Mathematics and Statistics 10(2): 358-365, 2022 http://www.hrpub.org
DOI: 10.13189/ms.2022.100210

A Branch and Bound Algorithm to Solve Travelling Salesman Problem (TSP) with Uncertain Parameters

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Received December 6, 2021; Revised February 25, 2022; Accepted March 15, 2022

Cite This Paper in the following Citation Styles

(a): *[1] S. Dhanasekar, Saroj Kumar Dash, Neena Uthaman , "A Branch and Bound Algorithm to Solve Travelling Salesman Problem (TSP) with Uncertain Parameters," Mathematics and Statistics, Vol. 10, No. 2, pp. 358 - 365, 2022. DOI: 10.13189/ms.2022.100210.*

(b): *S. Dhanasekar, Saroj Kumar Dash, Neena Uthaman (2022). A Branch and Bound Algorithm to Solve Travelling Salesman Problem (TSP) with Uncertain Parameters. Mathematics and Statistics, 10(2), 358 - 365. DOI: 10.13189/ms.2022.100210.*

Abstract The core of the theoretical Computing science and mathematics is computational complexity theory. It is usually concerned with the classification of computational problems in to P and NP problems by using their inherent challenges. There is no efficient algorithms for these problems. Travelling Salesman Problem is one of the most discussed problems in Combinatorial Mathematics. To deduct a Hamiltonian cycle in which the cost or time is minimum is the main objective of the TSP. There exist many algorithms to solve it. Since all the existing algorithms are not efficient to solve it, still many researchers are working to produce efficient algorithms. If the description of the parameters is vague, then fuzzy notions which include membership value are applied to model the parameters. Still the modeling does not give the exact representation of the vagueness. The Intuitionistic fuzzy set which includes non-membership value along with membership values in its domain is applied to model the parameters. The decision variables in the TSP, the cost, time or distance are modeled as intuitionistic fuzzy numbers, then the TSP is named as Intuitionistic fuzzy TSP (InFTSP). We develop the intuitionistic fuzzified version of littlewood's formula or branch and bound method to solve the Intuitionistic fuzzy TSP. This method is effective because it involves the simple arithmetic operation of Intuitionistic fuzzy numbers and ranking of intuitionistic fuzzy numbers. Ordering of intuitionistic fuzzy numbers is vital in optimization problems since it is equivalent to the ordering of alternatives. In this article, we used weighted arithmetic mean method to order the fuzzy numbers. Weighted arithmetic mean method satisfies linear property which is a very important characteristic of ranking function. Numerical examples are solved to validate the given algorithm and the results are discussed.

Keywords Intuitionistic Fuzzy Numbers, Intuitionistic Fuzzy Ranking, Intuitionistic Fuzzy Arithmetic, Intuitionistic Fuzzy TSP, Branch and Bound Algorithm

1. Introduction

A Traveling Salesman has to visit a number of given cities, starting and ending at the same city. Finding the optimal tour which is the Hamiltonian tour is the main purpose of the Travelling Salesman Problem (TSP). If the number of cities increases, deducting optimal tour becomes more complex. The TSP is classified as the NP-Complete problem and also one of the most studied problem. The description of the decision parameters in the TSP is uncertain or vague, the concept of fuzzy notions introduced by Zadeh [1] has been used to fix it. A Fuzzy Travelling Salesman Problem (FTSP) is a Travelling salesman problem in which the cost or distance will be modeled in fuzzy numbers in the place of crisp numbers. Various researchers [2-11, 14] presented various algorithms to solve FTSP. Since FTSP is a polynomial time problem, many algorithms are still proposed to get the minimum Hamiltonian cycle. Even though fuzzy notions are effective to handle the uncertainty by evolving membership values to certain objects, but in some situations in which the membership values cannot be determined due to the insufficient information up to the satisfaction level of the decision maker. Due to the same determination of non-membership values also cannot be evaluated. Consequently there exists a hesitation factor in determining the membership and non-membership values. Fuzzy set theory is not a sufficient tool to deal with these problems. The Intuitionistic fuzzy notions which are a generalized version fuzziness are instigated by Atanassov [20] in 1986. It gives more freedom in modeling the vagueness. The two characteristic functions the degree of membership (belongingness) and the degree of non-membership (non-belongingness) of elements of the universe are used in the intuitionistic fuzzy set. In this article the decision parameters of TSP are modeled as intuitionistic fuzzy notions. Very few authors [21-23] researched in this InFTSP. Ordering of Intuitionistic fuzzy numbers plays a crucial role in optimization problems. Many researches worked in this ordering and proposed many algorithms. Some rankings are specific to particular task. There is no ranking technique proposed for all. In this article weighted arithmetic mean method is applied to order the intuitionistic fuzzy numbers. In this paper, Branch and Bound algorithm is used to obtain the optimal InFTSP. The proposed algorithm is validated through examples. In this paper, Section 2 presents the basics of intuitionistic fuzzy numbers, arithmetic operations and their ranking techniques. Branch and bound algorithm for solving InFTSP and a model is furnished in Section 3. Conclusions are provided in 4th section.

2. Preliminaries

Definition 2.1 An intuitionistic fuzzy set $\tilde{\boldsymbol{A}}^{\boldsymbol{I}}$ in the universe of discourse $\boldsymbol{X}$ is the set of ordered triples $\tilde{\boldsymbol{A}}^{\boldsymbol{I}} = \{\langle \boldsymbol{x}, \boldsymbol{\mu}_{\tilde{A}}(\boldsymbol{x}), \boldsymbol{\vartheta}_{\tilde{A}}(\boldsymbol{x}); \boldsymbol{x\epsilon X}\rangle\}$ where $\boldsymbol{\mu}_{\tilde{A}}(\boldsymbol{x}), \boldsymbol{\vartheta}_{\tilde{A}}(\boldsymbol{x})$ are mapping function from X into [0,1] such that $\boldsymbol{0 \leq \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) \leq 1}$ where $\boldsymbol{\mu_{\tilde{A}}(x)}$ ***and*** $\boldsymbol{\vartheta_{\tilde{A}}(x)}$ represent the degree of membership and the degree of non-membership of the element $\boldsymbol{x\epsilon X}$ to $\boldsymbol{A \subset X}$.

Definition 2.2 The value of $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \vartheta_{\tilde{A}}(x)$ is called the degree of non-determinacy of the element $x\epsilon X$ to the InFS $\tilde{A}^I$.

Definition 2.3 An Intuitionistic fuzzy subset $\tilde{A}^I$ of the real line $\mathcal{R}$ is called an intuitionistic fuzzy number (InFN) if the following holds:

- There exist $m\epsilon\, \mathcal{R}\; \mu_{\tilde{A}}(m) = 1$ and $\vartheta_{\tilde{A}}(m) = 0$ (m is called the mean value of A)
- $\mu_{\tilde{A}}$ is a continuous mapping from $\mathcal{R}$ to the closed interval [0,1] and $\forall x \in \mathcal{R}$, the relation $0 \leq \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) \leq 1$.

Definition 2.4 A Intuitionistic fuzzy number $\tilde{A} = (r_1, r_2, r_3)(r_1', r_2', r_3')$ with membership function of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-r_1}{r_2-r_1} & r_1 \leq x \leq r_2 \\ 1 & r_2 \leq x \leq r_3 \\ \frac{r_4-x}{r_4-r_3} & r_3 \leq x \leq r_4 \\ 0 & otherwise \end{cases},$$

$$\vartheta_{\tilde{A}}(x) = \begin{cases} \frac{r_2'-x}{r_2'-r_1'} & r_1' \leq x \leq r_2' \\ 0 & r_2' \leq x \leq r_3' \\ \frac{x-r_4'}{r_4'-r_3'} & r_3' \leq x \leq r_4' \\ 0 & otherwise \end{cases}$$

Where $r_1 \leq r_2 \leq r_3 \leq r_4$, $r_1' \leq r_2' \leq r_3' \leq r_4'$ and $r_1' \leq r_1, r_2' \leq r_2, r_3' \leq r_3$, $r_4' \leq r_4$ If $r_2 = r_3$ *and* $r_2' = r_3'$ then $\tilde{A}^I = (r_1, r_2, r_3)\,(r_1', r_2', r_3')$ is called Intuitionistic triangular fuzzy number. Pictorially it can represented as

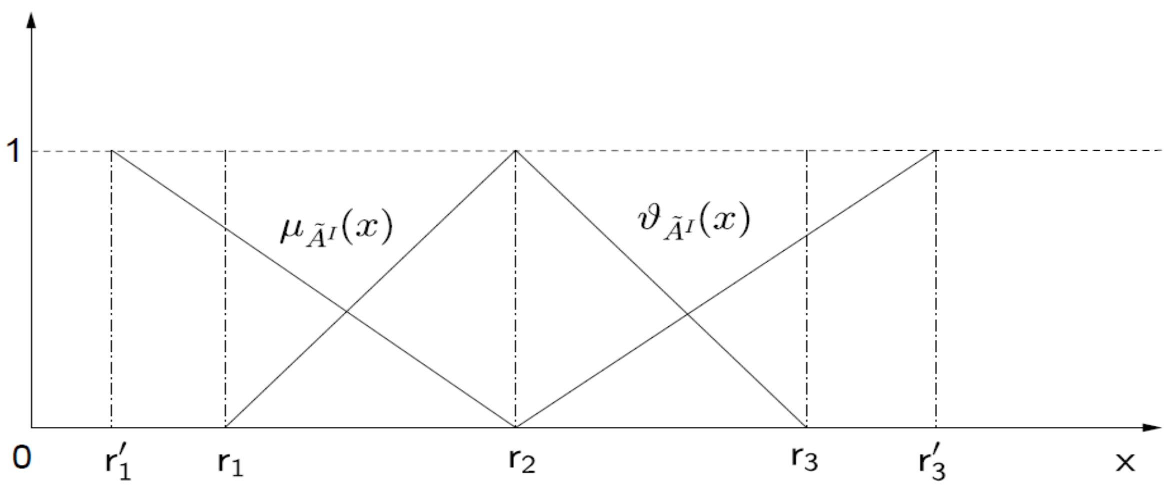

Figure 1. Membership function diagram Intutionistic Triangular Fuzzy Number.

Definition 2.5 The Addition and subtraction of Intuitionistic fuzzy parameters are

Intuitionistic Fuzzy Addition:

$$(r_1, r_2, r_3, r_4)(r_1', r_2', r_3', r_4') + (s_1, s_2, s_3, s_4)(s_1', s_2', s_3', s_4') = (r_1 + s_1, r_2 + s_2, r_3 + s_3, r_4 + s_4)(r_1' + s_1', r_2' + s_2', r_3' + s_3', r_4' + s_4').$$

$$(r_1, r_2, r_3)(r_1', r_2', r_3') + (s_1, s_2, s_3)(s_1', s_2', s_3') = (r_1 + s_1, r_2 + s_2, r_3 + s_3)(r_1' + s_1', r_2' + s_2', r_3' + s_3')$$

Intuitionistic Fuzzy Subtraction:

$$(r_1, r_2, r_3, r_4)(r_1', r_2', r_3', r_4') - (s_1, s_2, s_3, s_4)(s_1', s_2', s_3', s_4') = (r_1 - s_4, r_2 - s_3, r_3 - s_2, r_4 - s_1)(r_1' - s_4', r_2' - s_3', r_3' - s_2', r_4' - s_1').$$

$$(r_1, r_2, r_3)(r_1', r_2', r_3') - (s_1, s_2, s_3)(s_1', s_2', s_3') = (r_1 - s_3, r_2 - s_2, r_3 - s_{11})(r_1' - s_3', r_2' - s_2', r_3' - s_1')$$

Definition 2.6 let $\widetilde{A^I} = (r_1, r_2, r_3)(r_1', r_2', r_3')$ be an intuitionistic triangular fuzzy number. The combined arithmetic mean ranking of $\widetilde{A^I}$ is $R(\widetilde{A^I}) = \frac{3*\bar{A}_1 + 3*\bar{A}_2}{3+3}$ where $\bar{A}_1 = \frac{(r_1+r_2+r_3)}{3}, \bar{A}_2 = \frac{(r_1'+r_2'+r_3')}{3}$.

- If $R(\widetilde{A^I}) < R(\widetilde{B^I})$ then $\widetilde{A^I} \prec \widetilde{B^I}$
- If $R(\widetilde{A^I}) > R(\widetilde{B^I})$ then $\widetilde{A^I} \succ \widetilde{B^I}$
- If $R(\widetilde{A^I}) = R(\widetilde{B^I})$ then $\widetilde{A^I} \approx \widetilde{B^I}$

Definition 2.7 Linearity:

Let $\widetilde{A^I} = (r_1, r_2, r_3)(r_1', r_2', r_3')$ and $\widetilde{B^I} = (s_1, s_2, s_3)(s_1', s_2', s_3')$ be two intuitionistic triangular fuzzy numbers then $R(k_1\widetilde{A^I} + k_2\widetilde{B^I}) = k_1 R(\widetilde{A^I}) + k_2 R(\widetilde{B^I})$.

Proof: let $\widetilde{A^I} = (r_1, r_2, r_3)(r_1', r_2', r_3')$ and $\widetilde{B^I} = (s_1, s_2, s_3)(s_1', s_2', s_3')$ be two intuitionistic triangular fuzzy numbers. Then

$$k_1\widetilde{A^I} + k_2\widetilde{B^I} = (k_1 r_1 + k_2 s_1, k_1 r_2 + k_2 s_2, k_1 r_3 + k_2 s_3)(k_1 r_1' + k_2 s_1', k_1 r_2' + k_2 s_2', k_1 r_3 + k_2 s_3')$$

$$R(k_1\widetilde{A^I} + k_2\widetilde{B^I}) = \frac{3}{6}(k_1 r_1 + k_2 s_1 + k_1 r_2 + k_2 s_2 + k_1 r_3 + k_2 s_3 + k_1 r_1' + k_2 s_1' + k_1 r_2' + k_2 s_2' + k_1 r_3' + k_2 s_3')$$

$$\Rightarrow \frac{3}{6}(k_1 r_1 + k_1 r_2 + k_1 r_3 + k_1 r_1' + k_1 r_2' + k_1 r_3') + \frac{3}{6}(k_2 s_1 + k_2 s_2 + k_2 s_3 + k_2 s_1' + k_2 s_2' + k_2 s_3')$$

$$\Rightarrow k_1 R(\widetilde{A^I}) + k_2 R(\widetilde{B^I}).$$

Definition 2.8 An InFTSP is modeled as

$$\min \tilde{z}^I = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{C}_{ij}^I x_{ij}$$

subject to

(1) $\sum_{i=1}^{n} x_{ij} = 1, \sum_{j=1}^{n} x_{ij} = 1$

(2) $x_{ij} + x_{ji} \le 1, 1 \le i \ne j \le n$

(3) $x_{ij} + x_{jk} + x_{ki} \le 2, 1 \le i \ne j \ne k \le n$

(4) $x_{ip_1} + x_{p_1 p_2} + \cdots + x_{p_{n-2} i} \le n-2, 1 \le i \ne p_1 \ne p_2 \ne \ldots \ne p_{n-2} \le n$

The matrix representation is

Table 1. Cost matrix of Intuitionistic fuzzy TSP

	Town 1	Town 2	Town 3	…….	Town n
Town 1	$\tilde{\infty}^I$	$\tilde{C}_{12}^I$	$\tilde{C}_{13}^I$	……	$\tilde{C}_{1n}^I$
Town 2	$\tilde{C}_{21}^I$	$\tilde{\infty}^I$	$\tilde{C}_{23}^I$	……	$\tilde{C}_{2n}^I$
Town 3	$\tilde{C}_{31}^I$	$\tilde{C}_{32}^I$	$\tilde{\infty}^I$	……	$\tilde{C}_{3n}^I$
……	…..	……	…….	……	…….
Town n	$\tilde{C}_{n1}^I$	$\tilde{C}_{n2}^I$	$\tilde{C}_{n3}^I$	……	$\tilde{\infty}^I$

Constraints (1) guarantee that all places are visited exactly once. Each 2- places sub tours are eliminated by Constraint (2). Constraint (3) getting rid of all 3-places sub tours. Every (n-2) place sub-tours are excluded by constraint (4).

Definition 2.9 The feasible solution of the Intuitionistic fuzzy TSP is the solution which excludes all sub tours.

For example an intuitionistic travelling salesman problem with five cities the solution should exclude all the sub tours of length 2, 3 and 4.

Definition 2.10 An optimal solution of the intuitionistic fuzzy travelling salesman problem is the feasible solution which optimizes.

3. Branch and Bound Algorithm

Step 1

Select (new) the lowest intuitionistic fuzzy parameter in each row and subtract it from the other intuitionistic fuzzy parameter in the corresponding row and do the same along the columns till there is atleast one Intuitionistic fuzzy zero parameter in every row and every column. The total reduction $\tilde{r}^I$ is equal to the sum of the fuzzy intuitionistic elements subtracted. Let the resulting intuitionistic fuzzy parameter matrix be $\tilde{C}_{ij}^I$.

Step 2

Find the penalty of not using each intuitionistic fuzzy zero cell in $\tilde{C}_{ij}^I$. We argue that if we do not use the link (h, k), we must use one of the element in h th row and one of the elements kth column. Thus the intuitionistic fuzzy cost of not using (h, k), is at least equal to the sum of the smallest element in row h and the smallest element in column$k$. Assign these intuitionistic fuzzy penalties in the corresponding cell of the zero intuitionistic fuzzy cells in the intuitionistic fuzzy parameter matrix $\tilde{C}^I_{ij}$.

Step 3

Let (h,k) be the intuitionistic fuzzy zero entry with the highest penalty. In case of a tie, select arbitrarily. Divide the given set $'S'$ into two subsets. $\tilde{S}^I(\bar{h},\bar{k})$which does not contain the link (h,k), and $\tilde{S}^I(h,k)$ which contains the link (h,k) , then calculate the lower bounds on the fuzzy costs of all routes in each subset. (i.e.) $\tilde{\theta}^I(\bar{h},\bar{k}) \approx \tilde{r}^I + \tilde{P}^I_{hk}$. This $\tilde{P}^I_{hk}$ is the highest intuitionistic fuzzy penalty.

Step 4

Find the total fuzzy cost of route containing the highest penalty (i.e) lower bound for $\tilde{S}^I(h,k)$. This is done as follows: if the link (h,k), has been chosen then the link (k,h) is prohibited. Also if the link (h,k), has been chosen, then there should not be any other links in h th row and k th column. Delete the h th row and k th column. Find $\tilde{r}^I_{hk}$. That is $\tilde{r}^I_{hk} \approx$ intuitionistic fuzzy cost will be at least the amount by which the remaining intuitionistic fuzzy cost matrix $\tilde{C}^I_{ij}$ is reduced. This implies the lower bound of $\tilde{S}^I(\bar{h},\bar{k})$ is $\tilde{\theta}^I(h,k) \approx \tilde{r}^I + \tilde{r}^I_{hk}$.

Step 5

- If $\tilde{\theta}^I(\bar{h},\bar{k}) \prec \tilde{\theta}^I(h,k)$ then return to the original reduced intuitionistic fuzzy cost matrix and put cost $\tilde{\infty}^I$ in the (h,k) cell and repeat the steps from 2 onwards.
- If $\tilde{\theta}^I(\bar{h},\bar{k}) \succ \tilde{\theta}^I(h,k)$ then return to step 2 and repeat it on the intuitionistic fuzzy cost matrix obtained in step 4. (Ensure that the intuitionistic fuzzy cost matrix have at least one fuzzy zero element in each row and column).

Step 6

Repeat these steps until the optimal solution is obtained.

4. Advantages of Proposed Algorithm

The nature of this algorithm enables us to understand it easily and apply since it has resemblance with classical branch and bound algorithm.

This algorithm ensures that the decision maker gets all the results in terms of Intuitionistic fuzzy numbers and the decision can be made accordingly.

By this algorithm we directly get the optimal solution which satisfies route conditions. It is not the case for other existing methods.

Convergence of this proposed method validates its advantages over the other existing methods.

5. Numerical Examples

Example 5.1

Let the InFTSP be

Table 2. Cost matrix of the Example 5.1

	Town A	Town B	Town C	Town D
Town A	$\tilde{\infty}^I$	(1,2,3)(0,2,4)	(8,9,10)(7,9,11)	(9,10,11)(8,10,12)
Town B	(0,1,2)(0,1,2)	$\tilde{\infty}^I$	(5,6,7)(4,6,8)	(3,4,5)(2,4,6)
Town C	(14,15,16)(13,15,17)	(6,7,8)(5,7,9)	$\tilde{\infty}^I$	(7,8,9)(6,8,10)
Town D	(5,6,7)(4,6,8)	(2,3,4)(1,3,5)	(11,12,13)(10,12,14)	$\tilde{\infty}^I$

After applying step 1 we get the following matrix

Table 3. Reduced Cost matrix-I of the Example 5.1

	Town A	Town B	Town C	Town D
Town A	$\tilde{\infty}^I$	(−2,0,2)(−4,0,4)	(−2,2,6)(−5,2,9)	(3,7,11)(−1,7,15)
Town B	(−2,0,2)(−2,0,2)	$\tilde{\infty}^I$	(−4,0,4)(−6,0,6)	(−2,2,6)(−5,2,9)
Town C	(6,8,10)(4,8,12)	(−2,0,2)(−4,0,4)	$\tilde{\infty}^I$	(−4,0,4)(−8,0,8)
Town D	(1,3,5)(−1,3,7)	(−2,0,2)(−4,0,4)	(0,4,8)(−3,4,11)	$\tilde{\infty}^I$

The intuitionistic fuzzy reduction cost is $\approx (1,2,3)(0,2,4) + (0,1,2)(0,1,2) + (2,3,4)(1,3,5) + (6,7,8)(5,7,9) + (3,5,7)(2,5,8) + (-1,1,3)(-3,1,5) \approx (11,19,27)(5,19,33)$.

Table 4. Cost matrix-I with fuzzy penalty of the Example 5.1

$\tilde{\infty}^I$	$(\mathbf{-2,0,2})(\mathbf{-4,0,4})^{(-4,2,8)(-9,2,13)}$	$(-2,2,6)(-5,2,9)$	$(3,7,11)(-1,7,15)$
$(\mathbf{-2,0,2})(\mathbf{-2,0,2})^{(-3,3,9)(-7,3,13)}$	$\tilde{\infty}^I$	$(\mathbf{-4,0,4})(\mathbf{-6,0,6})^{(-4,2,8)(-7,2,11)}$	$(-2,2,6)(-5,2,9)$
$(6,8,10)(4,8,12)$	$(\mathbf{-2,0,2})(\mathbf{-4,0,4})^{(-6,0,6)(-12,0,12)}$	$\tilde{\infty}^I$	$(\mathbf{-4,0,4})(\mathbf{-8,0,8})^{(-4,2,8)(-9,2,3)}$
$(1,3,5)(-1,3,7)$	$(\mathbf{-2,0,2})(\mathbf{-4,0,4})^{(-1,3,7)(-5,3,11)}$	$(0,4,8)(-3,4,11)$	$\tilde{\infty}^I$

In this intuitionistic fuzzy cost matrix the highest intuitionistic fuzzy penalty is $(-1,3,7)(-5,3,11)$ which is in the cell $(4,2)$ (i.e)DB . The total intuitionistic fuzzy cost not containing the highest intuitionistic fuzzy penalty link is

$$\tilde{\theta}^I(\bar{D}\ ,\bar{B}\ ) \approx\ (11,19,27)(5,19,33) + (-1,3,7)(-5,3,11)\ \approx\ (10,22,34)(0,22,44)$$

The total fuzzy cost of route containing the highest intuitionistic fuzzy penalty is

$$\tilde{\theta}^I(D,B\ ) \approx\ (11,19,27)(5,19,33) + (-2,0,2)(-4,0,4\ ) \approx\ (9,19,29)(1,19,37)$$

Since $\tilde{\theta}^I(D,B\ ) \prec \tilde{\theta}^I(\bar{D}\ ,\bar{B}\ )$, we put the cell (B,D) as $\tilde{\tilde{\tilde{\infty}}}^I$ and omit the row $'D'$ and column $'B'$. The remaining intuitionistic fuzzy parameter matrix is

Table 5. Cost matrix of the Example 5.1 after deleting the row D and Column B

	Town A	Town C	Town D
Town A	$\tilde{\infty}^I$	$(-2,2,6)(-5,2,9)$	$(3,7,11)(-1,7,15)$
Town B	$(-2,0,2)(-2,0,2)$	$(-4,0,4)(-6,0,6)$	$(-2,2,6)(-5,2,9)$
Town C	$(6,8,10)(4,8,12)$	$\tilde{\infty}^I$	$(-4,0,4)(-8,0,8)$

In this Intuitionistic fuzzy parameter matrix the first row does not have intuitionistic fuzzy zero. Following the step 1, we get

Table 6. Reduced Cost matrix-II of the Example 5.1

	Town A	Town C	Town D
Town A	$\tilde{\infty}^I$	$(-8,0,8)(-14,0,14)$	$(-3,5,13)(-10,5,20)$
Town B	$(-2,0,2)(-2,0,2)$	$(-4,0,4)(-6,0,6)$	$\tilde{\infty}^I$
Town C	$(6,8,10)(4,8,12)$	$\tilde{\infty}^I$	$(-4,0,4)(-8,0,8)$

The intuitionistic fuzzy cost matrix with fuzzy intuitionistic penalty is given as follows

Table 7. Cost matrix-II with fuzzy penalty of the Example 5.1

	Town A	Town C	Town D
Town A	$\tilde{\infty}^I$	$(-8,0,8)(-14,0,14)^{(-1,7,15)(-7,7,21)}$	$(-3,5,13)(-10,5,20)$
Town B	$(-2,0,2)(-2,0,2)^{(2.8.14)(-2,8,18)}$	$(-4,0,4)(-6,0,6)^{(-10,0,10)(-16,0,16)}$	$\tilde{\infty}^I$
Town C	$(6,8,10)(4,8,12)$	$\tilde{\infty}^I$	$(-4,0,4)(-8,0,8)^{(313,23)(-6,13,32)}$

The intuitionistic fuzzy reduction cost $\approx\ (11,19,27)(5,19,33) + (-2,2,6)(-5,2,9)\ \approx\ (9,21,33)(0,21,42)$.

In this intuitionistic fuzzy cost matrix the highest intuitionistic fuzzy penalty is $(9,15,21)(3,15,27)$ which is in the cell $(3,4)$ (i.e)CD. The total intuitionistic fuzzy cost not containing the highest intuitionistic fuzzy penalty link is

$$\tilde{\theta}^I(\bar{C}\ ,\bar{D}\ ) \approx (9,21,33)(0,21,42) + (3,13,23)(-6,13,32)\ \approx\ (12,34,56)(-6,34,74)$$

The total fuzzy cost of route containing the highest intuitionistic fuzzy penalty is

$$\tilde{\theta}^I(C,D\ ) \approx\ (9,21,33)(0,21,42) + (-4,0,4)(-8,0,8\ ) \approx (5,21,37)(-8,21,50)$$

Since $\tilde{\theta}^I(C,D\ ) \prec \tilde{\theta}^I(\bar{C}\ ,\bar{D}\ )$, we put the cell (D,C) as $\tilde{\tilde{\tilde{\infty}}}^I$ and omit the row $'C'$ and column $'D'$. The remaining intuitionistic fuzzy parameter matrix is

Table 8. Cost matrix of the Example 5.1 after deleting the row C and Column D

	Place A	Place C
Place A	$\tilde{\infty}^I$	$(-8,0,8)(-14,0,14)$
Place B	$(-2,0,2)(-2,0,2)$	$(-4,0,4)(-6,0,6)$

From the given intuitionistic fuzzy cost matrix it is evident that, the cell $(1,3)$ should be fixed. (i.e.) (A, C).

After omitting the $'A'$ row and $'C'$ column, we are left with cell $(2,1)$ (i.e.) (B, A). Therefore the optimal route is $A \rightarrow C \rightarrow D \rightarrow B \rightarrow A$.

The intuitionistic fuzzy optimal value is

$$\approx (8,9,10)(7,9,11) + (7,8,9)(6,8,10) + (2,3,4)(1,3,5) + (0,1,2)(0,1,2)$$
$$\approx (17,21,25)(14,21,28).$$

6. Results and Discussions

The degree of acceptance of the travelling cost increases if the cost increases from $Rs\,17$ to $s\,21$. The acceptance level decreases when the decision parameter increases from $Rs\,21$ to $Rs\,25$. The acceptance level of the DM is zero after $Rs\,(17,25)$. The Decision maker's acceptance level is full is if the travelling cost is $Rs.21$. The non-acceptance level of the travelling cost for the decision maker inclines if the cost creeps from $Rs\,14$ to $Rs\,21$ while it climbs if the cost creeping from $Rs\,21$ to $Rs\,28$. After the limit of $Rs\,14$ to $Rs\,28$ cost is totally un-acceptable.

Assuming that $\mu_{\tilde{Z}^I}\ (c)$ is the grade of membership value (measure of acceptance or satisfaction) and $\vartheta_{\tilde{Z}^I}$ (c) is the grade of non-membership value (measure of non-acceptance) of travelling cost parameter$c$. Then the grade of acceptance of the travelling cost c is $100\,\mu_{\tilde{Z}^I}\ (c)$% for the Decision Maker and the grade of non-acceptance is $100\,\vartheta_{\tilde{Z}^I}\ \ (c)$ % for the Decision Maker. The degree of hesitation for the acceptance of the travelling cost c is given by $100\,\pi_{\tilde{Z}^I}(c)$ where $\pi_{\tilde{Z}^I}(c) = (1 - \mu_{\tilde{Z}^I}(c) - \vartheta_{\tilde{Z}^I}\ (c))$ denotes the index of hesitation. Values of $\mu_{\tilde{Z}^I}\ (c)$ and $\vartheta_{\tilde{Z}^I}\ (c)$ at different values of c can be obtained using membership function are given as.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-17}{4} & 17 \le x \le 21 \\ 1 & x = 21 \\ \frac{25-x}{4} & 21 \le x \le 25 \\ 0 & otherwise \end{cases}, \quad \vartheta_{\tilde{A}}(x) = \begin{cases} \frac{21-x}{7} & 14 \le x \le 21 \\ 0 & x = 21 \\ \frac{x-28}{7} & 21 \le x \le 28 \\ 0 & otherwise \end{cases}$$

Remark-1

Using the Dynamic Programming method [24], the optimum solution of the above problem is given as $(17,21,25)$. This problem is converted in to intuitionistic fuzzy TSP and solved it by the proposed method. There is no evidence of applying the dynamic programming method to prohibited routes TSP. The proposed method can be applied for all kinds TSP.

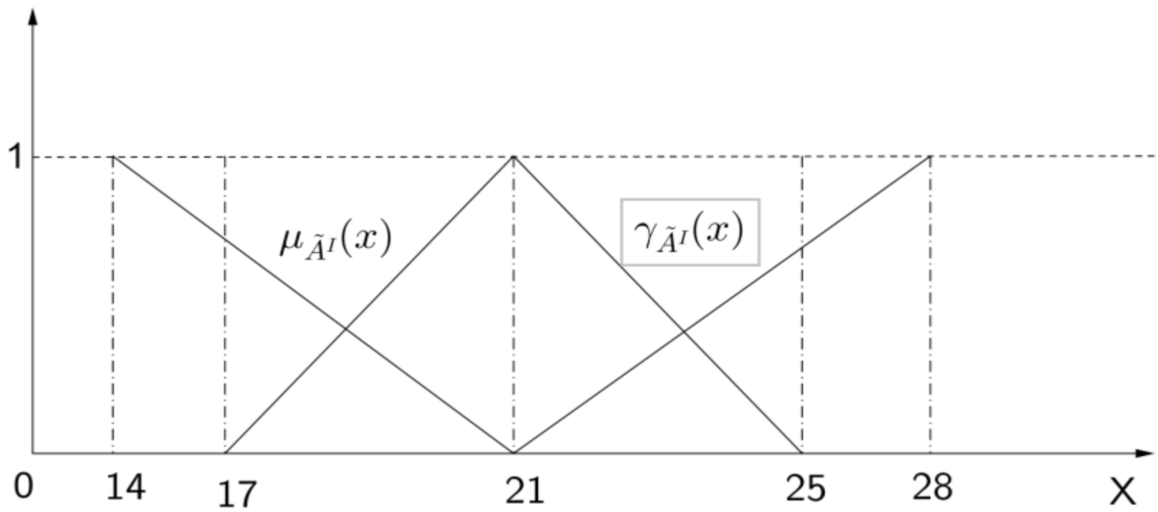

Figure 2. Membership Diagram of the Intuitionistic Optimal Solution of Example 5.1

Example 5.2

Let the InFTSP be

Table 9. Cost matrix of the Example 5.2

	Town A	Town B	Town C	Town D	Town E
Town A	∞^I	(23,25,27)(21,25,29)	(38,40,42)(36,40,44)	(8,10,12)(6,10,14)	(10,12,14)(8,12,16)
Town B	(23,25,27)(21,25,29)	∞^I	(18,20,22)(16,20,24)	(21,23,25)(19,23,27)	(9,11,13)(7,11,15)
Town C	(38,40,42)(36,40,44)	(18,20,22)(16,20,24)	∞^I	(19,23,27)(17,23,29)	(31,33,35)(29,33,39)
Town D	(8,10,12)(6,10,14)	(21,23,25)(19,23,27)	(19,23,27)(17,23,29)	∞^I	(18,20,22)(16,20,24)
Town E	(10,12,14)(8,12,16)	(9,11,13)(7,11,15)	(31,33,35)(29,33,39)	(18,20,22)(16,20,24)	∞^I

When you apply the proposed approach the optimal route is $A \rightarrow D \rightarrow C \rightarrow B \rightarrow E \rightarrow A$. The intuitionistic fuzzy optimal value is $\approx (8,10,12)(6,10,14) + (19,23,27)(17,23,29) + (18,20,22)(16,20,24) + (9,11,13)(7,11,15) + (10,12,14)(8,12,16) = (64,76,88)(54,76,98)$.

The degree of acceptance of the travelling cost increases if the cost increases from $Rs\ 64$ to 76. The acceptance level decreases when the decision parameter increases from $Rs\ 76$ to $Rs\ 88$. The acceptance level of the DM is zero after $Rs\ (64,88)$. The Decision maker's acceptance level is full is if the travelling cost is $Rs. 76$. The non-acceptance level of the travelling cost for the decision maker inclines if the cost creeps from $Rs\ 54$ to $Rs\ 76$ while it climbs if the cost creeping from $Rs\ 76$ to $Rs\ 98$. After the limit of $Rs\ 54$ to $Rs\ 98$ cost is totally un-acceptable.

Assuming that $\mu_{\tilde{Z}^I}\ (c)$ is the grade of membership value (measure of acceptance or satisfaction) and $\vartheta_{\tilde{Z}^I}\ (c)$ is the grade of non-membership value (measure of non-acceptance) of travelling cost parameter c. Then the grade of acceptance of the travelling cost c is $100\ \mu_{\tilde{Z}^I}\ (c)\%$ for the Decision Maker and the grade of non-acceptance is $100\ \vartheta_{\tilde{Z}^I}\ (c)\ \%$ for the Decision Maker. The degree of hesitation for the acceptance of the travelling cost c is given by $100\ \pi_{\tilde{Z}^I}(c)$ where $\pi_{\tilde{Z}^I}(c) = (1 - \mu_{\tilde{Z}^I}(c) - \vartheta_{\tilde{Z}^I}\ (c)\ )$ denotes the index of hesitation. Values of $\mu_{\tilde{Z}^I}\ (c)$ and $\vartheta_{\tilde{Z}^I}\ (c)$ at different values of c can be obtained using membership function are given as.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-64}{12}\ 64 \leq x \leq 76 \\ 1\ x = 76 \\ \frac{88-x}{12}\ 76 \leq x \leq 88 \\ 0\ otherwise \end{cases},$$

$$\vartheta_{\tilde{A}}(x) = \begin{cases} \frac{76-x}{22}\ 54 \leq x \leq 76 \\ 0\ x = 21 \\ \frac{x-98}{22}\ 76 \leq x \leq 98 \\ 0\ otherwise \end{cases}$$

Remark-2

In [25] this problem is solved by Hungarian method and got the solution as (70, 76, 82) (72, 76, 80) which is not a intuitionistic fuzzy number. It violates the condition for an intuitionistic fuzzy number. i.e., $(a,b,c)(a',b',c')$ if $a' \leq a, c \leq c'$. In our proposed method we got it as $(64,76,88)(54,76,98)$ which is an intuitionistic fuzzy number.

In the Hungarian method optimal solution which satisfies route conditions cannot be obtained directly. In the proposed method the solution obtained satisfies route conditions directly.

The spread of the solution obtained in the proposed method is more than the solution obtained in the Hungarian method.

Comparison Table

Table 10. Comparison Table

Example	Existing Method	Proposed Method
Example 5.1	(17,21,25)(14,21,28).	(17,21,25)(14,21,28).
Example 5.2	(70, 76, 82) (72, 76, 80) Fuzzy Hungarian method. It is not an intuitionistic fuzzy number. The solution obtained may not satisfy route condition.	(64,76,88)(54,76,98) The spread is more. It can be applied to all kinds of TSP.

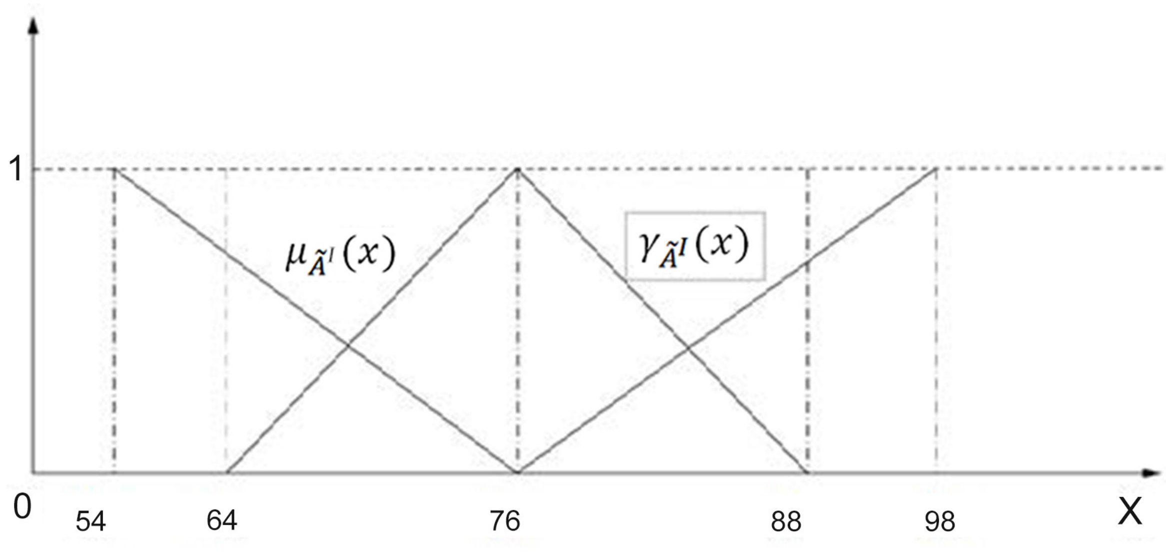

Figure 3. Membership Diagram of the Intuitionistic Optimal Solution of Example 5.2

5. Conclusion

In this paper, we deployed branch and bound algorithm to obtain the optimal route in Intuitionistic fuzzy Travelling Salesman Problem. This algorithm is efficient and easily understandable because of its resemblance to Branch and Bound algorithm. The examples presented in this paper ensure the authentication of the corrective and effective nature of the algorithm. The results are validated with the existing technique, further it satisfies the route conditions of the TSP.

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