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On Invariants of Surfaces with Isometric on Sections

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Abstract In one of the directions of classical differential geometry, the properties of geometric objects are studied in their entire range, which is called geometry "in large". Many problems of geometry "in large" are connected with the existence and uniqueness of surfaces with given characteristics. Geometric features can be intrinsic curvature, extrinsic or Gaussian curvature, and other features associated with the surface. The existence of a polyhedron with given curvatures of vertices or with a given development is also a problem of geometry "in large". Therefore, the problem of finding invariants of polyhedra of a certain class and the solution of the problem of the existence and uniqueness of a polyhedra with given values of the invariant are relevant. This work is devoted to finding invariants, surfaces isometric on sections. In particular, we study the expansion properties of convex polyhedra that preserve isometry on sections. For such polyhedra, an invariant associated with the vertex of a convex polyhedral angle is found. Using this invariant, we can consider the question of restoring a convex polyhedron with given values of conditional curvature at the vertices. The isometry on section differs from the isometry of surfaces. The isometry of surfaces does not imply the isometry in sections, and vice versa. One of the invariants of surfaces isometric in cross sections is the area of the cylindrical image. This paper presents the properties of the area of a cylindrical image.

Keywords Isometry on Sections, Isometry, Spherical Image, Cylindrical Image, Completely Additive Function, Development, Invariant

1 Introduction

In one of the directions in classical differential geometry, the properties of geometric objects are studied along their entire range, which is called geometry "in large". In 1813, O. Cauchy proved that two closed polyhedra, which are equally composed of congruent faces, are equal. This result is one of the first among the solved problems of geometry "in large".

As a result of seminal studies on geometry "in large", a number of topical problems have been solved in recent years, in particular, the following series of results has been obtained: the existence and uniqueness theorems for pointwise slant immersions of Riemannian manifolds M^n into a complex space form $\tilde{M}^n(c)$ of constant holomorphic sectional curvature have been established [1], new invariants such as arc-length, curvature and torsion with a fractional-order have been introduced, and the problem of reconstruction of the curve in terms of the new invariants has been solved [2], the geometric invariant properties of a normal curve on a smooth immersed surface under conformal transformation have been established [3].

Many problems in geometry "in large" are related to the isometry of surfaces. If the surfaces are isometric, you can select the coordinate lines so that the surfaces have the same metric. The results of one of the authors are connected with the isometry of a foliated manifold. The map isometry on sections is a special case of the map isometry of foliated manifold, in other words, isometry on sections, each section of one surface is associated with a section of another surface. In [4] proved that there is a foliation for which there exists an element of the isometry group of the foliated manifold, which is not an element of the isometry group. The work [5] introduced some topology on the group $G_F(M)$ of all isometries of foliated manifold (M, F), which depends on a foliation F and coincides with compact-open topology when F is an n-dimensional foliation. If the codimension of F is equal to n, convergence in our topology coincides with point wise convergence, when

$n = dimM$. It has been proved that the group $G_F(M)$ is a topological group with compact-open topology. The paper [6] is devoted to the study of the properties of isometries of foliated manifolds, and properties on the limit of geodesic lines of foliated manifolds are proved. In addition to this was showed some properties of group $G(M)$ with F- compact-open topology. In [7] proved that some subgroups of the group $Diff_F(M)$ are topological groups with the F-compact-open topology. In [8] on geometry of foliated manifolds are stated and results on geometry of Riemannian (metric) foliations are discussed.

Proceeding from this, G. Weil posed and outlined a solution to the problem of the existence of a closed convex surface with a given metric. This problem received an exhaustive solution in the most general formulation for metrics of positive curvature in the works of A.D.Aleksandrov, A.V.Pogorelov and their students [9]. In 1951, A.V. Pogorelov proved that a closed convex surface is uniquely determined by its metric in the class of general closed convex surfaces. That is, closed isometric convex surfaces are equal.

2 Surfaces with isometric on sections

In three-dimensional Euclidean R^3 space, consider the surface F and the non-zero vector $\overrightarrow{e}$, the surface F is intersected by all possible planes π^j perpendicular to the vector $\overrightarrow{e}$. The set of cross - section points is denoted by $\gamma^j = F \cap \pi^j$. The class of surfaces for which the section γ^j. is homeomorphic to a segment, a straight line or a circle, we is denoted by $P \in W\{\overrightarrow{e}\}$ [10].

Definition 1. Surfaces F_1 and F_2 are called isometric on sections if there are directions $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ perpendicular to the given sections, and a homeomorphism f of the surfaces satisfies the following conditions:

$a)$ Points on the surface F_1 that belong to the same sections are mapped to points that belong to the same section of the surface F_2. Images of points that lie on different sections lie on different sections;

$b)$ The distances between the planes containing curves γ^1 and γ^2 and the planes containing curves $f(\gamma^1)$ and $f(\gamma^2)$are equal;

$c)$ The length of the arc of the curve $\gamma \subset \pi^j$ between two points is equal to the length of the arc of the curve $f(\gamma)$ between the corresponding points [10].

The isometry of surfaces does not follow isometry on section, and vice versa.

Example 1.

Cylinder $\begin{cases} x = \cos u \\ y = \sin u \\ z = v \end{cases}$ and plane $\begin{cases} x = x_0 + u \\ y = y_0 + v \\ z = z_0. \end{cases}$

They are locally isometric, but there are no directions in which they are isometric on sections.

Example 2.

Elliptic paraboloid $\begin{cases} x = u \\ y = v \\ z = u^2 + v^2 \end{cases}$ and hyperbolic paraboloid $\begin{cases} x = u \\ y = v \\ z = u^2 - v^2. \end{cases}$

There are directions in which they are isometric on sections, but they are not locally isometric.

The question naturally arises: under what conditions will surfaces that are isometric on sections be isometric to each other?

The answer to this question is given in [11], and it is formed as follows:

Let F_1 and F_2 be surfaces that are isometric on sections with respect to three non-planar directions. This means that there is a pair of systems of three non-planar vectors $\{\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}\}$ and $\left\{\overrightarrow{e_1}', \overrightarrow{e_2}', \overrightarrow{e_3}'\right\}$ such that $F_1\{\overrightarrow{e_1}\}$ is isometric to sections $F_2\left\{\overrightarrow{e_1}'\right\}$, $F_1\{\overrightarrow{e_2}\}$ is isometric to sections $F_2\left\{\overrightarrow{e_2}'\right\}$, and $F_1\{\overrightarrow{e_3}\}$ is isometric to sections $F_2\left\{\overrightarrow{e_3}'\right\}$. That is, the conditions for determining isometricity on sections are met with respect to three non-planar directions separately, and sections $\gamma_1^i,\ \gamma_2^i,\ \gamma_3^i$ form a network of coordinate lines on the surface F_i in pairs.

Theorem 1. *Surfaces of class C^2 that are isometric on sections with respect to three directions are isometric* [11].

We give an example of surfaces that are isometric on sections with respect to two non-collinear directions, but not isometric to each other.

Example 3. Ellipsoid $\frac{x^2}{\lambda^2 a^2} + \frac{y^2}{a^2} + \frac{\lambda^2 z^2}{a^2} = 1$ is isometric on sections with two respect to spheres $\overrightarrow{e_1}\{1, 0, \lambda\}$ and $\overrightarrow{e_2}\{1, 0, -\lambda\}$ with radius a, where $|\lambda| < 1,\ \ a > b > c > 0$.

The introduced concept of surfaces that are isometric on sections is equivalent to the isometry of surfaces in a space with a degenerate metric, in particular, the Galilean space [12].

The plane P is called the reference of some set Q lying in the space R^3, if P and Q have common points and the entire set Q lies on one side of P.

Let some set M be allocated on the convex surface F. Take the unit sphere whose center is at the origin of the coordinate. We draw all possible reference planes to the surface F, through each point of the set M, we will lay aside the unit vectors of external normals to these planes from the center of the sphere. The geometric location of the ends of these normals is called the spherical image set M. We denote the spherical image of the set M by M^* [13].

Consider a cylinder whose basic circle is the larger circumference of the unit sphere

$$\begin{cases} x^2 + y^2 + z^2 = 1 \\ x = 0 \end{cases}$$

and the generators are parallel to vector $\overrightarrow{i}$.

With the help of a central project, we will project the area M^* into a cylinder. The central projection of the set M^* on the cylinder is called the cylindrical image of the set M and denoted by M'. A similar map is considered in Galilean space.

[10] proved following results:

Proposition. *The area of a region on a surface F is invariant with respect to the transformation*

$$\begin{cases} x = x' + a \\ y = \alpha x' + y' \cos\phi + z \sin\phi \\ z = \beta x' - y' \sin\phi + z' \cos\phi \end{cases}$$

where $a, \alpha, \beta \in R$.

Theorem 2. *Let the surfaces* $F_1,\ F_2 \in W\{\overrightarrow{e}\}$ *be given by the equations*

$$\overrightarrow{r}_1(u,v) = u\overrightarrow{i} + y(u,v)\overrightarrow{j} + z(u,v)\overrightarrow{k}$$

and

$$\overrightarrow{r}_2 = u\overrightarrow{i} + [\alpha u + y(u,v)\cos\varphi + z(u,v)\sin\varphi]\overrightarrow{j} +$$

$$+ [\beta u - y(u,v)\sin\varphi + z(u,v)\cos\varphi]\overrightarrow{k},$$

then they are isometric on sections and the corresponding areas of the cylindrical images that are equal to each other.

Theorem 3. *The area of a cylindrical image is a completely additive function of a set on a convex surface* $F \in W\{\vec{e}\}$*, defined for all Boral sets.*

3 On Invariant of Polyhedra Isometric on Sections

In [14] differential invariants of Lie group of one parametric transformations of the space of two independent and three dependent variables are studied. It shows method of construction of invariant differential operator. Obtained results are applied for finding differential invariants of surfaces. In [15] determined what kind of geometrical meaning the identified properties have in the Euclidean spase. It is shown that the Galilean movement gives surface deformation in the Euclidean sense. Deformation of the surface is indicated by the fact that the Gaussian curvature remains unchanged. In the sense of Alexandrov [16], by a polyhedron we mean a surface composed of a finite number of polygons. Since a polyhedron is also a surface, the definition of isometricity on sections is also applicable for polyhedra. In three - dimensional Euclidean space, we consider a polyhedral surface $P \in W\{\overrightarrow{e}\}$ hat does not have edges and reference planes perpendicular to vector $\overrightarrow{e}$. Planes perpendicular to vector $\overrightarrow{e}$ can be reference only at the edge points of polyhedra.

To study the properties of polyhedra, it is necessary to consider their developments. Therefore, we now introduce the concept of a polyhedron development preserving isometry on sections. By a development preserving isometry on sections, we will understand the development of a polyhedron onto a plane in which points lying on the same section retain belonging to this section.

Since polyhedra from class $W\{\overrightarrow{e}\}$ are considered, the boundary sections of these polyhedra can be a point, a segment, or a polygon. Moreover, the inner part of the polygon does not belong to the polyhedron in question. In order to keep the polyhedron points belonging to the same section, it is necessary to find out how to cut the faces when constructing the development.

A development that preserves isometry on sections differs from a Euclidean development. Therefore, the method of cutting faces, as well as the method of gluing edges, differs from Euclidean and, of course, depends on the direction of vector $\overrightarrow{e}$. It is known that the Euclidean development [16] does not depend on which plane we deploy the polyhedron to, so the development of a polygon is a polygon equal to the one being deployed. That is, it makes no sense to talk about the Euclidean development of the polygon.

In our case, the development is considered only on planes parallel to vector $\overrightarrow{e}$. If the plane containing the polygon is parallel to the vector $\overrightarrow{e}$, then the development of the polygon preserving the isometry on sections in the direction $\overrightarrow{e}$ will be a polygon equal to it or different from it by a shift. Let's find out how to deploy polygons lying on planes that are not parallel to vector $\overrightarrow{e}$.

Let the plane π_1 containing the polygon be angle $\alpha \neq 0$ with victor $\overrightarrow{e}$. Then we take a plane π_2 parallel to vector $\overrightarrow{e}$ and such that its intersection with plane π_1 is perpendicular to vector $\overrightarrow{e}$. From the points of the polygon, we lower the perpendiculars to the plane π_2. The set of bases of the perpendiculars is an orthogonal projection of the polygon onto the plane π_2. The resulting projection of the polygon is a development that preserves the isometry on sections in the direction $\overrightarrow{e}$. If you rotate the plane π_2 around some straight line parallel to vector $\overrightarrow{e}$, you get polygons that are isometric on sections to the development, that is, when you rotate the plane around a straight line parallel to vector $\overrightarrow{e}$, the development of the polygon does not change. In addition, shifts in the direction orthogonal to vector $\overrightarrow{e}$ are allowed.

For the convenience of reasoning, we will direct the axis OX along the vector $\overrightarrow{e}$, in this case we will talk about a development that preserves the isometry on sections to the plane XOY. In the future, we will consider the developments only on the XOY plane.

Similarly, [16], we require the following from the development:

1. From each polygon to another, you can go by walking along polygons that have glued sides. This condition means that the development does not break up into completely unrelated parts.

2. Each side of the polygon is either not glued to any side, or glued to only one side.

3. Any segments of the sides identified during gluing always have equal lengths in the direction $\overrightarrow{e}$.

Here are some examples of polyhedron development.

a) The development of the dihedral angle is the entire plane. Consider the development of a polygon lying on both half-planes that make up a dihedral angle. The development of the polygon depends on the location of the edge of the dihedral angle and vector $\overrightarrow{e}$.

The edge of the dihedral angle is parallel to vector $\overrightarrow{e}$. Then the development of a polygon is a polygon equal to it or different from it by a shift. In this case, the areas of a polygon lying on a dihedral angle and its development, which preserves isometry on sections, are equal to each other.

Let the faces of a dihedral angle be angles α and with vector $\overrightarrow{e}$ and its edge is not parallel to vector $\overrightarrow{e}$. We will expand the polygon onto the plane π, containing the edge of the dihedral angle and parallel to the vector $\overrightarrow{e}$. Then the development will be the same polygon, this polygon is obtained by projection onto the π plane.

b) Consider the development of the cube relative to the direction of the diagonal of the base.

Obviously, the development of the cube depends on the location of the cube relative to vector $\overrightarrow{e}$ (Figure 1).

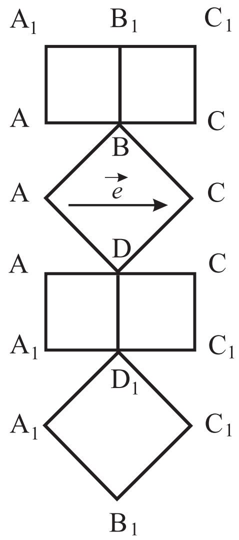

Figure 1. Development of the cube in direction of vector $\vec{e}$.

c) In the drawing, the vertices to be glued are marked with the same letters and the direction of vector $\overrightarrow{e}$ is indicated. Consider the development of the parallelepiped relative to vector $\overrightarrow{e}$ (Figure 2).

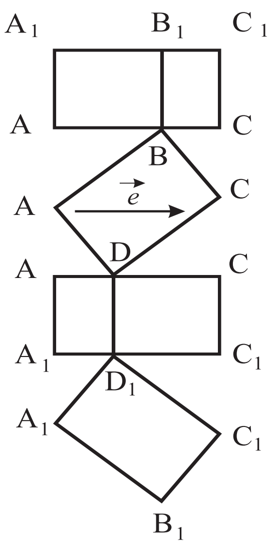

Figure 2. Development of the parallelepiped in direction of vector $\vec{e}$.

Let us be given a triangle ABC.

Definition 2. The defect of the sides of the triangle ABC relative to the angle A is called the number

$$\omega_{ABC} = AB + AC - BC.$$

Similar defect was considered in the work of Zhiqiang Xu, Guoliang Xu [17], where they studied the approximation of the Gaussian curvature using the Gauss-Bonnet scheme.

Using the definition of the defect of the sides of a triangle, we derive formulas for calculating the defect of the sides of a certain class of polygons. The choice of this class of polygons depends on the sections of the convex polyhedron with planes perpendicular to this vector. In addition, the polygon defect must have some curvature properties of a convex polyhedron. Therefore, the formula for the defect of the sides of the polygon will be determined inductively. First, we consider for quadrilaterals of various types of the class under consideration, then for polygons.

1. Consider a non-convex quadrilateral $ABDC$. Drawing the diagonal BC, we get two triangles ABC and DBC. Since triangle ABC contains triangle DBC, the defect of the sides of quadrilateral $ABDC$ relative to angle A will be defined as the difference between the defects of the sides of triangles ABC and DBC, this gives positive certainty and additivity of the defect. Hence we get

$$\omega_{ABDC} = \omega_{ABC} - \omega_{DBC} =$$
$$= AB + AC - BC - (DB + DC - BC) =$$
$$= AB + AC - DB - DC.$$

2. Let the figure consist of two triangles that have one vertex in common and the angles at this vertex are vertical. By connecting points B and C, we get triangle CAB which contains triangle OAB, on the other hand we get triangle BDC which contains triangle ODC. Reasoning similarly, we get

$$\omega_{OABDC} = \omega_{OAB} + \omega_{ODC} =$$
$$= \omega_{CAB} + \omega_{COB} + \omega_{BDC} + \omega_{BOC} =$$
$$= CA + BD - AB - DC.$$

3. For a convex quadrilateral $ABDC$, the defect of the sides relative to the angle A is determined exactly the same as for a non-convex quadrilateral. Continuing the sides AB and AC, we take some point E in the continuation of AB and draw a straight ED through the points E and D. The intersection point of the line ED with the continuation AC is denoted by F. The defect of the sides of the polygon $ABDC$ relative to the angle A is defined as the difference of the defects of the sides of the triangles

$$\omega_{ABDC} = \omega_{AEF} - \omega_{BED} - \omega_{CDF} = AB + AC - BD - DC.$$

4. Let's generalize the concept of a defect of sides for a nonconvex polygon. The defect of the sides of the polygon relative to the angle O is determined by the formula

$$\omega = OA_1 + OA_n - P_{A_1A_2...A_n} +$$
$$+ OA_{n+1} + OA_m - P_{A_{n+1}A_{n+2}...A_m},$$

where $P_{A_1A_2...A_n}$ is the perimeter of the polyline $A_1A_2...A_n$, $P_{A_{n+1}A_{n+2}...A_m}$- is the perimeter of the polyline $A_{n+1}A_{n+2}...A_m$ (Figure 3).

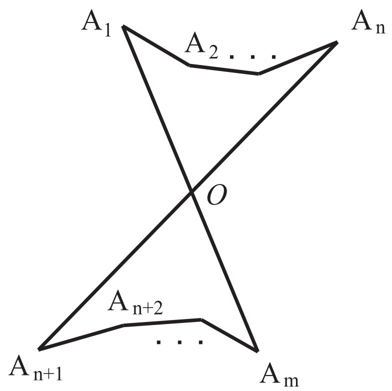

Figure 3. Nonconvex polygon $OA_1A_2...A_nA_{n+1}A_{n+2}...A_m$.

Obviously, the vertex of the trihedron is at the origin of coordinates. Assume that the plane $x = const$ is not the reference plane of the trihedron. The S trihedron is crossed by $x = \pm 1$ planes. Then one of them intersects one of the edges of the trihedron, the other intersects the other two. The intersection points of planes $x = \pm 1$ with edges are denoted by A', B, C. Point A' is symmetrically displayed relative to the origin and we get point A.

Consider triangle ABC.

Definition 3. The conditional full angle of the triangular angle in the direction $\overrightarrow{e}\,(OX)$ is called the defect of the sides of the triangle ABC relative to the angle A.

Obviously $\omega_{ABC} > 0$. Using the definition of a conditional full angle for a three-sided angle, we find a conditional full angle for a four-sided angle. Let the vertex of the tetrahedral angle be at the origin and have no reference planes and edges perpendicular to vector $\overrightarrow{e}\,(OX)$. The polyhedron is intersected by $x = \pm 1$ planes.

Consider the following possible cases.

a) One of the planes intersects one of the edges of the polyhedron, the other intersects the other three. The intersection points of the planes $x = \pm 1$ with the edges are denoted by $A'B'DC$. Similarly, we obtain a non-convex (or convex) quadrilateral with vertices $ABDC$. The defect of the sides of the quadrilateral obtained above in case 1 or in case 3 will be taken as a conditional full angle of the quadrilateral.

b) Each of the planes is intersected by two edges of the polyhedron. The intersection points of the planes with the edges are denoted by $A'B'DC$. Then the conditional full angle is the defect of the sides of the polygon defined in case 2 above.

We generalize the concept of a conditional full angle in the direction $\overrightarrow{e}$ for any polyhedral angle from class $W\{\overrightarrow{e}\}$.

Let us be given a polyhedral angle whose vertex is at the origin, having no edges and reference planes perpendicular to vector $\overrightarrow{e}$. The polyhedron is intersected by planes $x = \pm 1$. The intersection points of the planes with the edges are denoted by $A'_1, A'_2, ..., A'_{m-1}, A'_m, A'_{m+1}, ..., A'_n$.

Connecting points $A'_1, A'_2, ..., A'_{m-1}, A'_m$, we get a polyline with boundary points A'_1 and A'_m. The polyline $A'_{m+1}, A'_{m+2}, ..., A'_n$ is constructed in exactly the same way. The polyline with boundary points A'_1, A'_m is symmetrically displayed relative to the origin and we get a polygon. Then the conditional full angle corresponds to the defect of the sides of the polygon and is calculated by the formula

$$\omega = OA_1 + OA_m + OA_{m+1} + OA_n - P_1 - P_2,$$

where P_1 and P_2 are the lengths of the polylines with the beginning A_1 and A_{m+1} and with the ends A_m and A_n, respectively, here O is the intersection point of the segments A_1A_n and A_mA_{m+1}.

On the coordinate plane XOY, consider a convex n - gon $A_1A_2...A_n$, inside which the origin of coordinates is located, and having no vertices lying on the axis OY. On the axis OZ, select the point $S\,(0,0,h)\,,\quad h \neq 0$, and write down the equation of the edge SA_k, where $1 \leq k \leq n$, denote the coordinates of the vertices of the polygon by $A_k\,(a_k, b_k, 0)$ and find the intersection point of the edge SA_k with the planes $x = \pm 1$:

$$\begin{cases} \frac{x}{a_k} = \frac{y}{b_k} = \frac{z-h}{h} \\ x = \pm 1 \end{cases} \Rightarrow \begin{cases} y = \pm \frac{b_k}{a_k} \\ x = \pm 1 \end{cases};\; z = h\left(1 \pm \frac{1}{a_k}\right).$$

Obviously, the abscissae and ordinates of the intersection points do not depend on h. The length of the polyline that appears when calculating the conditional total angle at vertex S in the direction of vector $\overrightarrow{e}(OX)$ can be calculated using the formula

$$l = \sum_{k=1}^{n-1} A_kA_{k+1} =$$

$$= \sum_{k=1}^{n-1} \sqrt{\left(\frac{b_{k+1}}{a_{k+1}} - \frac{b_k}{a_k}\right)^2 + h^2\left(\frac{1}{a_{k+1}} - \frac{1}{a_k}\right)^2}.$$

It is not difficult to prove the following theorem.

Theorem 4. *The conditional total angle ω is invariant under transformations of the form:*

$$\begin{cases} x' = x + a \\ y' = \alpha x + y\cos\varphi - z\sin\varphi + b \\ z' = \beta x + y\sin\varphi + z\cos\varphi + c\,. \end{cases}$$

4 Conclusions

Many questions of geometry "in large" are connected with the existence and uniqueness of surfaces with given geometric characteristics. The found invariants are the geometric characteristics of the reconstructed surface. After proving the positive definiteness and monotonicity of the found invariants, the problem of restoring the surface by the method of A. D. Aleksandrov [13] can be solved using the properties of the expansion of convex polyhedra, which preserves isometry on sections.

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