give the conditions that are both necessary and sufficient for the function $\mathcal{F}_m(z)$ belongs to the classes $\mathcal{MSP}_p(\eta,\zeta), \mathcal{UCM}_p(\eta,\zeta)$ and also we discuss the relation of these subclasses with $\mathcal{R}^\tau(\mathcal{A},\mathcal{B})$. Also we consider the function $\mathcal{G}_m f(z) = \int_0^z \frac{\mathcal{F}_m(t)}{t} dt$, and obtain the above conditions.

The following lemmas are necesssary to obtain the major results:

Lemma 1.3. [17] If f is of the form (2). Then $f \in \mathcal{MSP}_p(\eta,\zeta)$ if and only if it satisfies the following inequality

$$\sum_{k=2}^{\infty} (2k - \cos\eta - \zeta)|a_k| \leq \cos\eta - \zeta. \tag{3}$$

Where $|\eta| < \frac{\pi}{2}$; $0 \leq \zeta < 1$. Also, when $\zeta = 0$ the above inequality is obtained as

$$\sum_{k=2}^{\infty} (2k - \cos\eta)|a_k| \leq \cos\eta. \tag{4}$$

Lemma 1.4. [17] If f is of the form (2). Then $f \in \mathcal{UCM}_p(\eta,\zeta)$ if and only if

$$\sum_{k=2}^{\infty} k(2k - \cos\eta - \zeta)|a_k| \leq \cos\eta - \zeta. \tag{5}$$

Where $|\eta| < \frac{\pi}{2}$; $0 \leq \zeta < 1$. Let f be the function defined in (2). Then $f \in \mathcal{UCM}_p(\eta)$, when $\zeta = 0$ if and only if

$$\sum_{k=2}^{\infty} k(2k - \cos\eta)|a_k| \leq \cos\eta. \tag{6}$$

Lemma 1.5. [2] Let f be the function of the form given in (1). If $f \in \mathcal{R}^\tau(\mathcal{X},\mathcal{Y})$ then

$$|a_k| \leq (\mathcal{X}-\mathcal{Y})\frac{|\tau|}{k}. \tag{7}$$

The result is sharp.

2 Main Results

In this section, we obtain the conditions which are both necessary and sufficient for $\mathcal{F}_m(z) \in \mathcal{MSP}_p(\eta,\zeta)$. For our convenience, we use the notations of the series given below, throughout this paper.

$$\sum_{k=2}^{\infty} \frac{m^{k-1}}{(k-1)!} = e^m - 1,$$

$$\sum_{k=2}^{\infty} \frac{m^{k-1}}{(k-2)!} = me^m,$$

$$\sum_{k=2}^{\infty} \frac{m^{k-1}}{(k-3)!} = m^2e^m.$$

Further, we assume that $|\eta| < \frac{\pi}{2}; 0 \le \zeta < 1$ and $m > 0$.

Theorem 2.1. The function $\mathcal{F}_m(z) \in \mathcal{MSP}_p(\eta, \zeta)$ if and only if

$$2m + (2 - \cos\eta - \zeta)(1 - e^{-m}) \le \cos\eta - \zeta. \quad (8)$$

Proof. Since

$$\mathcal{F}_m(z) = z - \sum_{k=2}^{\infty} \frac{m^{k-1}}{(k-1)!} e^{-m} z^k \quad (9)$$

In light of lemma 1.3, it is sufficient to demonstrate that

$$\sum_{k=2}^{\infty} \frac{m^{k-1}}{(k-1)!}(2k - \cos\eta - \zeta)e^{-m} \le \cos\eta - \zeta$$

Now, we have

$$\begin{aligned} & \sum_{k=2}^{\infty} \tfrac{m^{k-1}}{(k-1)!}(2k - \cos\eta - \zeta)e^{-m} \\ = \quad & 2\sum_{k=2}^{\infty} \tfrac{m^{k-1}}{(k-1)!}(k-1)e^{-m} + 2\sum_{k=2}^{\infty} \tfrac{m^{k-1}}{(k-1)!}e^{-m} \\ & +(-\cos\eta - \zeta)\sum_{k=2}^{\infty} \tfrac{m^{k-1}}{(k-1)!}e^{-m}, \\ = \quad & 2\sum_{k=2}^{\infty} \tfrac{m^{k-1}}{(k-1)!}(k-1)e^{-m} \\ & +(2 - \cos\eta - \zeta)\sum_{k=2}^{\infty} \tfrac{m^{k-1}}{(k-1)!}e^{-m}. \end{aligned}$$

Using our notation, we get

$$= 2m + (2 - \cos\eta - \zeta)(1 - e^{-m})$$

Now, from the above equation it is clear that the maximum of the series is attained at $\cos\eta - \zeta$ iff the inequality (8) holds.

Theorem 2.2. The function $\mathcal{F}_m(z) \in \mathcal{UCM}_p(\eta, \zeta)$ if and only if

$$\begin{aligned} 2m^2 + m(6 - \cos\eta - \zeta) + (2 - \cos\eta - \zeta)(1 - e^m) \\ \le \cos\eta - \zeta. \quad (10) \end{aligned}$$

Proof. We establish that in view of lemma 1.4

$$\sum_{k=2}^{\infty} k\frac{m^{k-1}}{(k-1)!}(2k - \cos\eta - \zeta)e^{-m} \le \cos\eta - \zeta.$$

Now,

$$\begin{aligned} & \sum_{k=2}^{\infty} k\tfrac{m^{k-1}}{(k-1)!}(2k - \cos\eta - \zeta)e^{-m} \\ = \quad & \sum_{k=2}^{\infty}[2((k-1)(k-2) + 3(n-1) + 1) \\ & +[((n-1)+1)(-\cos\eta - \zeta)]]\tfrac{m^{k-1}}{(k-1)!}e^{-m}, \\ = \quad & \sum_{k=2}^{\infty}\Big\{2(k-1)(k-2)\tfrac{m^{k-1}}{(k-1)!}e^{-m} \\ & +6(k-1)\tfrac{m^{k-1}}{(k-1)!}e^{-m} + 2\tfrac{m^{k-1}}{(k-1)!}e^{-m}\Big\} \\ & +\sum_{k=2}^{\infty}\Big\{(k-1)(-\cos\eta - \zeta)\tfrac{m^{k-1}}{(k-1)!}e^{-m} \\ & +(-\cos\eta - \zeta)\tfrac{m^{k-1}}{(k-1)!}e^{-m}\Big\}, \\ = \quad & 2\sum_{k=2}^{\infty}\tfrac{m^{k-1}}{(k-3)!}e^{-m} + 6\sum_{k=2}^{\infty}\tfrac{m^{k-1}}{(k-2)!}e^{-m} \\ & +2\sum_{k=2}^{\infty}\tfrac{m^{k-1}}{(k-1)!}e^{-m} + (-\cos\eta - \zeta)\sum_{k=2}^{\infty}\tfrac{m^{k-1}}{(k-2)!}e^{-m} \\ & +(-\cos\eta - \zeta)\sum_{k=2}^{\infty}\tfrac{m^{k-1}}{(k-1)!}e^{-m}. \end{aligned}$$

Then we obtain, after simple calculations

$$= 2m^2 + m(6 - \cos\eta - \zeta) + (2 - \cos\eta - \zeta)(1 - e^m)$$

The above expression is maximum by $\cos\eta - \zeta$. Hence the inequality (10) is obtained.

In Theorem 2.1 and Theorem 2.2 we found the upper bound of coefficients of the function $\mathcal{F}_m(z)$ to be in the classes $\mathcal{MSP}_p(\eta, \zeta)$ and $\mathcal{UCM}_p(\eta, \zeta)$.

3 Inclusion Properties

In this section, the class $\mathcal{MSP}_p(\eta, \zeta)$ is investigated by using the lemma 1.5.

Theorem 3.1. If $f \in \mathcal{R}^\tau(\mathcal{X}, \mathcal{Y})$ is true then $\mathcal{I}_m f(z) \in \mathcal{MSP}_p(\eta, \zeta)$ if the following inequality holds.

$$\begin{aligned} (\mathcal{X}-\mathcal{Y})|\tau|\left[2(1 - e^{-m}) - \frac{1}{m}(\cos\eta + \zeta)(1 - (1+m)e^{-m})\right] \\ \le \cos\eta - \zeta. \quad (11) \end{aligned}$$

Proof. It suffices to establish that in light of lemma 1.3.

$$\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}(2k-\cos\eta-\zeta)e^{-m}|a_k|\leq\cos\eta-\zeta.$$

Since $f\in\mathcal{R}^{\tau}(\mathcal{X},\mathcal{Y})$, then by lemma 1.5, we obtain

$$|a_k|\leq(\mathcal{X}-\mathcal{Y})\frac{|\tau|}{k}. \qquad (12)$$

We have

$$\begin{aligned}
&\sum_{k=2}^{\infty}\tfrac{m^{k-1}}{(k-1)!}(2k-\cos\eta-\zeta)e^{-m}|a_k|\\
\leq\ &(\mathcal{X}-\mathcal{Y})|\tau|\left[\sum_{k=2}^{\infty}\tfrac{1}{k}\tfrac{m^{k-1}}{(k-1)!}(2k-\cos\eta-\zeta)e^{-m}\right],\\
=\ &(\mathcal{X}-\mathcal{Y})|\tau|\left[2\sum_{k=2}^{\infty}\tfrac{m^{k-1}}{(k-1)!}e^{-m}\right.\\
&\left.-(\cos\eta+\zeta)\sum_{k=2}^{\infty}\tfrac{1}{k}\tfrac{m^{k-1}}{(k-1)!}e^{-m}\right],\\
=\ &(\mathcal{X}-\mathcal{Y})|\tau|\left[2(e^m-1)e^{-m}-\tfrac{1}{m}(\cos\eta+\zeta)\right.\\
&\left.(e^m-1-m)e^{-m}\right],\\
=\ &(\mathcal{X}-\mathcal{Y})|\tau|\left[2(1-e^{-m})-\tfrac{1}{m}(\cos\eta+\zeta)\right.\\
&\left.(1-(1+m)e^{-m})\right].
\end{aligned}$$

Hence from the last expression the maximum value is obtained as $\cos\eta-\zeta$. This completes the proof.

Theorem 3.2. If $f\in\mathcal{R}^{\tau}(\mathcal{X},\mathcal{Y})$ is true then $\mathcal{I}_m f(z)\in\mathcal{UCM}_p(\eta,\zeta)$ if

$$(\mathcal{X}-\mathcal{Y})|\tau|\left[2m+(1-e^{-m})(2-\cos\eta-\zeta)\right]\leq\cos\eta-\zeta \qquad (13)$$

Using the lemma 1.4 and applying a similar procedure given to Theorem 3.1, we obtain the above result. Hence the proof is omitted.

4 Integral Theorem

In this section, we define the integral function $\mathcal{G}_m(z)$ and the inclusion of this function to the classes $\mathcal{UCM}_p(\eta,\zeta)$, $\mathcal{MSP}_p(\eta,\zeta)$ are investigated based on some known inequality.

Theorem 4.1. If $\mathcal{G}_m$ is defined as

$$\mathcal{G}_m(z)=\int_0^z\frac{\mathcal{F}_m(t)}{t}dt, \qquad (14)$$

then $\mathcal{G}_m\in\mathcal{UCM}_p(\eta,\zeta)$ iff it satisfies the inequality (8).

Proof. Since

$$\mathcal{G}_m(z)=z-\sum_{k=2}^{\infty}\frac{1}{k}\frac{m^{k-1}}{(k-1)!}e^{-m}z^k$$

Then, according to lemma 1.4, we just need to demonstrate that

$$\sum_{k=2}^{\infty}k(2k-\cos\eta-\zeta)\times\frac{1}{k}\frac{m^{k-1}}{(k-1)!}e^{-m}\leq\cos\eta-\zeta$$

or, equivalently

$$\sum_{k=2}^{\infty}(2k-\cos\eta-\zeta)\frac{m^{k-1}}{(k-1)!}e^{-m}\leq\cos\eta-\zeta$$

We skip the specifics because the remainder is similar to Theorem 2.1.

Theorem 4.2. If $m\geq 0$, then the function $\mathcal{G}_m\in\mathcal{MSP}_p(\eta,\zeta)$ if and only if

$$2(1-e^{-m})-m^{-1}(\cos\eta+\zeta)(1-e^{-m}(1-m))\leq\cos\eta-\zeta \qquad (15)$$

We excluded Theorem 4.2's proof because it is very similar to Theorem 4.1's proof.

5 Special cases

In the following corollaries some of the imporatant inqualities are obtained by specialising the parameter $\zeta=0$ in Theorems 2.1 to 4.2.

Corrollary 5.1. We have $\mathcal{F}_m(z)\in\mathcal{MSP}_p(\eta)$ if and only if

$$2m+(2-\cos\eta)(1-e^{-m})\leq\cos\eta.$$

Corollary 5.2. We have $\mathcal{F}_m(z)\in\mathcal{UCM}_p(\eta)$ if and only if

$$2m^2+m(6-\cos\eta)+(2-\cos\eta)(1-e^{m})\leq\cos\eta.$$

Corollary 5.3. If $f\in\mathcal{R}^{\tau}(\mathcal{X},\mathcal{Y})$ then for $\mathcal{I}_m f(z)\in\mathcal{MSP}_p(\eta)$ if

$$(\mathcal{X}-\mathcal{Y})|\tau|\left[2(1-e^{-m})-\frac{1}{m}\cos\eta(1-e^{-m}(1+m))\right]\leq\cos\eta.$$

Corollary 5.4. If $f \in \mathcal{R}^\tau(\mathcal{X},\mathcal{Y})$ then for $\mathcal{I}_m f(z) \in \mathcal{UCM}_p(\eta)$ if

$$(\mathcal{X}-\mathcal{Y})|\tau|\left[2m+(1-e^{-m})(2-\cos\eta)\right] \leq \cos\eta.$$

Corollary 5.5. The function $\mathcal{G}_m \in \mathcal{UCM}_p(\eta)$ if and only if

$$2m+(2-\cos\eta)(1-e^{-m}) \leq \cos\eta.$$

Corollary 5.6. The function $\mathcal{G}_m \in \mathcal{MSP}_p(\eta)$ if and only if

$$2(1-e^{-m})-\frac{1}{m}\cos\eta(1-e^{-m}(1-m)) \leq \cos\eta.$$

By specialising the parameter $\zeta = 0$ and $\eta = 0$ in Theorems 2.1 to 4.2, we get the following results.
Corrollary 5.7. We have $\mathcal{F}_m(z) \in \mathcal{MSP}_p(\eta)$ if and only if

$$2m+(1-e^{-m}) \leq 1.$$

Corollary 5.8. We have $\mathcal{F}_m(z) \in \mathcal{UCM}_p(\eta)$ if and only if

$$2m^2+5m+(1-e^{m}) \leq 1.$$

Corollary 5.9. Let $f \in \mathcal{R}^\tau(\mathcal{X},\mathcal{Y})$ then $\mathcal{I}_m f(z) \in \mathcal{MSP}_p(\eta)$ if

$$(\mathcal{X}-\mathcal{Y})|\tau|\left[2(1-e^{-m})-\frac{1}{m}(1-e^{-m}(1+m))\right] \leq 1.$$

Corollary 5.10. Let $f \in \mathcal{R}^\tau(\mathcal{X},\mathcal{Y})$ then $\mathcal{I}_m f(z) \in \mathcal{UCM}_p(\eta)$ if

$$(\mathcal{X}-\mathcal{Y})|\tau|\left[2m+(1-e^{-m})\right] \leq 1.$$

Corollary 5.11. The function $\mathcal{G}_m \in \mathcal{UCM}_p(\eta)$ if and only if we have the same result of Corollary 5.7.

Corollary 5.12. The condition which are both necessary and sufficient for the function $\mathcal{G}_m$ to belong to the class $\mathcal{MSP}_p(\eta)$ is

$$2(1-e^{-m})-m^{-1}(1-e^{-m}(1+m)) \leq 1.$$

6 Conclusion

This paper deals with the different subclasses of univalent function associted with poison distribution series. Here we have obtained the results of integral theorem, inclusion properties and many interesting special cases. In future, partial sums, Turan-type inequalities and neighbourhood problems are to be investigated in such classes.

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